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Bonding Heterogeneity in Mixed-Anion Compounds Realizes Ultralow Lattice Thermal Conductivity†

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Crystalline materials with intrinsically low lattice thermal conductivity (*κ*lat) pave the way towards high performance in various energy applications, including thermoelectrics. Here we demonstrate a strategy to realize ultralow *κ*lat using mixedanion compounds. Our calculations reveal that locally distorted structures in chalcohalides MnPnS₂Cl (Pn = Sb, Bi) derives a bonding heterogeneity, which in turn causes a peak splitting of the phonon density of states. This splitting induces a large amount of scattering phase space. Consequently, κ_{lat} of MnPnS₂Cl is significantly lower than that of a single-anion sulfide CuTaS₃ with a similar crystal structure. Experimental *κ*_{lat} of MnPnS₂Cl takes an ultralow value of about 0.5 W m⁻¹ K⁻¹ at 300 K. Our findings will encourage the exploration of thermal transport in mixed-anion compounds, which remain a vast unexplored space, especially regarding unexpectedly low *κ*lat in lightweight materials derived from the bonding heterogeneity.

1. Introduction

Searches for low lattice thermal conductivity ($κ$ _{lat}) materials have been of great importance in achieving high performance in thermoelectrics, $1-3$ thermal barrier coatings, 4 and thermal insulation.⁵ In the past decade, the validity of some extrinsic effects for reducing *κ*_{lat}, such as point defects,^{6,7} fine grain boundary, 8 nanoscale precipitates, 9 and all-scale hierarchical microstructures,¹⁰ has been successfully demonstrated. However, these strategies usually affect charge carrier mobility, which is detrimental to thermoelectric performance. Thus, understanding the intrinsic mechanisms of thermal transport realizing low *κ*lat which are associated closely with chemical bonding, crystal structure, and lattice anharmonicity, can be more helpful in some cases, as it provides us with efficient ways to manage both electrical and thermal properties independently.

In recent years, several structural origins bringing intrinsically low *κ*_{lat} like complex crystal structure,¹¹⁻²⁰ resonant bonding,²¹⁻²³ stereochemically active lone-pair electrons,²⁴⁻³¹ rattling and its reminiscent anharmonic vibration, $32-41$ superionic transition, $42-46$ partial occupancy, $47,48$ and bonding heterogeneity^{27,40,49-57} have been unveiled. Among these, we focus on the bonding heterogeneity, namely the coexistence of strong and weak bonding, as it is more likely to be observed in many classes of crystalline solids and can be utilized more easily to realize low κ_{lat}. The bonding heterogeneity is frequently observed in Zintl phases, which consist of coexisting ionic and covalent sublattices. In addition, local structures containing multiple anions should form a distorted coordination environment, which gives the bonding heterogeneity.

Mixed-anion compounds, solid-state materials containing more than one anionic species in a single phase, have recently attracted much attention because they have potential to offer novel and attractive functionalities which were not observed in conventional "single-anion" materials.⁵⁸ Some superior properties in the mixed-anion compounds have already been discovered, such as visible-light photocatalysis, pleochroism, and battery applications.⁵⁸ In the thermoelectrics field, anionic solid solutions like PbTe-PbSe^{59,60} and $Bi_2Te_3-Bi_2Se_3^{61}$ have been regarded as an effective way to realize carrier concentration tuning, band engineering, and reduction of *κ*lat via phonon-alloy scattering. In addition, a mixed-anion BiCuSeO consisting of $(Cu_2Se_2)^{2-}$ and $(Bi_2O_2)^{2-}$ layers has been a hot material because of its extremely low *κ*lat and promising figure of merit.62–64 The origin of low *κ*lat is derived from large anharmonicity due to the anisotropic layered structure. It is a typical example of mixed-anion compounds containing multiple anions in separate building blocks. On the other hand, the mixed-anion are also likely to form locally distorted structures in which more than one anions are bonded to a cation. This class of materials is less explored in terms of thermal transport.

