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1 **Response surface optimization and Artificial neural network modeling of**  
2 **biodiesel production from crude Mahua (*Madhuca indica*) oil under**  
3 **supercritical ethanol condition using CO<sub>2</sub> as co-solvent**

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## 25 Abstract

26 The present study describes the renewable, environment-friendly approach for the  
27 production of biodiesel from low cost, high acid value mahua oil under supercritical ethanol  
28 condition using carbon dioxide (CO<sub>2</sub>) as a co-solvent. CO<sub>2</sub> was employed to decrease the  
29 supercritical temperature and pressure of ethanol. Response surface method (RSM) is the most  
30 preferred method for optimization of biodiesel so far. In last decade, artificial neural network  
31 (ANN) has come up as one of the most efficient method for empirical modeling and  
32 optimization, especially for non-linear systems. This paper presents the comparative studies  
33 between RSM and ANN for their predictive, generalization capabilities, parametric effects and  
34 sensitivity analysis. Experimental data were evaluated by applying RSM integrating with  
35 desirability function approach. The importance of each independent variable on the response was  
36 investigated by using sensitivity analysis. The optimum conditions were found to be temperature  
37 (304 °C), ethanol to oil molar ratio (29:1), reaction time (36 min), and initial CO<sub>2</sub> pressure (40  
38 bar). For these conditions, experimental fatty acid ethyl ester (FAEE) content of 97.42% was  
39 obtained, which was in reasonable agreement with predicted one. The sensitivity analysis  
40 confirmed that temperature was the main factors affecting the FAEE content with the relative  
41 importance of 39.24%. The lower values of correlation coefficient ( $R^2 = 0.868$ ), root mean square  
42 error (RMSE = 4.185), standard error of prediction (SEP = 5.81) and absolute average deviation  
43 (AAD = 5.239) for ANN compared to those  $R^2$  (0.658), RMSE (7.691), SEP (10.67) and AAD  
44 (8.574) for RSM proved better prediction capability of ANN in predicting the FAEE content.

45 **Keywords:** Mahua oil, Supercritical ethanol, Biodiesel, Response surface method, Artificial  
46 neural network

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## 50 1. Introduction

51 To solve the problems of global warming, fluctuating oil prices, CO<sub>2</sub> emissions, and  
52 possibly create new job opportunities, extensive research and development programs are in  
53 search of renewable energy sources are actively pursued. Biofuels have become an important  
54 renewable energy source, particularly for the transportation applications. As a renewable energy  
55 source, biomass is one of the superior sources of energy and industrial-scale biomass based  
56 energy production could improve the socioeconomics of many underdeveloped societies and  
57 countries along with the raising awareness of environmental protection.<sup>1</sup> Biodiesel composed of  
58 mixture of alkyl esters that mostly produced by esterification and transesterification of various  
59 lipid feedstocks such as vegetable oils and animal fats with methanol or ethanol as the reacting  
60 alcohol.<sup>2</sup> One of the major problems for the use of biodiesel is the poor availability the  
61 economic raw material. Therefore, different alternative feedstocks for biodiesel production need  
62 to be explored. In India, with abundance of forest resources, there are number of non-edible tree  
63 borne oilseeds with an estimated annual production of more than 20 million tones, which have  
64 great potential for making biodiesel to supplement other conventional sources. Among the  
65 available feedstocks, mahua (*Madhuca indica*) is an economically important oilseed tree of the  
66 Sapotaceae family, which grows in several parts of India. Mahua (*Madhuca indica*) is one such  
67 non-edible tree based seed oil, which has an estimated annual production potential of 181,000 Mt  
68 in India.<sup>3</sup> Fatty acid composition of mahua oil has earlier been reported in literature<sup>4,5</sup> and is  
69 given in Table S1. The major constituents are palmitic, stearic, oleic, linoleic, arachidic which  
70 contribute about 96% of total fatty acids present in mahua oil.

71 The studies about biodiesel production from mahua oil via alkali catalysed  
72 transesterification have been reported in the literatures.<sup>5,6</sup> Kumari et al.<sup>4</sup> employed lipase  
73 catalyst (*Pseudomonas cepacia immobilized on CLEAs and PCMCs*) for synthesis of biodiesel  
74 from mahua oil with high free fatty acid content (20%) and reported 92 % conversion in 2.5 h  
75 with (cross-linked enzyme aggregates) CLEAs, 99% conversion in 2.5 h with (protein-coated  
76 microcrystals) PCMCs. However, the catalytic biodiesel production has several drawbacks such  
77 as time-consuming due to low reaction rate and corrosion risk for acid-catalyzed, high sensitivity  
78 to free fatty acid (FFA) and water contents in the oil feedstock and difficulty in the separation of  
79 biodiesel and catalyst from soap for alkaline catalyst, high cost and deactivation risk of the active  
80 sites for both enzyme and heterogeneous catalysts. Moreover, biodiesel production employing

81 liquid catalysts requires complex and costly treatments for the acidic or alkaline wastewater.<sup>2</sup>  
82 Recently, supercritical transesterification (SC-TE) has been highlighted as an emerging  
83 technology for the biodiesel synthesis from various feedstocks. This method is able to  
84 completely convert fatty acids in feedstocks to alkyl ester with high purity without involving any  
85 catalysts and offers simple product separation from the mixture.<sup>7</sup> Other advantages of this route  
86 include fast reaction kinetics, ease separation of products, and tolerance to FFA and water  
87 contents in the oil feedstock because esterification of free fatty acid (FFA) and transesterification  
88 of triglycerides proceed simultaneously.<sup>2</sup> Apart from the advantages, it has some drawbacks like  
89 high equipment cost and high energy consumption due to high temperature and pressure  
90 conditions. This limits the supercritical transesterification process to be viable for large scale  
91 industrial applications. However, the introduction of co-solvents like hexane, carbon dioxide,  
92 propane into the reaction mixture decreases the severity of the reaction parameters and can make  
93 this process practical. The addition of co-solvents can decrease the critical point of alcohol and  
94 allow the supercritical reactions to be carried out at milder temperatures.<sup>8</sup> Different alcohols  
95 such as methanol, ethanol, propanol, butanol and amyl alcohol can be used for the  
96 transesterification. Ethanol is preferred in present study because it is renewable, non-toxic, eco-  
97 friendly and can be produced from agricultural resources. Also, fatty acid ethyl ester are better  
98 than fatty acid methyl esters in term of fuel properties, including cetane number, oxidation  
99 stability and cold flow properties.<sup>1</sup>

100 In the last decades the different mathematical tools, useful for modeling and optimization  
101 of biodiesel synthesis, have been progressively developed. For instance, response surface  
102 methodology (RSM) and artificial neural networks (ANN) are powerful mathematical methods  
103 suitable for modeling and simulation of various processes in real applications. Both  
104 methodologies do not need the explicit expressions of the physical meaning of the system or  
105 process under investigation. Therefore, RSM as well as ANN belong to modeling techniques  
106 dealing with the development of non-parametric simulative models. Such models have a wide  
107 applicability in various disciplines of science. In fact, these models approximate the functional  
108 relationships between input variables and the output (response) of the process using experimental  
109 data. Afterwards, the models are used to estimate the optimal settings of input variables to  
110 maximize or minimize the response.<sup>9</sup>

111 Biodiesel production has been predicted and optimized using several topologies of ANN  
112 in a few studies.<sup>10-14</sup> Yuste and Dorado,<sup>12</sup> accurately simulated base-catalyzed waste olive oil  
113 transesterification for biodiesel production using an ANN model introducing a tool for making a  
114 decision in the experimental process. They showed that the ANN is a capable alternative tool for  
115 experimentally testing of the process optimization. Ramadhas et al.<sup>15</sup> developed different ANN  
116 models based on multi-layer feed forward, radial base, generalized regression and recurrent  
117 network for predicting the cetane number (CN) of biodiesel fuel. The predicted CN of biodiesel  
118 is comparable to that of actual CN of the biodiesel. Rodríguez et al.<sup>16</sup> applied multiple linear  
119 regression and artificial neural networks for obtaining a model for predicting cetane number and  
120 validated the model using data from literature. The models based on multiple linear regressions  
121 cannot predict cetane number with similar accuracy as the obtained for the selected neural  
122 network. The biodiesel production from rapeseed soapstock and methanol in the presence of the  
123 *candida-rugosa* lipase immobilized on chitosan was analyzed by Ying et al.<sup>17</sup> using an ANN,  
124 showing desirable correspondence between predicted and experimental values of the FAME  
125 yield. Basri et al.<sup>18</sup> has reported the comparison of ANN and RSM in lipase-catalyzed synthesis  
126 of palm-based wax ester, which also suggest the superiority of ANN over RSM for both data  
127 fitting and estimation capabilities.

