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Numerical study of thermal and solutal advancements in ZnO–SAE50 nanolubricant flow past a convergent/divergent channel with the effects of thermophoretic particle deposition

Shilpa B.,[a] Pudhari Srilatha,[b] Umair Khan, [ID] *[c] Naveen Kumar R.,[d] Samia Ben Ahmed[e] and Raman Kumar[f]

The present research focuses on the significance of thermophoretic particle deposition on a ZnO–SAE50 nanolubricant flow in a stretchable/shrinkable convergent/divergent channel in the presence of an applied magnetic field and nonlinear heat radiation. A parameter in the governing differential equations and wall boundary conditions defines the physical mechanism of the model. The Galerkin finite element method, combined with similarity transformation, is adopted to solve the governing equations. The Levenberg–Marquardt backpropagating algorithm of an artificial neural network model forecasts heat and mass transfer properties. The results reveal that by stretching/shrinking the walls enough, the classical flow and heat properties are significantly affected. The stretching of the convergent or divergent channel is observed to increase the velocity profiles, whilst shrinking results in backflow regions. In terms of the temperature field, stretching causes more heat to be produced in the flow; nevertheless, the thermal layer is decreased and cooling is attained by channel shrinkage, which may have important technical implications.

1. Introduction

Fluid flow may often occur in various technical and natural systems and in both convergent and divergent channels (CDCs). These channels include alterations in the cross-sectional area along the flow direction, which substantially influences the behaviour of the fluid moving through the channel. CDCs play essential roles in various technical applications, including nozzles and diffusers in water jets, jet engines and hydraulic systems. To build effective and secure systems, it is necessary to have a solid understanding of the factors that control the flow of fluids *via* these channels. Analyzing and predicting fluid flow behaviour in CDCs represent a common task for engineers, who often utilize various mathematical models to do so. The significance of physical factors on the thermal attributes of nanofluid (NF) flow in CDCs was evaluated by Hamedani *et al.*[1] Utilizing the novel hybrid model, Khan *et al.*[2] studied the heat transmission attributes of the NF flow past a CDC. The consequence of heat production on the flow of NF past a permeable CDC was explained by Mishra *et al.*[3] The flow of magnetic NFs *via* a stretchy CDC was elaborated by Hashim and Chu.[4] The heat transmission analysis of a non-Newtonian NF flow past CDCs was examined by Rehman *et al.*[5] using numerical modelling. Turkyilmazoglu[6] has analyzed the extended Jeffery–Hamel flow in the stretching/shrinking of a CDC.

Nanoparticles significantly enhance heat transmission in nanoliquids, which are nanoparticle suspensions in a base liquid such as water or oil. This growing topic of study has attracted substantial interest because of its potential to boost the efficiency of many heat transport applications. Recently, Razaq *et al.*[7] explored a flow with three nanoparticle suspensions. Khan *et al.*[8] explored the heat transmission features in nanoliquid flow past the Homann stagnation region with heat radiation. Sarfraj *et al.*[9] explored the flow of fluid with two different nanoparticle suspensions. Recently, Sarfraz and Khan[10–13] explored the thermal and mass transmission features in different nanoliquids with various nanoparticle suspensions past different surfaces with varied physical characteristics. Qureshi *et al.*[14] examined the morphological nanolayer effect on the flow of hybrid nanoliquids generated by the dispersion of

[a] Department of Mathematics, Dayananda Sagar College of Engineering, Bengaluru, Karnataka, India. E-mail: abshilpa1297@gmail.com

[b] Department of Mathematics, Institute of Aeronautical Engineering, Hyderabad, India. E-mail: pudhari.srilatha@gmail.com

[c] Department of Computer Science and Mathematics, Lebanese American University, Byblos, Lebanon. E-mail: umair.khan@lau.edu.lb

[d] Department of Mathematics, Amrita School of Engineering, Amrita Vishwa Vidyapeetham, Bengaluru, India. E-mail: nkrmaths@gmail.com

[e] Department of Chemistry, College of Sciences, King Khalid University, Abha, P.O. Box 9004, Saudi Arabia. E-mail: sbahmad@kku.edu.sa

[f] Department of Mechanical Engineering, University Centre for Research & Development, Chandigarh University, Mohali, Punjab, 140413, India. E-mail: raman.me@cumail.in

a polymer/CNT nanocomposite material. The heat transfer in hybrid micropolar ferrofluid flow across a non-linearly stretched sheet was investigated by Rauf et al.[15]

Nanolubricants are a subcategory of advanced lubricants that make use of nanoparticles as a means of improving the lubricating qualities of the product. The ZnO (zinc oxide)–SAE50 nanolubricant is a particular form of nanolubricant intended to fulfil the SAE50 viscosity grade and includes zinc oxide nanoparticles. This viscosity grade is often used in engine oils for various industrial and automotive applications. Recently, Nayak et al.[16] investigated the flow of ZnO–SAE50 nanolubricant (NL) via a moving Riga surface in the presence of heat radiation. Muhammad and Ahmed[17] debriefed an unsteady ZnO–SAE50 NL flow in a permeable channel using the subdomain method. Utilizing the Cattaneo–Christov model, Riaz et al.[18] explored variable attributes of the ZnO–SAE50 NL flow via a Riga plate. Riaz et al.[19] inspected the behaviour of ZnO–SAE50 NL flow across a Riga plate. Utilizing the Buongiorno model, Nagapavani et al.[20] explored variable attributes of the ZnO–SAE50 NL flow via a heated disk.

