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A new form of equivalent stress for combined axial-torsional loading considering the tension-compression asymmetry of polymeric material

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Jianwei ZHANG, Han JIANG*, Guozheng Kang, Chengkai JIANG, and Fucong LU

Although the Mises criterion has been generally adopted to obtain the equivalent stress for materials under multiaxial loading, it does not consider the influence of tension-compression asymmetry of the pressure-sensitive material. For the polymeric materials, to include the non-negligible effect of tension-compression asymmetry, this work proposed a new form of equivalent stress in which the tensile and compressive yield strengths are presented. This form was compared with the work of Mises, Bai and Christensen in the support of experiment data. It was found that the proposed form is more suitable for polymeric material under combined axial-torsional loading condition. Implications of the present findings for designing the stress-controlled combined axial-torsional loading experiment were also discussed.

1. Introduction

With the basic assumption of isotropy, independence on both pressure and the sign of external stresses, the Mises yield criterion has been generally accepted to describe the yielding behaviour of materials under any loading condition using only uniaxial tensile yield strength.[1] For many ductile metals insensitive to pressure, e.g. copper, aluminium, and alloy steels etc., the Mises yield criterion has been shown in excellent agreement with experimental results.[2] However, for the materials whose tensile yield strength is significantly less than compressive yield strength, which is true for most polymeric materials, the form of Mises equivalent stress fails to describe their yield behaviours under multiaxial loading.[3, 4] Substantially depending on temperature, loading rate, and pressure, et al., the yield behaviour of polymer has drawn much attention.[5-11] Bowden et al.[12] proposed modified forms of Tresca and Mises criterion by linearly relating the first invariant of stress tensor, I_1, to the maximum shear stress and the square root of the second deviatoric stress invariant $\sqrt{J_2}$. Raghava et al.[13] suggested a yield criterion in the form of the combination of I_1 and J_2 with only the tensile and compressive yield strengths as the material parameters in the yield criterion. Pae's work[14] combined $\sqrt{J_2}$ with a polynomial form of I_1. Furthermore, Farrokh et al.[15] introduced the dependency of strain rate to yield criterion of polymer. Ghorbel[16] considered the effect of the third invariant of the deviatoric stress tensor J_3. Altenbach et al.[17] gave a detail review on the phenomenological yield criteria.

Although many works have been conducted on the yield criterion for polymeric materials, little attention has been paid to the equivalent stress for combined axial-torsional loading considering the effect of tension-compression asymmetry. The form of Mises equivalent stress, $\sqrt{\sigma^2+3\tau^2}$, was used for combined axial-torsional loading while neglecting the tension-compression asymmetry.[18-20] This is no longer valid for many polymeric materials. Since the Mises equivalent stress overestimates the shear stress under tension-torsion loading condition, Mittal et al.[21] suggested an empirical equivalent stress as $\sqrt{\sigma^2+2\tau^2}$ for PMMA under monotonic tension-torsion loading case. Furthermore, Sittner et al.[22] proposed $\sqrt{\sigma^2+k\tau^2}$ for shape memory alloy. Mittal's and Sittner's work only focused on the influence of shear stress, while the tension-compression asymmetry property of polymeric materials was not included.

In this paper, a modified form of Mises equivalent stress considering the tension-compression asymmetry property of polymeric material is proposed. It is compared with the work of Mises, Bai[23] and Christensen[24, 25] with the help of experiment data of three polymeric materials which show different levels of sensitivity on tension-compression asymmetry, i.e., polycarbonate (PC), polymethyl methacrylate (PMMA) and polystyrene (PS). As the proposed form of equivalent stress is shown more suitable for polymeric material, it can be used to guide the experiment design for combined axial-torsional loading.

Applied Mechanics and Structure Safety Key Laboratory of Sichuan Province, School of Mechanics and Engineering, Southwest Jiaotong University, Chengdu, Sichuan 610031, China.
**Corresponding author. Tel: +86-28-87601442; fax: +86-28-87600797*
E-mail address: jianghan@home.swjtu.edu.cn

2. Theoretical foundation

2.1 Yield criterion considering tension-compression asymmetry

Assuming a material begins to yield when the elastic energy of distortion reaches a critical value, Mises criterion is written as:

$$\bar{\sigma}_{Mises}=\sqrt{\frac{(\sigma_1-\sigma_2)^2+(\sigma_1-\sigma_3)^2+(\sigma_2-\sigma_3)^2}{2}}=\sigma_{yld} \quad (1)$$

Here σ_{yld} is equal to the tensile yield strength T. Compressive yield strength C is not considered in Equation 1. In other words, here T=C.

