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In situ polymer flocculation and growth in Taylor-Couette flows

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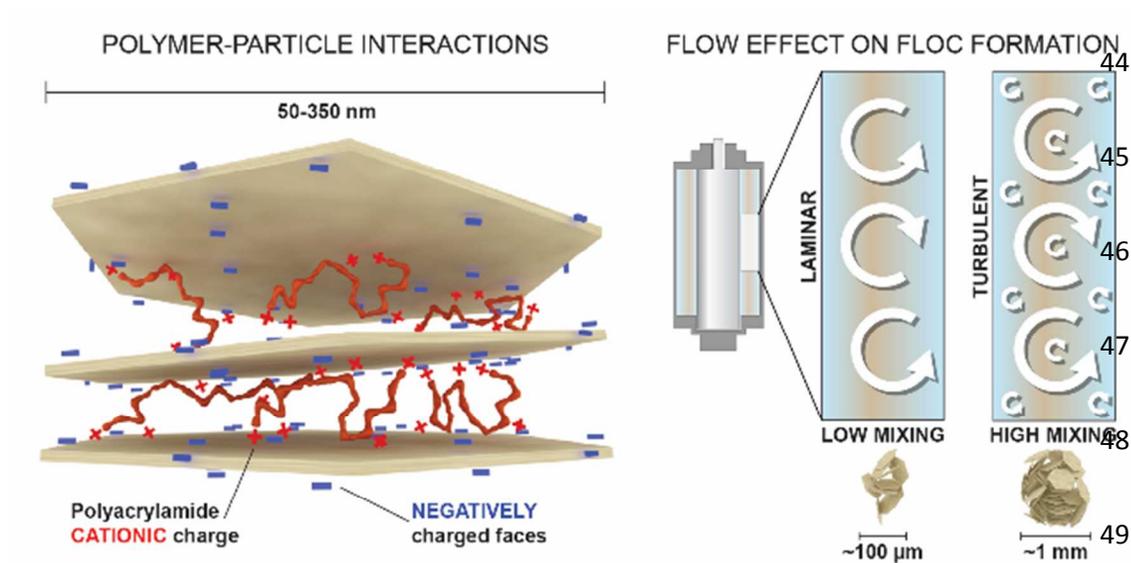
26 Abstract

27

28 Flocculation of small particulates suspended in solution is a key process in many industries,
29 including drinking water treatment. The particles are aggregated during mixing to form larger
30 aggregates, known as flocs, through use of a polyelectrolyte flocculant. The flocculation of these
31 particulates in water treatment, however, are subject to a wide spatial variation of hydrodynamic
32 flow states, which has consequences for floc size, growth rate, and microstructure. Floc assembly
33 dynamics are explored here using a commercially available cationic polyacrylamide, commonly
34 used in water treatment, and anisotropic Na-bentonite clay particles under a variety of
35 hydrodynamic mixing conditions. A Taylor-Couette cell with the unique ability to radially inject
36 fluid into the rotating annulus was used to study how specific hydrodynamic flow fields affect
37 assembly and structure of these materials during the flocculation process. Faster floc growth
38 rates and decreased floc fractal dimensions were observed for higher order flow states, indicating
39 improved mass transfer of the polymer flocculant and breakage at the edges of the flocs (shear
40 rounding), respectively. This work sheds more light on the complexities of polymer-induced
41 flocculation, towards improving dosing and efficiency of large-scale operations.

42

43 Graphical Abstract



50 Graphical abstract sentence summary: Transient, in-situ polymer-particle flocculation and
 51 growth was studied as a function of hydrodynamic flow state using a modified Taylor-Couette
 52 cell.

53

54

55 1 Introduction

56

57 The removal of solid particulates using polyelectrolyte-driven flocculation is a common
58 process in a variety of applications including paper manufacturing and drinking water
59 treatment.¹⁻³ Despite its widespread use, the process remains poorly understood due to the
60 complexity of the system, and treatment plants therefore rely on highly empirical methods to
61 determine polymer flocculant dosing levels. Charged polymers are added to a fluid system
62 during mixing in order to bind to suspended particulates, forming flocs. Rapid floc growth is
63 then enabled by orthokinetic aggregation, where the polymer-coated particulates collide due to
64 fluid motion, and the velocity gradient during the mixing process depends on the nature of the
65 fluid flow.⁴⁻⁶ The local hydrodynamics around a growing floc is one of the key factors that
66 affects the conformation of the polymer on the surface. Extensional forces in particular can
67 stretch the polymer from the surface to lengths greater than the radius of gyration, R_g , of the free
68 polymer in solution, increasing the bridging capability of the polymer.^{7,8} As the flocs are formed
69 and grow during the flocculation process, they can eventually break due to fluid shear forces.^{9,10}
70 After some time, the flocs will approach a steady state between floc growth and breakage.¹¹⁻¹³

71 There are several different molecular-level mechanisms for polymer-particle adsorption and
72 flocculation events, including charge neutralization, polymer bridging, charge-patch flocculation,
73 and polymer depletion.^{1,3,14-16} The aqueous environment affects the underlying flocculation
74 mechanism through highly interrelated physicochemical properties including the suspension
75 composition (pH, ionic strength, and salt valency),⁸ coagulant type,^{17,18} particle surface
76 properties,¹⁹ and dosing and mixing conditions.^{9,12,20} In this study, bentonite was used as the
77 solid particulate of interest. Bentonite is an anisotropic smectite clay commonly found in surface

78 waters with a high adsorption capacity for water and has been used in other flocculation
79 studies.²¹⁻²⁴ Individual bentonite particles can be visualized as thin sheets with negatively
80 charged faces and either positively or negatively charged edges depending on the solution
81 pH.^{25,26} As a result, bentonite sheets can be arranged in a porous, edge-face arrangement, an
82 edge-edge arrangement, or a dense, face-face structure.^{27,28} Variations in aggregate surface
83 morphology more than likely result in different polyelectrolyte adsorption capacities and
84 interaction potentials due to the uneven charge distribution of the functional edge groups.

