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## Directional-dependent pockets drive columnar–columnar coexistence†

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**The rational design of materials requires a fundamental understanding of the mechanisms driving their self-assembly. This may be particularly challenging in highly dense and shape-asymmetric systems. Here we show how the addition of tiny non-adsorbing spheres (depletants) to a dense system of hard disc-like particles (discotics) leads to coexistence between two distinct, highly dense (liquid)–crystalline columnar phases. This coexistence emerges due to the directional-dependent free-volume pockets for depletants. Theoretical results are confirmed by simulations explicitly accounting for the binary mixture of interest. We define the stability limits of this columnar–columnar coexistence and quantify the directional-dependent depletant partitioning.**

Compartmentalization, the distribution of components over different domains, occurs at multiple length scales. Competition for space, which grants access to resources of different kinds, leads to the emergence of patterns, for example, in the canopies of trees,<sup>1</sup> in crowds of humans,<sup>2</sup> and in predator-prey fish schools.<sup>3</sup> Inside a prokaryotic cell, the shape and size of organelles influences their final distribution;<sup>4</sup> this effect is termed macromolecular crowding.<sup>5</sup> Therefore, understanding the fundamentals underlying the simple question ‘what goes where’ is paramount in the rational design of materials. In this Communication, we extend the concept of geometrical free volume fractions<sup>6</sup> to highly dense systems containing disc-like (discotic) particles in the presence of tiny non-adsorbing spheres. For the reader who is interested in more details we refer to the ESI† and will provide our Mathematica<sup>7</sup> scripts upon a reasonable request.

Size and shape asymmetry play a crucial role in the distribution of compounds in multi-component systems:<sup>3</sup> small compounds only fit in the free volume pockets available in between the larger ones. The partitioning of non-adsorbing components (depletants) leads to a depletion zone around the bigger entities, where the depletant concentration is lower than in the bulk. This depletion zone relates to the excluded volume between the species:<sup>8</sup> the space inaccessible to a second particle due to the presence of the first one.<sup>9</sup> In a mixture of anisotropic particles and non-adsorbing spheres interacting solely *via* excluded volume, the spheres induce attraction patches between the anisotropic particles due to their optimal entropic gain upon maximum overlap of the depletion zones.<sup>10</sup> Colloidal systems have been proposed as candidates to isolate the role of excluded volume in highly size- and shape-asymmetric, dense environments.<sup>11–13</sup> The role of entropy in self-assembly, termed shape-entropy,<sup>14,15</sup> has received substantial attention *via* controlled theoretical,<sup>16</sup> simulational,<sup>17,18</sup> and experimental<sup>19,20</sup> studies. Discotic colloids are widespread in natural and human-made products spanning from blood,<sup>21</sup> laquer coatings,<sup>22</sup> clays,<sup>23</sup> paints,<sup>24</sup> cosmetics,<sup>25</sup> and nacre-mimetic materials<sup>26</sup> to coloration-change mechanisms.<sup>27</sup> In many of these examples, discotics are not the only compound present. Thus, a better understanding of partitioning in simple discotic–depletant mixtures will provide guidelines towards a smarter material design. Here, we quantify the distribution of tiny non-adsorbing spheres in dense discotic systems. To this end, we developed a geometrical free-volume theory (FVT) whose predictions are confirmed *via* direct coexistence Monte Carlo simulations accounting for the binary mixture.

We focus on the columnar phase of discotics, containing a one-dimensional stacking of hexagonally-arranged particles.<sup>28,29</sup> The system parameters are the disc aspect ratio (*i.e.*, the relative thickness of the platelet)  $A \equiv L/D$ , where  $L$  is the platelet's thickness and  $D$  is its diameter, and the relative size of the depletant,  $q \equiv 2\delta/D$ , where  $\delta$  is the radius of the depletant [Fig. 1(a)]. We consider a system (S) with volume  $V$  containing  $N_c$  discotics with volume  $v_c$  at volume fraction  $\phi_c = (N_c v_c)/V$ . The depletant volume fraction is  $\phi_d = (N_d v_d)/V$ , where  $N_d$  is the number of