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Although a few previous reports $52-54,65$ deal with this type mixed-anion materials, unique features derived from the mixed-anions are still ambiguous because they are not compared with a single-anion counterpart. We can expect that the locally distorted structures in mixed-anion materials bring the bonding heterogeneity, which effectively realize low *κ*_{lat}.

Herein, we demonstrate an effective guideline that the mixed-anion framework with distorted local structures is a promising series of materials to realize ultralow $κ_{lat}$. We chose Mn-based chalcohalides MnPnS₂Cl (Pn = Sb, Bi) as model materials to test the validity of our strategy using mixed-anion compounds. To extract unique features derived from the mixedanion, a single-anion sulfide $CuTaS₃$, which has some structural similarities to the mixed-anion chalcohalides, was chosen for comparison. By performing calculations based on density functional theory (DFT), we explore lattice dynamics and thermal transport properties of these compounds. We find a remarkable variety of second order interatomic force constants (IFC) within the distorted coordination polyhedra containing multiple anions, which shows significant bonding heterogeneity in this series. Some peak-splitting in the phonon density of states (DOS) originating from the bonding heterogeneity significantly increases the 3-phonon scattering phase space (SPS), which results in a much smaller phonon relaxation time of MnPnS₂Cl than CuTaS₃. Consequently, the calculated *κ*_{lat} of $MnPnS₂Cl$ is strikingly about seven times lower than CuTaS₃. The synthesized polycrystalline sample of $MnPnS₂Cl$ is actually demonstrated to exhibit the ultralow κ_{lat} of about 0.5 W m⁻¹ K⁻¹ at room temperature. Considering the relatively low density and averaged atomic mass of $MnSbS₂Cl$, this mixed-anion approach can give a route to uncover lightweight materials with low K_{lat} .

The crystal structures of mixed-anion chalcohalides MnPnCh₂X $(Ph = Sh, Bi, Ch = S, Se, X = Cl, Br, I)$ series, including our target materials, were originally investigated by single-crystal X-ray and neutron diffraction. $66-68$ Fig. 1a shows the crystal structure of the mixed-anion chalcohalides visualized by VESTA 3.69 $MnSbS₂Cl$ and $MnBiS₂Cl$ crystallize in the orthorhombic structure (space group *Pnma*).66,67 The primitive cell contains five crystallographically inequivalent atoms, Mn, Sb/Bi, S(1), S(2), and Cl. All the atoms occupy 4c Wyckoff position. These compounds are composed of locally distorted structures derived from constituent multiple anions. The coordination environments around Mn and Sb/Bi atoms are shown in Fig. 1b and 1c, respectively. The Mn atom is octahedrally coordinated by four S and two Cl atoms with *cis*-geometry. The Sb/Bi atom is coordinated by five S and three Cl atoms. The interatomic distances between Sb/Bi and Cl are much longer than those between Sb/Bi and S atoms. These polyhedra share their edges and vertices and form infinite chains along the *b*-axis. The locally distorted structures containing different bond lengths and chemical environments should result in a significant bonding heterogeneity, which in turn can cause interesting lattice dynamics and thermal transport properties. It is necessary to make a comparison between the mixed-anion compounds and a single-anion counterpart which resembles each other, in order to extract unique features originating from the mixed-anion state. CuTaS₃, a single-anion sulfide, has some structural similarity to $MnPnS_2Cl.$ Fig. 1d shows the crystal structure of CuTaS₃. CuTaS₃ crystallizes in the orthorhombic structure with space group *Pnma*, and the primitive cell contains five crystallographically inequivalent atoms, Cu, Ta, S(1), S(2), and S(3).⁷⁰ All the atoms occupy 4c Wyckoff position. The coordination environments around Cu and Ta atoms are shown in Fig. 1e and 1f, respectively. In the ICSD database, $MnPnCh₂X$ is classified as the same structure-type as $CuTaS₃$ though the local structures are quite different. The Cu atom is tetrahedrally

Fig. 1 (a) The unit cells of MnPnS₂Cl (Pn =Sb, Bi) and the coordination environments around (b) Mn and (c) Sb/Bi atoms. (d) The unit cell of CuTaS₃ and the coordination environment around (e) Cu and (f) Ta atoms. The interatomic distances of atom pairs are shown along each bond. The value shown in (b) and (c) correspond to MnSbS2Cl.