For the materials sensitive to pressure, Bai et al.[23] proposed

$$\bar{\sigma}_{Bai}=\bar{\sigma}-\alpha I_1=(1-3\alpha\eta)\bar{\sigma}_0=\sigma_{yld} \quad (2)$$

I_1 is the first invariant of the stress tensor. $\bar{\sigma}_0$ is the stress under zero pressure. $\eta=\frac{I_1}{3\sqrt{3J_2}}$ is the normalized pressure, which is also referred as the triaxiality parameter. J_2 is the second invariant of the deviatoric stress tensor. α is a material constant to represent the pressure sensitivity of material. According to the literature[26], α is taken in the form of

$$\alpha=\frac{3(C-T)}{T+C} \quad (3)$$

Here α is a good index to represent the magnitude of tension-compression asymmetry.

Equation 2 can be reformed in terms of principle stress as

$$\bar{\sigma}_{Bai}=\sqrt{\frac{(\sigma_1-\sigma_2)^2+(\sigma_1-\sigma_3)^2+(\sigma_2-\sigma_3)^2}{2}}+\frac{(C-T)}{T+C}\cdot(\sigma_1+\sigma_2+\sigma_3)=\frac{2TC}{T+C} \quad (4)$$

The absolute value of C is adopted here. For $T=C$, Equation 4 reduces to Mises criterion.

Christensen[24, 25] proposed "a two-property yield for homogeneous, isotropic materials"

$$\frac{1}{2TC}\left[(\sigma_1-\sigma_2)^2+(\sigma_1-\sigma_3)^2+(\sigma_2-\sigma_3)^2\right]+\left(\frac{1}{T}-\frac{1}{C}\right)\cdot(\sigma_1+\sigma_2+\sigma_3)=1 \quad (5)$$

which is quite similar to Raghava's work[13].

2.2 Equivalent stress for combined axial-torsional loading

With extensive works on the yield criterion for polymeric materials, little attention has been paid to the equivalent stress for combined axial-torsional loading. In this section, a modified form of equivalent stress is proposed to consider the tension-compression asymmetry.

For combined axial-torsional experiment, the Mises equivalent stress can be written as

$$\bar{\sigma}_{Mises}=\sqrt{\sigma^2+3\tau^2}=T \quad (6)$$

in which σ is the axial stress, τ is the torsional stress

Bai's work (Equation 4) can be reformed for combined axial-torsional experiment as

$$\bar{\sigma}_{Bai}=\sqrt{\sigma^2+3\tau^2}+\frac{C-T}{C+T}\sigma=\frac{2TC}{T+C} \quad (7)$$

Based on Equation 5, the equivalent stress for Christensen's work under combined axial-torsional loading is written as

$$\bar{\sigma}_{Christensen}=\frac{2}{T+C}*(\sigma^2+3\tau^2)+\frac{2(C-T)}{T+C}\sigma=\frac{2TC}{T+C} \quad (8)$$

While the Mises equivalent stress gives the simplest form of equation, Bai's and Christensen's work take into consideration of tension-compression asymmetry but with relatively complicated forms.

We can reform Equation 2 as,

$$\sqrt{3J_2}+\frac{C-T}{C+T}I_1=\frac{2TC}{T+C} \quad (9)$$

Squaring both sides of Equation 9 gives,

$$3J_2+2\sqrt{3J_2}\frac{C-T}{C+T}I_1+\left(\frac{C-T}{C+T}\right)^2 I_1^{\,2}=\left(\frac{2TC}{T+C}\right)^2 \quad (10)$$

For combined axial-torsional loading, it can be rewritten as

$$\left[1+\left(\frac{C-T}{C+T}\right)^2\right]\sigma^2+2\frac{C-T}{C+T}\sqrt{\sigma^2+3\tau^2}\sigma+3\tau^2=\left(\frac{2TC}{T+C}\right)^2 \quad (11)$$

For relatively ductile polymers, $|C-T|\ll|C+T|$, thus $\left(\frac{C-T}{C+T}\right)^2\sim(0)^2$. Noticing that $\sqrt{\sigma^2+3\tau^2}\approx\frac{C+T}{2}$, we take the square root of both sides of Equation 11 to get

$$\bar{\sigma}_P=\sqrt{\sigma^2+(C-T)\sigma+3\tau^2}=\frac{2TC}{T+C} \quad (12)$$

Equation 12 can reduce to the form of Mises equivalent stress as Equation 6 in the tension-compression symmetry case, i.e. T=C. As one can see in Table 1, quite appealing, the proposed equivalent stress not only takes consideration of the tension-compression asymmetry as Bai and Christensen did, but also owns a more concise form than both of them.