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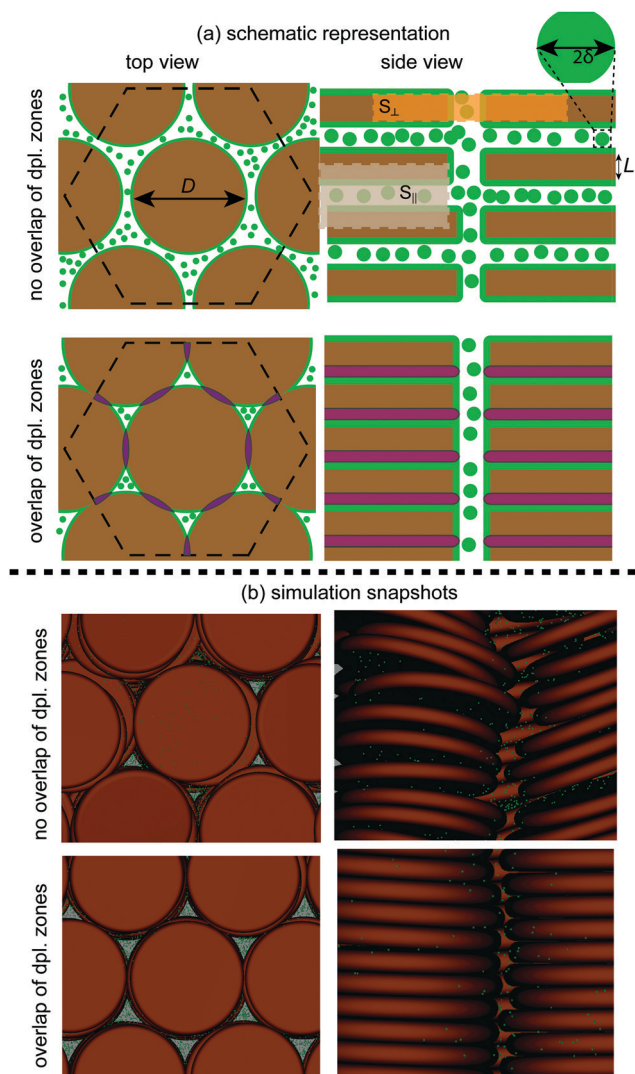


Fig. 1 Top panels: Representation of the inter- and intra-columnar depletion zone overlaps (purple) present when discotics (brown) are mixed with tiny depletants (green);  $q \equiv 2\delta/D = 0.05$ . Depletion zones around platelets indicated in green. The effective small systems for depletants in the inter-columnar ( $S_{\perp}$ ) and intra-columnar ( $S_{\parallel}$ ) directions are shown as orange and grey boxes. Bottom panels: Snapshots of discotic-depletant mixtures in the intra- and inter-columnar directions;  $\Lambda \equiv L/D = 0.1$ ,  $q = 0.01$ .

depletants with volume  $v_d$  in  $S$ . We consider depletants as penetrable hard spheres (PHSSs);<sup>30</sup> they do not interact with each other but are hard for the discotics. Theoretically, we account for platelets as hard cylinders;<sup>31</sup> in simulations we consider oblate hard spherocylinders (OHSCs).<sup>29,32</sup> We apply free volume theory (FVT;<sup>33</sup> see ESI†) to discotic-depletant mixtures. This FVT accounts for the partitioning of depletants over the different phases present in the system.<sup>34</sup> Unless otherwise indicated, we focus on  $\Lambda = 0.1$  and  $q = 0.01$ .

In Fig. 2, the free volume fraction for depletants in the columnar phase  $\alpha_c$  is presented. Contrary to common FVT, this function is calculated on geometrical grounds by analyzing the overlap of the depletion zones within the columnar unit cell. If there is no overlap of the depletion zones,  $\alpha_c$  is the volume