A complicated and significant phenomenon in fluid mechanics with respect to heat transfer is fluid flow with thermal radiation. The interaction of fluid flow and thermal radiation is an essential component of heat transport in a number of different technical and natural systems. To improve energy efficiency, develop better systems, and solve difficulties associated with climate change, it is vital to understand this coupled phenomenon and optimize it as much as possible. The complexity of the flow of fluids with heat radiation is being addressed in a wide variety of applications, and researchers and engineers are continuing to investigate novel methods for doing so. The influence of radiation on the Maxwell liquid flow past an SS was elaborated by Khan et al.[21] utilizing a numerical scheme. The flow of Casson fluid past an SS with the effect of thermal radiation was inspected by Eswaramoorthi et al.[22] Srilatha et al.[23] debriefed the flow of non-Newtonian fluid via an SS with the impact of radiation. The flow of nanoliquid across an SS with the consequence of thermal radiation was probed by Khan et al.[24] The flow of Reiner–Rivlin fluid past an SS with heat radiation was analyzed by Khan et al.[25]

The movement of suspended particles in a fluid (liquid or gas) and their deposition onto surfaces due to thermophoresis are described as a complicated process known as fluid flow with thermophoretic particle deposition. This method is essential in various applications, including the management of air pollution, the field of nanotechnology, and the deposition of aerosols. This is a process in which temperature gradients cause particles to be guided toward surfaces and deposited there. It is crucial to understand and be able to regulate this process in distinct scientific and technical domains since it has numerous applications, from the regulation of air pollution to the development of nanotechnology. The flow of Oldroyd-B fluid past a spinning disk subjected to thermophoretic particle deposition (TPD) was analyzed by Hafeez et al.[26] The consequence of TPD on the flow of fluid via a revolving disk was scrutinized by Prasannakumara and Gowda.[27] Bashir et al.[28] analyzed the effect of TPD on the flow of Oldroyd-B liquid via an SS. Kumar et al.[29]

investigated the effect of TPD on Maxwell liquid flow through an SS. Ramesh et al.[30] examined TPD's impact on fluid flow through a stretchable surface to determine its relevance.

In several areas of artificial intelligence and machine learning, artificial neural networks (ANNs), have shown success. One of the main challenges is optimizing ANN parameters to reduce loss function. There may be issues with gradient descent and other traditional optimization strategies, such as sluggish convergence or convergence to local minima. The Levenberg–Marquardt (LM) methodology is a widely used method for ANN optimization. This approach was developed to solve the problems mentioned above. The LM algorithm combines Gauss–Newton and the steepest descent techniques. It is now used to train ANNs. However, it was first developed to handle nonlinear least-squares issues. Mishra and Chaudhuri[31] examined the flow of third-grade fluid on heated plates using an ANN approach to explore their results. Shafiq et al.[32] used the ANN technique to examine the Williamson fluid flow via a radiative surface. The analysis of entropy generation in the fluid flow past the rotating disk was evaluated by Zhao et al.[33] using the ANN scheme. Srinivasacharya and Kumar[34] examined Casson fluid flow past a radial SS using the ANN approach. The heat transport attributes of the fluid flow using the ANN scheme were studied by Sedani et al.[35] Recently,[36-40] have utilized the LM backpropagating algorithm to analyze the thermal and mass transmission characteristics in vertical geometries such as a vertical channel, vertical annulus and inclined annulus under significant effects such as heat generation/absorption, heat sources/sinks, chemical reactions and thermal non-equilibrium conditions.

To improve the functionality of modern lubricants like ZnO–SAE50 NL, researchers are delving into the complexities of stretching/shrinking of CDCs. This novel technology uses radiation's ability to generate changes at the nanoscale, which are then used to fine-tune the characteristics of the lubricant. Scientists are at the forefront of creating nanoscale lubrication solutions that have the potential to revolutionize industries dependent on environmentally friendly and efficient machinery. They are doing this by utilizing advanced techniques such as thermophoretic particle deposition and utilizing the power of ANNs and finite element modelling (FEM). The advancement of lubrication science is being driven toward new levels of efficiency and sustainability by the synergy of various professions that are complementary to one another.

2. Mathematical formulation

The steady two-dimensional flow of an incompressible, laminar transmitting viscous ZnO–SAE50 NL from a source or sink of two walls that can expand or contract at an angle of 2α at channel walls that intersects on the z-axis and lies in planes is considered in a system of cylindrical polar coordinates (r, θ, z). Fig. 1 depicts the problem's schematic diagram.

The vector form of the equations is given by

$$\nabla \cdot v_i = 0, \tag{1}$$

$$\rho_{nf}[v_j v_{i,j}] = -\nabla p_i + \mu_{nf}\nabla^2 v_i + J \times B, \tag{2}$$

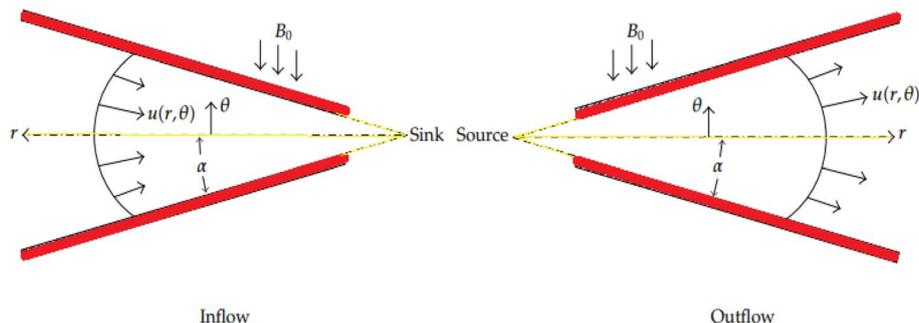Fig. 1   Geometrical configuration.

$$v_j T_{i,j} = \frac{k_{nf}}{(\rho c_p)_{nf}} \nabla^2 T_i - \frac{1}{(\rho c_p)_{nf}} \nabla(r q_{i,\mathrm{rad}}), \tag{3}$$

$$v_j C_{i,j} = D_B \nabla^2 C_i - \nabla(C V_T)_j. \tag{4}$$

We assumed that $u_\theta = 0$, it indicates that there have been no modifications to the z-direction, and thus, the flow is exclusively in the radial direction and solely depends on r and θ. Along the z-axis, there is a negligible magnetic field. A uniform magnetic field is exerted perpendicular to the direction of flow. The mass, linear momentum, thermal, and solutal balance laws are the foundation of the governing equations for this issue. They have been adjusted to consider the existence of the magnetic field and the effects of nonlinear heat radiation in this convergent/divergent channel. The governing equations are (see ref. 3–6 and 18)

$$\frac{1}{r} \frac{\partial(ur)}{\partial r} = 0 \tag{5}$$

$$u\frac{\partial u}{\partial r} = -\frac{1}{\rho_{nf}}\frac{\partial p}{\partial r} + v_{nf}\left[\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r} + \frac{1}{r^2}\frac{\partial^2 u}{\partial \theta^2} - \frac{u}{r^2}\right] - \frac{\sigma_{nf} B_0^2 u}{r^2 \rho_{nf}} \tag{6}$$

