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Indirect-to-direct band gap transition and optical properties of metal alloys of $\text{Cs}_2\text{Te}_{1-x}\text{Ti}_x\text{I}_6$: a theoretical study

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In recent years, double perovskites have attracted considerable attention as potential candidates for photovoltaic applications. However, most double perovskites are not suitable for single-junction solar cells due to their large band gaps (over 2.0 eV). In the present study, we have investigated the structural, mechanical, electronic and optical properties of the $\text{Cs}_2\text{Te}_{1-x}\text{Ti}_x\text{I}_6$ solid solutions using first-principles calculations based on density functional theory. These compounds exhibit good structural stability compared to $\text{CH}_3\text{NH}_3\text{PbI}_3$. The results suggest that Cs_2TeI_6 is an indirect band gap semiconductor, and it can become a direct band gap semiconductor with the value of 1.09 eV when the doping concentration of Ti^{4+} is 0.50. Moreover, an ideal direct band gap of 1.31 eV is obtained for $\text{Cs}_2\text{Te}_{0.75}\text{Ti}_{0.25}\text{I}_6$. The calculated results indicate that all the structures are ductile materials except for $\text{Cs}_2\text{Te}_{0.50}\text{Ti}_{0.50}\text{I}_6$. Our results also show that these materials possess large absorption coefficients in the visible light region. Our work can provide a route to explore stable, environmentally friendly and high-efficiency light absorbers for use in optoelectronic applications.

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1. Introduction

Lead-based halide perovskites have attracted great attention for optoelectronic applications in the past ten years.^{1–3} The power conversion efficiency (PCE) of lead-based halide perovskite solar cells increased to a high value of 25.2% in 2019.⁴ However, the toxicity of lead and poor stability still remain significant challenges for commercial applications. Extensive efforts have been paid to exploring lead-free or lead-less and stable halide perovskite materials for solar cells. Sn^{2+} and Ge^{2+} metal ions have been expected to replace Pb^{2+} ions in perovskites.^{5–9} Sn^{2+} ion is highly unstable and can be easily oxidized to Sn^{4+} ,¹⁰ which results in decreased photovoltaic performance. To date, the highest PCE of Ge-based halide perovskites is only 4.94%.¹¹ Two dimensional (2D) layered perovskites have been also explored such as $\text{A}_3\text{B}_2\text{X}_9$ ($\text{A} = \text{Cs}, \text{Rb}$; $\text{B} = \text{Bi}, \text{Sb}$; $\text{X} = \text{I}, \text{Br}, \text{Cl}$).^{12–17} However, these materials have large band gaps over 2.0 eV, which are not suitable for perovskite solar cells.

In recent years, halide double perovskites $\text{A}_2\text{B}^+\text{B}^{3+}\text{X}_6$ have been developed as stable and non-toxic alternatives to lead-based perovskite materials, where two toxic lead ions are substituted by using monovalent B^+ and trivalent B^{3+} , such as Ag^+ , Bi^{3+} , and Sb^{3+} . Although most of the reported double perovskites show excellent stability, their band gaps are too large for application in single-junction solar cells.^{18–21} A low band gap (1.66 eV) of double perovskites $\text{Cs}_2\text{NaBiI}_6$ has been synthesized, which possesses excellent stability and strong light absorption performance.²² However, its PCE is only 0.42%, which is much lower than those of lead-based hybrid perovskites.

The vacancy-ordered double perovskites with formula of A_2BX_6 can be regarded as a derivative structure of the traditional ABX_3 , where two toxic lead ions are substituted by one tetravalent ion to keep charge conservation as the conventional perovskites. Cs_2SnI_6 is one of the most representative perovskite materials, which belongs to a cubic structure with space group $Fm\bar{3}m$.²³ The previous study classified Cs_2SnI_6 as zero dimensional (0D) structure.²⁴ However, this compound still maintains three dimensional (3D) structure and presents 3D materials' properties, so it is still considered as 3D material.²⁵ Cs_2SnI_6 exhibits air and moisture stability, and strong visible light absorption, which are advantageous properties for photovoltaic applications.^{26,27} The first tested solar cell using Cs_2SnI_6 as absorber material in air showed a small PCE of 0.96%.²⁶ Meanwhile, MA_2SnI_6 was found to have a direct band gap of 1.81 eV with a strong absorption coefficient of $\sim 7 \times 10^4 \text{ cm}^{-1}$.²⁸