2. Results and Discussion

coordinated by four S atoms, and the Ta atom is octahedrally coordinated by six S atoms. The bond lengths within the coordination polyhedra vary despite the single anion, but to a lesser extent than the mixed-anion $MnPnS_2Cl$. In addition, $MnBiS₂Cl$ and CuTaS₃ have the similar theoretical density of 5.32 g cm⁻³ and 5.82 g cm⁻³, respectively, and the same percentage of heavy atoms where 20% of the constituent elements are heavy (Sb, Bi, Ta) and 80% are relatively light (Mn, Cu, S, Cl). Thus, CuTaS₃ is a suitable material to compare the lattice dynamics with MnPnS₂Cl to gain insight into the crucial roles of the mixed-anion.

We performed DFT-based calculations to investigate lattice dynamics and thermal transport properties of the mixed-anion $MnPnS₂Cl$ and the single-anion CuTaS₃. Fig. 2 shows the norm of calculated second-order IFC matrix, |Φ*ij*| as a function of interatomic distance. The IFC represents the strength of interaction between each atomic pair, namely bond strength. The IFCs decay rapidly with increasing interatomic distance though some non-negligible components are observed for Sb-S, Bi-S, and Ta-S bonds at around 6 Å. The IFCs of Cu-S bondsin the coordination tetrahedron of $CuTaS₃$ with almost identical bond length range from 4 to 5 eV Å^{−2}, while those of Ta-S bonds range from 4 to 7 eV Å⁻² due to slightly different bond length and distorted coordination. On the other hand, the IFCs of the nearest and second nearest Mn-S and Mn-Cl differ by a factor of about 3–6. For the nearest Sb-S (Bi-S) and Sb-Cl (Bi-Cl), the IFCs differ by a factor of 23 (18). These indicate that the local structures in the mixed-anion $MnPnS_2Cl$ hold the remarkable bonding heterogeneity compared to CuTaS₃. This significant difference of bonding environment between the mixed- and single-anion compounds can cause interesting features in lattice dynamics. To gain further insight into the electronic origin of the bonding heterogeneity, we examined the charge density and electron localization function (ELF), as shown in Fig. S1 in the Supplementary Information. The Mn-S shows a bit more covalency than the Mn-Cl bond both in $MnSbS₂Cl$ and $MnBiS₂Cl.$ This behavior can be understood by the larger electronegativity of Cl than S. The bonding environment

surrounding Sb in MnSbS₂Cl and Bi in MnBiS₂Cl looks different. The Sb-S exhibits covalency while the valence electrons of Sb in the Sb-Cl direction are strongly localized according to ELF. On the other hand, in MnBiS₂Cl, the electrons surrounding Cl in the Bi-Cl direction are localized instead of Bi. However, both bonding character results in the bonding heterogeneity, namely the significant difference in the IFC between Pn-S and Pn-Cl bonding.

The calculated phonon dispersion curves together with the phonon DOS are shown in Fig. 3a, 3b, and 3c. These compounds are dynamically stable as no imaginary modes are observed in the phonon dispersion. As shown in the atom-decomposed phonon DOS, the acoustic modes mainly originate from the heaviest atom (Sb, Bi, and Ta for $MnSbS₂Cl$, MnBiS₂Cl, and CuTaS₃, respectively). For CuTaS₃, high-frequency optical modes (> 250 cm−1) mainly derive from S atom, while heavier Cu and Ta atoms mainly contribute to the lower modes (< 180 cm−1). As a result, a gap-like opened structure is observed in the phonon DOS at around 180–250 cm⁻¹. On the other hand, the gap-like structure is not observed in the phonon DOS of MnPnS₂Cl. For MnPnS₂Cl, high-frequency region (> 200 cm⁻¹) is like CuTaS₃, namely large contribution from S atom. However, Cl atom contributes to the lower frequency region (70–200 cm−1) despite its almost the same atomic mass as S (S: 32.07 g mol⁻¹, Cl: 35.45 g mol⁻¹). This splitting of the peak in the phonon DOS between S and Cl is derived from the bonding heterogeneity. The phonon frequency *ω* is related to the atomic mass and the second-order IFC:

$$
\omega^2 \propto \frac{1}{\sqrt{M_A M_B}} \sum \Phi \quad (1)
$$

where M_A and M_B are the atomic mass of species A and B, respectively, and Φ is the second-order IFC. Thus, a significant variety of the IFC originating from the bonding heterogeneity results in the splitting of phonon frequency, even though the atomic mass of multiple anions is almost the same. Fig. 3d shows a schematic diagram representing the situation occurred

Fig. 2 Norm of IFC matrix, $|Φ^i|$, as a function of interatomic distance between atoms denoted by *i* and *j* for (a) MnSbS₂Cl, (b) MnBiS₂Cl, and (c) CuTaS₃.

Fig. 3 The calculated phonon dispersion and the total and atom-decomposed phonon DOS for (a) MnSbS₂Cl, (b) MnBiS₂Cl, and (c) CuTaS₃. A colored region in (c) represents a gaplike opened structure observed in the phonon DOS of CuTaS₃. (d) A schematic diagram representing the change of phonon DOS from single-anion to mixed-anion materials induced by the bonding heterogeneity. The gap-like structure of DOS observed in the single-anion is filled with additional states formed by peak splitting in the mixed-anion one.

Fig. 4 The calculated potential energy of each atomic site as a function of displacements along cartesian *x*-, *y*-, and *z*-directions for (a) MnSbS₂Cl, (b) MnBiS₂Cl, and (c) CuTaS₃. Solid lines are fitting curves using up to cubic term. (d) The calculated Grüneisen parameter as a function of the phonon frequency.

here. The gap-like structure observed in the phonon DOS of the single-anion material is filled with additional states generated from the peak splitting induced by the bonding heterogeneity in the mixed-anion material with locally distorted structures.

A potential energy surface that provides us with insight into anharmonicity at each atomic site can be visualized by plotting an energy change with respect to a displacement of the atom. Fig. 4a, 4b, and 4c show the potential energy of anion sites as a function of displacements along cartesian *x*-, *y*-, and *z*-directions for $MnSbS₂Cl$, $MnBiS₂Cl$, and $CuTaS₃$, respectively, with fitting curves using up to cubic term. The fitting using only quadratic term is shown in Fig. S2 in the Supplementary Information. The potential energy of all sites, including cation sites, is shown in Fig. S3 in the Supplementary Information. The anions in MnPnS₂Cl lie in almost harmonic potential while S3 site in $CuTaS₃$ deviates from harmonic behavior signigicantly. As shown in Fig. S3, cation sites both for $MnPnS_2Cl$ and $CuTaS_3$ exhibit more asymmetric and anharmonic features than anions. The Cl atoms of MnPnS₂Cl lie in a flatter potential than the S atoms, which is consistent with the weaker bonding environment and the lower phonon frequency of the Cl atoms. The difference in anisotropy of potential surrounding Cl atoms between $MnSbS₂Cl$ and $MnBiS₂Cl$, namely the shallower potential in the *y*-direction for MnBiS₂Cl, can be derived from the difference in the Mn-Cl bond length (2.492 Å in $MnSbS₂Cl$ and 2.549 Å in MnBiS₂Cl). The shallow potential well also corresponds to the large thermal displacement parameters represented by the mean square displacement, as shown in Fig. S4 in the Supplementary Information. To further gain insight into the anharmonicity of these compounds, we calculated the Grüneisen parameters as shown in Fig. 4d. The Grüneisen parameter is defined by the change of phonon frequency with respect to the change of volume of the unit cell. Some compounds possessing lone-pair electrons and low *κ*lat show anomalously large Grüneisen parameter exceeding 50.24,27 For the low frequency region of acoustic modes, the magnitude of the Grüneisen parameter is larger for CuTaS₃ than MnPnS₂Cl while MnPnS₂Cl has larger values at around 50 cm⁻¹ and 150 cm−1 . From the Grüneisen parameter, we can see that both $MnPnS₂Cl$ and $CuTaS₃$ has moderate anharmonicity compared to extremely anharmonic materials.^{24,27}