128 RSM and ANN have been widely applied for the modeling and optimization of biodiesel  
129 synthesis from various feedstocks using different methods such as classical base catalyzed  
130 transesterification,<sup>13,14,19,20</sup> heterogeneous base catalyzed transesterification,<sup>21</sup> conventional two-  
131 step acid catalyzed esterification and base catalyzed transesterification,<sup>22,23</sup> ultrasound assisted  
132 base catalyzed transesterification,<sup>24</sup> ultrasound assisted two-step acid catalyzed esterification  
133 and base catalyzed transesterification,<sup>25,26</sup> ultrasound assisted heterogeneous base catalyzed,<sup>27</sup>  
134 lipase catalyzed,<sup>18</sup> Infrared irradiation assisted esterification,<sup>28</sup>. Several papers took advantages  
135 of the genetic algorithm coupled with ANN to generate optimum operating variables for the  
136 studied process.<sup>11,17, 22</sup> Most of the reported literatures on comparison of RSM and ANN for  
137 biodiesel synthesis using different feedstocks were based on conventional, ultrasound, and  
138 infrared irradiation assisted techniques. To the best of our knowledge, there are no studies  
139 dealing with comparison of RSM and ANN modeling methods for the optimization of non-  
140 catalyzed biodiesel synthesis using non-edible oil.

141 Hence, the objective of the present work is to demonstrate the conversion of high free  
142 acid content crude mahua oil into biodiesel using single step supercritical ethanol process in  
143 presence of carbon dioxide (CO<sub>2</sub>). In order to assess and understand the effect of each variable  
144 on fatty acid ethyl ester (FAEE) content, statistical analysis was performed using the RSM.  
145 Moreover, the desirability function approach for optimization of FAEE content was employed in  
146 order to develop an efficient method for achieving maximum biodiesel production. In addition,  
147 the present work investigates the predictive and generalization capability of artificial neural  
148 network (ANN) to estimate the FAEE content. Furthermore, the efficiencies of both the models  
149 were statistically compared by the coefficient of determination (R<sup>2</sup>), root mean square error  
150 (RMSE), standard error of prediction (SEP), and absolute average deviation (AAD) based on the  
151 validation data set.

## 152 **2. Experimental Section**

### 153 *2.1. Materials*

154 Mahua oil was obtained from local market. Anhydrous ethanol (99.8%) was purchased  
155 from Mars scientific Inc. (Australia). Carbon dioxide (99%) was supplied by Bharti Gases  
156 (Nagpur, India). All chemicals including n-hexane (95%) and Sulfuric acid (99%) of analytical  
157 reagent (AR) grade was purchased from Merck Limited, Mumbai, India. Hexanoic acid (S.D.  
158 Fine Chem. Ltd. India) was used as an internal standard for the gas chromatographic analyses.  
159 The standards required for quantification of esters were procured from Sigma-Aldrich Co. Ltd.,  
160 Mumbai, India and were chromatographically pure. All the liquid chemicals were filtered  
161 through 2 µm pore size filter and the gases were passed through silica bed prior to use.

162 The acid value of oil was determined by acid base titration technique,<sup>29</sup> using the  
163 standard solution of KOH. Mahua oil had an initial acid value of 36 mg of KOH/g of oil. The  
164 critical temperature and pressure of ethanol is 516.2 K and 6.4 MPa. The properties of the  
165 ethanol favor the homogeneous mixing with oil at supercritical condition because they act as acid  
166 catalysts in the supercritical biodiesel synthesis.<sup>30</sup>

### 167 *2.2. Methods*

168 The supercritical ethanol process was carried out in a 50mL bench top AMAR 2630 high  
169 pressure reactor (SS 316) equipped with a E-3032 controller, magnetic stirrer, pressure gauge,  
170 external electric heater (Amar Equipments Pvt. Ltd., Mumbai, India). The instrument can be  
171 operated maximum up to 500 °C and 400 bars. The complete experimental set-up has been



172 shown in Fig. S1. A known amount of mahua oil and ethanol was charged to the reactor to give  
173 the different amount of ethanol to oil molar ratio ranging from 15:1 to 35:1. The reactor was then  
174 purged with known amount of CO<sub>2</sub> gas (10 - 50 bars) as co-solvent. Then, the mixture of mahua  
175 oil-ethanol-CO<sub>2</sub> was heated at the different temperatures (250 °C - 350 °C) for which the power  
176 was adjusted to give a heating rate 50 °C/ min and is defined as the zero reaction time  
177 (temperature and pressure reached the set value). The temperature was controlled within ± 2 °C  
178 and the pressure was monitored by pressure gauge in order to maintained the isothermal and  
179 isobar reaction conditions. After the set value reached, the mixture was stirred with magnetic  
180 stirrer for desired time (10 - 50 min). To stop the reaction after the predetermined reaction time,  
181 the reactor was quenched by immersing the reactor in a cold water bath. The product was  
182 collected, and hexane was used to elute any trace of product left in the reactor. The alcohol and  
183 hexane present with product was evaporated at 90 °C, leaving behind the mixture of unreacted  
184 oil, ester, and glycerol, and then taken for analysis by gas chromatography.

### 185 2.3. GC Analysis

186 The reaction samples were analyzed by gas chromatography (model GC-2010 plus,  
187 Shimadzu Corp., Tokyo, Japan) using a capillary column, MXT-Biodiesel TG (Restek, USA; 15  
188 m × 0.32 mm × 1µm film thickness of diphenyl dimethyl polysiloxane) and a flame ionization  
189 detector. Nitrogen was used as a carrier gas at a flow rate of 2.75 mL min<sup>-1</sup>. Hexanoic acid was  
190 used as an internal standard. Column oven temperature was initially maintained at 100 °C for 3  
191 minutes, then increases to 250 °C at the rate of 30 °C min<sup>-1</sup> and held here for 3 minutes. The  
192 injector and detector temperature were maintained at 270 °C. A sample volume of 1 µL mahua  
193 oil biodiesel (MOB) in hexane was injected using a split mode, with the split ratio of 1:50. The  
194 GC chromatograph of MOB is shown in Fig. S2.

