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## Minimum size for a nanoscale temperature discriminator based on a thermochemical system

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#### Abstract

What are the limits of size reduction for information processing

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devices based on chemical reactions? In this paper, we partially answer this question. We show that a thermochemical system can be used to design a discriminator of the parameters associated with oscillations of the ambient temperature. Depending on the amplitude and frequency of the oscillations, the system exhibits sharp transitions between different types of its time evolutions. This phenomenon can be used to discriminate between different parameter values describing the oscillating environment. We investigate the reliability of the thermochemical discriminator as a function of the number of molecules involved in the reactions. A stochastic model of chemical reactions and heat exchange with the neighborhood, in which the number of molecules explicitly appears, is introduced. For the selected values of the parameters, thermochemical discriminators operating with less than  $10^5$  molecules appear to be unreliable.

### 1 Introduction

According to a common opinion [1], the future information processing devices based on chemical reactions will be reduced to the nanoscale level, so that they can be incorporated into nano-capsules containing drugs. It is anticipated that nano-capsules will be small enough to be transported via blood vessels. The, yet smaller, sensing organs of nano-capsules will be able to scan the body for potential infections and release the drug exactly when necessary. Is such a scenario a fiction? What are the limits of size reduction for information processing devices based on chemical reactions? At the nanoscale, problems related to the stochasticity of chemical reactions are inevitable. In this paper we address the problem of size reduction for chemical information processing devices by considering a simple model of discriminator.

Different types of chemical information processing devices have been reported in the literature [2]. Sensors based on chemical reactions are especially interesting, because they imitate typical tasks performed by living organisms, that surely use biochemical reactions to gain information about the environment. A signal coming from a chemical sensor can be continuous or discrete. For example, one can get information on the distance to a source of excitations by comparing the frequency of pulses excited in a few identical excitable channels [3]. An excitable chemical medium can be also applied as a sensor of critical changes in time-dependent medium parameters. On the one hand, it has been demonstrated [4] that an excitation pulse can propagate in the medium with a slowly decreasing excitability level. On the other hand, if exactly the same decrease in system excitability rapidly occurs, then the excitation pulse vanishes. Therefore, by observing the distance traveled by a pulse, one is able to determine the rate of temporal changes in the medium excitability. The simplest sensors are binary discriminators (or binary classifiers). They are capable of determining if the conditions at which reactions proceed belong to a given class or not. For example, an excitable medium with a propagating pulse can be regarded as a discriminator if the rate of temporal changes in its excitability exceeds a given threshold value.

The two examples mentioned above illustrate the sensing potential of a spatially distributed medium. However, a homogeneous chemical medium can be also used as a discriminator of the conditions at which the reactions proceed. In a recent paper [5] we considered a dynamical system with hysteresis as a prototype of a decision making automaton. Systems with hysteresis, commonly observed in chemistry and biology [6, 7], are natural candidates for binary classifiers. In a system with hysteresis, one can distinguish two different classes of stable states  $S_1$  and  $S_2$ . System evolution toward a state from a particular class is determined by the value of the control parameter  $\lambda$  and the initial condition. Let us assume that the increase in the control parameter  $\lambda$  above a threshold  $\lambda_1$  triggers the transition from  $S_1$  to  $S_2$ . The reverse transition from  $S_2$  to  $S_1$  occurs if the value of the control parameter drops below  $\lambda_2$ . A system with such properties can be obviously used as a discriminator between the values of the control parameter. For example, if the initial state belongs to  $S_1$  and if after some time we observe the system in a state belonging to  $S_2$ , it means that at a certain moment the value of the control parameter necessarily exceeded  $\lambda_1$ . However, if only time monotonic changes in the value of the control parameter are considered, then the discriminating power of the system is reduced to only two values  $\lambda_1$  and  $\lambda_2$ .