To achieve further insight into the relationship between the bonding heterogeneity and the thermal transport, we calculated the 3-phonon SPS. The SPS is a measure of the amount of available scattering processes satisfying the energy and momentum conservation. The SPS is given as

$$
W_q^{\pm} = \frac{1}{N} \sum_{q',q'} \left\{ \begin{matrix} n_{q''} - n_{q'} \\ n_{q'} + n_{q''} + 1 \end{matrix} \right\} \delta(\omega_q - \omega_{q'} \pm \omega_{q'}) \tag{2}
$$

where W_q^+ and W_q^- are the SPS corresponding to absorption and emission processes of phonon mode *q*, respectively. Here, the variable *q* is defined by $q = (q, j)$ and $-q = (-q, j)$ where and *j* are the wave vector and the branch index of phonon modes. n_q is the Bose-Einstein distribution function. Fig. 5a and 5b show \overline{W}_q^+ and \overline{W}_q^- as a function of the phonon freuquency at 300 K, respectively. Notably, \overline{W}_{q}^{+} at the low-frequency region and W_q^- at the middle and high-frequency region are several

Fig. 5 The calculated 3-phonon SPS for (a) absorption and (b) emission processes, and (c) 3-phonon relaxation time at 300 K as a function of the phonon frequency.

times larger for $MnPnS_2Cl$ than CuTaS₃. These differences should originate from the peak splitting of phonon DOS discussed above, as represented by Fig. 3d. The gap-like structure observed in the phonon DOS causes the lower amount of 3-phonon scattering processes between acoustic-optical and optical-optical modes. The gap is filled with the additional states induced by the bonding heterogeneity, which results in the increase of acoustic-optical and optical-optical scattering processes. The SPS is proportional to the 3-phonon scattering rate, the inverse of phonon relaxation time *τ*. In other words, the larger the SPS, the smaller the *τ.* Thus, the larger SPS brings the lower *κ*_{lat}. Fig. 5c shows the 3-phonon relaxation time as a function of the phonon frequency at 300 K. At the overall frequency range, the *τ* of MnPnS₂Cl is significantly smaller than that of CuTaS₃. As shown in Fig. S5 in the Supplementary Information, the phonon group velocity of acoustic modes of $MnPnS₂Cl$ and $CuTaS₃$ is not essentially different, while the optical modes of $CuTaS₃$ at around 150–250 cm⁻¹, corresponding to the dispersive curve in Fig. 3c, have much larger value. However, this large group velocity of the optical modes of $CuTaS₃$ has a negligible contribution to the lattice thermal conductivity, as shown in the spectral thermal conductivity (Fig. S6, Supplementary Information). Also, the difference in the highest value of phonon frequency (~330 cm−1 for MnPnS₂Cl and ~420 cm⁻¹ for CuTaS₃) is not significantly relevant for *κ*lat because the contribution of high-frequency optical modes above 300 cm−1 to the thermal conductivity can be negligible. Therefore, the significant difference in the *τ* between $MnPnS_2Cl$ and $CuTaS_3$ is a dominant factor determining the difference in K_{lat} .