85 These complex interdependencies on the floc assembly kinetics and final floc microstructure
86 are not well understood and can result in poorly optimized polymeric dosing, which is one of the
87 major technical issues treatment plants consider. Under-dosing results in insufficient particulate
88 removal and over-dosing results in particulate restabilization, both undesirable outcomes which
89 can be cost-prohibitive in water treatment.²⁹ Flocs fragmented resulting from of shear forces can
90 be re-suspended and are often more difficult to remove than the initial particulate. For optimal
91 separation, large hydrodynamically robust flocs are desired for flocs to readily settle out due to
92 gravity, as well as withstand turbulent stresses in the flow with little to no fragmentation.^{1,10,30}

93 The effects of physicochemical and hydrodynamic conditions on final floc microstructure
94 have been examined using small angle light scattering, yielding final floc size and fractal
95 dimensions.³¹⁻³³ These studies found that the longer characterization length scales accessible by
96 static light scattering work well to determine mass fractal dimensions. However, interpretation of
97 the scattering patterns can be difficult, and static light scattering works best for smaller
98 aggregates with loosely packed structures. Recently, advanced image analysis techniques have
99 been used to study the dynamic flocculation behavior using jar tests and impellers, which offers a
100 more non-intrusive method of studying the flocs as compared to obtaining and preparing a

101 sample for light scattering experiments.⁹ Jar tests replicate scaled-down industrial water
102 treatment conditions, but lack homogeneous spatial and temporal flow features that are needed to
103 precisely study floc strength as a function of hydrodynamic flow. In addition to the lack of varied
104 flow states, the shear stresses within the fluid cannot be accurately controlled in a jar test to
105 determine the point where floc breakage occurs.^{30,34}

106 Unlike jar tests, Taylor-Couette (TC) cells, devices consisting of two concentric cylinders
107 with a specified gap width, offer an experimental method to study mixing events with the ability
108 to precisely control the hydrodynamics. TC cells can generate a wide variety of flow states as a
109 function of either or both cylinder speeds, and they range from laminar types to turbulent
110 types.³⁵⁻⁴⁰ Four different flow states were used in this study in order of increasing inner cylinder
111 speed: 1) laminar Taylor vortex flow (LTV), 2) laminar wavy vortex flow (LWV), 3) turbulent
112 wavy vortex flow (TWV), and 4) turbulent Taylor vortex flow (TTV). Laminar Taylor vortex
113 flow consists of axisymmetric, toroidal vortices with a characteristic spatial frequency but no
114 temporal frequency whereas laminar wavy vortex flow consists of a characteristic temporal and
115 spatial frequency.⁴¹ From a qualitative perspective, LTV flow look like a series of parallel bands
116 whereas LWV flow appears as parallel waves. The turbulent cases of these two flow states
117 physically resemble their laminar counterparts with additional turbulent features such as eddies.
118 The wide variety of flow states accessible by a TC cell, in addition to optical access via the
119 transparent outer cylinder has made it an ideal tool to study a variety of processes such as
120 polymer drag reduction, catalysis, filtration, and liquid-liquid mixing.⁴²⁻⁴⁴ TC cells have also
121 been used to study flocculation, although prior studies with TC cells were limited in that flocs
122 had to be pre-formed outside of the cell.^{10,45-47}

123 To study the entirety of the mixing process in an in-situ manner, a modified TC cell has
124 been designed and built by Wilkinson and Dutcher⁴⁸ to directly inject the polymer flocculant into
125 the annulus. Unlike other TC cells where the injection was in a single location or protruded into
126 the annulus, the cell built by Wilkinson and Dutcher injects flocculant at multiple, precisely
127 spaced axial and azimuthal locations to allow for larger volume injections and smoothing of any
128 azimuthal concentration gradients.^{43,48-50} The injection ports are built into the inner cylinder and
129 the port covers lie flush against the surface of the inner cylinder and are counter-matched to
130 prevent any alteration of the flow profile. Additional studies with this modified TC cell recently
131 published by Wilkinson and Dutcher have been conducted to determine the stability of flow
132 vortices to injection, and the flow states tested in this study are stable over a wide range of
133 cylinder speeds, injection drive pressures, and injection times.⁵¹ The advantages that this
134 modified TC cell offers over other cell designs is the ability to explore initial mixing effects in
135 flocculation of bentonite clay with a polyelectrolyte flocculant. Optical access allows for image
136 analysis techniques to calculate the size, morphology, and growth rate of flocs in a precise
137 hydrodynamic flow state.

138 2 Materials and Methods

139

140 2.1 Materials

141 The polymer flocculant, or polyelectrolyte, used in this study is a commercially available
142 cationic polyacrylamide (FLOPAM FO 4190 SH, SNF Polydyne) with 10% quaternary amine
143 monomer charge groups and a molecular weight of 6×10^6 g/mol. A 0.2 wt% polymer solution
144 was made by using a Jiffy mixer attachment to mix the solid polymer pellets into distilled water
145 for 30 min. The polymer rested in a refrigerator overnight prior to use and was remade every 2

146 weeks as necessary per supplier instructions. The distilled water used is from Premium Waters,
147 Inc. Powdered Na-Bentonite is ACS grade from Fisher Scientific and was used as received. The
148 kinematic viscosity of the 30 mg/L bentonite suspension was 1.00 ± 0.05 cSt from steady shear
149 experiments as shown in Figure S1, which showed Newtonian behavior in the frequency regions
150 relevant to the speeds tested in this study.

151 2.2 Methods: Sample Loading into TC Cell annulus and Spatial Calibration

152 The TC cell used in these experiments consist of a total of 16 injection ports evenly
153 distributed axially and azimuthally into the inner cylinder. Because the injection ports do not
154 protrude into the annulus and the port covers are contour-matched to the inner cylinder, the flow
155 profile of the resultant vortices are not modified during operation as shown by Wilkinson and
156 Dutcher.⁴⁸ Additional details on the TC cell design, with inner cylinder diameter of $13.5407 \pm$
157 0.0025 cm, gap width of 0.84 cm, inner cylinder radius to outer cylinder radius ratio of 0.891,
158 and injection assembly can be found elsewhere.⁴⁸ To make the bentonite suspensions, 30 mg of
159 bentonite was transferred to a 2 L beaker filled with 1 L of water. This process was repeated to
160 make a total of 2 L of 30 mg/L bentonite suspensions. The bentonite was dispersed using a
161 VELP Scientifica JTL4 Flocculator for 30 min at 300 RPM. The pH of the resultant suspensions
162 is approximately 6.6 due to the interaction of dissolved carbon dioxide in water and bentonite.⁵²
163 Since the size and morphology of bentonite, an anisotropic particle, is dependent on solution
164 ionic strength and pH, respectively, these two parameters were kept consistent between all
165 experiments in this study.^{53,54} Once finished, the bentonite suspensions were immediately
166 transferred to the annulus of the TC cell by way of tubing attached to the base of the cylinder
167 assembly as seen in Figure 1.

168 Once the bentonite was loaded, refractive index-matching paraffin oil was poured into the
169 Plexiglass tank holding the apparatus to eliminate the curved glass surface of the outer cylinder.
170 The cell was axially illuminated with a flicker-free LED light strip (Metaphase 19 in Exo2 Light)
171 to better visualize the injection port covers for spatial calibration for image analysis. A spatial
172 calibration image of the bottom-most port cover of the inner cylinder was captured using a
173 Basler Ace camera (1280 × 1024 pixels, 60 fps maximum frame rate) with a Tamron 25 mm c-
174 mount lens. ImageJ was then used to calculate the pixel-to-mm ratio of the spatial calibration
175 image.