We have recently demonstrated [5] that the suitability of a dynamical system with a sigmoidal kinetics for discrimination oriented tasks is enhanced if periodic perturbations are imposed. A periodically perturbed bistable system oscillates. The location of the system oscillations in the parameter space and the oscillation amplitude depend on the amplitude and frequency of the perturbation. In particular, oscillations between states in the class  $S_1$ are observed if the strength of the oscillating stimulus never exceeds  $\lambda_2$ . On the contrary, when the stimulus remains greater than  $\lambda_1$ , oscillations between states in the class  $S_2$  are obtained. The numerical simulations of the reaction kinetics defined by a third order rational function revealed a non-trivial property of a system with hysteresis: a marginal change of the inflow parameters (amplitude or frequency) can force a sharp transition between oscillations belonging to different classes. The transition occurs in a narrow range of perturbation parameters. The change between oscillation types can be easily distinguished and therefore, a chemical reaction exhibiting hysteresis can be used as a discriminator between the properties of the applied oscillatory perturbation.

In this paper we address the issue of the minimum size of a thermochemical discriminator. We are concerned with a thermochemical model that exhibits complex behaviors. Its time evolution changes depending on the rate constants of the reaction and the temperature of the environment. The model has been shown to possess oscillating behaviors, excitability and bistability [8, 9, 10]. In this work, the parameters have been selected so that the system has a single stationary state, but small changes in the ambient temperature

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can make it bistable. Therefore, information about the temperature of the environment can be extracted from the observed state of the system and we demonstrate that the system can be used as a discriminator between the values of the amplitude and frequency of the ambient temperature oscillations. This approach fits within the more general context of temperature estimation in small systems [11, 12] but does not call into question the standard definition of temperature in systems at local equilibrium.

The paper is organized as follows. In Section II, the ability of the considered thermochemical model to assess the parameters controlling the oscillations of the ambient temperature is proven in the framework of a deterministic approach. However, the deterministic description is valid in the thermodynamic limit, where the number of molecules tends to infinity. In systems containing a small number of molecules, the internal fluctuations can influence dynamics and reduce sensor reliability. In order to quantitatively estimate the effect of system size, we introduce a stochastic description in the Section III. The description of the internal fluctuations in isothermal chemical systems is well known. Reactions are considered as Markovian processes associated with discrete jumps for the number of particles. The master equation that describes these processes can be easily written [13, 14, 15] and solved [16]. In contrast, stochastic approaches including temperature as a random variable are not simple because temperature undergoes continuous, unbound jumps when the system exchanges energy with the surrounding. Different stochastic descriptions of thermochemical systems have been tempted [17, 18, 19]. Here, we use an original approach based on the kinetic theory [20] that we developed to define the transition probabilities associated with an exothermic reaction and the heat exchange with a thermostat [21]. The resulting integro-differential master equation is the basis for kinetic Monte-Carlo simulations of the system evolution. Performing simulations for different numbers of molecules, we can estimate how the performances of the discriminator depend on system size. The results of the numerical simulations are discussed in Section IV.

## 2 A temperature discriminator based on a thermochemical system

We consider a thermochemical system [8], inspired by a model introduced by Volter and Salnikov [22], that describes a perfect mixture composed of a reactant A, a product B, and a catalyst C, engaged in the following reactions:

$$A + C \quad \stackrel{k_1}{\to} \quad B + C + heat \ q \tag{1}$$

$$B \xrightarrow{k_2} A \tag{2}$$

The exothermic reaction (1) proceeds in bulk and is accompanied by a heat release q. The second reaction occurs on the surface and mimics unspecified mechanisms of reactant supply and product removal. Moreover, it is assumed that the system is open to energy exchange through its boundary with the environment represented by a thermostat at the temperature  $T_b$ . At the macroscopic level, the state of the system is characterized by two variables, the number density  $n_A$  of species A and the temperature, T. The number density,  $n_C$ , of the catalyst C does not change in time and the total number density,  $n = n_A + n_B + n_C$ , also remains constant. The deterministic dynamics is governed by the following balance equations for a system of N particles confined by volume V and surface S:

$$V\frac{dn_A}{dt'} = -k_1 n_A n_C V + k_2 n_B S \tag{3}$$

$$\frac{3}{2}nVk_B\frac{dT}{dt'} = k_1n_An_CVq - \kappa nSk_B(T - T_b) \tag{4}$$

where  $\kappa$  is the coefficient of heat exchange of Newton's law of cooling and  $k_B$  denotes the Boltzmann constant. Equation (3) describes the time evolution of the number density of A and Eq. (4) represents the time evolution of the total energy, equal to  $3/2Nk_BT$  as for an ideal gas containing N particles.

