



Resilience of Hund's rule in the chemical space of small organic molecules†

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We embark on a quest to identify small molecules in the chemical space that can potentially violate Hund's rule. Utilizing twelve TDDFT approximations and the ADC(2) many-body method, we report the energies of S_1 and T_1 excited states of 12 880 closed-shell organic molecules within the bigQM7₀ dataset with up to 7 CONF atoms. In this comprehensive dataset, none of the molecules, in their minimum energy geometry, exhibit a negative S_1 – T_1 energy gap at the ADC(2) level while several molecules display values < 0.1 eV. The spin-component-scaled double-hybrid method, SCS-PBE-QIDH, demonstrates the best agreement with ADC(2). Yet, at this level, a few molecules with a strained sp^3 -N center turn out as false-positives with the S_1 state lower in energy than T_1 . We investigate a prototypical cage molecule with an energy gap < -0.2 eV, which a closer examination revealed as another false positive. We conclude that in the chemical space of small closed-shell organic molecules, it is possible to identify geometric and electronic structural features giving rise to S_1 – T_1 degeneracy; still, there is no evidence of a negative gap. We share the dataset generated for this study as a module, to facilitate seamless molecular discovery through data mining.

1 Introduction

Studies from the 1970s onward, some even titled “violation of Hund's (multiplicity) rule...” have explored the possibility of

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† Electronic supplementary information (ESI) available: Contains the following: assessment of S_1 – T_1 gaps from ADC(2) and SCS-PBE-QIDH for 10 triangular molecules. Table S1 compares S_1/T_1 energetics calculated using the ADC(2) method with theoretical best estimates. Table S2 compares the S_1/T_1 energetics predicted by TDDFT and TDA. Table S3 contains ADC(2) and TDA energetics calculated using DFT-level geometries. Table S4 provides various error metrics for ADC(2) and SCS-PBE-QIDH predicted S_1/T_1 energetics. Fig. S1 illustrates the shifts in S_1 and T_1 due to SCS/SOS corrections to PBE-QIDH and RSX-QIDH methods. Fig. S2 shows a scatterplot of S_1 and T_1 energies with S_1 – T_1 gaps. Fig. S3–S12 offer screenshots of data mining exercises. Minimum energy geometries of structures 1–4 in Fig. 6 are also listed. Sample Python notebooks and further details are available at <https://github.com/moldis-group/pymoldis>.²⁶ See DOI: <https://doi.org/10.1039/d4cp00886c>

an electronic state of lower spin-multiplicity being more stable than its higher multiplicity counterpart.^{1–7} Although no experimentally known cases of Hund's rule violation exist among molecules in their ground state, *ab initio* wavefunction models have suggested that dynamic spin polarization effects may favor open-shell singlets over triplets.¹ In 1980, Leupin *et al.* suggested the likelihood of a more stable lowest excited singlet state, S_1 , compared to the triplet state, T_1 , based on fluorescence measurements of cycl[3.3.3]azines.⁸ Likewise, in 1985, certain non-alternant polycyclic hydrocarbons were considered to exhibit negative S_1 – T_1 gaps (STGs).² Typically, S_1 and T_1 are represented primarily by singly excited configurations, $^1\chi_{a \rightarrow r}$ and $^3\chi_{a \rightarrow r}$, where a and r are the occupied and virtual molecular orbitals (MOs) determined using the self-consistent-field (SCF) method for the ground state configuration, $^1\chi_0$. Hence, the expressions for the excitation energies are $E(^1\chi_{a \rightarrow r}) - E(^1\chi_0) = \varepsilon_r - \varepsilon_a - J_{ar} + 2K_{ar}$ and $E(^3\chi_{a \rightarrow r}) - E(^1\chi_0) = \varepsilon_r - \varepsilon_a - J_{ar}$, where ε is the MO energy, while J and K are Coulomb and exchange integrals, respectively. This indicates $STG = E(^1\chi_{a \rightarrow r}) - E(^3\chi_{a \rightarrow r}) = 2K_{ar}$ (twice the exchange integral). As the overlap between the densities of the a and r MOs diminishes, a reduction in K_{ar} leads to degenerate S_1 and T_1 .⁸ Along with vanishing K_{ar} , a decrease in the percentage contribution of the $^1\chi_{a \rightarrow r}$ configuration and an increased contribution from the doubly-excited configuration, $^1\chi_{aa \rightarrow rr}$, was hypothesized to be a factor to selectively stabilize the S_1 state over T_1 resulting in a negative STG.⁹

In 2019, two independent studies confirmed STGs < 0 in the prototypical cycl[3.3.3]azines—cyclazine¹⁰ and heptazine¹¹—using time-dependent density functional theory (TDDFT) approximations, as well as a few correlated wavefunction methods. Since then, there has been a renewed interest in exploring the historically significant inverse-STG candidates: N -containing triangular molecules^{12–20} and non-alternant hydrocarbons.^{21–23} Besides these classes of molecules, Bedogni *et al.*¹⁸ showed the possibility of $STG < 0$ in $C_nH_nN_n$ aza-rings.