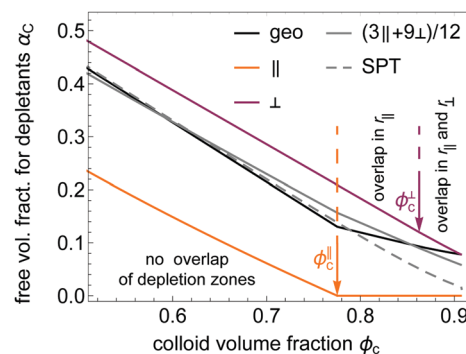
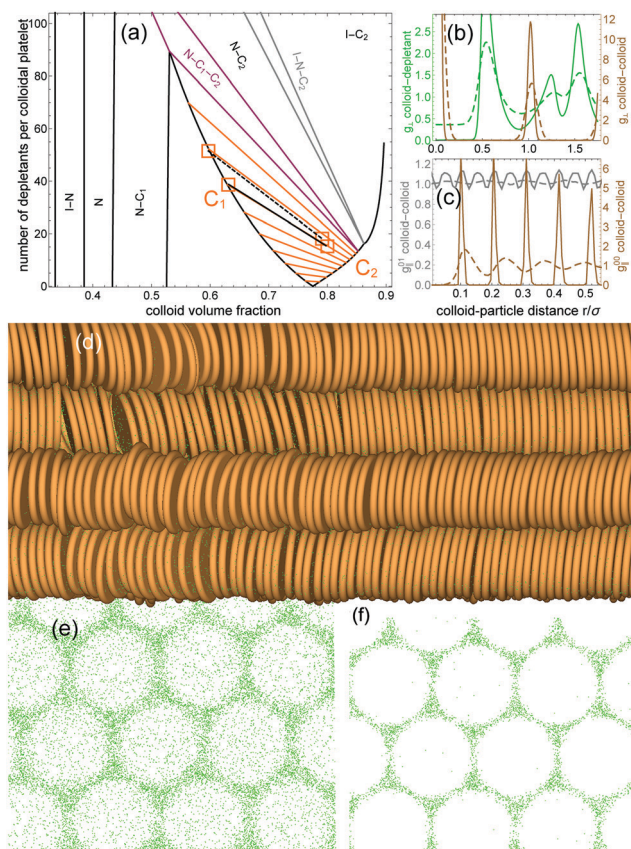


Fig. 2 Free volume fraction for depletants in the discotic columnar phase  $\alpha_c$  with increasing  $\phi_c$ ;  $\Lambda = 0.1$  and  $q = 0.01$ . Black curve is the expression used in the calculation of the binodals, orange and purple curves are the free volume fraction in the intra- and inter-columnar directions, and grey curve corresponds to weighted arithmetic mean. Dashed grey curve is the scaled particle theory (SPT) result for  $\alpha_c$ .<sup>31</sup> Arrows indicate the  $\phi_c$  at which the overlap of the depletion zones in the intra- and inter-columnar directions occurs.

unoccupied by the depletion zones and discotics over the system volume. Overlap of depletion zones leads to an increase in  $\alpha_c$ . This overlap occurs either in the intra- $r_{\parallel}$  or in the inter- $r_{\perp}$  columnar direction [Fig. 1(a)]. Overlap of depletion zones in  $r_{\parallel}$  occurs at lower  $\phi_c$  than in  $r_{\perp}$ .<sup>35</sup> The kink point in  $\alpha_c$  at  $\phi_c \approx 0.77$  marks the lowest discotic volume fraction  $\phi_c^{\parallel}$  at which depletion zone overlap in  $r_{\parallel}$  occurs. Due to the low  $q$ -value considered, for  $\phi_c < \phi_c^{\parallel}$  the  $\alpha_c$ -value calculated using the geometrical method proposed here or *via* the commonly applied scaled particle theory (SPT)<sup>34</sup> almost overlap. However, the SPT-derived  $\alpha_c$  underestimates the pockets for tiny depletants in the discotic columnar phase for  $\phi_c > \phi_c^{\parallel}$ : the partitioning of depletants is biased towards the low-density discotic phases when using an SPT-derived  $\alpha_c$ .