Equations (3,4) describe an activator-inhibitor system in which the temperature plays the role of the activator and the number density, of the inhibitor [8, 23]. For the following analysis we assume that reaction (1) is thermally activated. Its rate constant  $k_1$  is described by the Arrhenius law and involves an activation energy  $E_A$ . Reaction (2) is a non-activated process associated with the rate constant  $k_2$ . The kinetic theory of gases [20, 21] leads to the following dependence of the rate constants and coefficient of heat exchange on the temperature:

$$k_1 = k_1^0 \sqrt{T} \exp\left(\frac{-E_A}{k_B T}\right) \tag{5}$$

$$k_2 = p_2 \kappa \tag{6}$$

$$\kappa = \kappa^0 \sqrt{T} \tag{7}$$

where the constants  $k_1^0$  and  $\kappa^0$  are independent of temperature and  $p_2$  is the coefficient determining the probability of reaction (2) on the walls.

Equations (3,4) can be simplified by introducing scaled variables for the

concentration of A reagent,  $\alpha = n_A/(n_A + n_B)$  and the temperature,  $\theta = k_B T/E_A$ :

$$\frac{d\alpha}{dt} = \sqrt{\theta} \Big( -\alpha \exp(-1/\theta) + K_2(1-\alpha) \Big)$$
(8)

$$\frac{d\theta}{dt} = \frac{2}{3}Q\sqrt{\theta} \Big(\alpha \exp(-1/\theta) - K_0(\theta - \theta_b)\Big)$$
(9)

where  $t = k_1^0 n_c \sqrt{E_A/k_B} t'$ ,  $K_2 = p_2 \kappa^0 S/(k_1^0 n_C V)$ ,  $Q = q(n - n_C)/(E_A n)$ ,  $K_0 = \kappa^0 S/(Qk_1^0 n_C V)$ , and  $\theta_b = k_B T_b/E_A$ . Although simple, this model exhibits very rich dynamical behaviors [23, 24, 25, 26].

The nullclines  $\alpha_{\alpha}(\theta)$  and  $\alpha_{\theta}(\theta)$  associated with Eqs. (3,4) read:

$$\alpha_{\alpha}(\theta) = \frac{K_2}{K_2 + \exp(-1/\theta)}$$
(10)

$$\alpha_{\theta}(\theta) = K_0(\theta - \theta_b) \exp(1/\theta)) \tag{11}$$

The intersections of the nullclines define the steady states of the system. The nullcline  $\alpha_{\alpha}(\theta)$  does not depend on  $\theta_b$ . The function  $\alpha_{\alpha}(\theta)$  decreases monotonically for all values of  $K_2 > 0$ . The shape of  $\alpha_{\theta}(\theta)$  depends on  $\theta_b$ . For  $\theta_b \ge 0.25$ , the nullcline  $\alpha_{\theta}(\theta)$  monotonically increases, whereas for  $\theta_b \in [0, 0.25]$ , it has an N-shape such that its maximum  $\theta_{\text{max}}$  and minimum  $\theta_{\text{min}}$  are given by:

$$\theta_{\max} = \frac{1}{2}(1 - \sqrt{1 - 4\theta_b}) \tag{12}$$

$$\theta_{\min} = \frac{1}{2} (1 + \sqrt{1 - 4\theta_b})$$
(13)

Let us notice that  $\theta_{\text{max}} < 0.5$  and  $\theta_{\text{min}} > 0.5$ . Therefore, the value  $\theta_{sep} = 0.5$  seems to be a natural choice to separate steady states belonging to different classes.