Despite mounting computational evidence supporting the likelihood of Hund's rule violation, the credibility of negative STGs still attracts criticisms.²⁴ This skepticism arises from the

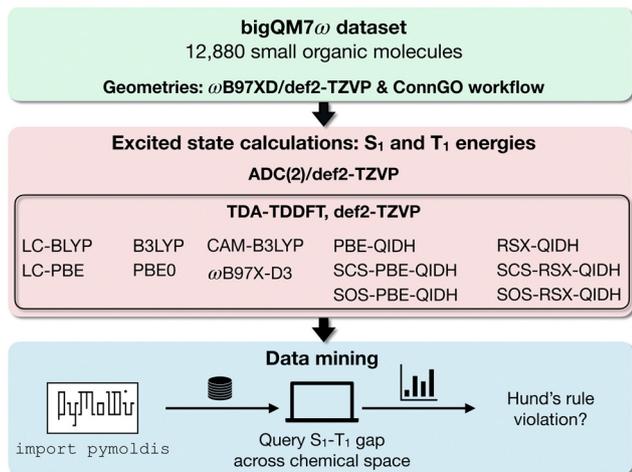


Fig. 1 Workflow outlining the design of the pymoldis module²⁶ for data mining S₁ and T₁ energies across 12 880 molecules with up to 7 CONF atoms in the bigQM7 ω dataset.²⁷ ADC(2) and TDDFT calculations were performed as a part of the present work. See (ESI[†]) for screenshots of example queries.

failure to account for the experimental conditions in computational modeling and the challenges posed by the large molecules for accurate *ab initio* calculations. More recently, Loos *et al.*²⁵ confirmed negative STGs in triangulene systems through composite excited states modeling and provided theoretical best estimates (TBES).

The present study aims to report STGs calculated using a many-body method and double-hybrid density functional theory (dh-DFT) models for 12 880 small organic molecules with systematically varying compositions and structures. Using this data, we verify the possibility of Hund's rule violation in the chemical space. Our workflow for data generation and the design of the module pymoldis for querying the reported data is illustrated in Fig. 1. The rest of this article discusses the qualitative aspects of the data, their analysis, and the technical details of the calculations.

2 Results and discussions

Fig. 2 shows the range spanned by the STGs of all the molecules in the bigQM7 ω dataset determined using various *ab initio* methods in the form of cumulative distributions. In both subplots, (a) and (b), of this figure, ADC(2) is used as the reference theory for evaluating the accuracy of other methods. In the (ESI[†]), we have discussed the accuracy of ADC(2) in combination with the def2-TZVP basis set for modeling STG of 10 triangle-shaped molecules using TBES reported in a previous study.²⁵ Additionally, we have benchmarked the performance of the TDDFT version of SCS-PBE-QIDH within the Tamm-Dancoff approximation (TDA), see Tables S1–S4 (ESI[†]).

Fig. 2(a) shows the cumulative count of STGs of 12 880 molecules predicted with ADC(2) and selected hybrid-DFT, long-range-corrected-DFT, and long-range-corrected-hybrid-DFT approximations. All the theoretical models featured in this plot predict positive STGs. Past studies^{17,28,29} have shown that explicit incorporation of electron correlation, for example, at the MP2-level (second-order many-body perturbation theory) as in dh-DFT, is a requirement to predict STG < 0. However, it has come as a surprise that at the ADC(2) level, which is the excited-state equivalent to MP2, none of the 12 880 molecules show a negative STG. Similar results from dh-DFT methods, along with their spin-component-scaled (SCS) and opposite-spin-component-scaled (SOS), are shown in Fig. 2(b). The zoomed-in inset shows the distribution of values from RSX-QIDH to shift towards the positive domain compared to ADC(2) values. Upon SCS/SOS corrections,³⁰ the distribution is shifted slightly to the negative domain. At the dh-DFT level, SCS-PBE-QIDH, an accurate method for modeling STGs of triangulenes, a few molecules exhibit STG < 0 eV; we give a detailed discussion of individual values later.