The geometrically-based  $\alpha_c$  enables quantification of the distribution of depletants into two effective small systems: (1) in  $r_{\parallel}$ , between the flat faces of the platelets; (2) in  $r_{\perp}$ , from the sides of the platelets [Fig. 1(a)]. There is always room for depletants in  $r_{\perp}$  ( $\alpha_c > 0 \forall \phi_c$ ), whilst  $\alpha_c$  vanishes upon the overlap of the depletion zones between the flat faces of the discotics (in  $r_{\parallel}$ ,  $\alpha_c = 0 \forall \phi_c > \phi_c^{\parallel}$ ). A weighted arithmetic mean of the three small systems in  $r_{\parallel}$  and of the nine ones present in  $r_{\perp}$  leads to a free volume fraction close to the geometrical  $\alpha_c$ -value. The amount of small systems present in each direction is the number of depletant-mediated discotic-discotic interactions [Fig. 1(a)].

In Fig. 3(a), we present a theoretical (equilibrium) phase diagram. All possible depletant-free phases are observed at different discotic  $\phi_c$  and depletant  $\phi_d$  volume fractions: isotropic (I), nematic (N), and columnar (C). Phase separation upon depletant addition is driven by partitioning of depletants and discotics over the different phases.<sup>33</sup> For the specified  $\{\Lambda, q\}$ , columnar-columnar ( $C_1$ - $C_2$ ) coexistence is revealed. The overall phase diagram topology and the order of the triple phase coexistences with increasing  $\phi_d$  (N- $C_1$ - $C_2$  and I-N- $C_2$ ) is in agreement with previous results.<sup>31</sup> This  $C_1$ - $C_2$  is reminiscent



**Fig. 3** (a) Phase diagram of discotics ( $L/D \equiv \lambda = 0.1$ ) mixed with tiny depletants ( $2\delta/D \equiv q = 0.01$ ) (tie-lines in orange). Orange stars are Monte Carlo simulations accounting for the binary mixture; tie-lines in black. Data and snapshots in (b–e) correspond to the points of the solid black tie-line. (b and c) Distribution functions in the inter- (b)  $g_{\perp}$  and intra- (c)  $g_{\parallel}$  columnar directions. Dashed curves:  $C_1$  phase; solid curves:  $C_2$  phase. The superscript 00 denotes correlation between particles in the same column, and 01 refers to correlation between particles in adjacent columns. (d–f) Simulation snapshots of discotics (brown) and depletants (green). (d) Snapshot from the (1100) plane of a direct-coexistence simulation. (e and f) Snapshots from the (0001) plane of the depletants present in the  $C_1$  (e) and  $C_2$  (f) phases.

of the solid–solid coexistence found in hard spheres (HSS) interacting *via* short-range (direct) attraction.<sup>36–38</sup> Here, the directionality of the attraction patches<sup>10,39</sup> induced by the disc-like shape drives the  $C_1$ – $C_2$  coexistence. Consequently, in the lower-density columnar phase ( $C_1$ ) there is no overlap of the depletion zones: pockets for depletants are available in both  $r_{\perp}$  and  $r_{\parallel}$ . However, in the higher-density columnar state ( $C_2$ ) depletion zones overlap between the flat faces of the platelets (in  $r_{\parallel}$ ), and the only pockets in the system are in  $r_{\perp}$ .

The maximum depletion attraction  $W_{AOV}^{\max}$  between discotics when  $q \rightarrow 0$  scales as  $W_{AOV}^{\max} \propto -\phi_d^R q^{-2}$ , stronger than between HSS ( $W_{AOV}^{\max} \propto -\phi_d^R q^{-1}$ ). Here,  $\phi_d^R$  is the depletant bulk concentration (see ESI†). The tendency of flat faces to align is enhanced by the presence of the depletants. Furthermore, the theoretically-predicted  $C_1$ – $C_2$  coexistence terminates at  $\phi_c = \phi_c^{\parallel}$  with  $\phi_d \approx 0$ , manifesting the two effective systems that tiny depletants access in the columnar phase. This vanishing

depletant concentration at the  $C_1$ – $C_2$  critical point (CP) contrasts with the finite value obtained for solid–solid coexistence between HSS:<sup>6</sup> there is no directionality for the pockets present in the HS solid. Next, the stability of this  $C_1$ – $C_2$  coexistence is tested against Monte Carlo (MC) simulations.

