Hydrogenation-Induced Large-Gap Quantum-Spin-Hall Insulator States in Germanium-Tin Dumbbell Structure

Xin Chen, Linyang Li, Mingwen Zhao*

School of Physics and State Key Laboratory of Crystal Materials, Shandong University, Jinan, Shandong, 250100, China

Abstract: The quantum spin Hall (QSH) effect in two-dimensional topological insulators (2D TIs) is promising for building nanoscaled devices with low energy consumption. The 2D TIs with a bulk band gap larger than the atomic thermal motion energy at room temperature (~26 meV) are essential for achieving room-temperature QSH effects. We proved from first-principles that this goal may be reached in a hydrogenated germanium-tin (Sn₆Ge₄H₄) dumbbell (DB) structure, where spin-orbit coupling (SOC) opens a bulk band gap of 235 meV. The topological nontriviality is related to the band inversion of s-pₓᵧ of Sn atoms induced by surface hydrogenation and can be characterized by a topological invariant of Z₂=1. This work offers a promising candidate material for achieving long-desired room-temperature QSH effect.

Corresponding author, E-mail: zmw@sdu.edu.cn
Introduction

Topological insulators (TIs) are a new quantum state of matter with a topologically nontrivial bulk band gap due to spin-orbit coupling (SOC) and gapless surface or edge states. Protected by time-reversal symmetry, low-energy scattering of the edge states in two-dimensional (2D) TIs (also called quantum spin Hall (QSH) insulators) is enjoined, resulting in dissipationless transport edge channels\(^1\)-\(^9\), which are quite promising for application in spintronics and quantum computations. To achieve such application, the bulk band gap in TIs should be larger than the atomic thermal motion energy. Although many materials have been predicted to be QSH insulators, only HgTe/CdTe\(^10\) and InAs/GaSb\(^11\) quantum wells were verified by transport experiments at extremely low temperature, due to their small bulk gap. Increasing the critical temperature is highly desirable for achieving the QSH effect at room-temperature. This has motivated an intensive search for large-gap 2D QSH insulators, such as metal-organic frameworks\(^12\)-\(^15\) and those containing heavy metal atoms\(^16\)-\(^20\). Additionally, our recent work showed that tensile strain can serve as an effective means to drive a normal insulator to a QSH insulator\(^21\).

Inspired by the successful discovery and application of graphene, extensive research into the 2D group-IV materials has been conducted, and many exotic electronic properties have been revealed.\(^22\)-\(^29\) For example, low-buckled (LB) silicene, germanene and stanene\(^30\) and their derivatives\(^31\),\(^32\) have been reported as QSH insulators with relatively strong SOC. Some of them have a bulk gap larger than the thermal motion energy at room temperature (\(~26\) meV). Apart from the LB
configurations with weak \( \pi-\pi \) bonding, a dumbbell (DB) structure was proposed as a stable phase of group-IV films\(^{33-36} \). Due to the fourfold-coordinated atoms included in this DB configuration, it is energetically more favorable than the LB configuration for some 2D group-IV materials\(^{33,35,37-40} \), such as DB silicene and DB germanene. The plausibility of these DB configurations has also been evidenced by some experimental findings. For example, the \( \sqrt{3} \times \sqrt{3} \) silicene multi-layers grown on an Ag(111) substrate\(^{41,42,43} \) can be reproduced by stacking DB-based silicene derivatives\(^{35,44} \). The 2x2 superstructure of germanene grown on an Al(111) substrate resembles DB germanene very well\(^ {45} \).