### 195 2.4. Experimental Design

#### 196 2.4.1. Response surface method

197 A five-level four-factor central composite experimental design (CCD) was used in this  
198 study. Reaction temperature (A), ethanol to oil molar ratio (B), time (C), and initial CO<sub>2</sub> pressure  
199 (D) were the input variables, the factor levels were coded as -2 to +2 as shown in Table S2. In  
200 this work, the input variables (factors) and their levels were selected, based on preliminary  
201 experiments carried out in the laboratory. According to the CCD, experiments were performed in  
202 order to find out the optimum combination and study the effect of process parameters on FAEE



203 content using the supercritical ethanol (SC-ET) process, and the results are given in Table 1.  
 204 Experimental data from the CCD was analysed using regression (Design Expert™ 8.0) and  
 205 fitted to a second-order polynomial model in order to identify all possible interactions of selected  
 206 factors with response function as follows:

$$207 \quad Y = b_0 + \sum_{i=1}^k b_i x_i + \sum_{i=1}^k b_{ii} x_i^2 + \sum_{i=1}^k \sum_{i=j}^k b_{ij} x_i x_j + e \quad (1)$$

208 where,  $Y$  is response (FAEE content %),  $b_0$ ,  $b_i$ ,  $b_{ii}$  and  $b_{ij}$  are the regression coefficients obtained  
 209 for constant, linear, quadratic and interaction terms, respectively.  $x_i$  and  $x_j$  are independent  
 210 variables, whereas  $i$  and  $j$  are the linear and quadratic coefficients, respectively.  $b$  is the  
 211 regression coefficient,  $k$  is the number of factors studied and optimized in the experiment and  $e$  is  
 212 random error. Furthermore, RSM integrated with desirability function approach was used for  
 213 simultaneous optimization of FAEE content.

#### 214 2.4.2. Desirability function approach

215 The individual desirability ( $d$ ) for response was calculated by one side transformation method  
 216 (Eq. 2), followed by calculation of overall desirability ( $D$ ) using univariate technique (Eq. 3) as  
 217 follows:<sup>27,31,32</sup>

$$218 \quad d_i = \begin{cases} 0 & Y_i \leq Y_{i-\min} \\ \left[ \frac{Y_i - Y_{i-\min}}{Y_{i-\max} - Y_{i-\min}} \right]^r & Y_{i-\min} < Y_i < Y_{i-\max} \\ 0 & Y_i \geq Y_{i-\max} \end{cases} \quad (2)$$

$$219 \quad D = (d_1^{w_1} d_2^{w_2} d_3^{w_3} d_4^{w_4} d_5^{w_5})^{1/\sum w_i} \quad (3)$$

220 where  $d_i$  is individual response desirability,  $Y_i$  is the response values,  $Y_{i-\min}$  is the minimum  
 221 acceptable value for response  $i$ ,  $Y_{i-\max}$  is the maximum acceptable value for response  $i$ ,  $r$  is a  
 222 weight used to determine the scale of desirability,  $D$  is the overall desirability,  $d_i$  is individual  
 223 response desirability, and  $w_i$  is a weighted composite desirability.

#### 224 2.4.3. Artificial neural network

225 The design of experiments (DoE), which is used for training the network and respective  
 226 experimental response (FAEE content) are given in Table 1. In this work, the network inputs and  
 227 target have been normalized before training. To this end, both input variables and target  
 228 (experimental response) have been normalized ranging from -1 (minimum level) up to +1  
 229 (maximum level). The normalization in the limits (-1,+1) was carried out since the tangent  
 230 sigmoid function (tansig) employed for ANN modeling ranges from -1 and +1. For  
 231 normalization target data the following equation was used:<sup>33</sup>

$$232 \quad \text{Normalized} = \left[ \frac{2*(X_{Ac}-X_{min})}{(X_{max}-X_{min})} \right] - 1 \quad (4)$$

233 where  $X_{min}$ ,  $X_{max}$ , and  $X_{Ac}$  are the minimum, maximum and actual data, respectively. The  
 234 normalization of inputs and target was performed to avoid overflows that may appear due to very  
 235 large or very small weights.<sup>9</sup> In this study, a three-layered feed-forward neural network with  
 236 tangent sigmoid transfer function (tansig) at hidden layer and a linear transfer function (purelin)  
 237 at output layer was used. The sigmoid transfer function was given by (Eq. 5).

$$238 \quad f(x) = 2 \times \frac{1}{1+e^{-2x}} - 1 \quad (5)$$

239 and the linear activation function (Eq. 6) is used as the output layer activation function.<sup>34</sup>

$$240 \quad f(x) = x \quad (6)$$

241 The back propagation algorithm was used for network training. Sixty percent of the data  
 242 was taken for the training set, twenty percent for the validation set and rest of the data for the test  
 243 set. Neural Network Toolbox V4.0 of MATLAB mathematical software was used for FAEE  
 244 prediction. The performance of ANN was statistically measured by root mean square error  
 245 (RMSE), standard error of prediction (SEP), absolute average deviation (AAD), and correlation  
 246 coefficients ( $R^2$ ) was carried out between experimental and predicted data. The formulas used for  
 247 error analyses were calculated by Eq. (7) to Eq. (10) respectively.<sup>27,35,36</sup>

$$248 \quad R^2 = 1 - \frac{\sum_{i=1}^n (Y_{i,p} - Y_{i,e})^2}{\sum_{i=1}^n (Y_{i,p} - Y_e)^2} \quad (7)$$

$$249 \quad RMSE = \sqrt{\frac{\sum_{i=1}^n (Y_{i,e} - Y_{i,p})^2}{n}} \quad (8)$$

$$250 \quad SEP = \frac{RMSE}{Y_e} \times 100 \quad (9)$$

$$251 \quad AAD = \frac{100}{n} \sum_{i=1}^n \frac{|(Y_{i,p} - Y_{i,e})|}{|Y_{i,e}|} \quad (10)$$

252 where,  $Y_{i,e}$  is the experimental data,  $Y_{i,p}$  is the corresponding data predicted,  $Y_e$  is the mean value  
 253 of experimental data and n is the number of the experimental data.

#### 254 2.4.4. Sensitivity Analysis

255 ANN being a *black box model*, it does not give insights of the system directly. But there  
 256 are numerous methods available which gives the sensitivity analysis of the system using inherent  
 257 nature of ANN. In order to evaluate the relative importance of each Input variable on the  
 258 response, Garson<sup>37</sup> proposed an equation based on the partitioning of the connection of the  
 259 weights as:

$$I_j = \frac{\sum_{m=1}^{m=N_h} (|w_{jm}^{ih}| / \sum_{k=1}^{N_i} |w_{km}^{ih}|) \times |w_{mn}^{ho}|}{\sum_{k=1}^{k=N_i} \{ \sum_{m=1}^{m=N_h} (|w_{km}^{ih}| / \sum_{k=1}^{N_i} |w_{km}^{ih}|) \times |w_{mn}^{ho}| \}} \quad (11)$$

where,  $I_j$  is the relative significance of the  $j^{\text{th}}$  input variable on the output variable,  $N_i$  and  $N_h$  are the number of input and hidden neurons, respectively.  $W$  is connection weight, the superscripts  $i$ ,  $h$  and  $o$  represents the input, hidden, and output layers, respectively. while the subscripts  $k$ ,  $m$  and  $n$  refer to input, hidden, and output neurons, respectively.<sup>37</sup>

### 3. Results & Discussion

#### 3.1. RSM modeling and Desirability function approach for Optimization

The second-order polynomial equation was fitted with the experimental results obtained on the basis of CCD experimental design. The final equation obtained in terms of coded factors as follows:

$$\text{FAEE content (\%)} = 87.82 + 3.02A + 4.73B + 8.26C + 2.05D + 2AB - 1.46AC - 0.038AD + 0.085 BC - 1.09BD - 0.036CD - 12.58 A^2 - 4.06B^2 - 6.89C^2 - 0.64D^2 \quad (12)$$

The adequacy and fitness of the model was tested by analysis of variance (ANOVA), which is shown in Table 2. The regression analysis indicates that all the four parameters had significant influence on the fatty acid ethyl ester (FAEE) content, which is confirmed by the  $P$ -values. The  $P$ -value of the lack of fit analysis is 0.0825, which is more than the 0.05 (confidence level is 95%). The regression model provides accurate description of the experimental data indicating successful correlation among the four transesterification process parameters that affect the FAEE content. The value of  $R^2$  was calculated to be 0.9709, which indicated good agreement of model value with experiment. The model was then further process to generate response surface plots using Matlab Version 8.3 (R2014a).