In Fig. 3(a), two equilibrium  $C_1$ – $C_2$  phase coexistences from direct coexistence MC computer simulations are also shown. As far as we are aware, this is the first time that direct-coexistence in highly dense discotic systems is simulated explicitly accounting for a binary mixture (OHSCs and PHSS); see ESI† for simulation details and further results. A snapshot of an equilibrium direct  $C_1$ – $C_2$  coexistence of the binary mixture is shown in Fig. 3(d). The close-packing fraction for OHSCs with  $\lambda = 0.1$  is  $\phi_c^{\text{cp}} \approx 0.88$ ,<sup>32</sup> which partially explains the lower  $\phi_c$ -values on the  $C_2$ -branch of the simulations. Due to their rounded edges, the stacking of OHSCs in the columnar phase differs from that of hard cylinders.<sup>32</sup> Besides this offset in the  $C_2$  branch, MC results and FVT predicted tie-lines are in remarkable agreement. Snapshots of the different plate-depletant mixtures [Fig. 1(b), and 3(e), (f)] show that the depletant partitioning is in line with theoretical predictions. More importantly, the MC simulations show that  $C_1$ – $C_2$  coexistence exists and is stable against fluctuations.

Next, we pay attention to discotic–discotic and discotic–depletant distribution functions  $g$  from the MC simulations [Fig. 3(b) and (c)]. The discotic–depletant distribution function in the inter-columnar direction  $r_{\perp}$  is the most insightful [ $g_{\perp}^{c-d}$ ]. For the  $C_2$  phase,  $g_{\perp}^{c-d} \approx 0$  for  $r_{\perp} \lesssim 0.5D$ : there are barely any depletants present in the intra-columnar direction in  $C_2$ . On the contrary, there is a rather constant distribution of depletants on the top and bottom of the discotic flat faces in the  $C_1$  phase:  $g_{\perp}^{c-d} \approx 0.4$  for  $r_{\perp} \lesssim 0.5D$ . The first peak at  $r_{\perp} \approx 0.5D$  of  $g_{\perp}^{c-d}$ , present both in  $C_1$  and  $C_2$ , corresponds to the doughnut-like pockets. The second and third peaks of  $g_{\perp}^{c-d}$  indicate that depletants are present in the interstices of both columnar phases. Furthermore, the  $g_{\perp}$ -value of these peaks is significantly higher in the  $C_2$  phase than in the  $C_1$  phase: the lack of pockets in  $r_{\parallel}$  in  $C_2$  leads to the accumulation of depletants in the interstices. In  $r_{\perp}$ , the discotic–discotic distribution  $g_{\perp}^{c-c}$  shows peaks corresponding to the hexagonal (two-dimensional) arrangement.

Discotic–discotic distributions in the intra-columnar direction  $r_{\parallel}$  are solid-like in  $C_2$  and more fluid-like in  $C_1$  [Fig. 3(c)]. We deduce from the discotic–discotic and discotic–depletant distributions that: (i) the  $C_1$  phase is liquid crystalline, whereas  $C_2$  is crystalline;<sup>29</sup> and (ii) depletants distribute according to the pockets present. In  $C_1$ , pockets are available both in  $r_{\parallel}$  and  $r_{\perp}$ . Opposite to this, in  $C_2$  pockets are only in the interstices (*i.e.*, in  $r_{\perp}$ ).

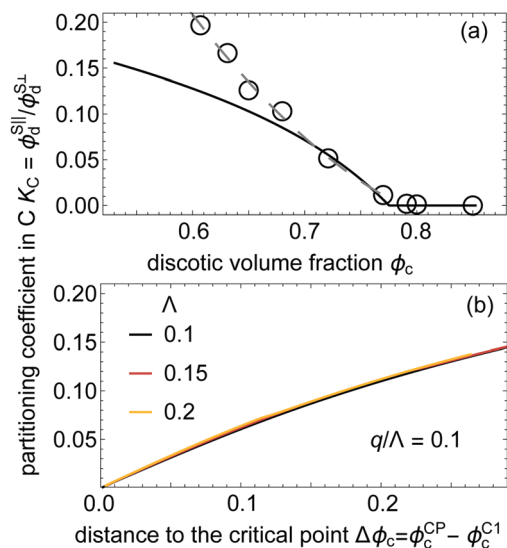
From our theoretical and simulation approaches we quantify how depletants partition in  $r_{\parallel}$  and  $r_{\perp}$  within each phase [Fig. 4(a)]. We define the partition coefficient of depletants in the columnar phase as:

$$K_C = \phi_d^{\parallel} / \phi_d^{\perp}. \quad (1)$$

This coefficient along the  $C_1$ – $C_2$  binodal from simulations is similar as predicted from theory. From simulations  $K_C$  is