More interestingly, the DB configuration of tin (DB stanene) has been predicted to be a QSH insulator with a large topologically nontrivial bulk band gap\(^ {33,46} \), especially when it is hydrogenated (DB stanane)\(^ {46} \). These works enrich the database of group-IV QSH insulators, beyond the LB honeycomb lattices, such as silicene, germanene, \( \delta \)-graphyne\(^ {47} \), graphene nanomesh\(^ {48} \), stanene\(^ {33} \), etc. However, the topological nontriviality is not retained in all the DB configurations of 2D group-IV materials. For example, the DB germanene proposed by Özçelik et al.\(^ {40} \), is a normal insulator. Moreover, group-IV elements can form abundant compounds with one another, e.g. silicon-carbide (SiC), silicon-germanium (SiGe), germanium-tin (SnGe), and so on. The honeycomb lattice of silicon carbide with a stoichiometry of SiC\(_3\) or Si\(_3\)C has been predicted to be QSH insulators\(^ {25} \). However, the group-IV binary compounds with DB configurations have never been investigated so far. Whether they are QSH insulators remains unclear.
In this contribution, we reported our systematic study on the electronic structures of group-IV DB binary compounds (as shown in Fig.1a) from first-principles calculations. We found that in contrast to DB stanene, none of them are QSH insulators. However, upon hydrogenation, germanium-tin (Sn\textsubscript{6}Ge\textsubscript{4}) can be tuned to a QSH insulator characterized by a topological invariant of \(Z_2 = 1\). The quantum phase transition was attributed to the band inversion of \(s-p_{xy}\) of Sn atoms induced by surface hydrogenation. The nontrivial bulk band gap in hydrogenated DB Sn\textsubscript{6}Ge\textsubscript{4} (Sn\textsubscript{6}Ge\textsubscript{4}H\textsubscript{4}) is about 235 meV, much larger than that of DB stanane\textsuperscript{46} and the thermal motion energy (~26 meV) at room temperature. The phonon spectrum confirms the high stability and plausibility of DB Sn\textsubscript{6}Ge\textsubscript{4}H\textsubscript{4}. This work offers a promising candidate material for applications of QSH effect at high temperature.

Methods

We performed first-principles calculations within density-functional theory (DFT) using the plane wave basis Vienna ab initio simulation package known as VASP code\textsuperscript{49-51}. The energy cutoff of the plane waves was set to 600 eV with an energy precision of \(10^{-5}\) eV. A generalized gradient approximation (GGA) in the form proposed by Perdew, Burke, and Ernzerhof (PBE)\textsuperscript{52} was adopted to treat the electron exchange-correlation functional. The atomic positions were relaxed until the maximum force on each atom was less than 0.01 eV/Å. The Brillouin zone (BZ) of the 2D materials was sampled by using an 11×11×1 Gamma-centered Monkhorst-Pack grid. A vacuum space of up to 20 Å was applied to minimize the artificial interactions between neighboring slabs. SOC was included by a second
variational procedure on a fully self-consistent basis. The phonon spectra were calculated using a supercell approach within the PHONON code\textsuperscript{53}.

Results and Discussion

The schematic representation of the DB configurations of group-IV binary compounds is shown in Fig. 1(a). Taking germanium-tin as an example, Sn and Ge have different coordination numbers, resulting in a stoichiometry of Sn\textsubscript{6}Ge\textsubscript{4} (denoted as DB Sn\textsubscript{6}Ge\textsubscript{4} hereafter). Sn is fourfold coordinated, similar to the case in stannane molecules (SnH\textsubscript{4}), while Ge is only threefold coordinated with a dangling bond as in the case of LB germanene. All the Sn atoms are on the middle plane sandwiched by two planes of Ge atoms. The buckling height of the 2D hexagonal structure is about 3.11 Å, shorter than that of DB stanene, 3.41 Å. The lattice constant (the length of the base vectors) is 8.83 Å. According to the electron localization function (ELF)\textsuperscript{54, 55} profile shown in Fig. 1(a), despite the Ge-Sn covalent bonds, there are obvious regions of electron density in between the Ge atoms of the up- and down-planes, which are beneficial to the stabilization of the system. The length of the Ge-Sn bond is about 2.72 Å. The distance between the two Ge planes is about 3.11 Å, longer than that in germanene, 2.44 Å.