176 2.3 Methods: Flocculation Experiment Protocol

177 The inner cylinder is rotated by a stepper motor (Applied Motion Products HT34-497 2 phase
178 stepper motor with a STAC5-S-E120 controller) equipped with a 7:1 gear reducer (Applied
179 Motion Products 34VL007) for inertial balance between the motor and the cylinder. To remove
180 unwanted flow dislocations in the vortices, flow priming was conducted by way of a motor
181 control script to ramp down from a speed higher than the required speed to the intended speed.⁵¹
182 A laser diode (Thorlabs, 450 nm, 1600 mW max) combined with a laser line generator was used
183 to create a laser light sheet tangential to the inner cylinder as shown in Figure 2A. The cationic
184 polyacrylamide injection process and movie recording were operated using LabView. For all
185 experiments, the drive pressure was set to 30 psi to inject cationic polyacrylamide from the
186 injection ports into the annulus at a calibrated injection rate of 1.115 g/s for 6 seconds as reported
187 in Wilkinson et al,⁴⁸ which corresponds to the optimal polymer dose for the bentonite-distilled
188 water of ~8 ppm as reported elsewhere.⁵³ The camera was vertically adjusted such that the field
189 of view was between the 3rd and 4th injection port covers from the bottom of the annulus. The
190 frame rate of the camera was set to 30 fps with an exposure time of 8 ms for all experiments.

191 As discussed later, flocculation mixing protocols typically have two speeds – an initial
 192 “Mix speed” (Stage 1), followed by a slower “Growth Speed” (Stage 2). However, it should be
 193 noted that these standard names are misleading – certainly some growth can occur in the early
 194 “Mix” stage, and mixing can occur in the later “Growth” stages. In this study, the early Stage 1
 195 steady “Mix” speeds were 0.04 s^{-1} , 0.17 s^{-1} , 0.50 s^{-1} , 1.10 s^{-1} , and 1.47 s^{-1} , and the later Stage 2
 196 steady “Growth” speed is fixed at 0.46 s^{-1} . After flow priming, the initial inner cylinder Stage 1
 197 angular velocity was set and the recording started. After a 30 second delay, the polymer was
 198 injected and mixed at this Stage 1 speed for 3 min to distribute the polymer throughout the
 199 annulus. After 3 min at the Stage 1 speed, the inner cylinder speed was then set to the Stage 2
 200 speed of 0.46 s^{-1} at a quick ramp rate of $0.92 \text{ rotations/s}^2$ for 30 min to allow the flocs to grow.
 201 The value of 0.46 s^{-1} was chosen as it was the slowest speed tested which was able to suspend
 202 enough flocs throughout the duration of the flocculation process for image analysis. Once each
 203 experiment was completed, the annulus was drained and the inner cylinder removed from the
 204 apparatus for cleaning with a dilute solution of Micro-90. The inner cylinder drive shaft and
 205 ports were re-primed with cationic polyacrylamide. Once reassembled, the annulus was refilled
 206 with distilled water prior to the next experiment.

207 2.4 Methods: Image Analysis of Flocculation Experiments

208 The movies were analyzed in MatLab using a process adapted from Vlieghe et al¹⁰ to
 209 obtain floc size and morphology information as a function of time. The raw images were
 210 converted to grayscale images and binarized for all experiments as shown in Figure 2C. Size was
 211 reported in terms of the radius of gyration, R_g , of the floc as calculated by the following equation,

$$R_g^2 = \frac{1}{N_p} \sum_{i=1}^{N_p} [(x_i - x_c)^2 + (y_i - y_c)^2] \quad \#(1)$$

212 where N_p is the number of pixels making up each floc, (x_i, y_i) are the individual pixel coordinate
213 pair, and (x_c, y_c) is the centroid coordinate pair. The area average for all flocs in a 10 second
214 interval (300 frames) was computed.

215 To quantify the morphology of the flocs, a surface-based fractal dimension, D_{sf} , was
216 calculated using the following relationship as reported by Vlieghe et al¹⁰

$$A \propto P^{2/D_{sf}} \quad \#(2)$$

217 where A is the cross-sectional floc area, and P is the floc perimeter. The fractal dimension varies
218 in value from 1 to 2, where a value of 1 indicates a circular shape and a value of 2 indicates a
219 rod-like shape. A linear regression was performed on each 10 second set of data (A, P) to
220 determine D_{sf} .

222 3 Results and Discussion

223

224 3.1 Effect of Hydrodynamics on Floc Size and Growth Rate

225 In water treatment processes, it is preferable to rapidly mix the flocculant for a short
226 period of time with the solid particulate of interest to disperse the polymer throughout the
227 suspension as quickly and evenly as possible. This faster mixing step (Stage 1) is followed by a
228 slower mixing step (Stage 2) for a longer time period to allow for continued floc growth.⁵⁵ To
229 test the effect of both mixture speed and vortex flow on the growth of the flocs, four different
230 flow regimes were selected for Stage 1 mixing. A set of laminar flow states with corresponding
231 Taylor and wavy vortex types (LTV and LWV, respectively), and a set of turbulent flow states
232 with corresponding Taylor and wavy vortex types (labeled TTV and TWV, respectively) were

233 used for this study as shown in Figure 2B. The flow states were reported in terms of the inner
 234 cylinder Reynolds number, Re_i . The Re for TC flow is defined by the following equation

$$Re_i = \frac{\Omega_i R_i d}{\nu} \quad \#(3)$$

235 where R_i and Ω_i are the inner cylinder radius and rotational speed, respectively, d is the gap width
 236 between the inner and outer cylinders, and ν is the kinematic viscosity of the sample.⁵⁶ The Stage
 237 2 mixing speed was kept constant for all experiments at Re of 1680 (TWV). For this particular
 238 TC cell, the ranges for the flow states in terms of Re are 124 to 167 for LTV, 251 to 870 for
 239 LWV, 1400 to 2924 for TWV, and above 2924 for TTV.

240 Figure 3 depicts the time-dependent radius of gyration, R_g , for the bentonite flocs as a
 241 function of time and vortex flow type. The radius of gyration of the flocs increases in all runs
 242 after the initial injection of cationic polyacrylamide at time = 30 seconds, until a plateau value of
 243 R_g is obtained. The plateau values of R_g were approximately 1.1 mm in the LWV, TWV, and
 244 TTV cases while the plateau value was approximately 0.7 mm in the LTV case. Shear breakage
 245 of the flocs due to velocity gradients present in the flow limit the maximum floc size.^{6,9,17,45}
 246 Interestingly, the curves for the Re of 4150 and 5530 cases show nearly identical growth
 247 behavior, most likely due to their having the same vortex type.