Fig. 3 illustrates the effect of SCS corrections to PBE-QIDH predicted values of STG in the form of probability densities of the shift in S₁ and T₁ energies with the inclusion of SCS. Overall, the SCS corrections lower the S₁ energies while raising

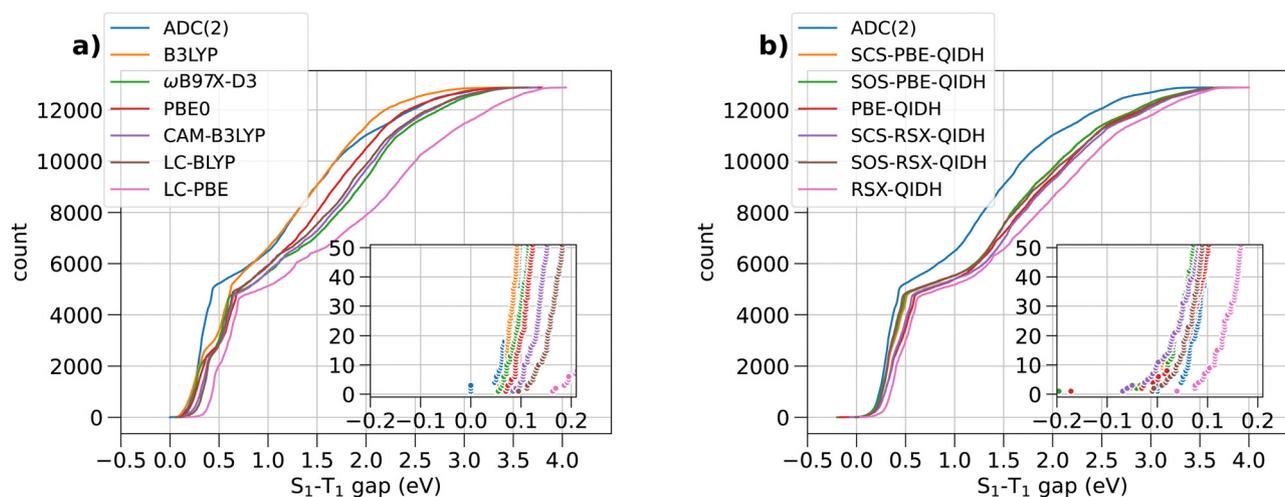


Fig. 2 Cumulative distribution of S₁-T₁ energy gaps of 12 880 bigQM7 ω molecules calculated with various *ab initio* methods. The inset shows the distribution in the range of -0.2 to 0.2 eV.

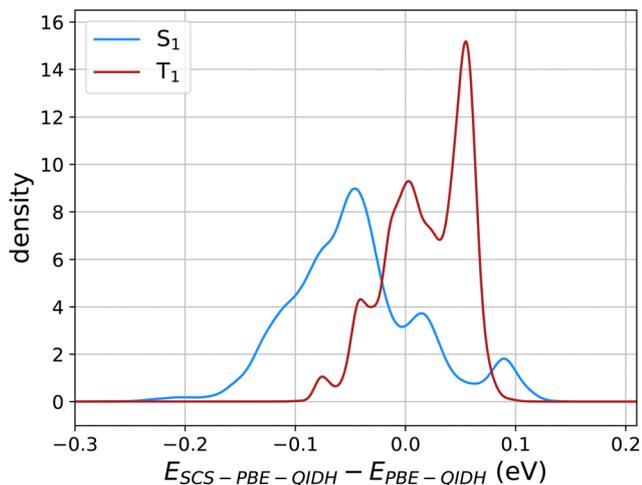


Fig. 3 Probability density of the shift in S_1 and T_1 energies (in eV) for 12 880 molecules upon the inclusion of spin-component-scaling (SCS) in PBE-QIDH.

the T_1 values, illustrating why SCS-PBE-QIDH favors smaller STGs than the unscaled method, PBE-QIDH. Fig. S1 of the ESI[†] displays similar plots for SOS-PBE-QIDH, SCS-RSX-QIDH, and SOS-RSX-QIDH. While the SCS/SOS variants of RSX-QIDH shift the S_1 and T_1 energies further apart compared to the PBE-QIDH variants, previous benchmark studies^{25,31} have shown SCS-PBE-QIDH to be more accurate for modeling molecules with negative STGs. Hence, one can conclude scaled-RSX-QIDH to result in more false-positive predictions (*i.e.* spurious predictions of $STG < 0$ eV) than scaled-PBE-QIDH. Notably, the SCS/SOS corrections applied to the PBE-QIDH and RSX-QIDH dh-DFT methods are specifically tailored for the TD-DFT framework.³⁰ Hence, STGs predicted using the Δ SCF approach will be similar in unscaled and scaled DFT methods.