To evaluate the energetic stability of DB Sn\textsubscript{6}Ge\textsubscript{4}, we calculated the formation energy (\(E_{\text{form}}\)) with respect to LB germanene and stanene, according to the expression:

\[
E_{\text{form}} = (E_{\text{total}} - n_{\text{Ge}} \times \mu_{\text{Ge}} - n_{\text{Sn}} \mu_{\text{Sn}})/(n_{\text{Ge}} + n_{\text{Sn}}),
\]

where \(E_{\text{total}}\) is the total energy per unit cell of DB Sn\textsubscript{6}Ge\textsubscript{4}, \(\mu_{\text{Ge}}\) and \(\mu_{\text{Sn}}\) are the chemical potentials of Ge and Sn atoms in the already-synthesized LB germanene and stanene\textsuperscript{30}, \(n_{\text{Ge}}\) and \(n_{\text{Sn}}\) are the respective numbers of...
the Ge and Sn atoms in one unit cell of DB Sn₆Ge₄. Our DFT calculations showed that DB Sn₆Ge₄ has a negative formation energy of about -0.14 eV/atom, suggesting the energetically favorability over LB germanene and stanene. The energetic stability of DB Sn₆Ge₄ is related to its unique coordination. The fourfold coordinated Sn and the interaction between the up- and down-planes of the threefold coordinated Ge atoms contribute to the stability of DB Sn₆Ge₄ over LB germanene and stanene composing exclusively of threefold coordinated Sn or Ge atoms. By ab initio phonon spectrum calculations, we further demonstrated the dynamic stability of DB Sn₆Ge₄, as shown in Fig. 1(b). No modes with imaginary frequencies were found in the spectrum and the structure was therefore proved to be dynamically stable.

The electronic band structures of DB Sn₆Ge₄ were then calculated. Without considering SOC, the band lines in the vicinity of the Fermi level, as indicated in Fig. 2(a), exhibit a direct band gap of 242 meV at the Γ point. When SOC was switched on, the direct-band-gap features were preserved, but the band gap was narrowed to 64 meV, as shown in Fig 2(b). Parity analysis indicates that this is topological trivial band gap with a zero topological invariant (Z₂=0). Pristine DB Sn₆Ge₄ is therefore a normal insulator, similar to the bulk counterpart. We also calculated the electronic band structures of other group-IV binary compounds with DB configurations, DB X₆Y₄ (X,Y=C, Si, Ge, and Sn), and found that one of them are QSH insulators (see Supporting Information).

Ge atoms can be easily bonded with H atoms, for instance, in the case of germane (GeH₄). We tried to passivate the dangling bonds of Ge atoms in DB Sn₆Ge₄ with H
atoms. As shown in Fig. 3(a), all the Sn and Ge atoms in hydrogenated DB Sn₆Ge₄ (denoted as Sn₆Ge₄H₄ hereafter) are fully-coordinated. The H-Ge distance in the optimized Sn₆Ge₄H₄ is 1.56 Å, close to the value 1.55 Å in germane molecule. The distance between Ge planes is enlarged to 3.25 Å. Both the lattice constant (8.75 Å) and the Ge-Sn distance (2.70 Å) are shorter than the corresponding values of pristine DB Sn₆Ge₄ (8.83 Å and 2.72 Å), respectively. The binding energy of hydrogen atoms with DB Sn₆Ge₄ is about -2.17 eV, lower than that in DB stanane, -1.92 eV. These results confirmed that hydrogen atom can chemically bind to Ge atoms, in consistent with the ELF profile shown in Fig. 3(a). The Sn-Ge covalent bonds are retained in DB Sn₆Ge₄H₄, while the interaction between the up- and down-planes of Ge atoms is weakened as indicated by ELF profiles and the enlarged spacing between Ge planes.