#### 3.2. Effect of process parameters

Three-dimensional response surface plots are shown in Figs. 1(a-c) revealing the predicted effects of factors on the response. Fig. 1a shows the influence of reaction temperatures and ethanol/oil molar ratio on FAEE content for fixed levels of reaction time of 30 min and initial  $\text{CO}_2$  pressure of 30 bar. Fig. 1a shows that FAEE content increased with the increase in temperature from 275 to 305 °C, afterwards the trend is reversed. This may be due to the partial thermal degradation of mono- and polyunsaturated fatty acids present in mahua oil.<sup>38</sup> Imahara et al.<sup>39</sup> reported that unsaturated fatty acids tend to decompose at high temperature and pressure conditions due to the isomerization of double bond functional group from cis-type carbon

290 bonding into trans type carbon bonding which are naturally unstable fatty acids. Similar  
291 observation has been reported by other researchers who investigated the production of biodiesel  
292 from wet algal biomass using methanol<sup>40</sup> and ethanol conditions.<sup>1</sup>

293 Fig. 1a also depicts the effect of the ethanol/oil molar ratio on FAEE content.  
294 Stoichiometrically, ethanol to oil molar ratio of 3:1 is required to form three moles of fatty acid  
295 ethyl esters (biodiesel) and one mole of glycerol. In supercritical transesterification process, the  
296 molar ratio of ethanol to oil used is significantly higher than the stoichiometric amount. This can  
297 be explained on the basis that a large excess molar ratio of ethanol to oil is required to bring the  
298 reaction system to homogeneous supercritical state. Moreover, a large excess molar ratio of  
299 ethanol to oil was purposely used to drive the chemical equilibrium to the right-hand side based  
300 on Le Chatelier's principle and ensures high conversion of triglycerides within short time, and  
301 also high amount of ethanol act as a solvent, acid catalyst and reactant for oil to ester conversion.  
302 The conversion of triglycerides into fatty acid ethyl esters takes place sequentially as follows: (i)  
303 the reaction between ethoxide anion and the carbonyl carbon of triglyceride to form ethyl ester  
304 and diglyceride; (ii) the reaction between ethoxide anion and the carbonyl carbon of diglyceride  
305 to form ethyl ester and monoglycerides; and (iii) the reaction between ethoxide anion and the  
306 carbonyl carbon of monoglycerides to form ethyl ester and glycerol.<sup>2,41</sup>

307 Fig. 1a shows that increase in ethanol to oil molar ratio increases the FAEE content up to  
308 30:1 and further increases in ethanol to oil molar ratio decreases the FAEE content. Initially, the  
309 increase in the FAEE content is due to the increased contact area between ethanol and oil and the  
310 increased mutual solubility in the presence of co-solvent CO<sub>2</sub>. Later, excess ethanol started to  
311 interfere with the glycerin separation due to increased solubility, which resulted in lower FAEE  
312 content.<sup>8,42</sup> According to He et al.<sup>43</sup>, after the mixture of alcohol and oil changes into a  
313 homogenous state, continuing raising the molar ratio of alcohol to oil cannot help to increase the  
314 fatty acid alkyl ester yield, as the reaction is restrained by the reaction equilibrium, which also  
315 makes the increase dosage of alcohol do not lead to any obvious effect on the fatty acid alkyl  
316 ester yield after a certain value of the molar ratio. The maximum FAEE content was achieved at  
317 temperature of 304 °C and ethanol to oil molar ratio of 29:1.

318 Reaction time plays a crucial role in the economy of the process and productivity.  
319 Conventional transesterification reactions take hours to complete while supercritical alcohol  
320 transesterification can be achieved in much shorter time periods.<sup>1</sup> Balat conducted experiments

321 under condition 280 °C, molar ratio of 40:1 of sunflower to ethanol. The fatty acid ester content  
322 (FAEE) content was 80% after 300 s. <sup>44</sup> Muppaneni et al. <sup>8</sup> reported 91% of FAEE yield at 300  
323 °C, 33:1 ethanol to oil molar ratio, and 20 min of reaction time. Fig. 1b shows the influence of  
324 reaction temperatures and time on FAEE content for fixed levels of ethanol/oil molar ratio of  
325 25:1 and initial CO<sub>2</sub> pressure of 30 bar. It can be seen from Fig. 1b that reaction time has positive  
326 effects on FAEE content up to 36 min, and thereafter it shows negative impacts on FAEE  
327 content. The optimal residence time in our experiment was somewhat different from those in  
328 other reports, <sup>8,44</sup> which may be explained by the difference between their experimental  
329 conditions, and ours as well as varied nature of oil. The major reason for the decrease of the  
330 FAEE content after the critical point of residence time at high reaction temperatures is the loss of  
331 unsaturated FAEE. In the reaction conditions of high reaction temperatures, namely above 304  
332 °C, there were other side reactions already, such as thermal decomposition reactions and  
333 dehydrogenation reactions <sup>1,40,43,45</sup> consuming the unsaturated FAEE, especially the C18:1 and  
334 C18:2. At the beginning of a transesterification reaction, the rate of FAEE production is higher  
335 than that of FAEE consumption, and therefore the content of FAEE increases before reaching the  
336 equilibrium point between the transesterification reaction and the side reactions. However, after  
337 this point, the rate of FAEE consumption is higher than that of FAEE production, and with the  
338 increase of residence time, the FAEE decreases.

339 Fig. 1c shows the effects of temperature and initial CO<sub>2</sub> pressure on FAEE content, when  
340 ethanol/oil molar ratio and reaction time were maintained constant as 25:1 and 30 min,  
341 respectively. As one can see, with the increment of initial CO<sub>2</sub> pressure up to the level of 40 bar,  
342 the FAEE content increases, beyond 40 bar there is no significant effect of initial CO<sub>2</sub> pressure  
343 on FAEE content. Han et al. <sup>46</sup>, in the alcoholysis of soyabean oil in methanol with the addition  
344 of co-solvent CO<sub>2</sub>, found that significant decrease in the severity of the process conditions  
345 required for supercritical reaction. Yin et al. <sup>47</sup> reported that esters yield for the reaction using  
346 supercritical methanol increased when using carbon dioxide as co-solvent. Tsai et al. <sup>48</sup> reported  
347 that addition of CO<sub>2</sub> in supercritical transesterification of sunflower oil using methanol is  
348 insignificant on FAEE yield at higher pressures above 10 MPa. For stating the significance of  
349 CO<sub>2</sub> pressure on the reaction, separate reactions were carried at optimum condition such as by  
350 using CO<sub>2</sub> and without using CO<sub>2</sub>. The FAEE content observed was 97.42% for CO<sub>2</sub> pressurized  
351 reaction and 76.83% for without CO<sub>2</sub>. This experiment analysis shows that CO<sub>2</sub> has the

352 significant effect on reaction kinetics. The possible reason may be the fact that increasing the  
353 reaction pressure simultaneously increases the density of the reaction mixture. The  
354 transesterification conversion is enhanced with an increased reaction mixture density.<sup>43</sup>

355 The RSM integrated with desirability function approach was used for simultaneous  
356 optimization of FAEE content, owing to its potential over conventional RSM.<sup>27</sup> The global  
357 optimized conditions for FAEE content were found to be temperature (A) = 304 °C, ethanol to  
358 oil molar ratio (B) = 29:1, reaction time (C) = 36 min, initial CO<sub>2</sub> pressure (D) = 40 bar. The  
359 predicted response of FAEE content at optimized conditions was 95.08 % (wt), with D value of  
360 0.9316. The maximum FAEE content of 97.42% (wt) was obtained at optimized conditions,  
361 representing only 2.34% difference between estimated and actual FAEE content. Results  
362 suggested that the optimal conditions attained had the least error and can be practically applied to  
363 produce biodiesel from mahua oil.

