

# Second Order Jahn-Teller Interactions at Unusually High Molecular Orbital Energy Separations

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2	Separations
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10	Abstract
11	Second order Jahn-Teller (SOJT) effects arise from interactions between filled and empty molecular

12 orbitals of like symmetry. These interactions often lead to structural distortions whose extent is inversely proportional to the energy difference between the interacting orbitals. The main objectives of the work 13 14 described here are (1) the calculation (using density functional theory methods) of the energies of the 15 valence molecular orbitals in the species EH<sub>3</sub> (E = N, P, As or Sb), HEEH (E = C, Si, Ge or Sn), and H<sub>2</sub>EEH<sub>2</sub>, (E 16 = C, Si, Ge or Sn) and (2) the correlation of these energies with barriers for planarization or linearization. 17 The calculations suggest an upper limit of about 12 eV energy separation of the interacting levels for SOJT 18 effects to be significant, which is considerably larger than previously thought and implies that SOJT effects 19 may be more common than currently realized.

#### 1 Introduction

2 The process by which pyramidal molecules of the general formula  $ER_3$  (E=N–Bi; R=H, alkyl, aryl, silyl, 3 halide, etc., or the corresponding anionic group 14 and cationic group 16 species) are transformed into their inverted conformation (Figure 1) has been studied both experimentally<sup>1</sup> and theoretically<sup>2-8</sup> for many 4 5 decades. A detailed understanding of this process is of fundamental interest with respect to bonding in 6 these molecules and chemical bonding in general. Classically, the inversion process can be regarded as a 7 (fundamental) vibrational mode that interconverts the two pyramidal conformations via a planar transition state structure.<sup>9</sup> The energy difference between the pyramidal minimum and the transition 8 9 state structure is the so-called inversion barrier. The magnitude of the inversion barrier is related to several factors.<sup>1</sup> These include the relative electronegativity (EN) of the substituents (R) and central atom 10 11 (E), conjugative interactions between E and R, the steric properties of R, the presence of lone pairs on R 12 and geometrical constraints at E. The influence of electronic factors, e.g., the EN values of the central and substituent atoms, has been rationalized in terms of the Bent<sup>11</sup>/Walsh<sup>12,13</sup> rules and differences in 13 hybridization tendencies between the lightest and heavier elements within each group (Kutzelnigg).<sup>14</sup> 14 15 Conjugation can lower the relative energy of the planar transition state structure and geometrical confinement can increase the relative strain of the minimum or transition state structure.<sup>15</sup> 16





2 **Figure 1.** Inversion/linearization processes examined herein.

A less prominent view of the inversion process is based on second order Jahn-Teller (SOJT) effects<sup>16-23</sup> involving the extent of HOMO-LUMO interactions and their dependence on symmetries of the frontier orbitals and the electronegativities of E and R. These effects are ubiquitous in chemistry, and in molecular species they generally involve a symmetry governed interaction between a filled and an empty molecular orbital that occurs during a molecular vibration. This interaction can produce a change in the geometry that usually far exceeds those produced by the better known but arguably less important (for chemistry) first order Jahn-Teller effect. The extent of the SOJT distortion is inversely proportional to the

energy difference between the interacting levels. It was pointed out recently that there was no
 information available on the energy separations between the levels whose interactions produced the SOJT
 distortion,<sup>26</sup> and it is generally thought that for SOJT effects to be observable there is an upper limit of ca.
 4 eV in the energy separation of the levels.<sup>17,18</sup>

5 The relative energies of the frontier orbitals for the structures in Figure 1 are reflected in the 6 magnitude of the inversion barrier, yet relatively few calculations<sup>8</sup> that provide energies of the 7 symmetrically relevant interacting frontier orbitals for a series of ER<sub>3</sub> species with varying E or R groups 8 have been published to date.<sup>24</sup> Inversion or linearization processes for pyramidal or bent geometries 9 found for the heavier main group element analogues of alkenes and alkynes are closely related to the ER<sub>3</sub> 10 inversion process (Figure 1b,c),<sup>19-21</sup> and theoretical studies that relate these processes to SOJT effects are 11 also scarce.<sup>25,26</sup>