To demonstrate the energetic stability of DB Sn₆Ge₄H₄, we compared the energy of DB Sn₆Ge₄H₄ with the sum of the energies of isolated DB Sn₆Ge₄ and atomic hydrogen atoms in hydrogen plasma environment⁵⁶,⁵⁷. Our DFT calculations showed that the former is lower than the later by 0.62 eV/atom. The energetic favorability of DB Sn₆Ge₄H₄ can also be demonstrated by a hypothetical reaction: 6SnH₄ + 4GeH₄ → Sn₆Ge₄H₄ + 18H₂. Our calculation indicates that it is an exothermal reaction with an energy release of 0.15 eV/atom. The phonon spectrum of DB Sn₆Ge₄H₄ is plotted in Fig. 3(b). Obviously, there are no modes with imaginary frequencies, implying the dynamic stability of DB Sn₆Ge₄H₄.

The electronic band structure of DB Sn₆Ge₄H₄ differs significantly from that of pristine DB Sn₆Ge₄. Without considering SOC, DB Sn₆Ge₄H₄ is a gapless
semiconductor with the valence band maximum (VBM) and conduction band minimum (CBM) energetically degenerate at the Γ point, as shown in Fig. 4(a). The electronic states in the vicinity of the Fermi level originate from different atomic orbitals. The orbital-resolved electron density of states (DOS) showed that both VBM and CBM arise mainly from the p_{x,y} atomic orbitals of the Sn atoms, as shown in Fig. 4(b), in good consistent with the density profiles of the electron wavefunctions (WFs) shown in Fig. 4(d). For the valence band next to VBM (denoted as VBM-1), the WFs exhibit clear features of the s-orbital of Sn atoms and p_z-orbital of Ge atoms. The degeneracy of the two p_{x,y} bands at the Γ point is a direct consequence of the C_{3v} symmetry of the lattice. Such band alignment represents a common feature of a QSH insulator family, including DB stanane\(^{46}\) and the derivatives of germanene\(^{31, 32}\), which is closely related to topological nontriviality. Further studies on other group-IV binary compounds indicated that none of them exhibit such type of band alignment (see Supporting Information). DB Sn_{6}Ge_{4}H_{4} stands out from the group-IV binary compounds as a candidate QSH insulator.

When SOC was switched on in our DFT calculations, a band gap of 275 meV is opened up at the Γ point of DB Sn_{6}Ge_{4}H_{4}, as shown in the down panel of Fig. 4(a). The global indirect band gap is about 166 meV, slightly larger than that of DB stanane, 160 meV\(^{46}\). Such a large bulk gap suggests that DB Sn_{6}Ge_{4}H_{4} is quite promising for achieving room-temperature QSH effect.

A comparison study of the orbital-resolved band structures of DB Sn_{6}Ge_{4} and DB Sn_{6}Ge_{4}H_{4} was then conducted to determine their topological aspects. For convenience,
we denoted the electronic states arising from s, p_{x,y}, p_z atomic orbitals of Sn as |s^±>, |p_{x,y}^±>, and |p_z^±>, in which the parity of the states are labeled by “+/−”. For pristine DB Sn_6Ge_4, no matter whether SOC is included or not, the energies of the electronic states near the Fermi level are successively in the same order: |p_{x,y}^+>, |p_z^-> and |s^->, as shown in Fig. 5(a). But the order of |p_{x,y}^+> and |s^-> is reversed in DB Sn_6Ge_4H_4 without SOC, as shown Fig. 5(b), leading to a s-p_xy band inversion. When SOC was included, the degeneracy of the partial occupied |p_{x,y}^+> state in DB Sn_6Ge_4H_4 was lifted, resulting in a large nontrivial band gap. This implies that surface hydrogenation induces the s-p_xy band inversion and drives DB Sn_6Ge_4 to a QSH insulator, as visually revealed in Fig. 5(c). Such s-p-type band inversion mechanism also holds in other QSH insulators, such as HgTe quantum well and fluorinated stanene.