248 The floc growth rate can be quantified from the data by using a modified version of the
 249 logistic growth equation

$$\frac{R_g(t)}{R_{g,max}} = \frac{1}{1 + \left(\frac{1}{\frac{R_{g,0}}{R_{g,max}} - 1} \right) e^{-rt}} \quad \#(4)$$

250 where R_g is the radius of gyration at a given time in mm, $R_{g,max}$ is the maximum value of R_g in the
251 fitting range in mm, $R_{g,0}$ is the initial value of R_g at the beginning of the fitting range in mm, r is
252 the growth rate in mm/s, and t is time in seconds. While this model is typically used for
253 quantifying growth rates in microbial and ecological studies, it has also been used to quantify
254 floc growth as the flocs tend to reach a “carrying capacity” in their population in the form of a
255 maximum floc size.^{57–60}

256 The logistic growth equation was fit to the portion of the plot where there was a marked
257 increase in the floc size with time. The fits to the relevant portion of the data are presented in
258 Figure 4 and Figures S2-S5. In addition to the fits, the residuals between the measured values
259 and calculated values from the logistic growth equation are present to show how well the logistic
260 growth equation fits the data points. For the LWV, TWV, and TTV experiments, the model fits
261 the data well based on the residuals. The model was not a good fit for the LTV experiment based
262 on the residuals. This was most likely due to the relatively poor mass transfer of the polymer
263 flocculant associated with the LTV flow state, which is discussed in the following section.

264 Previous investigations have shown that maximum floc size is on the order of the
265 Kolmogorov microscale, and floc size decreases with increasing global velocity gradient via the
266 global dissipation rate of turbulent kinetic energy.^{9,45,61} What was interesting to note about the
267 floc size results from the present study is that maximum floc size did not differ much between
268 the different inner cylinder speeds with the exception of the lowest speed. Prior flocculation
269 studies in the literature, however, explored mixing hydrodynamics on flocculation in stirred
270 tanks with impellers or in TC cells under uncontrolled mixing conditions, which have been
271 shown to possess inhomogeneities over spatial and temporal flow features.^{10,45}

272 3.2 Effect of Hydrodynamics on Polymer Flocculant Mass Transfer

273 Figure 3 also shows a delay in the growth of the flocs as a function of inner cylinder
 274 speed. The lag time is defined as the point in time where the slope of the growth curve
 275 instantaneously increased rapidly from zero slope. This delay, or lag time, in growth can be
 276 attributed to the intervortex mass transfer of the cationic polyacrylamide. Intervortex mass
 277 transfer can be quantified by an effective dispersion coefficient recently determined for this
 278 particular TC cell by Wilkinson and Dutcher

$$D_z^* = 2\lambda k_{cb} \#(5)$$

279 where λ is the axial wavelength of the vortex, and k_{cb} is the intermixing coefficient.^{51,62}
 280 Intervortex mass transfer refers to mass transfer of the polyacrylamide between individual vortex
 281 bands versus intravortex mass transfer, which refers to mass transfer of the polyacrylamide
 282 within a single vortex. To compare the effect of intervortex mixing during the flocculation
 283 process, the lag time was plotted against the effective dispersion coefficient for the initial mixing
 284 step as shown in Figure 5. The lag times varied from 40 seconds for the highest order flow state
 285 (TTV) up to 1600 seconds for the lowest order flow state (LTV). D_z^* increases approximately an
 286 order of magnitude from $1.12 \times 10^{-5} \text{ m}^2/\text{s}$ to $2.16 \times 10^{-4} \text{ m}^2/\text{s}$ over the range of Re used in this
 287 study, indicating improved mass transfer of the polymer as the inner cylinder speed increased.

288 The lag time strongly depends on the initial Stage 1 speed. For the higher mix speeds of
 289 Re of 4150 and 5530 cases (TTV), most of the growth curve from Figure 3 is contained within
 290 the initial Stage 1 mixing portion of the plot, which are the data to the left of the solid gray line.
 291 The growth curve for the Re of 1870 (TWV) begins before the solid gray line in the Stage 1
 292 region and continues past the line into the Stage 2 mixing region to the right of the dark gray line.
 293 Unlike the turbulent cases, the growth for the Re of 160 (LTV) and Re of 650 (LWV) cases

294 commences well past the speed transition between Stage 1 and Stage 2. As the mass and therefore
295 global concentration of polymer injected into the system is consistent across all trials, this
296 difference in the initiation of floc growth is due to the reduced mass transfer abilities of the
297 laminar flow states compared to their corresponding turbulent flow states. The decrease in
298 effective dispersion coefficient and increase in lag time of the LTV and LWV states compared to
299 the TWV and TTV states are therefore largely due to the reduced ability of the LTV flow state to
300 intermix the polymer flocculant with the bentonite to form flocs. In addition to reduced mass
301 transfer abilities based on Re , the presence of an “unmixed core” in the laminar flow states
302 observed and discussed previously could also be an explanation for poorer mixing compared to
303 the corresponding TWV flow state.^{43,49–51,63} Evidence of this “unmixed core” was not present in
304 the turbulent flow states.

305 3.3 Effect of Hydrodynamics on Floc Morphology

306 In addition to floc size, hydrodynamics play a key role in floc morphology. Solid particulate
307 removal efficiency via sedimentation and filtration depends on the floc structure as morphology
308 is related to both floc size and density.⁶⁴ Velocity gradients in the mixing process can alter the
309 floc structure with time, which can also have consequences for removal efficiency. Floc
310 morphology can be quantified using fractal dimensions. Previous studies used light scattering
311 techniques to obtain a mass fractal dimension, which gives three-dimensional information about
312 the floc structure.^{11,65,66} However, using light scattering requires removal of sample flocs from
313 their original environment. In order to measure floc structure in a non-intrusive manner, image
314 analysis can be used to calculate a two-dimensional fractal dimension, D_{sf} .^{10,64,67}

315 The two-dimensional fractal dimension was obtained by linearly regressing a plot of the
316 log of the area versus log of the perimeter for each ten second data set. The closer the D_{sf} value is

317 to 1, the more circular the floc is in profile whereas the closer D_{sf} is to 2, the more linear and rod-
 318 like the floc is in profile. Figure 6 shows the evolution of D_{sf} with time at each Stage 1 mixing
 319 speed. In all cases, D_{sf} decreases with time until D_{sf} plateaus during the later Stage 2 portion of
 320 the plot. The plateau value of D_{sf} at the end of the flocculation experiment is largest for the
 321 slowest speed ($Re = 160$), and decreases with increasing inner cylinder speed. This decrease in
 322 D_{sf} with increasing inner cylinder speed is caused by the larger velocity gradients present in more
 323 turbulent flow types (higher speeds), which at longer mixing times shear the edges of the flocs
 324 and subsequently round them off.