Table 1 Comparison of different forms of equivalent stress

Item	Equivalent stress
Mises	$\sqrt{\sigma^2+3\tau^2}$
Bai	$\sqrt{\sigma^2+3\tau^2}+\frac{C-T}{C+T}\sigma$
Christensen	$\frac{2}{T+C}*(\sigma^2+3\tau^2)+\frac{2(C-T)}{T+C}\sigma$
Proposed	$\sqrt{\sigma^2+(C-T)\sigma+3\tau^2}$

3. Results and Discussion

3.1 Comparison of different forms of equivalent stress for combined axial and torsional loading

Four forms of equivalent stress, i.e., Mises, Bai, Christensen and the proposed one, are compared with experimental data of three kinds of polymer, i.e., PC, PMMA and PS, showing different levels of sensitivity to tension-compression asymmetry.

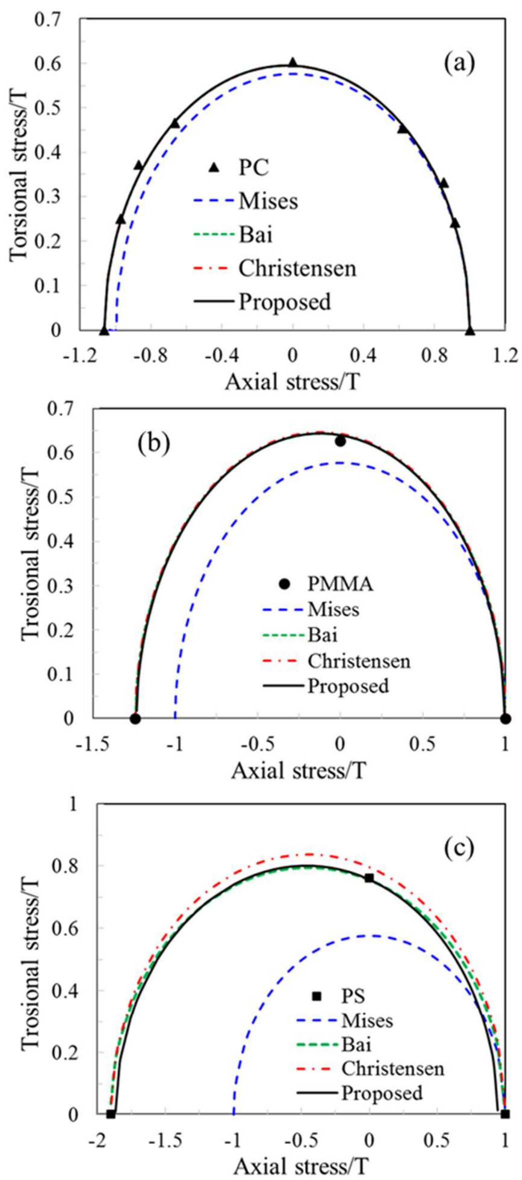

Fig.1 Comparison between the experiment data with four forms of equivalent stress under combined axial-torsional condition, i.e., Mises, Bai, Christensen and the proposed form: (a) PC, (b) PMMA (@60℃), (c) PS (@90℃). The experiment data of PMMA and PS are from the literature.[27]

For PC, which has the relatively low sensitivity to tension-compression asymmetry with α_{PC}=0.09, the four kinds of equivalent stress for combined axial-torsional loading of PC are shown in Fig 1a. The experimental results of PC were obtained from uniaxial tension, compression and a series of proportional combined axial-torsional experiment at a constant strain rate of 5e-3/s at room temperature in the authors' lab. The axial and torsional stresses are nondimensionalized by the tensile yield strength. It can be found from Fig. 1a that Mises equivalent stress cannot well describe the tension-compression asymmetry of PC, while the proposed form shows the capability as good as Bai and Christensen.

For PMMA (α_{PMMA}=0.32) and PS (α_{PS}=0.94), which are more sensitive to the tension-compression asymmetry than PC, the comparison between the four forms of equivalent stress and experiment data are given in Fig. 1b and 1c, respectively. It should be noted that PMMA and PS are brittle at room temperature, not suitable for the discussion about yield behaviour. However, at elevated temperature, e.g., PMMA @ 60℃ and PS @ 90℃ respectively, they do appear certain ductility and the yield strengths are measurable. So the experiment results of PMMA (@60℃) and PS (@90℃) are taken from the literature[27] to evaluate the proposed form of equivalent stress in this paper. For all the three polymers, the Mises equivalent stress cannot consider the tension-compression asymmetry. The Christensen's form gives overestimated results for torsional stress and this is more obvious for the materials owning higher sensitivity to tension-compression asymmetry such as PS. Bai's and the proposed form are more suitable under combined axial-torsional loading.