$$-\frac{1}{\rho_{nf}}\frac{1}{r}\frac{\partial p}{\partial \theta} + \frac{2}{r^2}v_{nf}\frac{\partial u}{\partial \theta} = 0 \tag{7}$$

$$u\frac{\partial T}{\partial r} = \frac{k_{nf}}{(\rho c_p)_{nf}}\left[\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r} + \frac{1}{r^2}\frac{\partial^2 T}{\partial \theta^2}\right] - \frac{1}{(\rho c_p)_{nf}}\frac{1}{r}\frac{\partial[r q_{r,\mathrm{rad}}]}{\partial r}$$
$$- \frac{1}{(\rho c_p)_{nf}}\frac{1}{r^2}\frac{\partial[r q_{\theta,\mathrm{rad}}]}{\partial \theta} \tag{8}$$

$$u\frac{\partial C}{\partial r} = D_B\left[\frac{\partial^2 C}{\partial r^2} + \frac{1}{r}\frac{\partial C}{\partial r} + \frac{1}{r^2}\frac{\partial^2 C}{\partial \theta^2}\right] - \frac{\partial[C V_T]}{\partial \theta} \tag{9}$$

Using Roseland approximation for radiation (see ref. 3–6 and 18)

$$q_{r,\mathrm{rad}} = -\frac{4\sigma^*}{3k^*}\frac{\partial T^4}{\partial r} \text{ and } q_{\theta,\mathrm{rad}} = -\frac{4\sigma^*}{3k^*}\frac{\partial T^4}{\partial \theta} \tag{10}$$

The thermophoretic deposition velocity in the θ – direction is of the form

$$V_T = -k\frac{V}{T_r}\frac{dT_\theta}{d\theta} \tag{11}$$

Because of the presumption of symmetry at the channel midline

$$\theta = 0, \frac{\partial C}{\partial \theta} = \frac{\partial T}{\partial \theta} = \frac{\partial u}{\partial \theta} = 0, u = \frac{u_c}{r} \tag{12}$$

Because of the plate stretching/shrinking convergent/divergent wall conditions,

$$\theta = \pm\alpha, u = \frac{S}{r}, T = \frac{T_w}{r^2}, C = \frac{C_w}{r^2} \tag{13}$$

The radial velocity can be expressed as

$$u(r, \theta) = \frac{f(\theta)}{r} \tag{14}$$

On integration of the azimuthal velocity expression the stretching pressure can be written as

$$\frac{p}{\rho} = \frac{2V}{r^2}f(\theta) + g \tag{15}$$

Similarity variables (see ref. 3–6 and 18)

$$F(\eta) = \frac{f(\theta)}{u_c}, \eta = \frac{\theta}{\alpha}, \Theta(\eta) = r^2\frac{T}{T_w}, \phi(\eta) = r^2\frac{C}{C_w}. \Big\} \tag{16}$$

Invoking ref. 41–45, the ZnO–SAE50 nanolubricant effective dynamic viscosity, effective density, heat capacitance, and effective thermal conductivity are determined by using

$$\mu_{nf} = \mu_{ZnO-SAE50} = \mu_{SAE50}(-0.002407T + 1.035 + 0.04336\phi), \tag{17}$$

$$\mu_{SAE50} = 2.174\,e^{-0.06062T}, \tag{18}$$

$$\rho_{nf} = \rho_{SAE50}(1 - \phi) + \rho_{ZnO}\phi = \rho_{ZnO-SAE50}, \tag{19}$$

$$(\rho C_p)_{nf} = (1 - \phi)(\rho C_p)_{SAE50} + (\rho C_p)_{ZnO}\phi = (\rho C_p)_{ZnO-SAE50}, \tag{20}$$

$$\frac{k_{nf}}{k_f} = 1 + \frac{k_{ZnO}A_{ZnO}}{k_{SAE50}A_{SAE50}} + ck_{ZnO}P_e\left(\frac{A_{ZnO}}{k_{SAE50}A_{SAE50}}\right) = \frac{k_{ZnO-SAE50}}{k_{SAE50}}, \tag{21}$$

$$c = 25\,000, P_e = \frac{d_{ZnO}u_{ZnO}}{\alpha_{SAE50}}, \frac{A_{ZnO}}{A_{SAE50}} = \frac{d_{SAE50}}{d_{ZnO}}\left(\frac{\phi}{1-\phi}\right), u_{ZnO}$$

$$= \frac{2k_B T}{\pi \mu_{SAE50} d_{ZnO}^2}. \tag{22}$$

The governing equations are:

$$F''' + 4\alpha^2 F' + M\alpha^2 F'(1-\phi)^{2.5}$$
$$+ FF'\alpha Re\left[(1-\phi) + \frac{\rho_s}{\rho_f}\right](1-\phi)^{2.5} = 0 \tag{23}$$

$$(A_2 + N)\Theta'' + 4\alpha^2(A_2 + N)\Theta + 2F Pr\alpha^2\Theta[1 + A_1\phi - \phi] = 0 \tag{24}$$

$$\alpha^2\left[4\phi + \frac{\phi'}{\alpha^2}\right] + 2F\alpha\phi ReSc + kScA_3\left[\phi'\Theta' + \phi\Theta''\right] = 0 \tag{25}$$

The boundary conditions are

$$F(0) = 1, F'(0) = 0, F(1) = \lambda, \Theta'(0) = 0, \Theta(1) = 1, \phi'(0) = 0, \phi(1) = 1. \tag{26}$$

where $\lambda = \frac{S}{u_c}$ is the stretching ($\lambda > 0$) or shrinking ($\lambda < 0$) parameter and

$$Re = \frac{\alpha u_c}{v_f}, Pr = \frac{u_c(\rho c_p)_f}{k_f}, M = \frac{B_0^2\sigma_f}{\mu_f}, N = \frac{16\sigma^*T_\infty^3}{3k^*k_f}, Sc = \frac{v_f}{D_B},$$
$$A_1 = \frac{(\rho c_p)_{ZnO}}{(\rho c_p)_{SAE50}}, A_2 = \frac{k_{nf}}{k_f}, A_3 = \frac{\left[(1-\phi) + \frac{\rho_{ZnO}}{\rho_{SAE50}}\right](1-\phi)^{2.5}}{v_{SAE50}}. \tag{27}$$