The scatterplot in Fig. 4 offers a detailed view of the distribution of STGs around 0 eV. We find that the predominant entries shown in this plot are in the blue region denoted “true negatives in SCS-PBE-QIDH”, implying that the TDDFT method agrees with ADC(2), both predicting these molecules as Hund’s rule obeying systems with positive STGs. Molecules shown in the red region of Fig. 4 are “false positives in SCS-PBE-QIDH” as these systems show $STG > 0.04$ eV according to ADC(2), while in the TDDFT formalism they show $STG < 0$ eV. As already highlighted in Fig. 2, there are no “true positives” or “false negatives” as none of the 12 880 molecules in the bigQM7 ω dataset exhibit a negative STG at the ADC(2) level.

The boundary separating positives and negatives in Fig. 4 is not sharp as both ADC(2) and SCS-PBE-QIDH have uncertainties > 0.1 eV in their predictions (see benchmarks in the ESI[†]). Fig. S2 (ESI[†]) shows scatterplots of the joint distributions of STG with the S_1 and T_1 energies at both the SCS-PBE-QIDH and ADC(2) methods. At both levels of theories, one finds the small STG systems to have S_1 and T_1 energies in the range of 6–8 eV. Additionally, we find a molecule with S_1 and T_1 energies in the 3–4 eV range to have a small STG at the SCS-PBE-QIDH level (see Fig. S2c and d, ESI[†]). By querying the dataset using the pymoldis

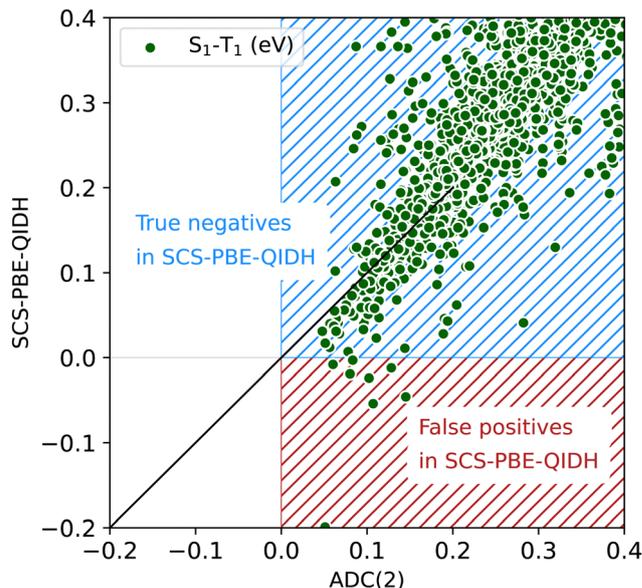


Fig. 4 Scatterplot of S_1-T_1 energy gap ≤ 0.4 eV from SCS-PBE-QIDH and ADC(2) for the bigQM7 ω molecules.

module (see Fig. S3–S12, ESI[†]), we found the corresponding molecule to be 2,6-dihydro-1H-pyridin-3-one (SMILES: O=C1CNCC=C1), which is cyclohexenone with an N atom at the δ -position.

To identify geometrical aspects common to the small STG molecules, we have queried the pymoldis database for ten molecules with the smallest STG according to TDA/SCS-PBE-QIDH. Fig. 5 shows a screenshot of the query. A geometric moiety common to all N-containing systems is a substantial deviation of the bonding environment of the sp^3 -N center from the ideal geometry. For the ten benchmark triangular systems, the S_1 and T_1 excitation energies are < 3 eV due to the possibility of a low-energy $n \rightarrow \pi^*$ transition. However, in the systems shown in Fig. 5, the S_1/T_1 excitations are of the $n \rightarrow \sigma^*$ type with excitation energies > 6 eV. Nine of the ten molecules shown in Fig. 5 contain a 3-membered heterocyclic ring, while one is a fluorinated cyclopropane derivative; the S_1 and T_1 excitation energies of the latter are > 8 eV, at both TDDFT and ADC(2) levels while the corresponding $STG = -0.01$ eV and 0.13 eV at TDDFT and ADC(2) levels, respectively. The molecule corresponding to SMILES, CC1C2CCN1C2, in Fig. 5 contains a propellane-type cage with $STG = -0.01$ eV at the TDDFT level and an STG of 0.06 eV at the ADC(2) level. The bigQM7 ω dataset comprises several such cage systems with small STGs.