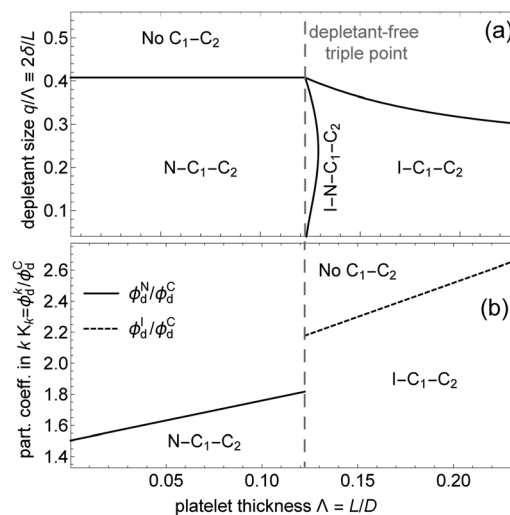


**Fig. 4** (a) Partition coefficients of depletants in the different columnar directions,  $K_C = \phi_d^{\parallel}/\phi_d^{\perp}$ , along the  $C_1$ - $C_2$  coexistence binodal for  $\{A, q\} = \{0.1, 0.01\}$ . Circles correspond to simulations; dashed gray curve guides the eye. (b)  $K_C$  along the low-density columnar phase  $C_1$  at fixed  $q/\Lambda$ ; distance to the critical point is  $\Delta\phi_c \equiv \phi_c^{CP} - \phi_c^{C1}$ .

computed from  $g_{\perp}^{c-d}$  (ESI†). The genuinely fluid-like nature in  $r_{\parallel}$  accounted for in simulations for the  $C_1$  phase partially explains the deviation from theory at  $\phi_c$  far from the CP.

This  $C_1$ - $C_2$  coexistence is not unique to  $\{A, q\} = \{0.1, 0.01\}$ . From theoretical predictions,  $K_C < 1$ :  $\phi_d^{\perp} > \phi_d^{\parallel} \forall \phi_c$  for any  $\{A, q\}$  (ESI†). In the denser columnar state  $C_2$ ,  $\phi_d^{\parallel} \approx 0$ . Therefore,  $K_C \approx 0 \forall \phi_c > \phi_c^{\parallel}$  (above the  $C_1$ - $C_2$  critical point). Hence, we focus on  $K_C$  only in  $C_1$ . In Fig. 4(b) it can be appreciated that at fixed  $q/\Lambda$ ,  $K_C$  along the  $C_1$ - $C_2$  binodal barely depends on  $\Lambda$  or on the nature of the triple-point (N- $C_1$ - $C_2$  or I- $C_1$ - $C_2$ , see ESI†). The size of the effective system in  $r_{\parallel}$  follows from the ratio of the depletant diameter to the thickness of the discotic,  $q/\Lambda$ . At fixed  $q$ ,  $K_C$  increases with increasing  $\Lambda$ : the thicker discotics, the larger the small system size in  $r_{\parallel}$ . Thus, for smaller  $q$  the directionality of the pockets is enhanced and the  $C_1$ - $C_2$  coexistence spans over larger phase space (ESI†).