We calculated the *κ*_{lat} by solving the phonon Boltzmann transport equation within the single mode relaxation time approximation. The calculation only considers the intrinsic 3 phonon scattering. Fig. 6a, 6b, and 6c show the temperature dependence of the *κ*lat (*xx*, *yy*, and *zz* components) for

 $MnSbS₂Cl$, $MnBiS₂Cl$, and $CuTaS₃$, respectively. The averaged value of each direction is also shown. In addition, we synthesized the polycrystalline samples of $MnSbS₂Cl$, $MnBiS₂Cl$, and CuTaS₃ by solid-state reaction, melting, and spark plasma sintering and measured the experimental *κ*_{lat} (see Experimental Procedure and measured X-ray diffraction patterns in Fig. S7 in the Supplementary Information). The sintered samples of MnPnS₂Cl contain a tiny amount of unknown impurity phases. We can regard the impurity phases to have little effects on the transport properties above room temperature. The temperature dependence of the experimental *κ*_{lat} is plotted in Fig. 6a, 6b, and 6c. The calculated *κ*lat shows a significant anisotropy for these compounds due to the anisotropic crystal structures. The

Fig. 7 *κ*lat at 300 K versus averaged atomic mass for various compounds including the mixed-anion MnPnS₂Cl (this work), CuTaS₂ (this work), h-BN (in-plane),⁷⁷ 4H-SiC,⁷⁸ Si,⁷⁹

GaN,⁸⁰ Mg₂Si,⁸¹ CoSb₃,⁸² Bi₂Te₃,⁸³ PbTe,⁸⁴ Yb₁₄MnSb₁₁,⁸⁵ YbZn₂Sb₂,⁸⁵ CaZn₂Sb₂,⁸⁵ Zn₄Sb₃,⁸⁶ MnSi_y,¹⁴ Cu₂S,⁴⁶ Ba₈Ga₁₆Ge₃₀,³⁶ BiCuSeO,⁶² SnSe (a-axis),⁸⁷ NaCo₂O₄,⁸⁸ and Cu₄TiSe₄.⁸⁹

averaged calculated values at 300 K are 1.1 W m⁻¹ K⁻¹, 1.0 W m^{-1} K⁻¹, and 7.2 W m^{-1} K⁻¹ for MnSbS₂Cl, MnBiS₂Cl, and CuTaS₃, respectively. The temperature dependence of the experimental κ _{lat} for MnPnS₂Cl is flat, while the calculated one obeys T⁻¹ dependence. The experimental values at 300 K are 0.50 W m−1 K⁻¹, 0.53 W m⁻¹ K⁻¹, and 3.5 W m⁻¹ K⁻¹, respectively. Both the calculated and experimental values differ strikingly by a factor of 7 between MnPnS₂Cl and CuTaS₃. The deviation between the absolute values of calculated and experimental *κ*lat can be due to several factors: the experimental *κ*lat contains the effect of grain boundary and chemical disorder. Moreover, some effects unveiled in recent years are beyond the scope of our calculations, such as the temperature dependent anharmonicity 71 and crystal/amorphous boundary behavior,72,73 which might contribute to the glass-like flat temperature dependence.

It is worth mentioning again that the significantly lower *κ*lat of $MnPnS_2Cl$ than CuTaS₃ is mainly due to the enhanced SPS, which is harmonic vibrational property. The Grüneisen parameter suggests no significant difference in the lattice anharmonicity, which is closely connected to the strength of each 3-phonon scattering process.⁷⁴ There are some other known systems in which the SPS is a good measure of *κ*lat. 75,76