325 The absence of a particle imaging velocimetry setup (PIV) does not allow for a precise
 326 distribution of the velocity gradient. However, a global velocity gradient term at each inner
 327 cylinder speed can be calculated using the following equations:

$$G = \sqrt{\frac{\langle \varepsilon \rangle}{\nu}} \#(6)$$

328 where G is the global velocity gradient, $\langle \varepsilon \rangle$ is the global viscous dissipation of the turbulent
 329 kinetic energy, and ν is the kinematic viscosity of the bentonite suspension.⁴⁵ The viscous
 330 dissipation of turbulent kinetic energy can be calculate using the following equation:

$$\langle \varepsilon \rangle = \nu \left(\frac{dU}{dx} \right)^2 \approx \nu \left(\frac{U_{\text{inner}} - U_{\text{outer}}}{d} \right)^2 \#(7)$$

331 Where U_{inner} is the angular velocity of the inner cylinder, U_{outer} is the angular velocity of the
 332 outer cylinder, and d is the gap width between the two cylinders. Since the outer cylinder is
 333 stationary for all experiments, U_{outer} reduces to zero. The global velocity gradient was calculated

334 for each Stage 1 mixing speed and are listed here in order of increasing mixing speed: 2.0 s^{-1} , 8.6
335 s^{-1} , 25.3 s^{-1} , 55.7 s^{-1} , 74.4 s^{-1} . The Stage 2 global velocity gradient is 23.3 s^{-1} .

336 During the flocculation process in water treatment, the flocculant is mixed at higher speeds
337 (which implies a higher global velocity gradient) into the suspension to rapidly disperse the
338 flocculant, which subsequently also allows for more aggregate-aggregate collisions to form the
339 initial flocs. Certainly increasing the mixing speed would initially result in aggregates with
340 morphologies that are less circular (D_{sf} values tending toward 2 rather than 1) due to increased
341 aggregate-aggregate collisions from orthokinetic aggregation compared to diffusion alone.^{12,67}
342 However, here we should note that with sufficient speeds and mixing times, the D_{sf} values
343 decrease asymptotically to unity (more circular floc morphology), suggesting that the shear-
344 induced rounding overwhelms aggregate-aggregate collisions in determining floc morphology in
345 turbulent flows at longer mixing times.

346 4 Conclusion

347

348 The unique ability for this TC cell to non-intrusively inject one fluid into another fluid offers
349 unprecedented access to the entire mixing process during flocculation. To summarize the overall
350 effects on flocculation of bentonite clay with cationic polyacrylamide in distilled water at four
351 different flow states generated by the TC cell, the floc growth rate and D_{sf} at the speed transition
352 have been plotted for each inner cylinder speed as shown in Figure 7. As the inner cylinder speed
353 increases, the flocs more rapidly increase in size and become more spherical in shape. The
354 differences in floc growth rate and morphology are largely attributed to the global velocity
355 gradients associated with each flow state, which also affect the dispersion of the polymer
356 flocculant. The intermixing coefficients calculated and shown in Figure 5, which describe

357 turbulent dispersion and not molecular diffusion, of the polyacrylamide in solution are several
358 orders of magnitude larger than the diffusion coefficients of polymers similar to the one used in
359 this study.⁶⁸ Even though at the highest speeds tested here the growth rate was the fastest, the
360 fractal dimension of the flocs were more circular, indicating more shear rounding and breakage
361 at the surfaces of the flocs. Based on the results collected in this study, a balance between faster
362 growth rate and shear rounding needs to be met to satisfy the criteria for large, hydrodynamically
363 robust flocs.

364 Flocculation is a largely non-equilibrium process that is dependent on a variety of
365 physicochemical and hydrodynamic properties. The unique ability to directly inject a flocculant
366 into a particulate-laden suspension in this TC cell offers the ability to study the entirety of the
367 flocculation process in a hydrodynamically controlled fashion. While only one type of polymer
368 flocculant was used here and the ionic strength and pH of the water were kept consistent, these
369 process variables critical to the flocculation process can be altered for future experimental
370 studies. Fundamental understanding of the effects of these process variables on flocculation can
371 potentially be used to optimize water treatment operations.

372 5 Conflicts of Interest

373

374 There are no conflicts to declare.

375

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388 7 References

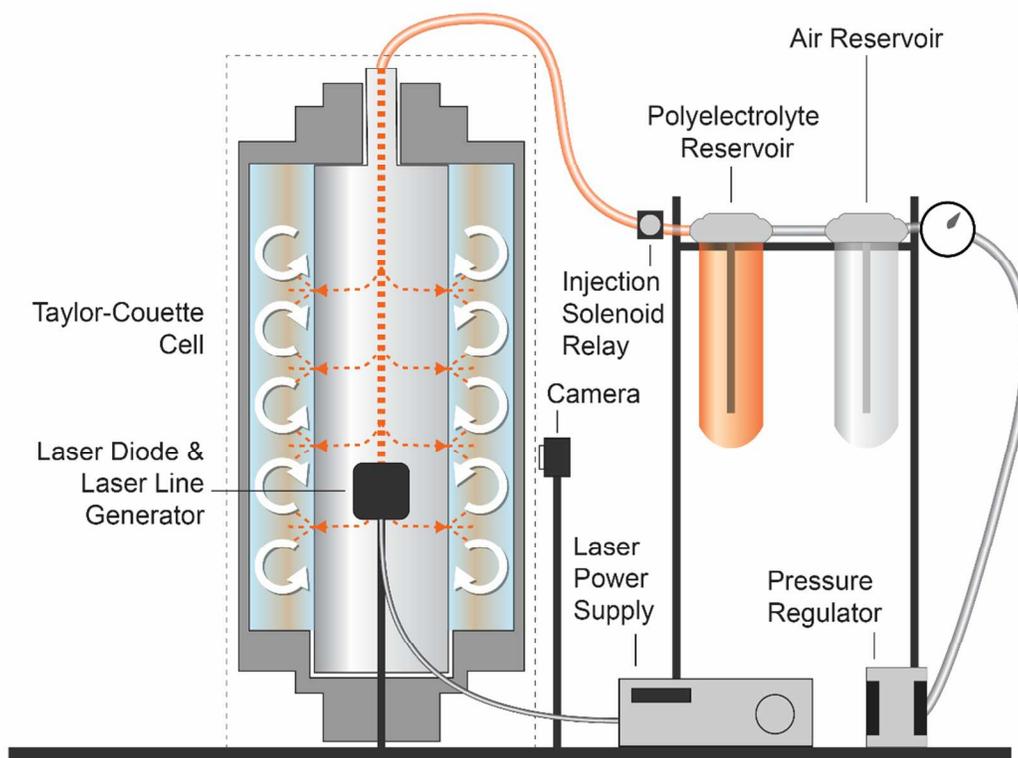
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554 8 Figures

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556 Figure 1: Experimental set-up for flocculation experiments in the TC cell. The bentonite
 557 suspension is loaded into the annulus by way of tubing attached to a Swagelok valve attached to
 558 the bottom of the cylinder base. The polyelectrolyte is held in a reservoir as pictured here and is
 559 injected at the top of the inner cylinder by way of a solenoid valve, which is controlled by a
 560 LabView program. The stepper motor for the inner cylinder is controlled separately by a motor
 561 controller. Under normal operating conditions, the Plexiglass tank surrounding the TC cell is
 562 filled with paraffin oil to match the index of refraction of the glass outer cylinder, which
 563 eliminates its curved surface during recording.