The topological nontriviality of DB Sn_6Ge_4H_4 can be confirmed by a nonzero topological invariant Z_2. Because both DB Sn_6Ge_4 and DB Sn_6Ge_4H_4 have an inversion symmetry, the Z_2 index can be calculated using the parity criteria proposed by Fu and Kane. According to this strategy, the Z_2 index can be determined from the parities of the four time-reversal and parity invariant points at BZ. The four time-reversal invariant momenta of the two lattices occur at the Γ and three M points, as shown in Fig. 5(d). The Z_2 invariant ν is defined by

\[ (-1)^\nu = \prod_i \delta_i \text{ with } \delta_i = \prod_{m=1}^{N} \xi_{2m}(\Gamma_i) \]

for 2N occupied bands. \( \xi_{2m}(\Gamma_i) = \pm 1 \) is the parity eigenvalue of the 2m-th occupied energy band at the time-reversal invariant momentum \( \Gamma_i \). Our DFT calculations
showed that $\delta_i$ has the values of (+), (+), (+), (+) in DB $\text{Sn}_6\text{Ge}_4$ and (+), (−), (−), (−) in DB $\text{Sn}_6\text{Ge}_4\text{H}_4$ at their four time-reversal invariant momenta, as indicated in Fig. 5(d). The topological invariants of pristine DB $\text{Sn}_6\text{Ge}_4$ and DB $\text{Sn}_6\text{Ge}_4\text{H}_4$ are therefore $Z_2=0$ and $Z_2=1$ respectively, suggesting the quantum phase transition from a normal insulator (DB $\text{Sn}_6\text{Ge}_4$) to a QSH insulator (DB $\text{Sn}_6\text{Ge}_4\text{H}_4$) induced by surface hydrogenation.

In view of the failure of PBE functional in determining electronic band gaps, we employed a hybrid functional in the form of Heyd-Scuseria-Ernzerhof (HSE)\textsuperscript{60} to recalculate the electronic band structure of DB $\text{Sn}_6\text{Ge}_4\text{H}_4$. By incorporating a potion of exact exchange from Hartree-Fock (HF) theory, HSE functional can give band gaps comparable to experimental values. We found that the s-p band inversion and topological nontriviality of DB $\text{Sn}_6\text{Ge}_4\text{H}_4$ are retained even under a tensile stain. The bulk band gaps opened due to SOC are about 235 meV (indirect) and 317 meV (direct band gap at the $\Gamma$), both of which are much larger than the PBE results.

Substrates are inevitable in the fabrication procedure of QSH-insulator-based devices. However, the coupling between QSH insulators with substrates may destroy their topological nontriviality\textsuperscript{20, 30}. For instance, when stanane is grown on a semiconducting substrate, it becomes a normal insulator or a metal without topological nontriviality\textsuperscript{30}. To determine the influence of substrate on the topological nontriviality of DB $\text{Sn}_6\text{Ge}_4\text{H}_4$, we built a superstructure of DB $\text{Sn}_6\text{Ge}_4\text{H}_4$ on a $(\sqrt{12} \times \sqrt{12})$ h-BN substrate, as shown in Fig. 6(a). The lattice of DB $\text{Sn}_6\text{Ge}_4\text{H}_4$ matches well with that of the h-BN substrate with a difference less than 0.6%. At the
equilibrium state, the interlayer spacing between DB Sn₆Ge₄H₄ and the substrate is about 2.29 Å, suggesting weak interaction between them. We calculated the electronic band structure of the superstructure and found that the band lines in the vicinity of the Fermi level are in accordance with the free-standing DB Sn₆Ge₄H₄, as shown in Fig. 6(b) and 6(c). Both s-p band inversion and topological nontriviality are intact, except that the SOC bulk gap is slightly improved. The robust topological nontriviality of DB Sn₆Ge₄H₄ against substrate effects is related to the origins of the bands near the Fermi level. These bands are mainly contributed by s and pₓᵧ atomic orbitals rather than pₓ orbital and thus are unlikely affected by substrates.