Starting with a cage-type molecule with a strained N, we have explored the possibility to design a molecule with the character of the lowest excitation as $n \rightarrow \pi^*$. We started with a symmetric cage system, quinuclidine, a [2.2.2]propellane with an axial CH group replaced by an N atom, structure 1 in Fig. 6. This molecule has an STG of 0.1 eV at the TDA-SCS-PBE-QIDH/def2-TZVP level. This small gap is because the excitation is primarily from non-bonding MO of N (highest occupied MO, HOMO) to C–H σ^* of the cage (lowest unoccupied MO, LUMO); their corresponding densities show poor overlap leading to a

```

import pymoldis
import pandas as pd

df=pymoldis.get_data('bigqm7w_S1T1')

S1T1_DFT=df['S1_SCSPBEQIDH(eV)'] - df['T1_SCSPBEQIDH(eV)']

NEntries=10

SmallGap_DFT_vals=S1T1_DFT.nsmallest(NEntries)

S1_DFT=df.iloc[SmallGap_DFT_vals.index]['S1_SCSPBEQIDH(eV)']
T1_DFT=df.iloc[SmallGap_DFT_vals.index]['T1_SCSPBEQIDH(eV)']
SMIs=df.iloc[SmallGap_DFT_vals.index]['SMI']

S1_ADC2=df.iloc[SmallGap_DFT_vals.index]['S1_ADC2(eV)']
T1_ADC2=df.iloc[SmallGap_DFT_vals.index]['T1_ADC2(eV)']
S1T1_ADC2=S1_ADC2-T1_ADC2

result = pd.concat([SMIs, S1_DFT, T1_DFT, SmallGap_DFT_vals, S1_ADC2, T1_ADC2, S1T1_ADC2], axis=1)
result.columns = ['SMI','DFT S1(eV)','DFT T1(eV)','DFT S1-T1(eV)','ADC2 S1(eV)','ADC2 T1(eV)','ADC2 S1-T1(eV)']

formatted_result = result.applymap(lambda x: f'{x:4.2f}' if isinstance(x, (int, float)) else x)

print(formatted_result.to_string(index=False))

```

SMI	DFT S1(eV)	DFT T1(eV)	DFT S1-T1(eV)	ADC2 S1(eV)	ADC2 T1(eV)	ADC2 S1-T1(eV)
'CC1(C)OC1(C)C'	8.08	8.28	-0.20	7.39	7.34	0.05
'CCC1(C)CN1C'	7.14	7.20	-0.05	6.73	6.62	0.11
'OC1CC2CN2C1'	7.31	7.36	-0.05	6.93	6.79	0.15
'CN1CC1(C)C'	7.14	7.16	-0.02	6.74	6.64	0.10
'CC1CCC2CN12'	7.42	7.44	-0.02	6.96	6.88	0.08
'CCC1(CF)CC1'	8.75	8.76	-0.01	8.56	8.44	0.13
'CC1C2CCN1C2'	7.21	7.22	-0.01	6.75	6.69	0.06
'CC1CCN2CC12'	7.41	7.41	-0.00	6.96	6.87	0.08
'CC1OC1(C)C'	8.48	8.47	0.01	7.64	7.56	0.08
'CN1CC1(C)CO'	7.32	7.31	0.01	6.97	6.82	0.14

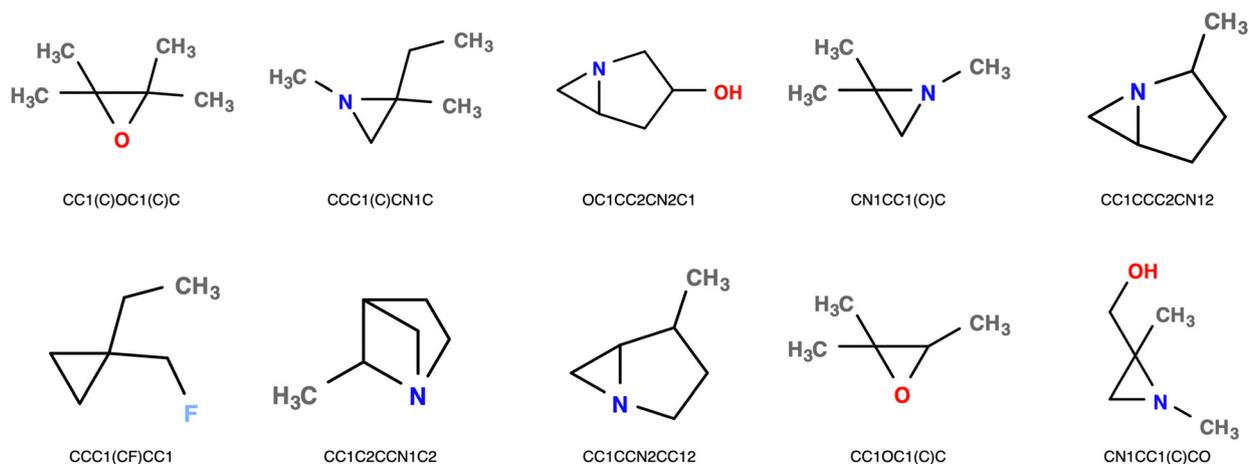


Fig. 5 Data-mining the bigQM7w dataset in pymoldis²⁶ to identify molecules with small S_1-T_1 gaps. For the ten molecules with the lowest gaps according to TDA-SCS-PBE-QIDH, SMILES strings and S_1/T_1 energies are displayed. The corresponding ADC(2) values are also given alongside. Molecular structures are displayed in cartoon format below. See the ESI† for more examples of data-mining exercises.