We have discovered that the mixed-anion $MnPnS_2Cl$ exhibits the ultralow *κ*_{lat} even at room temperature. Considering the relatively low density (MnSbS₂Cl: 4.12 g cm⁻³, MnBiS₂Cl: 5.32 g cm⁻³) and averaged atomic mass per atom (MnSbS₂Cl: 55.27 g mol^{−1} atom^{−1}, MnBiS₂Cl: 72.71 g mol^{−1} atom^{−1}), the κ_{lat} values of these compounds are unexpectedly low. Light atoms generally make stronger and more covalent bonding than heavier ones, which typically leads to high *κ*_{lat}. To highlight the anomaly of the mixed-anion compounds, we plotted the *κ*_{lat} versus the averaged atomic mass of various materials, including simple semiconductors and some known thermoelectric materials with low κ_{lat} in Fig. 7. Almost all materials with κ_{lat} < 1 W m⁻¹ K⁻¹ shown here have large averaged atomic mass larger than 80 g mol⁻¹ atom⁻¹, except for MnPnS₂Cl, Cu₂S,⁴⁶ and Cu₄TiSe₄.⁸⁹ Cu₂S undergoes two phase transitions and possesses liquid-like copper ions, which results in ultralow κ_{lat} .⁴⁶ The extremely low κ _{lat} of Cu₄TiSe₄, 0.19 W m^{−1} K^{−1} at 300 K, was very recently confirmed and attributed to its highly disordered structure and soft phonon modes.⁸⁹ Note that the lightweight and ultralow *κ*_{lat} $MnPnS₂Cl$ contains none of the phase transitions, structural instability, and extremely high anharmonicity but contains the bonding heterogeneity induced by the locally distorted structures.

A great advantage of the bonding heterogeneity by mixedanion concept to realize low *κ*lat, summarized in Fig. 3d, is that the phonon frequency of light atoms, like Cl in this work, can be lowered so that the number of optical modes whose frequency are close to acoustic modes increases. It means that the strategy can be utilized to explore lightweight and low *κ*lat materials. In particular, abundant mixed-anion materials unknown in terms of thermal conductivity, such as oxynitrides, oxysulfides, and chalcohalides, can be promising candidates.

3. Conclusions

In conclusion, we have demonstrated that the locally distorted structures observed in the mixed-anion compounds derive ultralow *κ*_{lat}. Using DFT-based calculations, we clarified the significant variety of the second-order IFC in the mixed-anion MnPnS₂Cl, which represents the bonding heterogeneity. This bonding environment causes the remarkable splitting of the phonon frequency, which essentially increasesthe amount of 3 phonon scattering processes. The increased SPS leads to the lower phonon relaxation time. Consequently, *κ*_{lat} of MnPnS₂Cl is significantly lower than that of a single-anion sulfide $CuTaS₃$ with the similar crystal structure and percentage of heavy atoms. Experimental *κ*_{lat} of MnPnS₂Cl shows an ultralow value of about 0.5 W m⁻¹ K⁻¹ at room temperature, which is unexpectedly low considering the relatively low averaged atomic mass per atom of these compounds. A possible future work will be to confirm whether the strategy found in this work can be applied to a broader range of the mixed-anion compounds. Our study decorates the mixed-anion compounds, which remain a vast unexplored space, with novel thermal functionality.

4 Experimental Section

Synthesis

Polycrystalline samples of MnPnS₂Cl were synthesized by solid state reaction of stoichiometric amounts of MnS, MnCl₂, and $Pn₂S₃$ (1:1:1). The starting materials were weighed and ground with an agate mortar in an Ar-filled glovebox, and then transferred to quartz tubes and sealed under a high vacuum. These ampoules were put into a box furnace and heated to 773 K for 2 hours, and kept for 7 days. The obtained ingots were ground into fine powders (< 45 μm) and consolidated by spark plasma sintering (SPS, SPS-515S, Fuji Electronic Industrial Co., Japan). To obtain dense and high-purity bulk samples, we applied high-pressure and low-temperature conditions for SPS. The powder samples were set in a 10 mm diameter cemented carbide die, heated to 813 K with a uniaxial pressure of 500 MPa under Ar atmosphere, and then held for 30 min. The dense bulk samples with relative density ~98% were obtained.

A polycrystalline sample of $CuTaS₃$ was synthesized by melting and following solid state reaction of element powders Cu, Ta, and S. To avoid S-poor impurity phases, the nominal composition was set to $CuTaS_{3.2}$. The starting materials were weighed and ground with the agate mortar, and then transferred to a quartz tube and sealed under a high vacuum. The ampoule was put into the box furnace and heated to 923 K for 24 hours, and kept for 7 days. The obtained ingot was ground into fine powders (< 45 μm) and consolidated by SPS. The powder sample was set in a 10 mm diameter graphite die, heated to 873 K with a uniaxial pressure of 100 MPa under Ar

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atmosphere, and then held for 10 min. The dense bulk sample with relative density ~98% was obtained.