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The material properties of $\text{Cs}_2\text{Sn}_{1-x}\text{Te}_x\text{I}_6$ were investigated to explore structure–property relationships with potential applications in photovoltaics.²⁹ In 2017, a promising family of all-inorganic based-Ti double perovskites were predicted to possess suitable band gaps, excellent optical absorption, and high stability.³⁰ In particular, the band gap of $\text{Cs}_2\text{TiI}_{6-x}\text{Br}_x$ can be tuned from 1.38 to 1.78 eV for single-junction and tandem solar cells.³⁰ A comprehensive and systematic study of halide perovskites has been performed by using density functional theory (DFT) calculations.³¹ The theoretical study on the material properties of Cs_2TiX_6 ($X = \text{I}, \text{Br}$) has been investigated.^{32–34} The PCE of 3.28% can be obtained for Cs_2TiBr_6 when it is incorporated into planar-heterojunction PSCs.³⁵ In 2018, Zhou *et al.* synthesized lead-free all-inorganic Cs_2PdBr_6 perovskite nanocrystals with a narrow band gap (1.69 eV) for the first time.³⁶ A series of lead-free Te-based double perovskites A_2TeX_6 ($A = \text{MA}, \text{FA}, \text{or BA}$; $X = \text{Br}^- \text{ or } \text{I}^-$; $\text{MA} = \text{CH}_3\text{NH}_3$; $\text{FA} = \text{CH}(\text{NH}_2)_2$; $\text{BA} = \text{benzylamine}$) were reported to be potential materials for optoelectronic devices.³⁷ These Te-based double perovskites exhibited a tunable band gap, a low trap density, and a high mobility. Recently, the structural stability and electronic and optical properties of Te-based double perovskite materials were investigated by using first-principles calculations.³⁸ Therefore, the design of new non-/low-toxic stable halide perovskites for solar cells is inevitable.

In this work, the structural stability, mechanical properties, electronic and optical properties of double perovskites $\text{Cs}_2\text{Te}_{1-x}\text{Ti}_x\text{I}_6$ were investigated for the first time. The results show that $\text{Cs}_2\text{Te}_{0.50}\text{Ti}_{0.50}\text{I}_6$ has excellent stability, suitable direct band gap, and strong optical absorption, which is a promising candidate material for optoelectronic device. The mechanical properties of these materials are discussed in detail, and the results indicate that they are ductile materials except for $\text{Cs}_2\text{Te}_{0.50}\text{Ti}_{0.50}\text{I}_6$.

2. Computational details

First-principles calculations were performed by using Vienna *ab initio* simulation package (VASP).³⁹ The interaction between the core and valence electrons was described by the projector-augmented wave (PAW) method.⁴⁰ The generalized gradient approximation (GGA) of Perdew–Burke–Ernzerhof (PBE) was employed to describe the exchange–correlation functional.⁴¹ A plane-wave energy cutoff of 500 eV was used. A $4 \times 4 \times 4$ k -point grid was adopted for geometric optimization until the convergence criteria of energy and forces were set to be 10^{-5} eV and $0.01 \text{ eV } \text{\AA}^{-1}$, respectively. The structures were fully relaxed without any constraints. A denser $8 \times 8 \times 8$ k -point grid was employed for electronic and optical properties calculations. Four k -points $X(0.5, 0, 0)$, $R(0.5, 0.5, 0.5)$, $M(0.5, 0.5, 0)$, and $G(0, 0, 0)$ were selected for the bandgap calculations. In order to gain accurate lattice parameters of the studied systems, the van der Waals (vdW) interactions were adopted within vdW-DFT in all calculations.⁴² For Cs_2TiI_6 , considering the underestimation of the band gap in the standard DFT calculation, we used the DFT+ U method⁴³ to obtain accurate electronic structure. Various values of U have been tested for Ti 3d orbitals of Cs_2TiI_6 .

$U = 2 \text{ eV}$ was finally chosen for the electronic and optical calculations for the $\text{Cs}_2\text{Te}_{1-x}\text{Ti}_x\text{I}_6$ solid solutions.