Here we describe the results of calculations on the trigonal pyramidal hydrides of group 15 atoms with emphasis on the relative energies of frontier orbitals and their implications for inversion barriers. We also describe parallel calculations on the group 14 heavier element ethene and ethyne analogues, which possess trans-pyramidalized (folded) or trans-bent structures in contrast to their planar or linear carbon analogues (Figure 1b,c).<sup>27</sup> We emphasize the symmetries and energies of the frontier orbitals during the inversion or linearization process and show that SOJT interactions occur even when HOMO-LUMO energy separations are as high as 12 eV, much higher than the previous estimate of 4 eV.<sup>28</sup>

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20 Methods

21 All computations were performed using the *Gaussian09* software suite.<sup>29</sup> Calculations were 22 performed with the  $\omega$ B97X-D functional and the Def2-TZVPP basis set.<sup>30</sup> All calculations utilized an

1 ultrafine quadrature grid (implemented using the "int=ultrafine" command). Some calculations were 2 carried out at the coupled cluster singles and doubles with perturbative triples (CCSD(T)) level,<sup>31</sup> with the 3 aug-cc-pVQZ basis set.<sup>32</sup> The results of these calculations, which confirmed, for NH<sub>3</sub> and PH<sub>3</sub>, that the 4  $\omega$ B97X-D/Def2-TZVPP model chemistry satisfactorily predicted orbital energies, are described in the 5 Supporting Information. *CylView*<sup>33</sup> and *Avogadro*<sup>34</sup> were used to generate ball-and-stick and orbital 6 (isovalue of 0.02000) images, respectively. To generate molecular orbital diagrams, a custom python script 7 utilizing *matplotlib* was employed.<sup>35</sup>

For the pnictogens, the calculated pyramidal geometries for all equilibrium structures feature  $C_{3v}$ point group symmetry. In our case, where R=H, all pnictogen transition state structures have  $D_{3h}$ symmetry. Nevertheless, recent calculations by Truhlar and coworkers have shown that when the substituent is a halogen and E is a heavier pnictogen (e.g., P-Bi), the transition state structures can be Tshaped with  $C_{2v}$  symmetry.<sup>9</sup> In addition, open-shell species can also participate in inversion.<sup>9</sup> However, we focus here on the closed shell  $C_{3v} \neq D_{3h} \neq C_{3v}$  process and the related processes in HEEH and H<sub>2</sub>EEH<sub>2</sub>.

14

## 15 Results and Discussion

#### 16 ER<sub>3</sub> Systems

A symmetry-directed approach to relate inversion and trans-bending barriers with the relative energies of the interacting molecular orbitals was used. First, we tested our computational approach by calculating molecular geometries, energies of valence orbitals, and inversion barriers for simple EH<sub>3</sub> (E = N, P, As and Sb) molecules.<sup>36</sup> The trigonal pyramidal minimum geometries have C<sub>3V</sub> symmetry and the planar transition state structures have D<sub>3h</sub> symmetry. In the D<sub>3h</sub> structures, the HOMO-2, HOMO-1, HOMO, LUMO, and LUMO+1 orbitals have the symmetries a<sub>1</sub>', e', a<sub>2</sub>'', a<sub>1</sub>', and e', respectively (Table 1 and Figure