Using the heat flux $q_w = -k\left(\frac{1}{r}\frac{\partial T}{\partial \theta}\right)$ the Nusselt number is given by

$$Nu = \frac{rq_w|_{\theta=\alpha}}{kT_w} = -\frac{1}{r^2\alpha}\Theta'(1) \tag{28}$$

3. Numerical method

The governing coupled nonlinear PDEs, eqn (5)–(9), are transformed into coupled nonlinear ODEs, eqn (23)–(25), by adopting apt similarity transformation eqn (16) and the relative boundary conditions provided in eqn (26). These equations are solved by adopting Galerkin FEM. The system of equations is transformed into a matrix form using GFEM, and the matrix elements are then merged to give the global matrix for the whole area. This global matrix is then iteratively solved to provide fluid
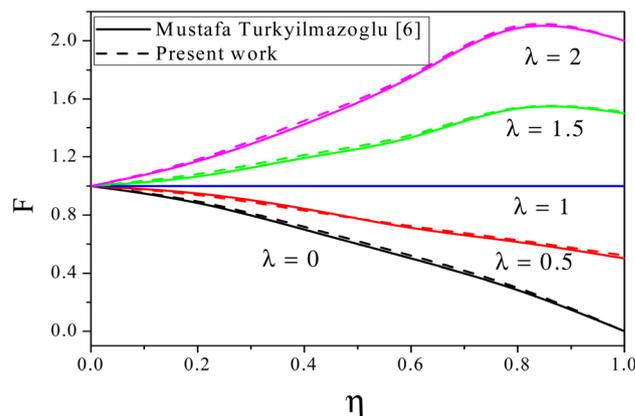

Fig. 2 Comparison of the present work with Turkyilmazoglu.[6]

flow, thermal and mass transmission fields. The computed solution error level is set at 0.00001 to ensure high precision in the results. When the disparity between the original state and the current state for the elements at all nodes reaches the aforementioned error level, the iterative process comes to an end.

Quadratic elements with three nodal points are used to determine the velocities, temperatures, concentrations, and heat and mass transport coefficients. The quadratic trail and shape functions are given below:

$$F(\eta) = N_1(\eta)F_1 + N_2(\eta)F_2 + N_3(\eta)F_3 \tag{29}$$

$$\Theta(\eta) = N_1(\eta)\Theta_1 + N_2(\eta)\Theta_2 + N_3(\eta)\Theta_3 \tag{30}$$

$$\phi(\eta) = N_1(\eta)\phi_1 + N_2(\eta)\phi_2 + N_3(\eta)\phi_3 \tag{31}$$

The shape functions are

$$N_1 = 1 - \frac{3\eta}{l} + \frac{2\eta^2}{l^2}, N_2 = \frac{4\eta}{l} - \frac{4\eta^2}{l^2}, N_3 = \frac{2\eta^2}{l^2} - \frac{\eta}{l} \tag{32}$$

The element stiffness matrix at one node can be written as $\tilde{K} = [K_1 K_2 K_3 K_4]$.

The stretching of the divergent channel on the velocity profile for the specific values $\alpha = 5$, $Re = 50$, $M = 0$, $\phi = 0$, $N = 0$, and $A_1 = A_2 = 1$ are compared with the existing Mustafa Turkyilmazoglu[6] results and a good concurrence is noticed between these two results (Fig. 2).