Characterization

The phase identification of the samples was performed by X-ray diffraction (XRD) measurements using Cu K*^α* radiation (SmartLab, Rigaku Co., Japan). The thermal conductivity *κ* was brought by *κ* = *DC*p*d*, where *D*, *C*p, and *d* are the thermal diffusivity, the specific heat, and the density, respectively. The thermal diffusivity and the specific heat were concurrently measured for the disk-shaped samples by laser flash method (LFA 467 Hyperflash, Netzsch, Germany) with a pyroceram disk as a reference sample. The sample density was measured by Archimedes method. The lattice thermal conductivity *κ*lat can be regarded as the same as the measured *κ* because MnPnS₂Cl and $CuTaS₃$ are insulators with high electrical resistivity.

Computational Method

We performed *ab initio* calculations based on density functional theory (DFT). The Quantum Espresso (QE) package^{90,91} with projector-augmented wave pseudopotentials^{92,93} was used for total energy calculations. The generalized gradient approximation functional with Perdew–Burke–Ernzerhof parametrization revised for solids (GGA-PBEsol)⁹⁴ was employed for the exchange and correlation potentials. For $MnPnS₂Cl$ (Pn = Sb, Bi), we employed spin-polarized settings with oppositely oriented magnetic moments on Mn atoms. Initial values of magnetic moments of Mn atoms were set to ±5 $\mu_{\rm B}$, and after structure relaxation, they were converged to ± 4.19 $\mu_{\rm B}$ and \pm 4.15 $\mu_{\rm B}$ for MnSbS₂Cl and MnBiS₂Cl, respectively. *k*point grids of $6 \times 14 \times 4$ and $6 \times 15 \times 4$ and energy cutoff of 100 Ry and 90 Ry for the primitive cell of $MnPnS_2Cl$ and $CuTaS_3$, respectively, were used for structure relaxation. The lattice parameters of the relaxed primitive cell of $MnPnS_2Cl$ and $CuTaS_3$ are listed in Table S1.

To extract the second and third order interatomic force constants (IFC), we used finite difference method with $2 \times 3 \times 1$ supercells containing 120 atoms based on the fully relaxed primitive cell to create displacement-force datasets. The magnitude of atomic displacements was set at 0.01 Å and 0.04 Å for calculating the second and third order IFCs, respectively. For the third order IFC calculation, we chose the cutoff radius as 14.2 Å for choosing triplets within which interaction is considered. The IFCs were obtained using ALAMODE⁹⁵ code. The phonon dispersion relation and density of states (DOS) were calculated from the second order IFC. The sampling grids used for summation were $7 \times 17 \times 5$ and $7 \times 18 \times 6$ for MnPnS₂Cl and CuTaS₃, respectively. Dielectric constant and Born effective charge were calculated in the perturbation framework implemented in the QE package for correcting phonon frequencies considering long-range interaction. ALAMODE code is also used to calculate the lattice thermal conductivity by solving the phonon Boltzmann transport equation within the single mode relaxation time approximation. The sampling grids $7 \times 17 \times 5$ and $7 \times 18 \times 6$ for MnSbS₂Cl and MnBiS₂Cl, respectively, were applied after confirming convergence of the

calculated lattice thermal conductivity with respect to the grid density.

Author Contributions

Naoki Sato: conceputualization, data curation, formal analysis, investigation, methodology, project administration, software, validation, visualization, writing – original draft, writing – review & editing. Norihide Kuroda: investigation. Shun Nakamura: investigation. Yukari Katsura: supervision. Ikuzo Kanazawa: supervision. Kaoru Kimura: supervision, writing – review & editing. Takao Mori: funding acquisition, project administration, supervision, writing – review & editing.

Conflicts of interest

There are no conflicts to declare.

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