1 2). Shown in Figure 2 is a representation of each molecular orbital for  $EH_3$  corresponding to the above-2 named MO symmetries. This sequence, which is in order of increasing energy, is observed in the four  $EH_3$ 3 molecules of the group. The energy spread of the highest and lowest energy levels reflects the E-H bond 4 strengths, and ranges from 30.62 eV in NH<sub>3</sub> to 20.96 eV in SbH<sub>3</sub>. The computed HOMO-LUMO energy separation for planar,  $D_{3h}$  NH<sub>3</sub> (11.85 eV) is higher by >2 eV than those of its heavier congeners PH<sub>3</sub> (9.73 5 6 eV), AsH<sub>3</sub> (8.83 eV) and SbH<sub>3</sub> (7.51 eV). Thus, when the undistorted, planar  $D_{3h}$  NH<sub>3</sub> molecule undergoes a 7 pyramidalization vibration of a<sub>2</sub>" symmetry, the triple direct product of the HOMO x vibration x LUMO 8 (i.e.,  $a_2^{"}x a_2^{"}x a_1' = a_1'$ ) is the totally symmetric irreducible representation of the point group  $D_{3h}$ . In other words, the mixing becomes symmetry-allowed upon the vibrational distortion from  $D_{3h}$  to  $C_{3V}$  and, in this 9 10 case, it produces a stabilization of the HOMO (lone pair) that correlates to the inversion barrier. Due to 11 the larger HOMO-LUMO energy difference in  $NH_3$  in comparison to those of its heavier congeners, the 12 interaction between these orbitals is relatively small and produces a low inversion barrier near 0.18 eV 13 (4.2 kcal/mol). In other words, since the energy gap in planarized ammonia compared to its heavier 14 congeners is large, there is less mixing and a smaller interaction between the two levels. In contrast, for 15 the heavier congeners with smaller HOMO-LUMO energy differences, the HOMO-LUMO interaction is 16 greater and affords much higher inversion barriers in the range 1.43 to 1.98 eV (33.0 to 45.7 kcal/mol). 17 The calculated HOMO-LUMO energy differences mirror, but are slightly larger than, those reported earlier by Schwertfeger, Laakkonen and Pyykkö; in addition, we calculate substantially larger energy differences 18 19 (ca. 1.2-2.1 eV) between the unoccupied LUMO and LUMO+1 levels (Table S1) than those (ca. 0.2 eV) calculated earlier.<sup>8</sup> Figure 3 shows that computed inversion barriers and orbital energy gaps correlate (R<sup>2</sup> 20 21 = 0.95).

22

- **Table 1.** The predicted energy differences between the interacting a<sub>2</sub>`` HOMO and the a<sub>1</sub>` LUMO in the
- $D_{3h}$  planar, geometry EH<sub>3</sub> systems and predicted barriers for inversion.

Molecule	∆E (eV)	Inversion Barrier (kcal/mol)
NH₃	11.85	4.2
PH <sub>3</sub>	9.73	33.0
AsH₃	8.83	39.0
SbH₃	7.51	45.7



Figure 2. Representations of the computed molecular orbitals of ammonia in D<sub>3h</sub> symmetry. Note that
 different molecular orientations are used for different orbitals to facilitate comprehension of orbital
 shapes.









In pyramidal ammonia with C<sub>3v</sub> symmetry, the valence orbitals have the symmetries a<sub>1</sub>, e, a<sub>1</sub>, a<sub>1</sub>,
and e, in order of increasing energy. This order parallels the sequence given above for NH<sub>3</sub> in D<sub>3h</sub> symmetry
(Figures 2 and 4). The energy range of the highest and lowest molecular orbital energy levels is 31.02 eV

1 which is just 0.40 eV greater than that of the  $D_{3h}$  structure. In contrast, for the heavier PH<sub>3</sub>, AsH<sub>3</sub>, and SbH<sub>3</sub> 2 analogues in C<sub>3v</sub> symmetry, the ordering of the two highest energy levels changes so that the sequence 3 becomes  $a_1$ , e,  $a_1$ , e,  $a_1$ . However, the energy differences between the highest e and  $a_1$  levels are relatively low: 0.34 eV (PH<sub>3</sub>), 0.22 eV (AsH<sub>3</sub>) and 1.02 eV (SbH<sub>3</sub>), compared to 2.3 eV in NH<sub>3</sub>. Nonetheless, we regard 4 5 the reversal of the order of the two highest energy orbitals as a consequence of the SOJT mixing of the HOMO and LUMO ( $a_2''$  and  $a_1'$  in  $D_{3h}$  symmetry), which produces two  $a_1$  levels in  $C_{3v}$  symmetry one of 6 7 which (the HOMO) is stabilized. These changes in energy are sufficiently small in NH<sub>3</sub> that the ordering of 8 the orbitals remains unchanged. In contrast, in the heavier PH<sub>3</sub>, AsH<sub>3</sub> and SbH<sub>3</sub>, the SOJT interactions are 9 significantly larger and the energy of the LUMO is increased sufficiently to become the LUMO+2 level for 10 the C<sub>3v</sub> geometry. Other considerations such as changing orbital overlap and bond strengths can affect the 11 ordering, but the increased SOJT interactions in the heavier element derivatives offer a simple and 12 straightforward model for the changed ordering.