4. Results and discussion

In this part, the effects of expanding or shrinking of CDCs *via* the factor are researched on the flow model being investigated to analyze the flow and heat transport features. This section aims to describe the graphical and numerical results of different critical factors of relevance on the pertinent characteristics of the ZnO–SAE50 NL. The modified system of ODEs and BCs is numerically addressed using GFEM. The data have
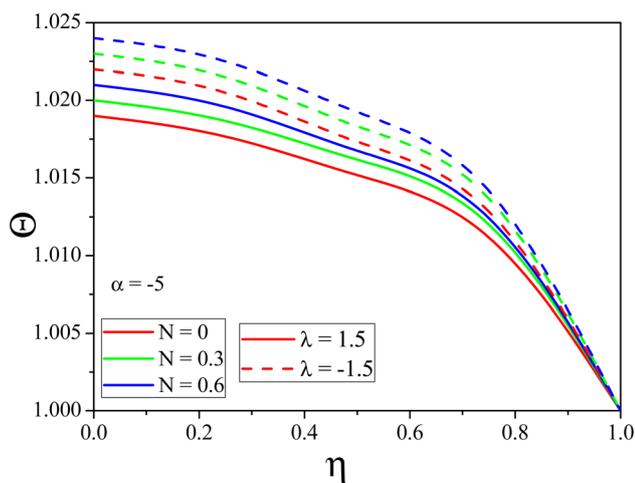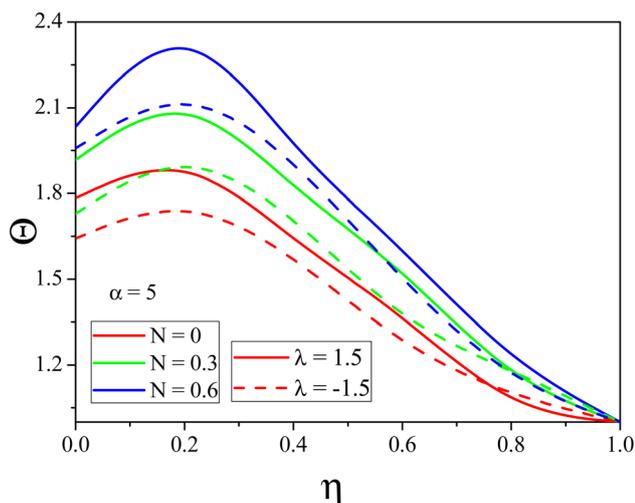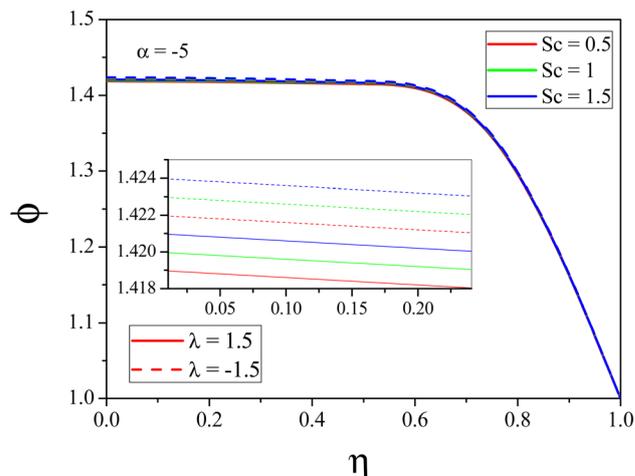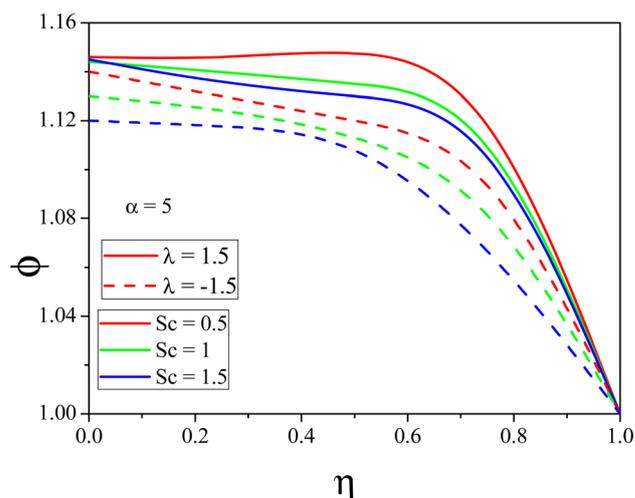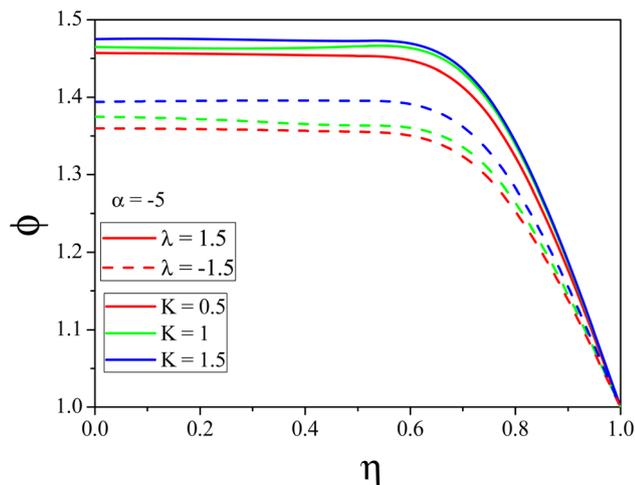
Fig. 7 $\phi(\eta)$ variation with distinct values of K for the convergent channel.
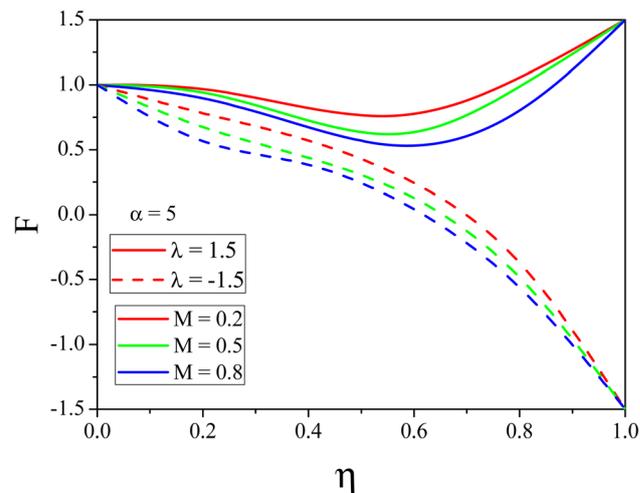

Fig. 9 $F(\eta)$ variation with distinct values of M for the divergent channel.

concentration. The upshots of M on $F(\eta)$ for both the cases are depicted in Fig. 9 and 10 respectively. As M increases the fluid velocity decreases in both CDCs. The Lorentz force is generated when a magnetic field is applied to conducting liquid. This force exerts its influence in the opposite direction of the flow, resulting in a slowing of the flow and, therefore, a reduction in the velocity of the fluid. This decrease in the flow speed, is accompanied by an increase in the temperature state of the fluid. This phenomenon is made abundantly obvious by the observable decrease in fluid velocity as well as the accompanying increase in fluid temperature. Fig. 11 and 12 represent the Reynolds number influence on $F(\eta)$. In Fig. 11 and 12, the momentum boundary layer exhibits a mixed pattern of expansion and decline. This can be seen in both figures. When either the α or the Re is increased, the thickness of the thermal boundary layer is shown to be significantly increased in Fig. 11 and 12. This occurs regardless of whether the boundary layer is being stretched or shrunk.
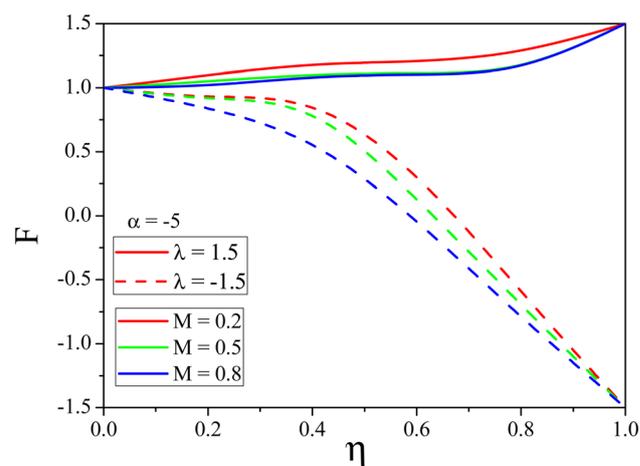

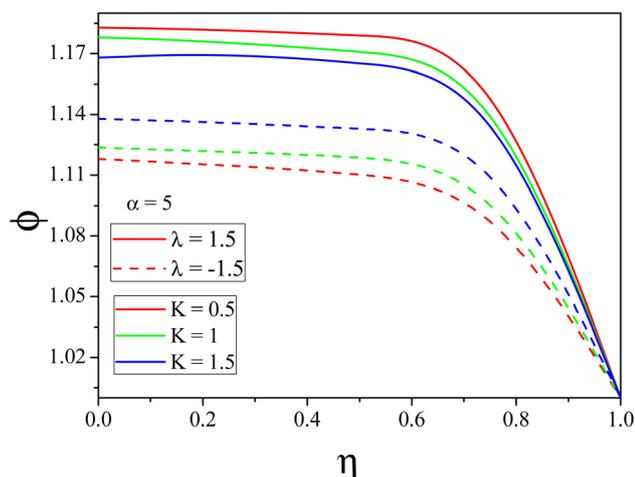Fig. 10 $F(\eta)$ variation with distinct values of M for the convergent channel.