3. Results and discussion

3.1 Structure properties

The crystal structures of both Cs_2TeI_6 and Cs_2TiI_6 belong to the cubic phase with space group $Fm\bar{3}m$.^{29,30} The calculated lattice constant of Cs_2TeI_6 is 11.66 Å, which is in good agreement with experimental value (11.71 Å).²⁹ The corresponding lattice constant of Cs_2TiI_6 is predicted to be 11.38 Å, which is much lower than its experimental value (11.67 Å).³⁰ The ion radii of Te^{4+} and Ti^{4+} are 0.97 Å and 0.61 Å, respectively. Interestingly, the experimental lattice constants of Cs_2TeI_6 and Cs_2TiI_6 are almost the same. It should be noted that the lattice parameter of Cs_2TiI_6 is estimated to be 11.67 Å according to the experimental XRD results. The optimized structures of the $\text{Cs}_2\text{Te}_{1-x}\text{Ti}_x\text{I}_6$ solid solutions are depicted in Fig. 1. x is the concentration of Ti^{4+} . The x values of 0.00, 0.25, 0.50, 0.75, and 1.00 are adopted in this study. For the species with $x = 0.25, 0.50$, and 0.75, all the possible arrangements are considered in this work. The results show that different structures with the same doping percentage

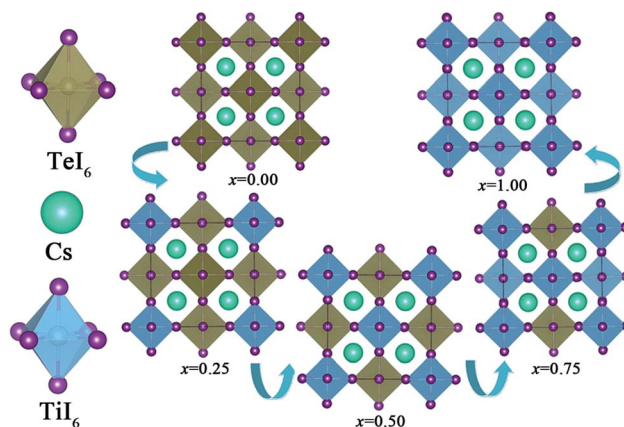


Fig. 1 The optimized structures of the $\text{Cs}_2\text{Te}_{1-x}\text{Ti}_x\text{I}_6$ solid solutions.

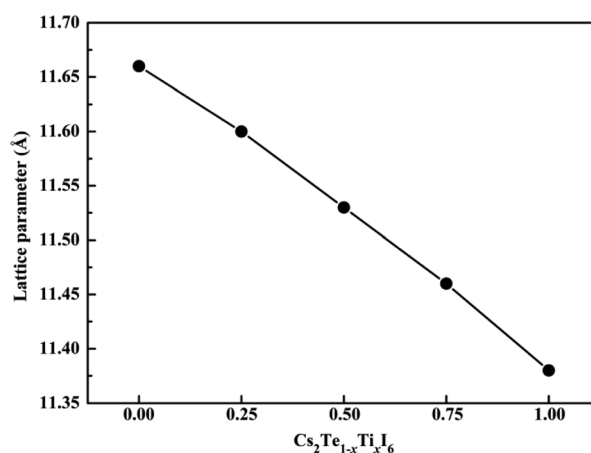


Fig. 2 The lattice constants of the $\text{Cs}_2\text{Te}_{1-x}\text{Ti}_x\text{I}_6$ solid solutions.



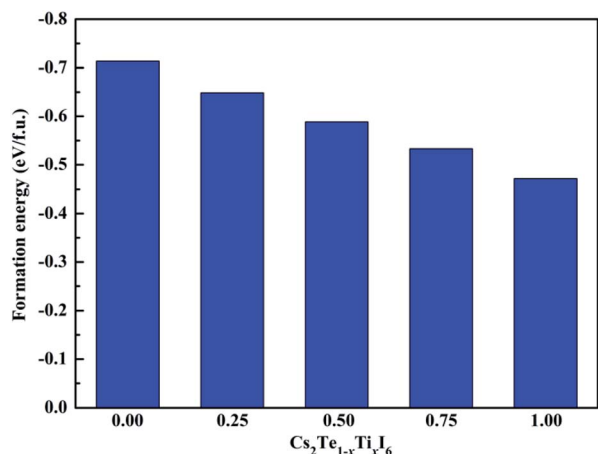


Fig. 3 Calculated formation energies of the Cs₂Te_{1-x}Ti_xI₆ solid solutions.

have the same stability, with calculated energy differences within less than 2 meV. The lattice constant of Cs₂Te_{1-x}Ti_xI₆ decreases gradually when the concentration of Ti⁴⁺, as shown in Fig. 2. Moreover, the bond length Te–I (2.96 Å) of Cs₂TeI₆ is larger than the bond length Ti–I (2.76 Å) of Cs₂TiI₆.