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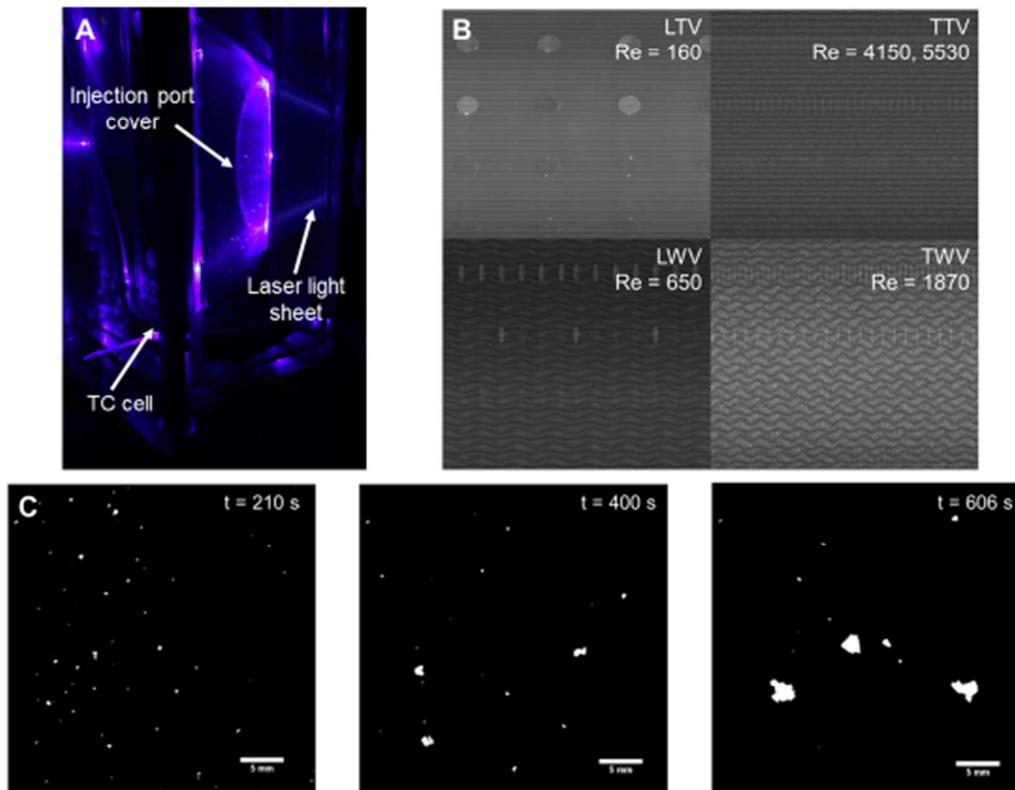
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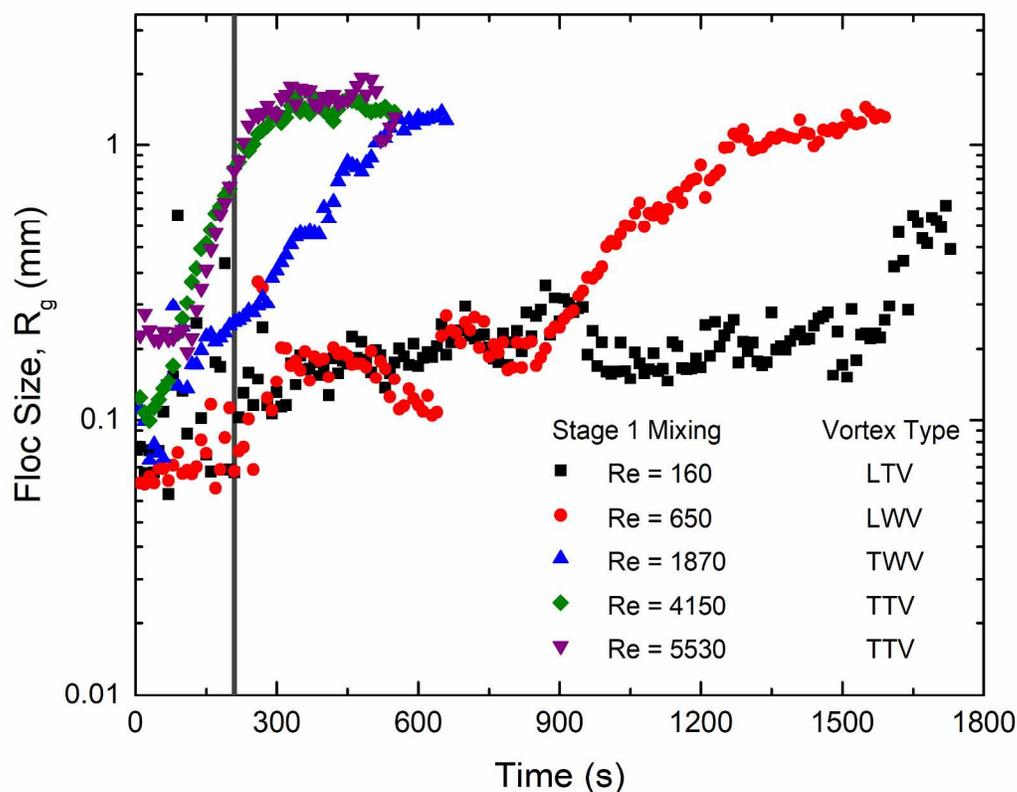


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576 Figure 2: A) Laser light sheet generated from laser diode illuminating the injection port cover
577 and bentonite flocs. B) The four wave states with their vortex type and speed in terms of Re .
578 There are two laminar types (Laminar Taylor Vortex, LTV and Laminar Wavy Vortex, LWV)
579 and two turbulent types (Turbulent Taylor Vortex, TTV and Turbulent Wavy Vortex, TWV).
580 The gray circles present in the image are the injection port covers. C) Binarized images of
581 bentonite flocs from at 210 s, 400 s, and 606 s into flocculation at an inner cylinder Growth
582 Speed of $Re = 4150$. Scale bars in C) are all 5 mm in length.