small value of exchange integral, K_{ar} . Further, to arrive at a local-geometric environment of the N atom as in cycl[3.3.3]-azines, we have introduced an additional cage to constrain the N-center to a plane (structure 2 in Fig. 6). This structure comprises perfectly co-planar C-N bonds resulting in degenerate S_1-T_1 levels. We have also modified quinuclidine by attaching three ethylene groups (structure 3). In this structure, the $S_0 \rightarrow S_1$ excitation has the character $n \rightarrow \pi^*$ (MO indices,

n : 52, which is the HOMO, and π^* : 53-55) while the $S_0 \rightarrow T_1$ excitation has the character $\pi \rightarrow \pi^*$ (MO indices, π : 49-51) with a large STG of 1.5 eV. Finally, we combined structural modifications introduced in structure 2 and structure 3 to arrive at structure 4 with a planar N interacting with π moieties through space. Interestingly, this system resulted in an STG of -0.21 eV at the DFT level. To verify this prediction of a negative STG, we have also performed ADC(2)/def2-TZVP calculations. For both

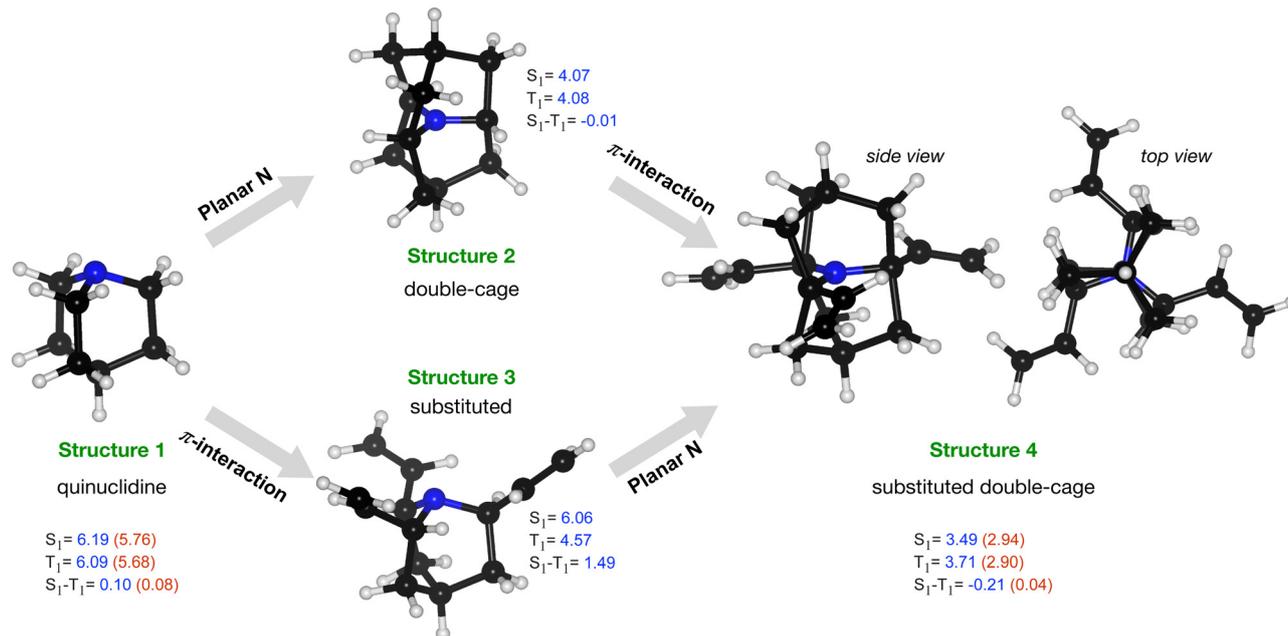


Fig. 6 TDA/SCS-PBE-QIDH predictions of S_1 , T_1 and $S_1 - T_1$ energies (in eV) of 1-azabicyclo[2.2.2]octane (aka quinuclidine) and its derivatives. For selected structures, additionally, ADC(2) values are in parentheses. White, black, and blue atoms correspond to H, C, and N atoms, respectively.

structure 1 and structure 4, we find the magnitudes of the ADC(2) excitation energies to be lower than the DFT values. While both energies drop by a similar magnitude in the former, in structure 4, the energy of T_1 drops more than the S_1 energy, giving rise to a nearly zero STG at the ADC(2) level. This case study indicates that for molecules such as structure 4, even some of the best double-hybrid DFT methods can spuriously predict a negative STG; hence, one may consider many-body methods such as ADC(2) as a baseline theory.