The tolerance factor (t) has been widely used to predict the structural stability of perovskite material. The formula is defined as follows:⁴⁴

$$t = \frac{R_A + R_X}{\sqrt{2}(R_B + R_X)} \quad (1)$$

where R_A , R_B , and R_X are the effective ionic radii for A, B, and X, respectively. In general, the perovskite structure is stable when the t value is in the range of 0.81–1.11.⁴⁵ Our calculated values of t are 0.86 and 0.97 for Cs₂TeI₆ and Cs₂TiI₆, respectively. These

results confirm that the substitution of Ti⁴⁺ for Te⁴⁺ in Cs₂TeI₆ is feasible. To further assess the thermodynamic stability of the Cs₂Te_{1-x}Ti_xI₆ solid solutions, we have calculated their formation energies. The formation energy can be obtained according to the following equation:

$$\Delta H = \Delta E(\text{Cs}_2\text{Te}_{1-x}\text{Ti}_x\text{I}_6) - 2\Delta E(\text{CsI}) - (1-x)\Delta E(\text{TeI}_4) - x\Delta E(\text{TiI}_4) \quad (2)$$

where $E(\text{Cs}_2\text{Te}_{1-x}\text{Ti}_x\text{I}_6)$, $E(\text{CsI})$, $E(\text{TeI}_4)$, and $E(\text{TiI}_4)$ are the total energy of Cs₂Te_{1-x}Ti_xI₆, CsI, TeI₄, and TiI₄, respectively. The calculated formation energies of the Cs₂Te_{1-x}Ti_xI₆ solid solutions are demonstrated in Fig. 3. Our calculated result shows that the ΔH value of MAPbI₃ is 0.01 eV per f.u., which is in good agreement with the previous reported data.⁴⁶ These results indicate that MAPbI₃ is marginally stable. It can be seen that all the compounds exhibit good stability with larger negative ΔH values compared to MAPbI₃. Moreover, it is observed that the structural stability of Cs₂Te_{1-x}Ti_xI₆ decreases gradually with the increasing concentration of Ti⁴⁺. In addition, the phonon spectrum of the mixed-metal double perovskites are also studied. As shown in Fig. 4, it can be seen that none of the imaginary phonon mode exists for Cs₂Te_{0.75}Ti_{0.25}I₆ and Cs₂Te_{0.25}Ti_{0.75}I₆, which indicate that they are kinetically stable. Interestingly, Cs₂Te_{0.50}Ti_{0.50}I₆ has very small imaginary frequencies. According to the previous study,⁴⁷ all small imaginary frequencies will disappear when it is at room temperature. Therefore, Cs₂Te_{0.50}Ti_{0.50}I₆ is considered to be a stable structure at room temperature.

3.2 Mechanical properties

To predict the mechanical stability of double perovskite Cs₂Te_{1-x}Ti_xI₆, the corresponding elastic constants were calculated