Fig. 8 $\phi(\eta)$ variation with distinct values of K for the divergent channel.
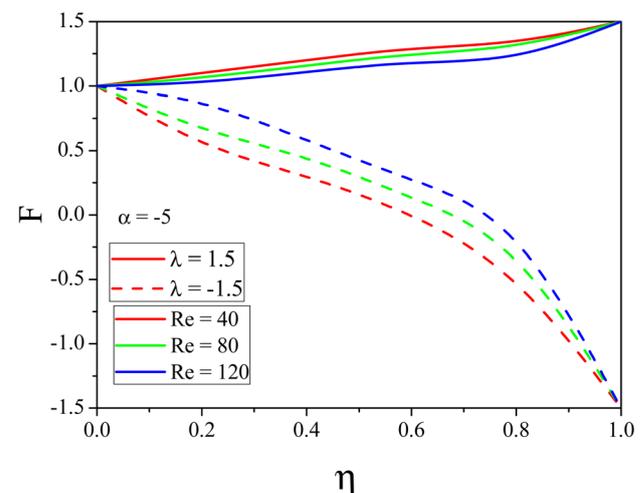

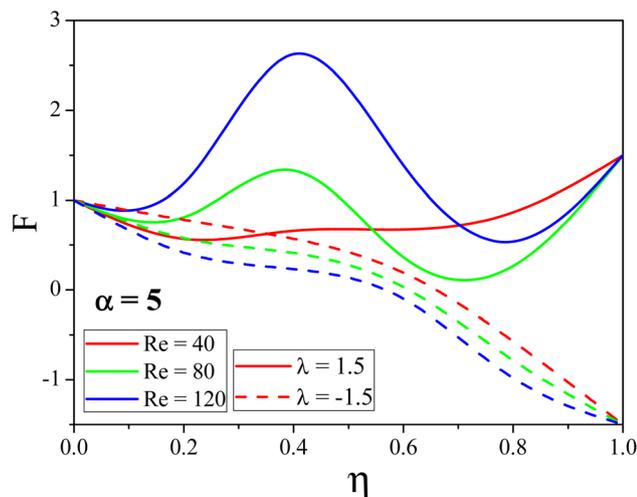Fig. 11 $F(\eta)$ variation with distinct values of Re for the convergent channel.

Fig. 12 $F(\eta)$ variation with distinct values of Re for the divergent channel.

5. ANN model

An artificial intelligence method was used to investigate the thermal and mass transport in the channel. This method was developed to serve as an alternative to the more traditional mathematical approaches. The ANN model is used as the analytical technique that is being taken here. With their complex logic, ANN models are an optimal tool that works well in simulations of intricate structures even when there is no functional relationship between them. This is because ANN models have sophisticated logic. The multi-layer perceptron (MLP) network model is used in the ANN model, which is designed to estimate the Nu and Sh values. Due to the MLP network model's robust construction, it has a reliable estimating capability. Each layer in MLP networks has a direct association with the one above it, since the design of these networks is layered. An MLP network's first layer, known as the

input layer, is the point at which new information is introduced into the system and in any MLP network, there is always a hidden layer. The very last layer of the MLPNN and the layer that is directly subsequent to the hidden layer is the output layer, where the results of simulations are collected. The underlying physical structure of this method is shown in Fig. 13(a) and (b).

The artificial neural network model is employed to analyze the thermal and mass transport features in the channel. The significant parameters such as M, Re, N, Pr, Sc, K, α and λ are considered as input parameters for the neural network. Nu and Sh are taken as output parameters. Fig. 13(a) and (b) show the neural network architecture for the considered ANN model. In this model four hidden layers are considered and in each hidden layer ten neurons are considered to forecast the heat and mass transport features. The input data are categorized into three parts, first part 70% is utilized for training, the second part 15% used for testing and 15% used for validation.

For the MLP network model, the Levenberg–Marquardt training (LMT) method was selected as the optimal option. The LMT approach is the one that is most often used in MLPNNs because of its excellent learning capabilities. The MLP network's hidden and output layers employ the tan-sig and purelin transport functions, respectively, as transfer functions. The transfer function models that were used are outlined below.

$$f(x) = \frac{1}{1 + e^{-x}}, \text{pureline}(x) = x$$

Fig. 14(a)–(e) and 15(a)–(e) represent the ANN model to forecast the heat and mass transport properties. Fig. 14(a) and 15(a) show the regression plot for the thermal and mass transport and it is clear that the root mean square and mean squared error values for heat and mass transfer are $R = 9.99999 \times 10^{-1}$ and $R = 9.99999 \times 10^{-1}$, MSE $= 5.8874 \times 10^{-9}$ and MSE $= 3.4265 \times 10^{-6}$ respectively. After training a feed forward neural network, an error histogram is created by creating
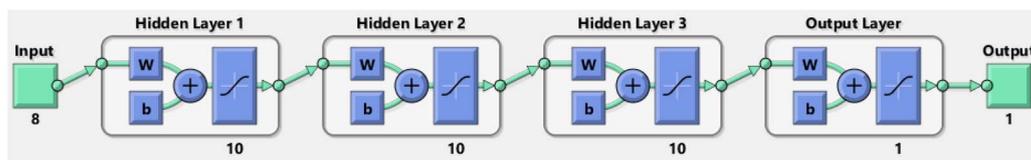
(a)
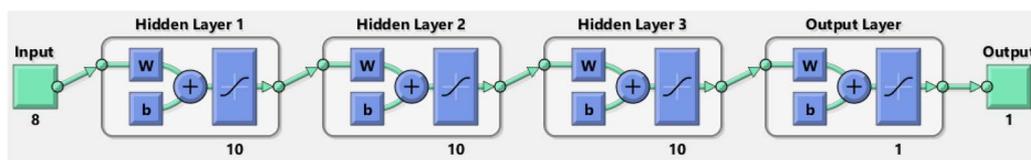
(b)

Fig. 13 Neural network for (a) heat transfer and (b) mass transfer.