While structure 4 is a minimum on the potential energy surface as verified through vibrational frequency analysis, we do not expect the system to be relevant to the thermally activated delayed fluorescence (TADF) applications.^{10,13,32–34}

On the other hand, it is a compelling computational chemistry exercise to modulate a molecule's STG by chemical modifications. Hence, even though structure 4 seems to be yet another false positive in the search for a Hund's rule-violating molecule, we have examined it further. We inspected the shape of the MOs involved in the S_1 and T_1 excitations and found the excitations to be primarily HOMO \rightarrow LUMO type. These MOs are on display in Fig. 7, from which we visually conclude that the densities of HOMOs and LUMOs do not overlap. For a more quantitative analysis, we calculated the \mathcal{A} -index³⁵ defined as $\int dr |\phi_a(\mathbf{r})| |\phi_r(\mathbf{r})|$ using Multiwfn,³⁶ and obtained the values: 0.37 and 0.40 for the S_1 and the T_1 states, respectively. The \mathcal{A} -index quantifies the degree of overlap between hole and electron in $S_0 \rightarrow S_1$ and $S_0 \rightarrow T_1$ excitations. In comparison, for both cyclazine and heptazine, the values of \mathcal{A} for the S_1 and T_1 states are nearly 0.50.

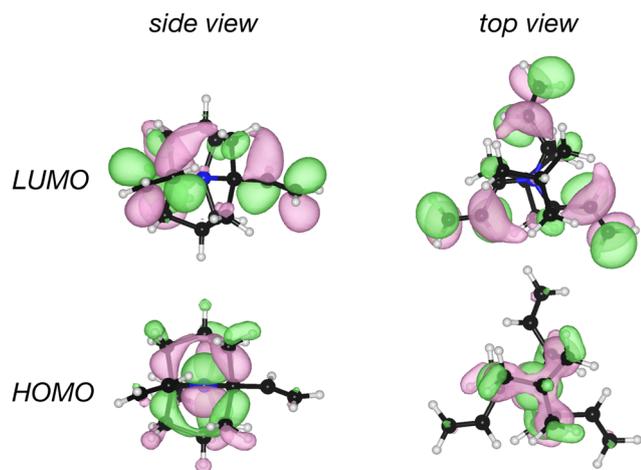


Fig. 7 Plots of HOMO and LUMO of the ethylene substituted double cage derivative of quinuclidine, structure 4, shown in Fig. 6.

3 Computational details

The bigQM7 ω dataset^{27,37} features 12 880 molecules with up to 7 CONF atoms with equilibrium geometries determined with the ω B97X-D DFT method and the def2-TZVP basis set. Molecular graphs of bigQM7 ω molecules, encoded as SMILES, were sourced from the GDB11³⁸ chemical space, which contains several common molecules, such as acetic acid and benzoic acid, which the subsequent databases GDB13³⁹ and GDB17⁴⁰ filtered out. Consequently, bigQM7 ω offers over thrice the number of molecules with up to 7 CONF atoms than the QM9 dataset⁴¹ derived from GDB17. The geometries of the bigQM7 ω molecules were optimized²⁷ using the connectivity preserving geometry optimization (ConnGO) workflow to prevent covalent bond

rearrangements during geometry optimization.⁴² Further, vibrational frequency analysis at the ω B97X-D/def2-TZVP level verified each structure in bigQM7 ω to be an energy minimum. Along with the minimum energy geometries, bigQM7 ω offers several ground-state properties ranging from partial charges to thermochemistry energies along with excited state properties.³⁷

For all 12 880 molecules in bigQM7 ω , we performed single-point vertical excited state calculations of the S_1 and T_1 energies using 12 DFT methods: PBE0,⁴³ B3LYP,⁴⁴ CAM-B3LYP,⁴⁵ ω B97X-D3,⁴⁶ LC-BLYP,⁴⁷ LC-PBE,⁴⁸ PBE-QIDH,⁴⁹ SCS-PBE-QIDH, SOS-PBE-QIDH,³⁰ RSX-QIDH,⁵⁰ SCS-RSX-QIDH³⁰ and SOS-RSX-QIDH.³⁰ We also calculated S_1 and T_1 energies using the correlated excited state method: second-order algebraic diagrammatic construction, ADC(2). The accuracy of ADC(2) and SCS-PBE-QIDH in combination with other settings is evaluated in the ESI† using previously reported²⁵ TBEs of 10 triangular systems as references. For these systems and the cage-type molecules, we performed geometry optimization – using the ω B97X-D3 DFT method with tightscf and tightopt keywords in combination with the def2-TZVP basis set. Minimum energy structures and S_1/T_1 energies of the bigQM7 ω molecules can be queried using the pymoldis module presented in this study (see Fig. S8 and S12 in the ESI†).