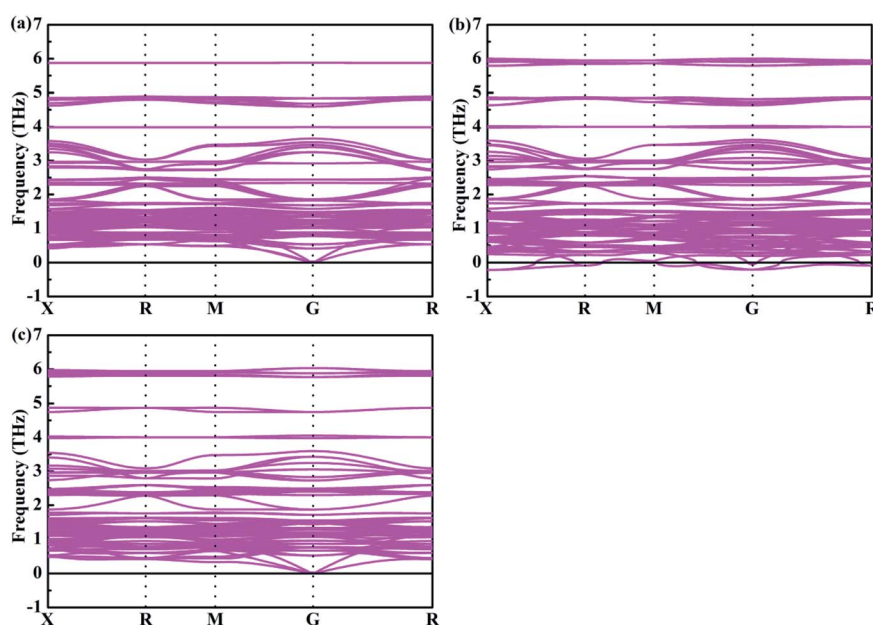


Fig. 4 Phonon spectrum of (a) Cs₂Te_{0.75}Ti_{0.25}I₆, (b) Cs₂Te_{0.50}Ti_{0.50}I₆, and (c) Cs₂Te_{0.25}Ti_{0.75}I₆.



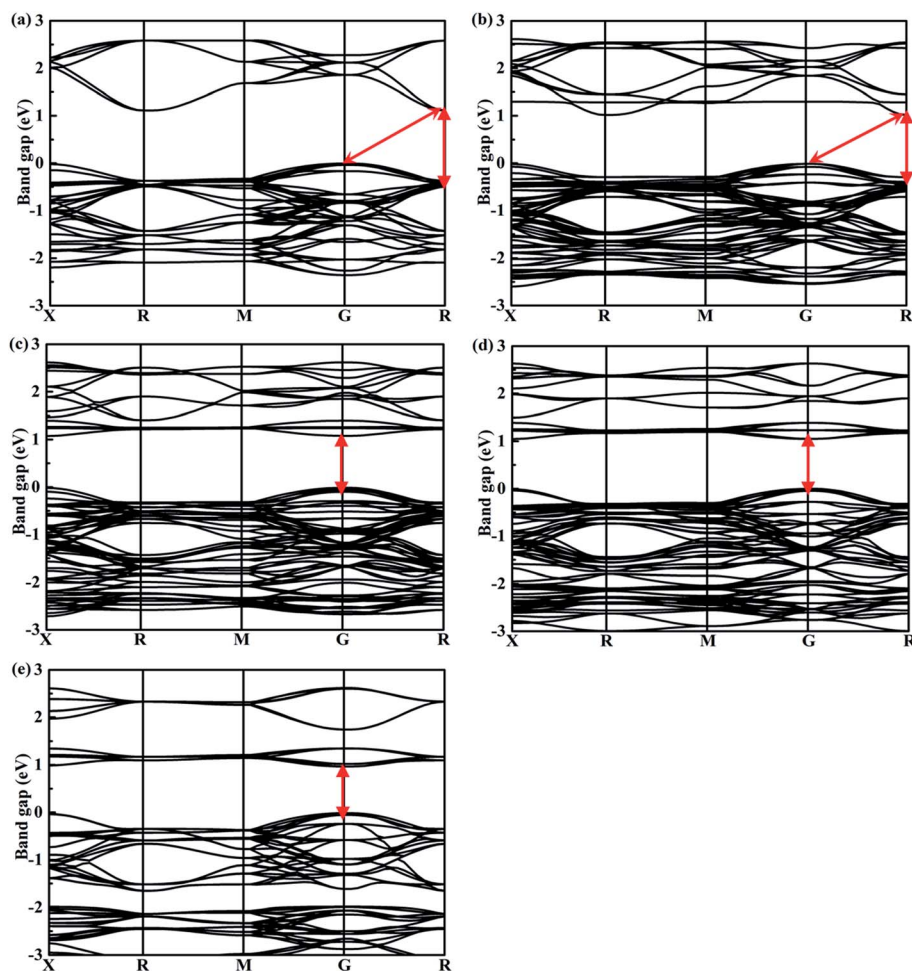
modulus and shear modulus by applying the following formula:⁵¹