Conclusions

Using first-principles calculations, we demonstrated theoretically that the unique coordinates of the 2D germanium-tin compound with a dumbbell structure (DB Sn₆Ge₄) make it energetically more stable than low-buckled stanene and germanene. DB Sn₆Ge₄ is a normal insulator with a direct gap of 242 meV. However, upon surface hydrogenation (resulting in DB Sn₆Ge₄H₄), it converts a QSH insulator characterized by a topological invariant of Z₂=1. The quantum phase transition from a normal insulator to a QSH insulator is related to the s-p band inversion induced by surface hydrogenation. The bulk band gap opened due to SOC can be as large as 235 meV which is much larger than the thermal motion energy at room temperature (~26 meV). The topological nontriviality is robust against substrate effects. The energetic stability and the phonon spectrum confirm the high stability and plausibility of DB Sn₆Ge₄H₄. These results are beneficial for achieving long-desired room-temperature QSH effect.
and fabrication of high-speed spintronics devices.

**Acknowledgements**

This work is supported by the National Basic Research Program of China (No. 2012CB932302), the National Natural Science Foundation of China (No. 91221101, 21433006), the 111 project (No. B13029), the Taishan Scholar Program of Shandong, and the National Super Computing Centre in Jinan.
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Figure caption

**Figure 1.** (a) Schematic representations (top and side views are presented in up and middle panels) and the ELF profiles (down panel) of DB Sn₆Ge₄. (b) Phonon spectrum of DB Sn₆Ge₄ along the high-symmetric points in the BZ.
Figure 2. Band structures and their enlarged views (right panel) of DB Sn6Ge4 (a) without and (b) with SOC. The energy at the Fermi level was set to zero.
Figure 3. (a) Top (top panel) and side views (middle panel) and the ELF profiles (down panel) of a DB Sn$_6$Ge$_4$H$_4$ unit cell. (b) Phonon spectrum of DB Sn$_6$Ge$_4$H$_4$ along the high-symmetric points in the BZ.
**Figure 4.** (a) The band structures without (top panel) and with SOC (down panel) of DB Sn₆Ge₄H₄, with their enlarged views of band lines shown in the right panel. (b) Orbital-resolved electron density states of DB Sn₆Ge₄H₄ without SOC. The DOS data were obtained by integrating the whole BZ. (c) and (d) are the isosurfaces of the electron wavefunctions of the VBM and VBM -1 at the Γ point. The isosurfaces of the CBM at the Γ point atom resemble those of the VBM and thus are not presented. The energy of the Fermi level was set to zero.
Figure 5. Comparison between the orbital-resolved band structures of (a) DB Sn₆Ge₄ and (b) DB Sn₆Ge₄H₄ in the vicinity of the Γ point obtained from DFT calculations without and with SOC. The superscript " + " and " − " denotes the parity of the wavefunctions. The contributions of different atomic orbitals are indicated by the dots in different colors and sizes. S-orbital of Sn (red), pₓᵧorbital of Sn (green), and pₓz-orbital of Sn (blue). (c) Schematic diagram of the electronic state evolution of DB Sn₆Ge₄ (left panel) and Sn₆Ge₄H₄ (middle and right panel) near the Fermi level. The red line denotes the Fermi level, and the effects of (I) hydrogenation and (II) SOC on the alignment of electronic states are indicated. (d) Brillouin zone of DB Sn₆Ge₄ and DB Sn₆Ge₄H₄. The signs (+/−) are associated with δᵢ at the time-reversal invariant momenta. b₁ and b₂ are the primitive reciprocal lattice vectors.
Figure 6. (a) Top and side views of a DB Sn₆Ge₄H₄ grown on a h-BN substrate. Electronic band structures of DB Sn₆Ge₄H₄ on h-BN substrate (b) without and (c) with SOC. The energy at the Fermi level was set to zero.