Fig. 14 ANN model for heat transfer analysis: (a) regression plot, (b) error histogram, (c) training state performance, (d) performance validation and (e) function fit plot.

a histogram of the discrepancies between the goal values and the forecast values. It is possible for these error figures to be in the negative since they indicate the amount by which expected values differ from goal values. Bins are used to denote the number of vertical bars that are present in the graph. The whole error range is divided up into thirty distinct buckets for easier
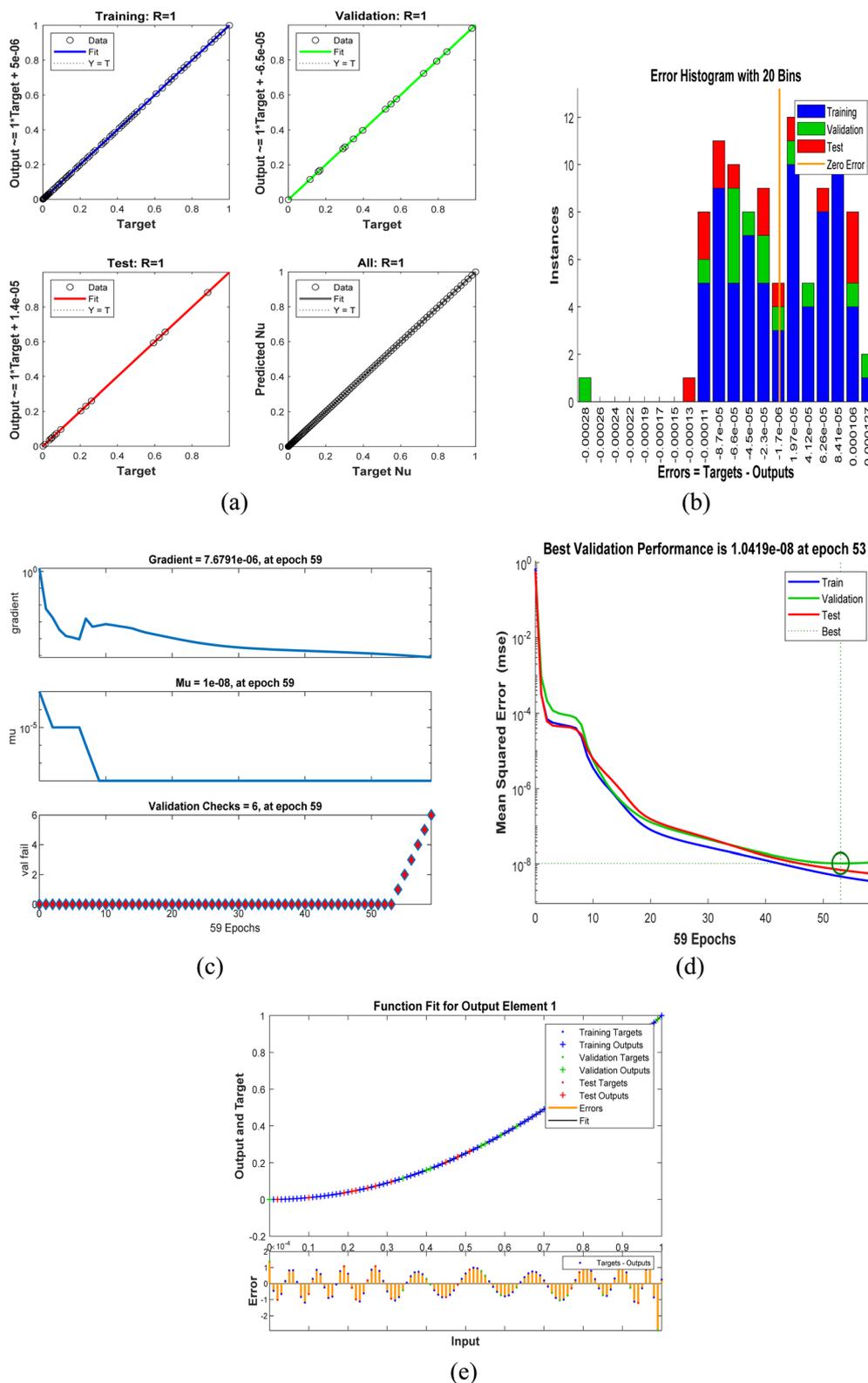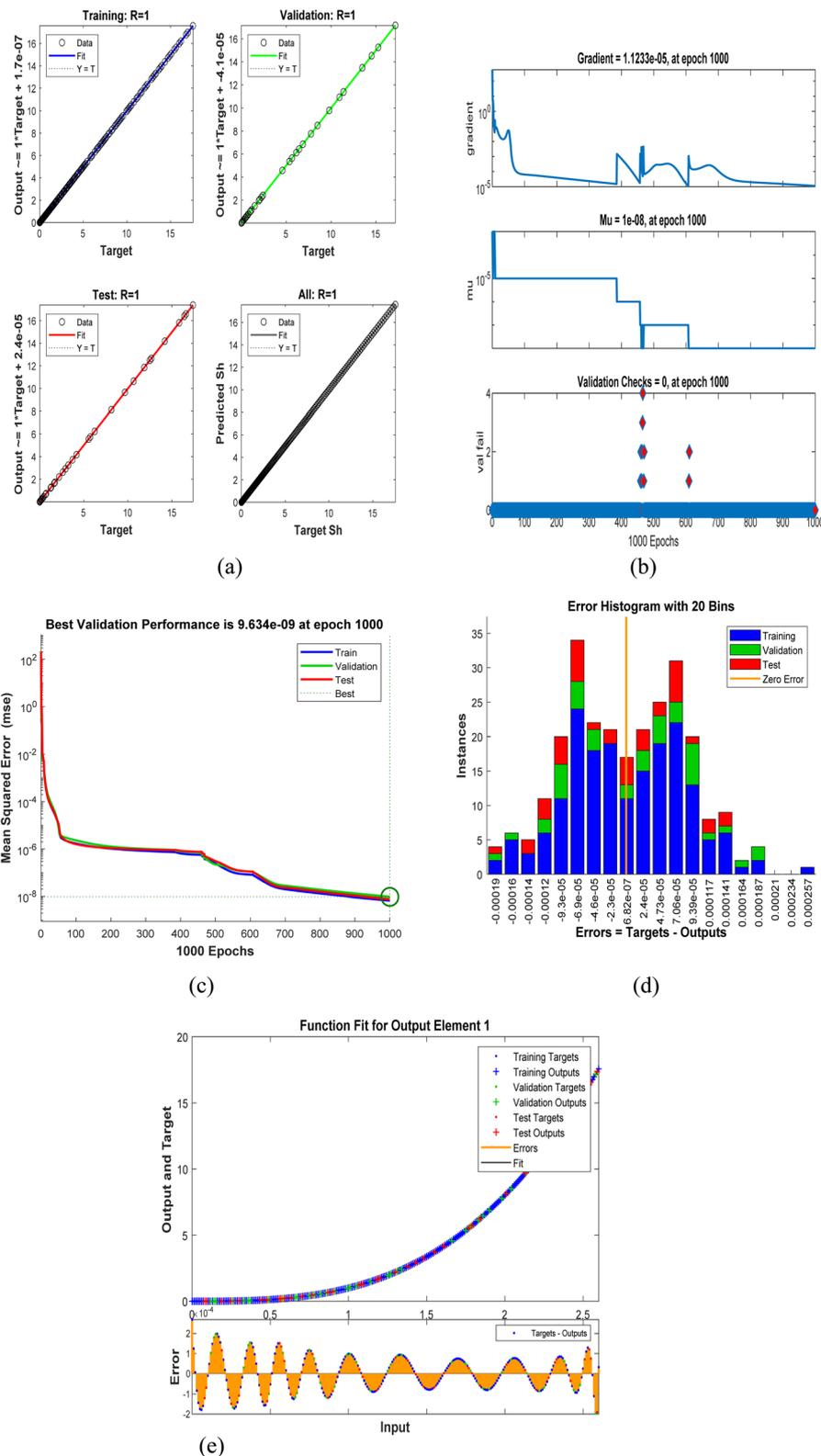
Fig. 15 ANN model for mass transfer analysis: (a) regression plot, (b) training state performance, (c) performance validation, (d) error histogram and (e) function fit plot.