### 364 3.3. Artificial Neural Network modeling

365 The optimum architecture of ANN model was determined based on three steps: (1)  
366 optimum number of neurons (2) selection of the best backpropagation training algorithm and (3)  
367 testing and validation of the model.<sup>27</sup> A number neural network architecture and topologies were  
368 selected and investigated for the estimation and prediction of FAEE content. This is due to the  
369 fact that the choice of an optimal neural network and architecture and topology is critical for  
370 successful application of ANN.<sup>49</sup>

371 The optimum number of neurons was determined based on the minimum value of mean  
372 square error (MSE) of the training and prediction set.<sup>50</sup> In optimization of the neural network,  
373 two neurons were used in hidden layer as an initial estimate. The training stops with MSE of  
374 0.00017 at 48 epoch, which are close to the acceptable limit for MSE to 0.001. The relation  
375 between MSE and number of neurons in the hidden layer is given in Fig. S3. As it can be seen,  
376 the MSE of was minimum just about 10 neurons. The best backpropagation algorithm was  
377 determined by studying ten different backpropagation algorithms using tansig transfer function at  
378 hidden layer and purelin transfer function at output layer and results are given in Table 3. Polak-  
379 Ribiere conjugate gradient backpropagation (CGPA) with smaller MSE was found to be the best  
380 of ten backpropagation algorithms. So, CGPA was considered as the training algorithm in this  
381 study. Hence, we used feed-forward CGPA with 10 artificial neurons in hidden layer for  
382 modeling of FAEE content. The optimum architecture of ANN (4:10:1) model in this case is

383 shown in Fig. 2. Fig. 2 consists of three layers as input layer with four input variables, hidden  
384 layer with ten hidden neurons, and output layer with single output variable. All neurons from  
385 hidden layer have tan-sigmoid transfer function (tansig) and the output layer neuron has linear  
386 transfer function (purelin). As can be seen from Fig. 2, the connections consist of weights and  
387 biases between inputs and neurons as well as between neurons from different layers.

388 The scatter diagrams that compare the experimental data versus the computed neural  
389 network data in both training, testing and validation networks are shown in Fig. 3. Fig. 3 shows  
390 the NN model with training, validation, test and all prediction set with very good values of R  
391 (0.9989, 0.9998, 0.9999, and 0.9994 respectively). Almost all data scatter around the 45° line that  
392 is the indication of excellent compatibility between the experimental results and ANN predicted  
393 data. These values of R between experimental response and ANN predicted response in all the  
394 cases suggests that the developed ANN model, which was trained using experimental data, was  
395 precise predicting FAEE content.

#### 396 3.4. Sensitivity Analysis

397 The ANN used in this study provided with weights listed in Table 4a. The relative  
398 significance of the four input variables calculated by Garson Eq. (11) shown in Table 4b. As may  
399 be seen from Table 4b, all of the four variables (temperature, ethanol/oil molar ratio, time, and  
400 initial CO<sub>2</sub> pressure with relative importance of 39.24, 19.61, 28.57 and 12.58 respectively) have  
401 strong effects on the FAEE content. Therefore, none of the variables studied in this work could  
402 have been neglected in the present analysis. The degree of effectiveness of variables was found  
403 in the order of.

404 temperature > reaction time > ethanol/oil molar ratio > initial CO<sub>2</sub> pressure

#### 405 3.5. Comparison between RSM and ANN models

406 The ANN and RSM model were compared for DoE, using which both the models were  
407 trained. The comparison was made on the basis of various parameters such as root mean square  
408 error (RMSE), standard error of prediction (SEP), absolute average deviation (AAD), and  
409 correlation coefficients (R<sup>2</sup>). The predicted values by ANN as well RSM model are tabulated in  
410 Table 1. ANN model had fitted the experimental data with an excellent accuracy. The  
411 generalization ability of both the models were judged only with unseen dataset. Thus, it was  
412 decided to test both the models using the separate unseen data (six runs) which does not belong  
413 to the training data sets. The experimental and predicted FAEE content are summarized in Table



414 5. The comparative values of RMSE, SEP, AAD and  $R^2$  are given in Table 6. The  $R^2$  for RSM  
415 and ANN was 0.658 and 0.868, and RMSE was 7.691 and 4.185, respectively. Table 6 indicates  
416 that both the models performed reasonably well, but ANN performed consistently better than  
417 RSM. The prediction performance of the ANN model for the validation data set confirmed its  
418 superior generalization capacity for the given case over the RSM. In addition, Fig. 4 shows the  
419 experimental and predicted values for each experimental run to obtained the FAEE content.  
420 From the Fig. 4, it is evident that the trained neural network has efficient approximated  
421 experimental values. The ANN model predictions lie much closer to the line of perfect prediction  
422 than the RSM model. Thus, the ANN model shows a significantly higher generalization capacity  
423 than the RSM model. This higher predictive accuracy of the ANN can be attributed to its  
424 universal ability to approximate the nonlinearity of the system, whereas the RSM is restricted to  
425 a second-order polynomial.<sup>51</sup> However, when using the ANN technique one must have in mind  
426 that its predictions are restricted on the process factors inside the ranges applied in the training  
427 process.<sup>14</sup>

### 428 *3.6. Fuel properties of Mahua oil Biodiesel*

429 A comparison of fuel properties are made between mahua oil, mahua oil ethyl ester,  
430 ASTM and DIN EN 14214 Biodiesel standard which are given in Table 7. The various properties  
431 of mahua oil biodiesel (MOB) are found to be comparable with that of the Diesel, American  
432 (ASTM) and European (DIN EN 14214) biodiesel standard. Cetane number is high, favorable for  
433 combustion. Flash point and Fire point are high, which is an advantage for fuel transportation.

#### 434 4. Conclusions

435 The production of biodiesel from crude mahua oil with high content of free fatty acid by  
436 ethanolysis reaction at supercritical condition using CO<sub>2</sub> as co-solvent has been investigated in  
437 this work. Experimental results demonstrated that use of co-solvent helps to increase FAEE  
438 content at lower process conditions, which facilitates less energy consumption for biodiesel  
439 conversion. In this work, RSM and ANN were applied for modeling and optimization of the  
440 supercritical biodiesel production process. Response surface method (RSM) integrated with  
441 desirability function approach was successfully applied for designing and optimizing the  
442 experiments with respect to the dependent variables. The regression equations in coded and  
443 actual terms were calculated by RSM to describe the empirical functional relationship between  
444 input variables and response (FAEE content). The biodiesel was found to content more than 97%  
445 FAEE content, which is well above EN 14214 limits of 96.4%. The sensitivity analysis of ANN  
446 confirmed that all the four variables have significant effects on FAEE content with the degree of  
447 effectiveness in order of temperature > reaction time > ethanol/oil molar ratio > initial CO<sub>2</sub>  
448 pressure. Based on the values of R<sup>2</sup>, RMSE, SEP, AAD for validation data sets, ANN model was  
449 demonstrated to be more efficient than RSM model both in data fitting and prediction  
450 capabilities. This renewable, eco-friendly process has the potential to provide a sustainable route  
451 for the production of high-quality biodiesel using low cost, high acid value, crude mahua oil.  
452 However, further exploration on this technology is necessary for scale up of process design,  
453 reaction kinetics and thermodynamics, storage stability, fuel analysis using the biodiesel fuel  
454 engine.