4 Figure 4. Molecular orbital diagram for  $NH_3$  in (a)  $C_{3v}$  symmetry and (b)  $D_{3h}$  symmetry. The ordering of the upper two energy levels is reversed for E = P, As and Sb. 5

## 1 REER and R<sub>2</sub>EER<sub>2</sub> Molecules

2 The bonding in the undistorted linear and planar HEEH alkyne and H<sub>2</sub>EEH<sub>2</sub> alkene and their heavier congeners (E = C, Si, Ge or Sn), respectively was also investigated.<sup>37</sup> For the silicon, germanium and tin 3 4 alkyne analogues, the lowest energy (non-hydrogen bridged) structure was found to have a planar, trans-5 bent geometry with C<sub>2h</sub> symmetry (Figure 1b; Table 2). In the case of carbon, the lowest energy structure 6 was calculated to be linear  $(D_{\infty h})$ , as expected. For the alkene analogues, carbon has the expected planar 7 geometry with D<sub>2h</sub> symmetry, whereas the trans-bent geometries with C<sub>2h</sub> symmetry (Figure 1c) were 8 predicted for the heavier element structures. Shown in Figure 5 is a representation of each molecular 9 orbital for representative heavier element alkyne and alkene analogues corresponding to their respective 10 MOs.



1

Figure 5. Orbital illustrations. Left to right: HGeGeH in  $D_{\infty h}$  symmetry, HGeGeH in  $C_{2h}$  symmetry, H<sub>2</sub>GeGeH<sub>2</sub> in  $D_{2h}$  symmetry, and H<sub>2</sub>GeGeH<sub>2</sub> in  $C_{2h}$  symmetry, with illustrations of the calculated orbitals. Note that different molecular orientations are used for different orbitals to facilitate comprehension of orbital shapes.

- 6
- 7

- 1 **Table 2.** The predicted energy differences between the interacting  $\pi_u$  HOMO and the  $\sigma_u$  LUMO, which lead
- 2 to a SOJT interaction upon undergoing a trans-bending vibration of C<sub>2h</sub> symmetry, and predicted barriers
- 3 for inversion.

Molecule	ΔΕ (eV)	Linearization Barrier (kcal/mol)
НССН	14.13	0.0
HSiSiH	7.71	20.2
HGeGeH	7.16	29.5
HSnSnH	5.82	48.9

5 In the case of ethyne, HCCH, we start with the  $D_{\scriptscriptstyle \infty h}$  point group (linear geometry) where we defined the HOMO-1, HOMO, LUMO and LUMO+1 as  $\sigma_{g_1}\pi_{u}$  (doubly degenerate),  $\pi_{g}$  (doubly degenerate) 6 and  $\sigma_u$  levels. However, in the heavier Si, Ge and Sn alkyne analogues, <sup>37</sup> in D<sub>wh</sub> linear geometry, we find 7 8 the sequence of orbital energies has changed to  $\sigma_{g}$ ,  $\pi_{u}$  (doubly degenerate),  $\sigma_{u}$  and  $\pi_{g}$  (doubly degenerate) 9 (Figure 6). This is due to the large increase in electronic repulsion generated by the heavier cores, especially for the valence s electrons.<sup>38</sup> The  $\pi_u$  and  $\pi_g$  orbitals are doubly degenerate in the linear 10 geometry, whereas, in the planar trans-bent geometry of  $C_{2h}$  symmetry, the degeneracy of the  $\pi$ -orbitals 11 12 is lifted and one of the original  $\pi$ -orbitals (the in-plane  $\pi$ -orbital) becomes an orbital of b<sub>u</sub> symmetry (the 13 out of plane  $\pi$ -orbital has a<sub>u</sub> symmetry) in the trans-bent species with non-bonding character as a result of its interaction with a  $\sigma^*$  orbital. In the C<sub>2h</sub> structure the energy levels corresponding to HOMO-2, 14 15 HOMO-1, HOMO, LUMO, LUMO+1, and LUMO+2 levels have the symmetries  $a_g$ ,  $b_u$ ,  $a_u$ ,  $a_g$ ,  $b_g$ ,  $b_u$ 16 respectively. This order holds for all studied alkyne analogues in C<sub>2h</sub> trans-bent geometry.





Figure 6. Correlation of the interacting valence orbitals of HGeGeH in linear  $D_{\infty h}$  (left) and trans-bent  $C_{2h}$ (right) geometries.