We performed ADC(2) calculations using QChem 6.0.2 and DFT calculations using ORCA 5.0.4.^{51,52} With in TDDFT, we calculated twelve energy eigenvalues—six singlets and six triplets—which we sorted separately to extract S_1 and T_1 excitation energies. In all calculations, we used the resolution-of-identity (RI) approximation.^{53,54} In DFT calculations, we used the ‘chain-of-spheres’ (COS) algorithm for exchange integrals (RIJ-COSX). In dh-DFT calculations, we used the universal fitting auxiliary basis sets by Weigend⁵⁵ (denoted def2/J) along with the def2-TZVP/C and aug-cc-pVTZ/C basis sets for the orbital basis sets def2-TZVP, and aug-cc-pVTZ, respectively.

4 Conclusions

We have probed the violation of Hund’s rule in the chemical space of about 13 000 small organic molecules with up to 7 atoms of C/O/N/F. We performed high-throughput calculations of excited states with various DFT methods and a more accurate theory, ADC(2). We selected these methods based on their accuracies when compared with the previously reported theoretical best estimates for the STGs of ten triangular molecules. ADC(2) with a triple-zeta basis set provides an effective cost-accuracy trade-off for generating large-scale data. Further, this method has been shown²⁵ to agree with composite excited state methods for predicting $STG < 0$ with an average error of < 0.05 eV. The critical result of the present study is that Hund’s rule⁵⁶ prevails across thousands of organic molecules with systematically varying structures covering almost all prototypical small organic molecules.

The data presented in this study is importable in Python code for data mining endeavors. Using this infrastructure, we identified molecules with vanishing STGs, some of which have negative values at the SCS-PBE-QIDH level while not violating

Hund’s rule as per ADC(2) predictions. A common geometric feature of these molecules was a substantial deviation of an N-atom from the typical sp^3 environment with both singlet and triplet excitations showing the $n \rightarrow \sigma^*$ character and potentially vanishing exchange interaction integral between the MOs involved in excitation.

We have selected a cage structure and attached ethylene groups to mimic the environment of the N atom as in the well-known cases of cycl[3.3.3]azines. The corresponding MOs involved in S_1/T_1 excitations exhibit characteristics seen in previously studied triangular negative-STG systems. Upon further scrutiny, we showed this molecule to obey Hund’s rule. Yet, introducing a polarizable environment in this system through donor–acceptor groups may selectively stabilize S_1 over T_1 .^{10,23} In this study, we did not investigate the practical utility of the small molecules studied here in the context of TADF. Such exploration necessitates meticulous consideration of adiabatic effects on a case-by-case basis, a task that exceeds the scope of our present investigation. The present study demonstrates that a data-driven approach allows for gaining insight into the molecular structural factors that can quench the singlet–triplet energy gap. We offered evidence that the chemical space of small closed-shell organic molecules lacks geometric and electronic structural factors that may be necessary for a negative S_1-T_1 energy gap.

Theoretical studies have identified only a few molecular fingerprints to favor negative STGs. Dynamic spin polarization, attributed to double excitation effects involving frontier orbitals, has emerged as a potential mechanism to induce a negative STG.^{1,57,58} While a quantitative relationship exists between molecular structural features and zero STG, a corresponding structure–property relation for negative STG remains elusive. Introducing functional groups is one promising avenue for designing large synthetically tractable molecules with negative STGs. For theoretical explorations, our research highlights the limitations of using DFT methods, which can result in false positives. Consequently, there is a pressing need for efficient strategies to accelerate predictions using correlated wavefunction methods. For large-scale investigations, data-driven approaches such as machine learning can complement first-principles modeling, especially when combined with inverse-design strategies, such as genetic algorithms.^{59,60}

Data availability

The data supporting the findings of this study are available within the article and its ESI.†

Author contributions

AM: conceptualization (equal); analysis (equal); data collection (equal); writing (equal); revision (equal). RR: conceptualization (equal); analysis (equal); data collection (equal); funding acquisition; project administration and supervision; resources; writing (equal); revision (equal).

Conflicts of interest

The author has no conflicts of interest to disclose.

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