analysis. The number of samples from the dataset that are included inside a certain bin is shown along the Y-axis. Fig. 14(b) illustrates a bin in the center of the plot, which

corresponds to an error of -1.7×10^{-5}, and the maximum height of that bin for the training dataset is less than but close to 3, but the maximum height of that bin for the validation

dataset and the test dataset is between 3 and 5. It denotes that a number of samples taken from a variety of datasets include an error that is located somewhere within the following range. The zero-error line illustrates the point at which the error axis (sometimes known as the X-axis) has a value of zero. In this particular illustration, the zero error point may be found within the bin that has a center value of -1.7×10^{-6}. In Fig. 15(d) the zero error line lies at 6.82×10^{-7} and also the validation and test dataset is between 12 and 18. Fig. 14(c) represents the performance plot of the training state in which the gradient is 7.6791×10^{-6} at epoch 59. The training continues until 6 at epoch 59. Similarly in Fig. 15(b) the gradient is 1.1233×10^{-5} at epoch 1000. Fig. 14(d)–(e) and 15(c) and (e) represent the best fit and function fit for the heat and mass transfer, respectively.

6. Conclusions

The current work analyses the significance of thermophoretic particle deposition on ZnO–SAE50 nanolubricant flow with magnetic field and nonlinear thermal radiations in a stretchable/shrinkable divergent/convergent channel. After converting the governing PDEs into nonlinear ODEs, the Galerkin finite element technique is used to solve these equations. The attained results are compared with the existing results and a good agreement is noticed. Also, the LM backpropagating ANN model is utilized to forecast the thermal and mass transport characteristics.

The following are conclusions drawn from the present work:

❖ The temperature of the nanolubricant increases in both convergent/divergent channels for increased N.

❖ As the Schmidt number goes up, the mass transfer rate goes up, and as a result, the mass transport goes down.

❖ In the convergent channel an increase in the thermophoretic coefficient enhances the concentration profile. Meanwhile in the stretching divergent channel the concentration profile decreases with k since when there is a substantial thermal difference, the thermophoretic force attracts more particles to the surface, which lowers the concentration.

❖ The liquid velocity in both the channel cases decreases as the Hartmann number rises. The Lorentz force is produced when a magnetic field is applied to an electrically conducting fluid. This force exerts its influence in the opposite direction of the flow, resulting in flow deceleration and, as a direct consequence of this, a reduction in the velocity of the fluid.

❖ The consequence of the Reynolds number on the liquid's speed of the stretching and shrinking of both the channels indicates a mixed characteristic of growth or decrease in the momentum boundary layer, depending on whether shrinking or stretching is applied to the channel.

Due to the many uses of flow through CDCs in engineering and industry, it is essential to understand how to construct relationships between flow, mass, and heat transport properties for various nanoliquids. Consequently, potential future research may centre on entropy production in addition to the influence of Stefan blowing, convective heat conditions, Newtonian heating, and non-uniform heat sources/sinks on various models of nanoliquid streams. This would allow researchers to

construct well-targeted numerical models and simulate distinct hydrodynamic and thermal interface constraints under different conditions, ultimately developing a mathematical model for non-Newtonian nanofluid flow.

Abbreviations

B_0	Transverse magnetic field
u	Velocity component (m s^{-1})
k_T	Thermal diffusion ratio
r	Radial direction
z	Axial direction
p	Pressure (Pa)
f	A function of θ
k	Thermophoretic coefficient
c_p	Specific heat (J kg^{-1} K^{-1})
T	Temperature (K)
q	Radiative heat flux
D_B	Brownian motion coefficient
C	Concentration
V_T	Non-dimensional thermophoretic velocity
u_c	Rate of movement in the radial direction (m^2 s^{-1})
S	Stretching/shrinking rate
T_w	Surface temperature of the channel (K)
C_w	Concentration at the surface
g	A function of r
V	Thermophoretic deposition velocity
F	Dimensionless radial velocity
M	Hartmann number
Re	Reynolds number
N	Radiation parameter
Pr	Prandtl number
Sc	Schmidt number
k	Thermal conductivity
q_w	Heat flux (W m^{-2})
Nu	Nusselt number

Greek letters

θ	Azimuthal direction
α	Elevation angle of convergent/divergent channels
ρ	Density
υ	Kinematic viscosity
σ	Electrical conductivity
η	Similarity variable
Θ	Temperature in non-dimensional form
ϕ	Volumetric expansion coefficient in dimensionless form
μ	Dynamic viscosity
λ	Stretching/shrinking parameter

Subscripts

nf	Nanofluid
f	Fluid

Conflicts of interest

There are no conflicts to declare.

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