5 For linear HCCH in  $D_{\infty h}$  symmetry, LUMO and LUMO+1 are the antibonding  $\pi_g^*$  and  $\sigma_u^*$  orbitals. 6 In the corresponding heavier element Si, Ge and Sn species in  $D_{\infty h}$  symmetry, this ordering is changed (see 7 above) and the LUMO becomes the  $\sigma_u^*$  orbital and the LUMO+1 becomes the  $\pi_g^*$  orbital. Upon trans-8 bending, the  $\sigma_u^*$  orbital becomes  $b_u$  in  $C_{2h}$  symmetry and one of the original  $\pi$ -bonding orbitals also 9 becomes  $b_u$  in symmetry. Thus, as the energy separations between the orbitals decrease in the heavier 10 element species, the interaction between the two b<sub>u</sub> orbitals increases and non-bonded electron density is generated at the group 14 atom, which favors the trans-bent geometry due to electron/electron 11 12 repulsion.

We note that for alkene analogues, the ordering of the orbitals upon changing from E=C to heavier elements is far less important than in the case of alkyne analogues (Figure 7). While we see changes in relative MO energies as we move through the alkene analogues, the changes mainly involve orbitals that are lower in energy than the HOMO, in contrast to the alkyne analogues where all energy differences mentioned involve orbitals that are very close to those involved in the interactions with which we are concerned here. While the HOMO is the interacting  $b_{3u}$  orbital for all the alkene analogues, we can see that the interacting unoccupied MO  $b_{1u}$ , is not the LUMO or LUMO+1 until we get to E=Ge and Sn. For E=Si, it is, in terms of energy, at the LUMO+2 level and for E=C, it is all the way at the LUMO+3 level. This also is reflected in the fact that while there is a trans-pyramidalization in the E=Si molecule, it is much smaller than the corresponding bends for E=Ge and Sn (Table 3).







10

- 1 **Table 3.** The interacting molecular orbitals (b<sub>2u</sub> HOMOs and b<sub>3u</sub> LUMOs) and their energies in the alkene
- 2 analogues. Inversion barriers are in kcal mol<sup>-1</sup>.

Molecule	ΔE (eV)	Planarization Barrier (kcal/mol)
$H_2CCH_2$	14.54	0.0
H <sub>2</sub> SiSiH <sub>2</sub>	9.78	0.9
H₂GeGeH₂	9.05	2.7
H₂SnSnH₂	7.44	10.1

4

5 Again, correlations between orbital energy gap and inversion barriers are observed for both

6 alkene and alkyne analogues (Figures 8 and 9).



Figure 8. Plot of the planarization barriers against the energy separation of the two interacting (occupied
 and unoccupied) MOs for ethylene and its heavier element congeners. Using a best fit line, an R<sup>2</sup> value of
 0.5529 is calculated. When carbon is excluded, an R<sup>2</sup> value of 0.9936 is calculated.



Figure 9. Plot of the linearization barriers in HEEH (E = C, Si, Ge or Sn) against the two interacting occupied
 and unoccupied MOs. Using a best-fit line, an R<sup>2</sup> value of 0.8399 is calculated. But when carbon is omitted,
 an R<sup>2</sup> value of 0.9989 is computed.

5

6 Conclusions

Here we examined the relationship between SOJT interactions and planarization/linearization
barriers. For the EH<sub>3</sub>, HEEH, and H<sub>2</sub>EEH<sub>2</sub> species that we examined, SOJT interactions are important even
when the energy difference between HOMO and the relevant interacting unoccupied orbital is as large as

ca. 12 eV, a much larger value than expected. For carbon-containing compounds HCCH, and H<sub>2</sub>CCH<sub>2</sub>, the
predicted occupied/unoccupied orbital energy differences are >14 eV and no SOJT effects are apparent.
In contrast, for the heavier element silicon, germanium, and tin analogues the energy differences tend to
range from 5-10 eV, and these energies are sufficiently low that orbital interactions lead to the
experimentally observed geometrical distortions, which, in some cases, lead to dissociation of the EE
bond.<sup>39</sup> The unexpectedly high magnitude of the energy separations found here implies that SOJT effects
of this type could be much more common than is currently realized.

8

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Calculations suggest an upper limit of approximately 12 eV energy separation of the interacting levels for second order Jahn-Teller effects to be significant, which is considerably larger than previously thought.