455

456

#### 457 Appendix A. Supplementary Material

458

459

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## Tables:

**Table 1.** CCD matrix of four independent variables along with experimental and predicted response

Temperature (° C)	Molar ratio	Reaction Time (min.)	Initial CO <sub>2</sub> pressure (bar)	FAEE content (%)		
				Experimental	RSM	ANN
275	20:1	20	20	48.48	47.24	48.48
325	20:1	20	20	51.79	52.27	52.29
275	30:1	20	20	53.04	50.35	53.04
325	30:1	20	20	67.71	63.4	67.71
275	20:1	40	20	66.59	66.57	66.59
325	20:1	40	20	65.91	65.76	65.91
275	30:1	40	20	73.84	70.03	74.04
325	30:1	40	20	79.12	77.23	79.13
275	20:1	20	40	49.14	49.31	48.85
325	20:1	20	40	53.44	54.19	53.69
275	30:1	20	40	59.69	56.78	59.69
325	30:1	20	40	71.37	69.67	71.37
275	20:1	40	40	67.25	68.5	67.24
325	20:1	40	40	66.57	67.54	66.39
275	30:1	40	40	78.5	76.31	78.5
325	30:1	40	40	85.18	83.37	85.18
250	25:1	30	30	28.12	31.46	28.55
350	25:1	30	30	42.11	43.55	42.28
300	15:1	30	30	65.62	62.12	65.63
300	35:1	30	30	72.79	81.06	72.79
300	25:1	10	30	40.41	43.75	40.4
300	25:1	50	30	75.34	76.78	75.35
300	25:1	30	10	76.74	81.17	76.94
300	25:1	30	50	89.03	89.38	89.44
300	25:1	30	30	88.56	87.82	87.46
300	25:1	30	30	86.32	87.82	87.46
300	25:1	30	30	88.59	87.82	87.46

**Table 2.** Analysis of Variance (ANOVA) for the fitted polynomial quadratic model of FAEE content

Source	Sum of Squares	df	Mean Square	F Value	p-value Prob > F	
Model	6620.61	14	472.9	28.5929	< 0.0001	significant
A:Catalyst Concentration	219.252	1	219.252	13.2566	0.0034	
B:Methanol/oil molar ratio	537.896	1	537.896	32.5227	< 0.0001	
C:Temperature	1636.14	1	1636.14	98.9258	< 0.0001	
D:time	101.024	1	101.024	6.1082	0.0294	
AB	64.2402	1	64.2402	3.88415	0.0723	
AC	34.1056	1	34.1056	2.06212	0.1766	
AD	0.0225	1	0.0225	0.00136	0.9712	
BC	0.1156	1	0.1156	0.00699	0.9348	
BD	18.9225	1	18.9225	1.14411	0.3058	
CD	0.02103	1	0.02103	0.00127	0.9721	
A <sup>2</sup>	3376.25	1	3376.25	204.138	< 0.0001	
B <sup>2</sup>	351.253	1	351.253	21.2378	0.0006	
C <sup>2</sup>	1012.8	1	1012.8	61.2367	< 0.0001	
D <sup>2</sup>	8.67567	1	8.67567	0.52456	0.4828	
Residual	198.469	12	16.5391			
Lack of Fit	195.079	10	19.5079	11.5075	0.0825	not significant
Pure Error	3.39047	2	1.69523			
Cor Total	6819.07	26				
<b>R<sup>2</sup> = 0.9709      Adj. R<sup>2</sup> = 0.9369      Pred. R<sup>2</sup> = 0.8341      CV = 6.13      S = 4.07</b>						



**Table 3.** Comparison of 10 backpropagation (BP) algorithm with 10 neurons in hidden layer

Backpropagation (BP) algorithms	Function	Mean squared error (MSE)	Iteration number	Correlation coefficient ( $R^2$ )	Best linear equation
Resilient backpropagation	trainrp	6.5245	35	0.981	$y = 0.983x + 0.240$
Fletcher-Reeves conjugate gradient backpropagation	traincgf	0.8231	64	0.927	$y = 0.962x + 2.325$
Polak-Ribiere conjugate gradient backpropagation	traincgp	0.00017	48	0.999	$y = 0.994x + 0.409$
Powell-Beale conjugate gradient backpropagation	traincgb	0.5166	26	0.979	$y = 0.978x + 1.384$
Levenberg-Marquardt backpropagation	trainlm	0.00527	15	0.998	$y = 0.992x + 0.403$
Scaled conjugate gradient backpropagation	trainscg	0.06355	63	0.989	$y = 0.989x + 0.791$
BFGS quasi-Newton backpropagation	trainbfg	0.0272	10	0.998	$y = 0.998x + 0.174$
One step secant backpropagation	trainoss	0.4323	103	0.977	$y = 0.985x + 1.117$
Variable learning rate backpropagation	traingdx	0.0667	55	0.976	$y = 0.979x + 1.425$
Gradient Descent with adaptive learning rate	traingda	0.0886	73	0.980	$y = 0.987x + 1.704$

**Table 4a.** Matrix of weights,  $W_1$ : weights between input and hidden layers;  $W_2$ : weights between hidden and output layers

$W_1$					$W_2$		
Neuron	Variable				Bias	Neuron	Weight
	Temp.	Molar ratio	Time	CO <sub>2</sub> pressure			
1	0.44089	-0.302	1.3715	-1.6845	-2.7861	1	-0.18261
2	1.6029	-1.8237	1.0114	0.12937	-1.7488	2	-0.55982
3	1.6085	1.2886	-1.2555	0.08211	-2.0685	3	1.2491
4	-1.4374	0.36731	-1.9491	-0.238	1.2039	4	0.03047
5	-0.433	-1.4371	0.83634	1.3732	-1.0401	5	-0.09378
6	2.6774	1.5293	-2.2877	-0.1141	-1.5536	6	-1.1185
7	2.3905	0.1609	-0.6757	1.2321	0.52144	7	0.35442
8	2.1526	0.28323	1.0811	-0.1826	1.8328	8	1.194
9	0.15168	0.14845	0.55727	1.6455	2.8863	9	-0.40243
10	-0.7973	-0.333	1.22	1.469	-3.0539	10	0.22069
						<b>Bias</b>	-0.35551

**Table 4b.** Relative importance of input variables on the output variable

Input variable	% Importance
Temperature	39.24
Methanol/oil molar ratio	19.61
Reaction time	28.57
Initial CO <sub>2</sub> pressure	12.58
Total	100.00