3.2 Comparison of different forms of equivalent stress for the plane strain compression

Fig.2 a-c illustrate the comparison between experimental results with the four forms of equivalent stress in the plane stress space. One can easily figure out that the Mises's form only can be used for certain stress status. Bai, Christensen and the proposed form show the same capability for PC, which is not quite sensitive to tension-compression asymmetry (α_{PC}=0.09). For the materials with higher tension-compression asymmetry sensitivity such as PMMA and PS, Bai overestimates the yield stress under the plane strain compression. The proposed form shows good predictive ability similar to Christensen.

Based on the discussions above, with a simple form, the proposed form of equivalent stress can be used for both combined axial-torsional loading and plane stress space. Only the tensile and compressive yield strengths, which can be easily obtained from uniaxial tests, are included in the proposed form of equivalent stress as Equation 12. No curve fitting procedures are required.

The stress-controlled axial-torsional combined loading experiment of polymeric material has drawn much attention because of its significance in safety assessment[18, 19, 28-30]. Equation 12 is very important for the designing of such loading condition for the polymeric materials with higher sensitivity to tension-compression asymmetry. The proposed equivalent stress can well describe the tension-compression asymmetry of polymer with a relatively simple form which shows clear physical meaning.

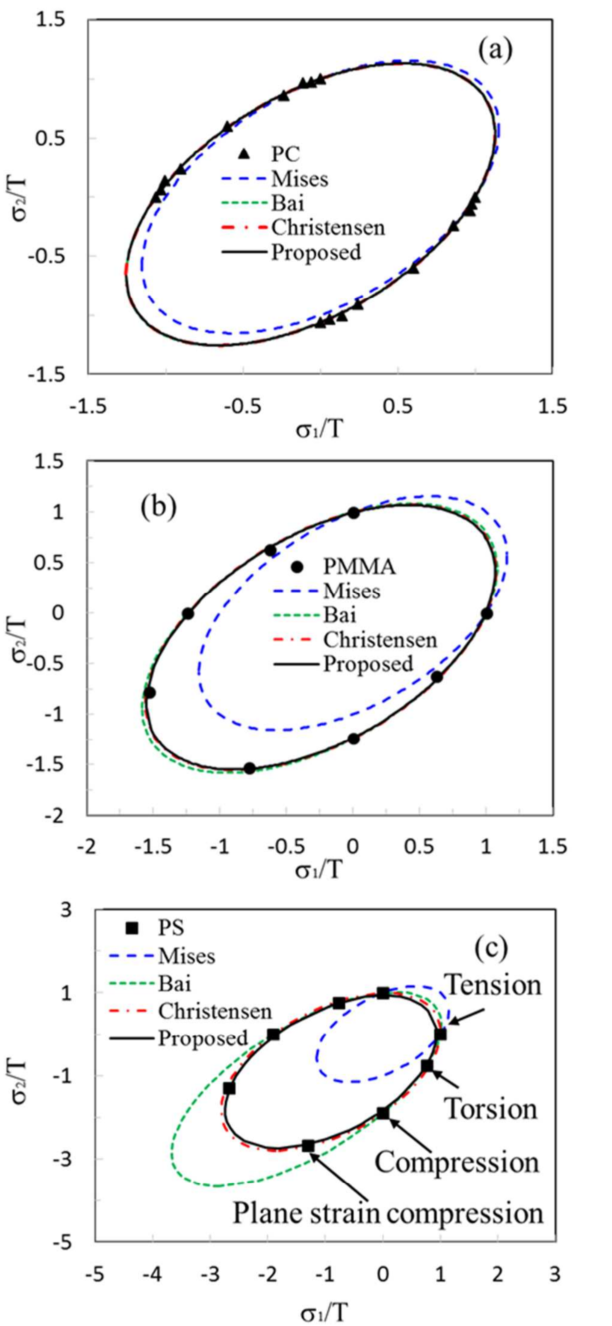

Fig.2 Comparison between the experiment data with four forms of equivalent stress in plane stress space, i.e., Mises, Bai, Christensen and the proposed form: (a) PC, (b) PMMA (@60℃), (c) PS (@90℃). The experiment data of PMMA and PS are from the literature.[27]

4. Summary

For combined axial-torsional loading, the tension-compression asymmetry of polymeric material cannot be ignored. A new form of equivalent stress is proposed considering the effect of the tension-compression asymmetry.