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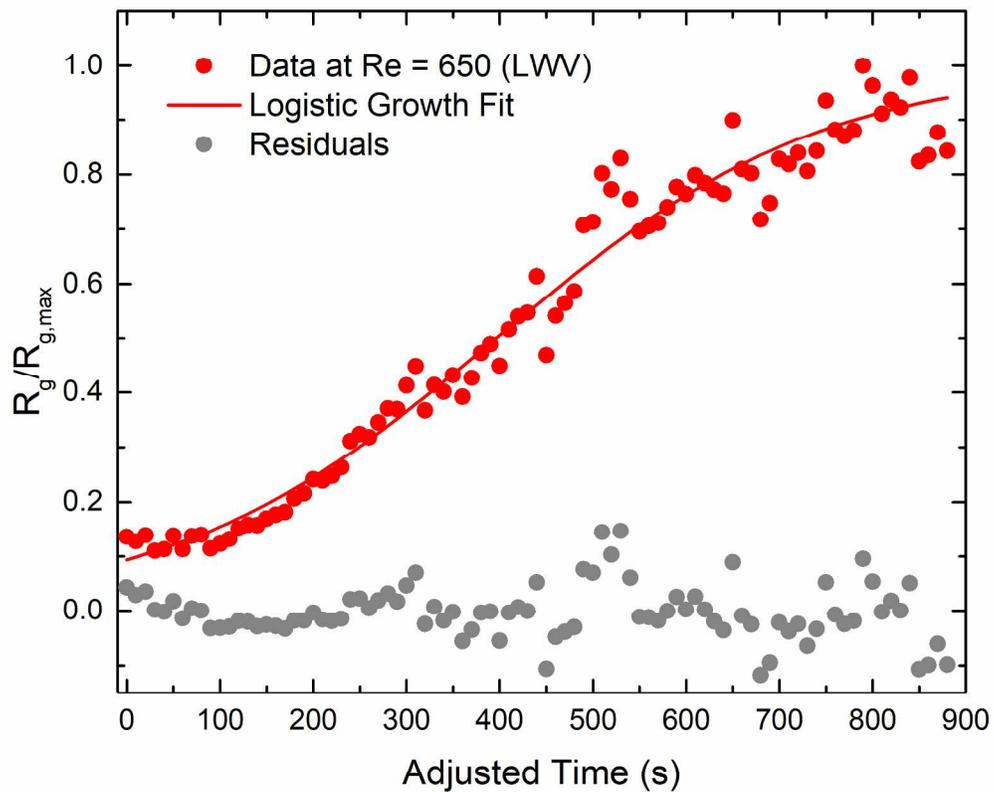
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Figure 3: Bentonite floc size as a function of time and Stage 1 mixing speed in terms of the inner cylinder rotational speed. The corresponding vortex type is listed next to the speed. The vertical, dark gray line denotes the point in time where the speed changes from the Stage 1 speed to Stage 2 speed, which is held constant for all experiments at 0.46 s^{-1} . Each point represents data averaged from 300 frames of the movie (10 s at a frame rate of 30 fps).

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Figure 4: Logistic growth fit to flocculation data at inner cylinder speed of 0.17 s^{-1} ($Re = 650$, LWV) over time. The red dots represent the R_g data points collected during the experiment normalized by the maximum value of R_g in the fitting range. The red line indicates the fit. The gray data points are the residuals of the fit, which is the data point calculated by the logistic growth model subtracted from the original data at the corresponding time point.

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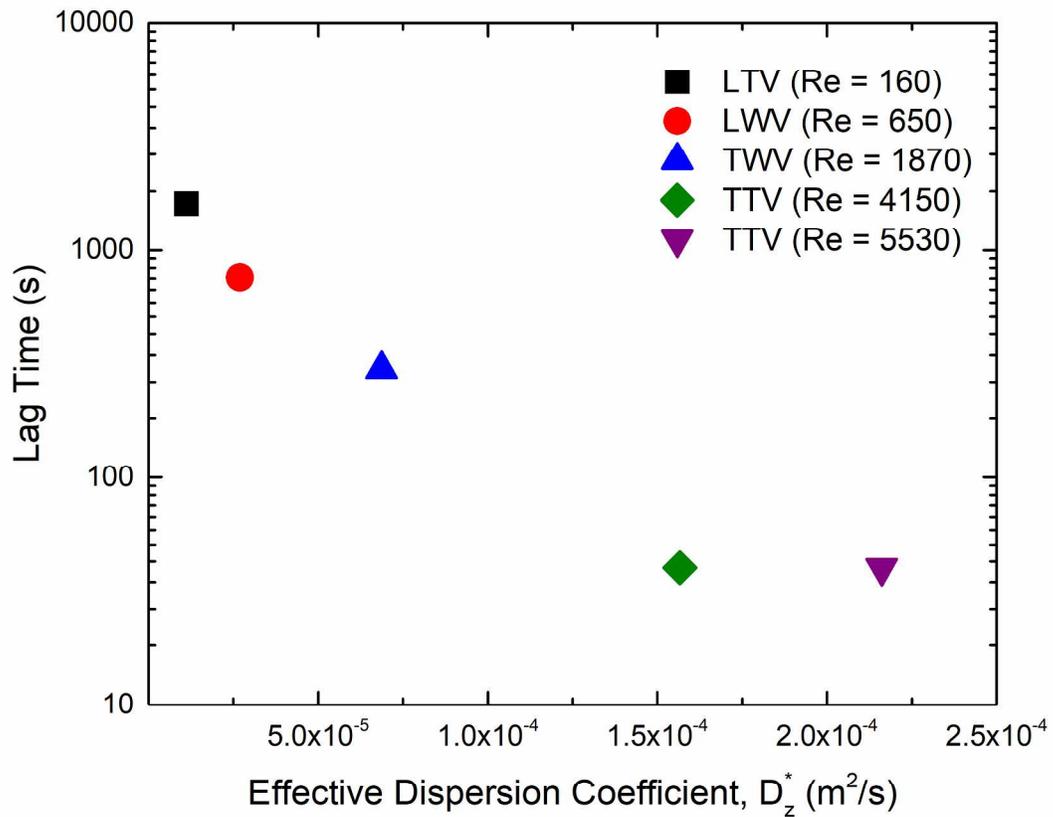
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623 Figure 5: Lag time of floc growth verse the effective dispersion coefficient, D_z^* . Lag time refers
624 to the point in time during flocculation where the flocs commence growth.