**Table 5.** Unseen validation data set for developed model

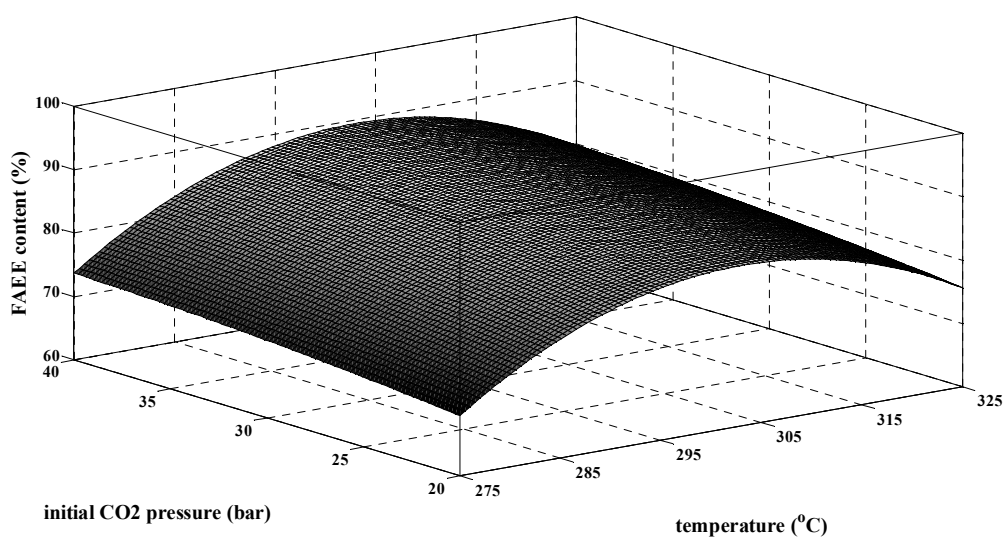
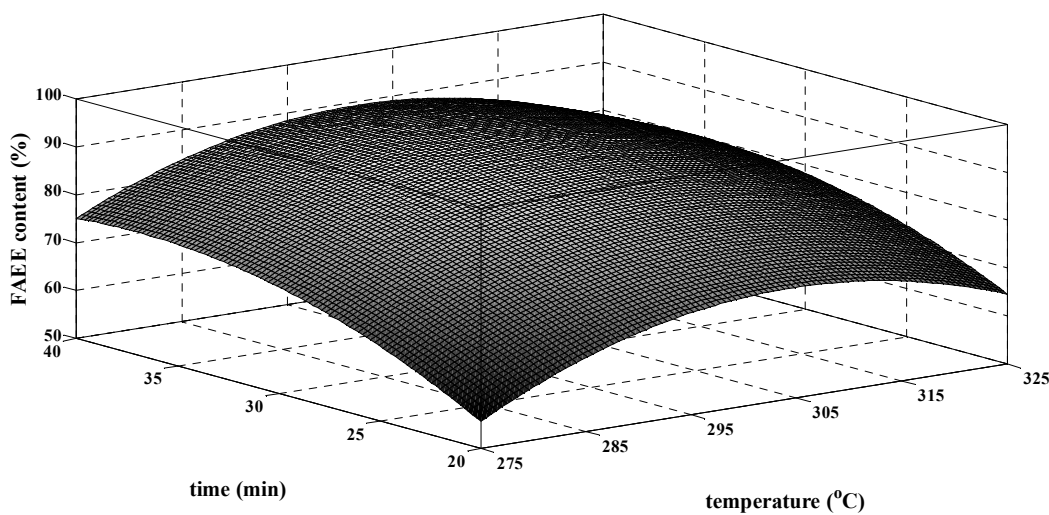
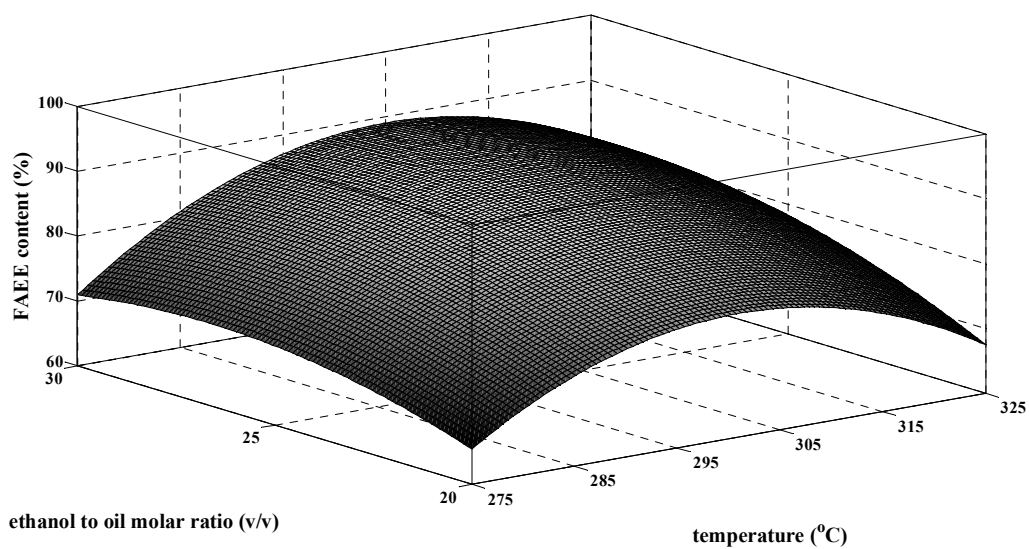
Sr. No.	Temperature (°C)	Molar ratio (v/v)	Reaction Time (min)	CO <sub>2</sub> pressure (bar)	FAEE content (%)		
					Experimental	RSM	ANN
1.	275	25	20	40	54.38	57.1	56.37
2.	325	25	40	20	78.12	75.55	79.66
3.	300	30	30	30	86.5	88.5	84.11
4.	300	20	40	40	71.43	80.6	75.48
5.	275	30	35	40	72.16	76.59	77.65
6.	325	30	35	40	69.54	85.11	76.66

**Table 6.** Comparison between predictive capabilities of RSM and ANN models

Performance parameters	DoE data		Validation data	
	RSM	ANN	RSM	ANN
Correlation coefficient ( $R^2$ )	0.971	0.999	0.658	0.868
Root mean square Error (RMSE)	2.711	0.416	7.691	4.185
Standard predicted deviation (SEP %)	4.087	0.627	10.67	5.81
Absolute average deviation (AAD %)	3.398	0.338	8.574	5.239

**Table 7.** Fuel properties of Mahua oil, Mahua oil biodiesel, Diesel, ASTM and DIN EN 14214

Sr. No.	Properties		Mahua oil	Mahua oil Biodiesel	Diesel	ASTM standard D-6751	DIN-EN 14214
1.	Density at 15 °C	(Kg/L)	0.954	0.871	0.846	-	0.86-0.9
2.	Viscosity at 40 °C	(mm <sup>2</sup> /s)	43.8	4.6	2.68	1.9-6.0	3.5-5.0
3.	Flash Point	(°C)	231	186	70	>130	>120
4.	Fire Point	(°C)	239	197	76		
5.	Pour Point	(°C)	15	3	-20	-	-
6.	Acid value	(mg of KOH/g)	38	0.29	-	<0.8	<0.5
7.	Calorific value	(MJ/Kg)	36	41	42.96	-	-
8.	Cetane number		48.57	50	47	Min. 47	Min. 51
9.	Cloud Point	(°C)	1	3	-13	-	-
10.	Water content	volume%	1.32	0.017	0.02	Max. 0.05	Max. 0.05
11.	Carbon Residue	Mass %	0.57	0.21	0.17	Max. 0.05	Max. 0.05
12.	Sulphur	Wt %	-	< 0.005	0.001	Max. 0.05	Max. 0.05
13.	Total Glycerin	ppm	-	47	-	-	-



**Figure 1.** 3D response surface plots showing the relative effect of process variables on FAEE content (%) (a) effect of temperature and ethanol/oil molar ratio; (b) effect of temperature and reaction time; (c) effect of temperature and initial CO<sub>2</sub> pressure