$$E = 9BG/(3B + G), \nu = (3B - 2G)/(6B + 2G) \quad (4)$$

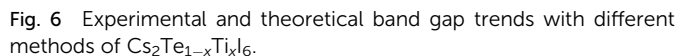
In order to investigate the ductility and brittleness of the studied double perovskites, we calculated the Pugh's and Poisson ratios. The Pugh's ratio is the ratio between bulk modulus and shear modulus (B/G), which is proposed by Pugh.⁵² The material is considered as ductile when the value of Pugh's ratio is larger than 1.75, otherwise it is brittle. The Poisson's ratio can also be used to separate the ductile materials from brittle materials. The ductile materials have the Poisson's ratio greater than 0.26, and it is smaller than 0.26 for brittle materials.⁵³ From Table 1, it can be seen that $\text{Cs}_2\text{Te}_{0.50}\text{Ti}_{0.50}\text{I}_6$ is a brittle material, while other four compounds are ductile materials. It should be noted that the Poisson's ratio values of the studied double perovskites are close to the required lowest limit. The smaller the value of Young's modulus, the better flexibility of a material. The results show that Young's modulus of Cs_2TeI_6 is the smallest among the five structures, which indicates that it is

$$C_{11} - C_{12} > 0, C_{11} + 2C > 120, C_{44} > 0 \quad (3)$$

The calculated results of three independent elastic constants are listed in Table 1. All the compounds satisfy the above Born stability criteria, indicating that they are mechanical stable. The bulk modulus (B) and shear modulus (G) can be obtained by using Hill approximations.⁵⁰ The Young's modulus (E) and Poisson ratio (ν) can be calculated from the values of bulk



RSC Adv., 2020, 10, 36734–36740 | 36737

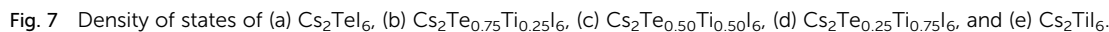


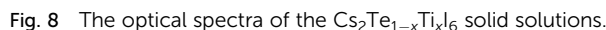
3.3 Electronic properties

The band structures calculated along high symmetry directions in the Brillouin zone are shown in Fig. 5. Cs₂TeI₆ has an indirect band gap between *G* and *R* point, and a direct band gap at *R* point. The indirect and direct band gaps of Cs₂TeI₆ are 1.12 and 1.48 eV, respectively. The previous reported band gap of Cs₂TeI₆ is about 1.5 eV^{54,55} and the recent experimentally observed

optical gap is 1.59 eV.²⁹ The band gap of Cs_2TiI_6 with the PBE method is 0.75 eV, which is lower than its experimental value (1.02 eV).³⁰ Therefore, the DFT+U method is employed to obtain accurate band gap of Cs_2TiI_6 . The result shows that Cs_2TiI_6 possesses a direct band gap with 0.98 eV at the G point, which is in good agreement with its experimental value.³⁰ The Spin-orbit coupling (SOC) effect on the electronic structures of Te- and Ti-based perovskites is small.^{30,37} It is observed that $\text{Cs}_2\text{Te}_{0.50}\text{Ti}_{0.50}\text{I}_6$ is a direct band gap semiconductor with the value of 1.09 eV, while $\text{Cs}_2\text{Te}_{0.75}\text{Ti}_{0.25}\text{I}_6$ is an indirect band gap semiconductor. The direct band gap of $\text{Cs}_2\text{Te}_{0.25}\text{Ti}_{0.75}\text{I}_6$ is 1.05 eV. The trend in band gap is displayed in Fig. 6. The band gap of Cs_2TeI_6 reduces gradually when the ratio of Ti^{4+} increases. The band gaps of the two calculations with PBE and PBE+U show the same tendency. The variation in band gap is only 0.11 eV when the concentration of Ti^{4+} changes from 0.50 to 1.00. It should be noted that the direct band gap of Cs_2TeI_6 is close to the experimental value. The direct band gap of $\text{Cs}_2\text{Te}_{0.75}\text{Ti}_{0.25}\text{I}_6$ is 1.31 eV, which is a potential candidate for single-junction solar cells according to the Shockley-Queisser theory.⁵⁶ An indirect-to-direct band gap transition can be observed when the doping content of Ti^{4+} increases from 0.25 to 0.50. Moreover, the direct band gap is significantly reduced by 0.22 eV. It can be seen that the band gaps of the $\text{Cs}_2\text{Te}_{1-x}\text{Ti}_x\text{I}_6$ solid solutions are apparently underestimated at the PBE method, as shown in Fig. 6.

Fig. 7 shows the calculated density of states of the $\text{Cs}_2\text{-Te}_{1-x}\text{Ti}_x\text{I}_6$ solid solutions. For Cs_2TeI_6 , the valence band maximum (VBM) is mainly contributed by the I-p orbitals, while





RSC Adv., 2020, 10, 36734-36740 | 36739

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