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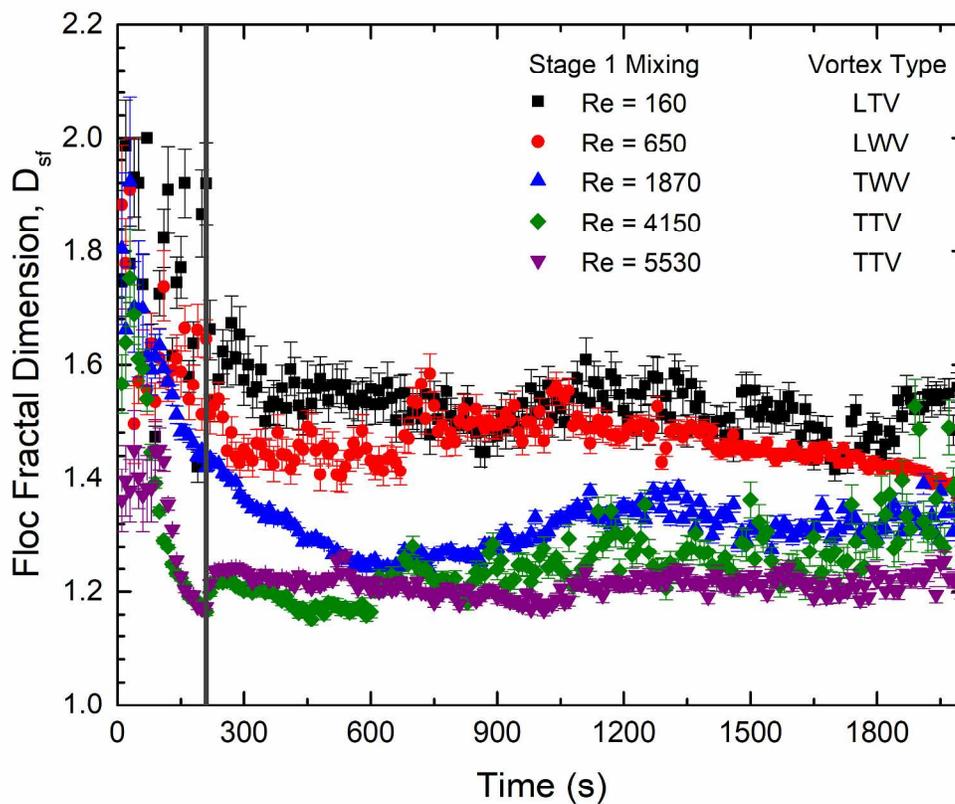
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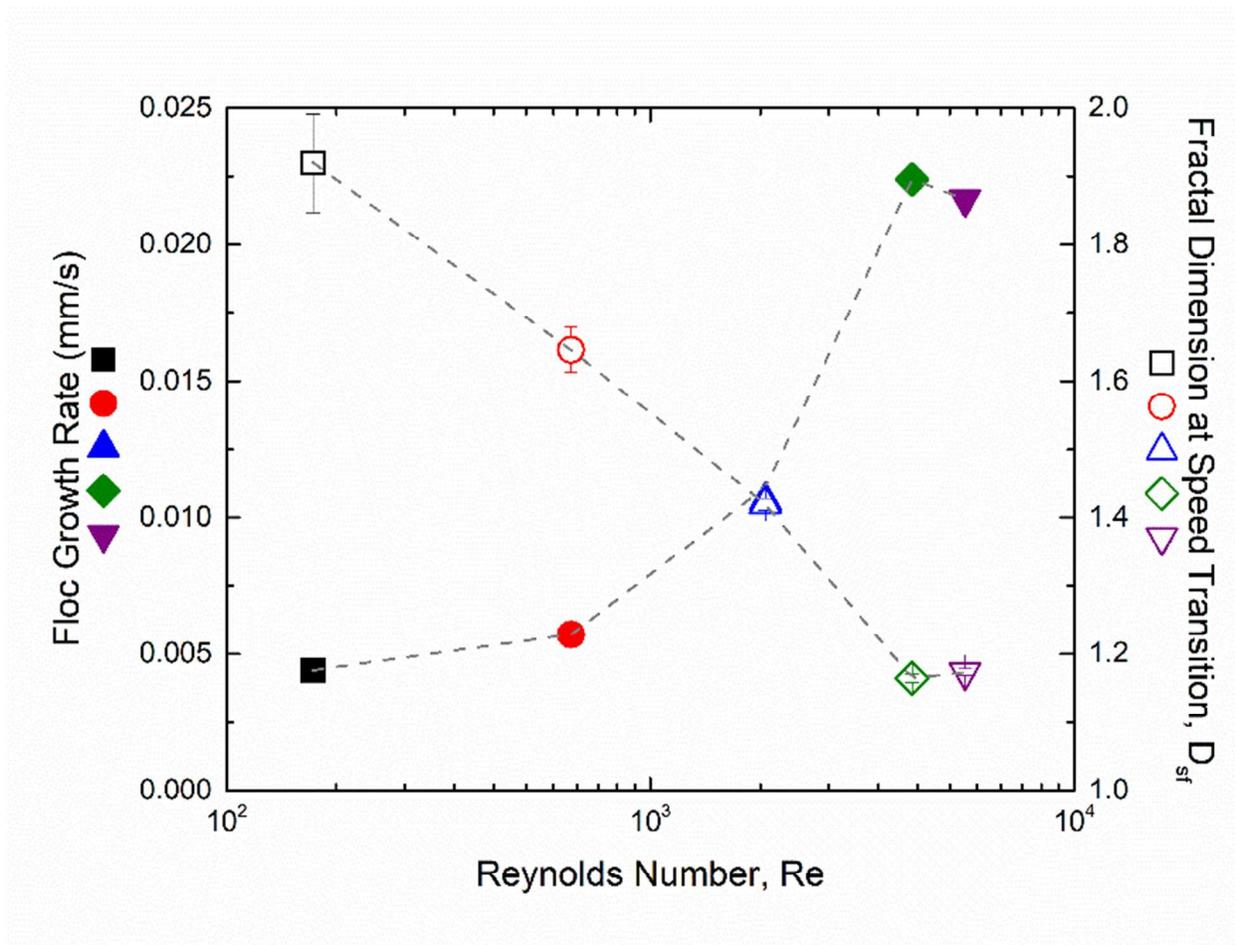
637 Figure 6: Bentonite morphology quantified as a 2-D perimeter-based fractal dimension as a
638 function of time and Stage 1 mixing speed. The corresponding vortex type is listed next to the
639 speed. The vertical, dark gray line denotes the point in time where the speed changes from the
640 Mix Speed to the Growth Speed, which is held constant for all experiments at 0.46 s^{-1} . Each
641 point represents data averaged from 300 frames of the movie (10 sec at a frame rate of 30 fps).

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652 Figure 7: Summary of floc growth rate (closed symbols) and floc fractal dimension (open
653 symbols) as functions of inner cylinder speed. The error bars on the floc fractal dimension
654 represent a 95% confidence interval, and the points were taken at the speed transition between
655 the “Mix Speed and the “Growth Speed.” The gray dashed line is there to guide the eye to
656 observe the increasing trend in flow growth rate and decreasing floc fractal dimension with
657 increasing Re . The solid blue triangle is behind the open blue triangle. Several of the error bars
658 are obscured by the symbols.