1. Using tensile (T) and compressive yield strength (C), no curve fitted parameter is needed in the proposed form. When T=C, the proposed form reduces to Mises equivalent stress. For $T \neq C$, it is validated with the experiment data of PC, PMMA and PS.
2. While Bai's form overstates the yield stress for plane strain compression, and Christensen's form overestimates the torsional stress, the proposed equivalent stress can describe the two loading conditions above well with a concise form;
3. For the polymeric material with tension-compression asymmetry property, the proposed form of equivalent stress is more suitable to guide the experiment design of combined axial-torsional loading.

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Notes and references

1 W. F. Hosford, *Fundamentals of engineering plasticity*, Cambridge University Press, Cambridge , 2013.
2 R. Hill, *The mathematical theory of plasticity*, Clarendon Press Oxford, Oxford, 1950.
3 I. M. Ward, *J. Mater. Sci.*, 1971, **6**, 1397.
4 Z. Stachurski, *Prog. Polym. Sci.*, 1997, **22**, 407.
5 C. Bauwens-Crowet, J. C. Bauwens and G. Homes, *J. Polym. Sci. A-2, Polym. Phys.*, 1969, **7**, 735.
6 P. B. Bowden, in *The Physics of Glassy Polymers*, ed. R. N. Haward, Springer, Netherlands, 1973, 5, 279.
7 I. Ward and J. Sweeney, *Mechanical Properties of Solid Polymers*, John Wiley & Sons, Chichester, 2012.
8 K. Hachour, F. Zaïri, M. Naït-Abdelaziz, J.-M. Gloaguen, M. Aberkane and J.-M. Lefebvre, *Int. J. Plasticity*, 2014, **54**, 1.
9 A. Argon, *Philos. Mag.*, 1973, **28**, 839.
10 J. Bauwens, J. Polym. Sci. A-2, Polym. Phys., 1970, **8**, 893.
11 Y. H. Li and X. S. Sun, *RSC Adv.*, 2015, **5**, 44009.
12 P. Bowden and J. Jukes, *J. Mater. Sci.*, 1972, **7**, 52.
13 R. Raghava, R. M. Caddell and G. S. Yeh, *J. Mater. Sci.*, 1973, **8**, 225.
14 K. D. Pae, *J. Mater. Sci.*, 1977, **12**, 1209.
15 B. Farrokh and A. S. Khan, *Eur. J. Mech. A-Solid.*, 2010, **29**, 274.
16 E. Ghorbel, *Int. J. Plasticity*, 2008, **24**, 2032.
17 H. Altenbach, A. Bolchoun and V. Kolupaev, in *Plasticity of Pressure-Sensitive Materials*, eds. H. Altenbach and A. Öchsner, Springer Berlin Heidelberg, 2014.
18 E. Guitton, G. Rio and H. Laurent, *Polym. Test.*, 2014, **36**, 32.
19 Z. Xia, X. Shen and F. Ellyin, *J. Mater. Sci.*, 2005, **40**, 643.

20 I. Yakimets, D. Lai and M. Guigon, *Mech. Time-Depend. Mat.*, 2007, **11**, 47.
21 R. Mittal and I. Singh, *Polym. Eng. Sci.*, 1982, **22**, 358.
22 P. Sittner, Y. Hara and M. Tokuda, *Metall. Mater. Trans. A*, 1995, **26**, 2923.
23 Y. Bai and T. Wierzbicki, *Int. J. Plasticity*, 2008, **24**, 1071.
24 R. M. Christensen, *J. Eng. Mater-T.*, 2004, **126**, 45.
25 R. M. Christensen, *J. Eng. Mater-T.*, 2007, **129**, 173.
26 J. Richeton, S. Ahzi, K. S. Vecchio, F. C. Jiang and R. R. Adharapurapu,Int. J. Solids Struct., 2006, **43**, 2318.
27 R. Quinson, J. Perez, M. Rink and A. Pavan, *J. Mater. Sci.*, 1997, **32**, 1371.
28 A. Berrehili, S. Castagnet and Y. Nadot, *Fatigue Fract. Eng. M.*, 2010, **33**, 345.
29 G. Chen, H.-q. Liang, L. Wang, Y.-h. Mei and X. Chen, *Polym. Eng. Sci.*, 2015, **55**, 664.
30 F. Z. Zhang, Y. L. Chen, C. Z. Sun, S. P. Wen and L. Liu, *RSC Adv.*, 2014, **4**, 26706.