It is possible to assess the triple (I-N-C, N- $C_1$ - $C_2$ , and I- $C_1$ - $C_2$ ) and critical  $C_1$ - $C_2$  points at any  $\{A, q\}$ .<sup>31</sup> This provides a stability overview of the  $C_1$ - $C_2$  coexistence [Fig. 5(a)]. The  $C_1$ - $C_2$  coexistence is found for a wide range of  $\{A, q\}$ -values. The depletant-free triple point sets the reference from which the N- $C_1$ - $C_2$  and I- $C_1$ - $C_2$  critical-end point (CEP) curves span. The CEP marks a critical point in coexistence with a distinctive third phase;<sup>40</sup> hence, it constitutes a powerful tool to identify the stability limit of the  $C_1$ - $C_2$  phase coexistence.<sup>6,31</sup> A quadruple I-N- $C_1$ - $C_2$  curve marks the transition from stable N- $C_1$ - $C_2$  to I- $C_1$ - $C_2$ .<sup>31</sup> Such quadruple coexistence in a two-component system is possible due to the extra field parameters  $\Lambda$  and  $q$ . In fact, already for the depletant-free discotic system an I-N-C coexistence is present<sup>32,35</sup> because  $\Lambda$  provides an extra field parameter; for hard spheres only two-phase fluid-solid coexistence emerges. A soft re-entrant behavior of the I- $C_1$ - $C_2$  at fixed  $\Lambda$  is revealed.



**Fig. 5** (a) Columnar coexistence overview in terms  $\Lambda$  and  $q/\Lambda = 2\delta/L$ . (b) Partitioning coefficient of depletants in the nematic (solid curve) or isotropic (dotted curve) phase relative to the columnar phase at the critical end point (CEP).

For the N-( $C_1$ - $C_2$ ) CEP curve,  $q^{CEP}/\Lambda \approx 0.4 \forall \Lambda \lesssim 0.12$ , with  $q^{CEP}$  the maximum depletant size at which  $C_1$ - $C_2$  coexistence is stable. For the I-( $C_1$ - $C_2$ ) CEP,  $q^{CEP}/\Lambda$  decreases with  $\Lambda$ ,  $q^{CEP}/\Lambda \approx 0.4$  for  $\Lambda \approx 0.12$  and  $q^{CEP}/\Lambda \approx 0.3$  for  $\Lambda \approx 0.23$ . To understand the dependencies with  $\Lambda$  of the N- $C_1$ - $C_2$  and of the I- $C_1$ - $C_2$  CEPs, the partitioning coefficient of depletants in phase  $k$  (with  $k = \{I, N\}$ ) relative to the columnar phase is defined:

$$K_k = \phi_d^k/\phi_d^C. \quad (2)$$

As more depletants fit in the isotropic than in the nematic phase, a higher concentration of depletants is present in the columnar phases when a N- $C_1$ - $C_2$  triple point occurs as compared to the I- $C_1$ - $C_2$  case. Hence  $K_N$  is lower than  $K_I$ .

A simple model to understand the partitioning of tiny non-adsorbed compounds (depletants) in dense discotic systems was developed and tested against Monte Carlo simulations explicitly accounting for the binary mixture. The tiny depletants can distribute over two distinct regions, corresponding to the intra- and inter-columnar directions. Partitioning of tiny depletants in the intra-columnar direction leads to columnar-columnar coexistence, whose critical point occurs precisely at the discotic concentration at which depletants do not fit into the intra-columnar direction. A geometrically-derived free volume fraction for depletants allows understanding of not only how non-adsorbing compounds distribute in dense systems, but also the stability limits of this columnar-columnar coexistence. By considering compounds which interact solely *via* excluded volume interactions, the role of entropy in concentrated and highly size- and shape-asymmetric mixtures is identified. If polydispersity could be accounted for, we expect the quadruple curves to broaden as there are more possibilities for the partitioning of the depletants, but the already large simulation equilibration times would dramatically increase. We note also that we are currently working on one-to-one comparisons between FVT and



experimental results for anisotropic particles, therefore moving towards a real application of the ideal FVT concepts.<sup>41</sup>

In summary, geometrical free volume fractions are a powerful tool for understanding partitioning over directional-dependent pockets in dense systems. The conceptual power of these geometrical free volume concepts has been recently exposed by applying the core of the idea here exposed to crowd control in public spaces following the new WHO regulations.<sup>42</sup> Moreover, we expect this geometrical free volume to deliver further interesting insights, for example in crowded biological environments (the cell) and in photonics. Further developments could include particle interactions beyond their excluded volume, polydispersity effects, and the presence of an external field (e.g., gravity or electric fields). However, quantifying what goes where is a natural first step.

## Conflicts of interest

There are no conflicts to declare.

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