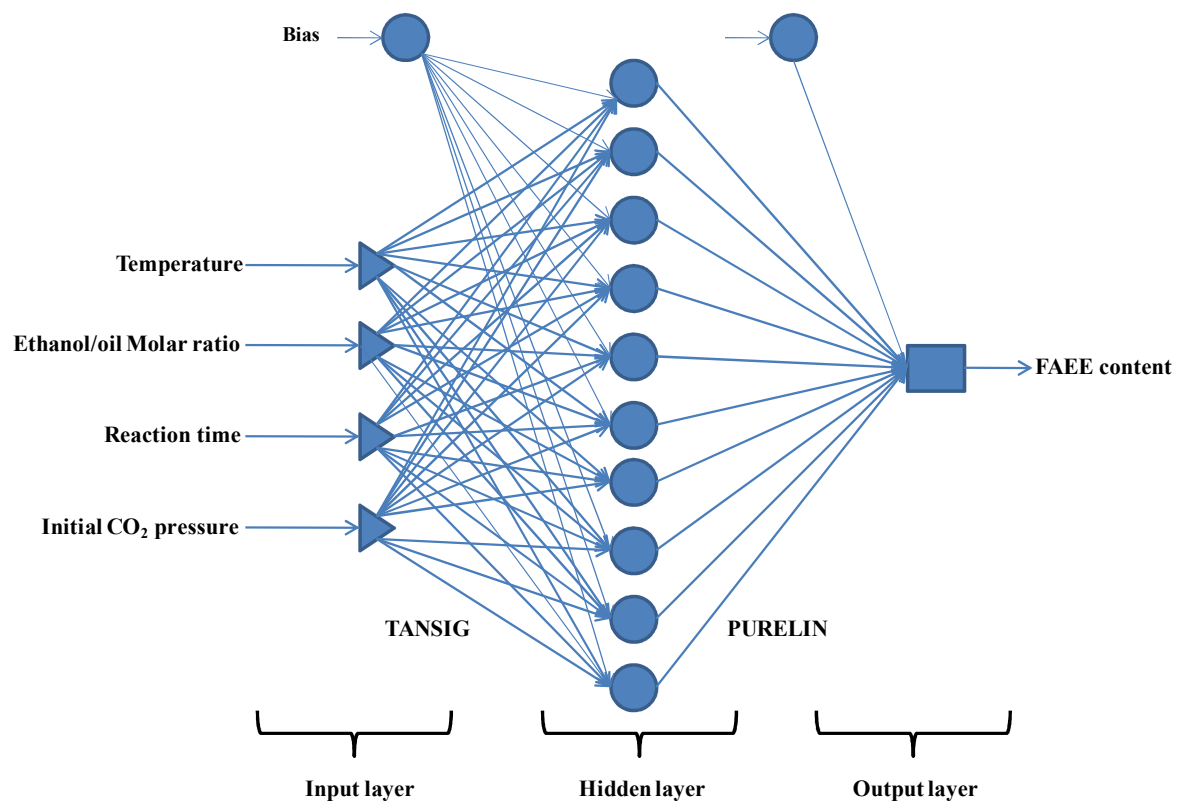
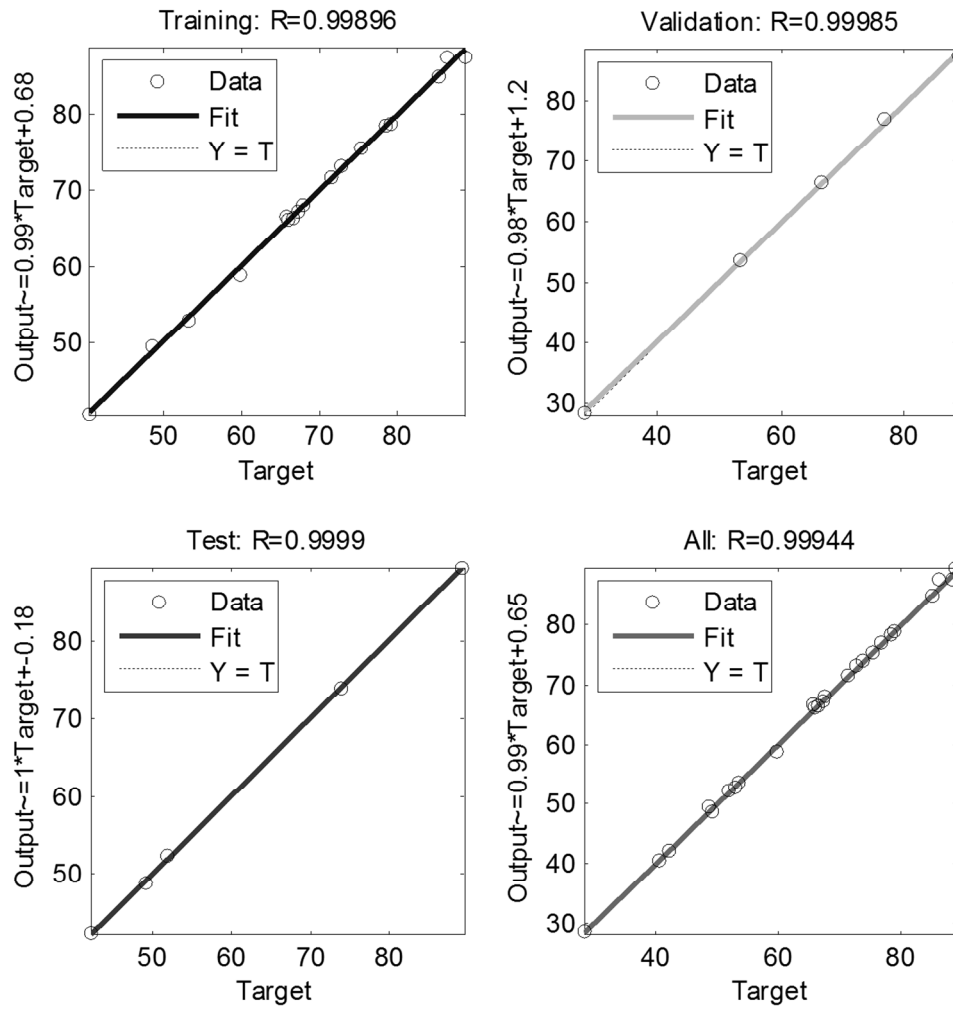
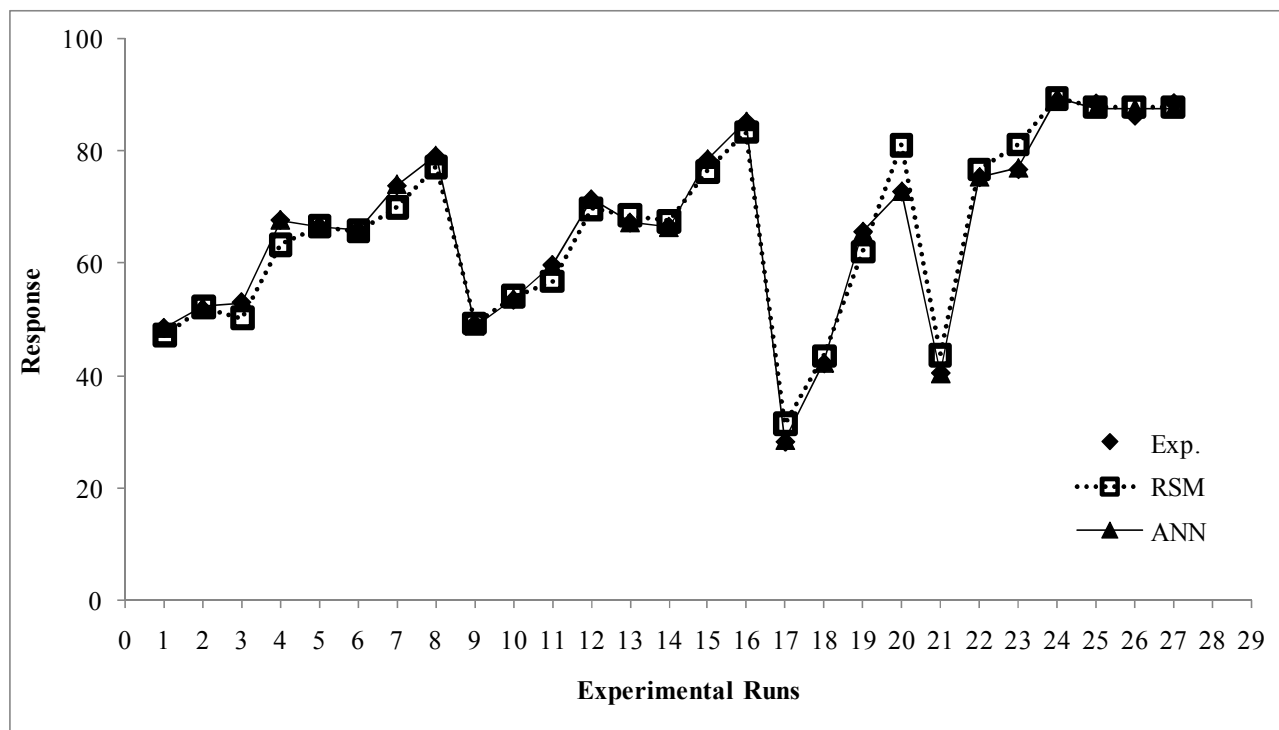


Fig. 2. Typical architecture of ANN with three layers





**Fig. 3.** Neural Network model with training, validation, test and all prediction set



**Fig. 4.** Comparison between experimental and predicted values by RSM and ANN for each experimental run to obtained the FAEE content