The following parameters are fixed at Q = 30,  $K_0 = 0.4$ , and  $K_2 = 1.2$ . Figure 1 shows the nullclines of the dynamical system given by Eqs. (10,11) for different values of  $\theta_b$  in the range [0.18, 0.21405]. In the considered range of  $\theta_b$  the system evolution qualitatively changes. For  $\theta_b < \theta_{c1} = 0.19185$  the system has a single stable stationary state associated with a low temperature smaller than 0.2161  $< \theta_{sep}$ . If  $\theta_{c1} = 0.19186 < \theta_b < \theta_{c2} = 0.21405$ , there are two stable states associated with a low and a high temperature and separated by an unstable state. Then, the time evolution depends on the initial condition and can terminate in each of the stable states. Finally, if  $\theta_b > \theta_{c2}$ , the system has a single stationary state associated with a high temperature larger than 0.76944  $> \theta_{sep}$ .

The system can be used for the discrimination between parameters characterizing the ambient temperature. Let us assume that changes in  $\theta_b$  occur on a much slower time scale than the relaxation in the thermochemical system. Let us fix the initial (at the time  $t_0$ ) temperature of the environment  $\theta_b(t_0)$  below  $\theta_{c1}$ . Initially, the system is in a steady state characterized by  $\theta(t_0) < \theta_{sep}$ . If, at a later time  $t_1$ , we observe the system in a state characterized by  $\theta(t_1) > \theta_{sep}$ , we conclude that, in the time interval  $[t_0, t_1]$ , the temperature of the environment exceeded  $\theta_{c2}$  during a sufficiently long time for the system to make a transition between the steady states. If the state observed at  $t_1$  is characterized by  $\theta(t_1) < \theta_{sep}$ , then the temperature of the environment never exceeded  $\theta_{c2}$  or dropped below  $\theta_{c1}$  before  $t_1$ . Therefore, the considered medium, like other systems with hysteresis, can be used to spot changes in the control parameter  $\theta_b$ .



Figure 1: Nullclines  $\alpha_{\alpha}(\theta)$  and  $\alpha_{\theta}(\theta)$  in the parameter space  $(\theta, \alpha)$  for a few selected values of the ambient temperature  $\theta_b$ . The solid line shows  $\alpha_{\alpha}(\theta)$ . The dashed lines from the top down illustrate  $\alpha_{\theta}(\theta)$  for  $\theta_b = 0.18, 0.19185, 0.20$  and 0.21405 respectively.

The functionality of the thermochemical discriminator based on Eqs. (8,9) can be significantly extended if we consider periodic changes in the temperature of the environment that occur on a time scale similar to the one characterizing the dynamics of the system. Specifically, we assume that  $\theta_b(t)$  is a continuous function of the form:

$$\theta_b(t) = \theta_{b0} \quad \text{for} \quad t < 0 \tag{14}$$

$$\theta_b(t) = \theta_{b0} + A_T (1 + \sin(2\pi f t + \phi_0)) \quad \text{for} \quad t \ge 0$$
(15)

The second term in Eq. (15) is not negative and  $\theta_b$  oscillates above  $\theta_{b0}$  with an amplitude of  $2A_T$  and a frequency f. For the value  $\phi_0 = 3\pi/2$  used in the numerical simulations, one has  $\theta_b(t=0) = \theta_{b0}$ .

We set  $\theta_{b0} = 0.18$  in such a way that, for t < 0, the system remains in the stable stationary state ( $\alpha_s = 0.9951$ ,  $\theta_s = 0.1946$ ) associated with a low temperature. We investigate how the thermochemical system answers to oscillations of the ambient temperature for a fixed value of the amplitude  $A_T$ . For  $A_T < 0.017$ , one has  $\theta_{b0} + 2A_T < \theta_{c2}$  and the steady state stays on the lower temperature branch of the  $\alpha_{\alpha}(\theta) = 0$  nullcline: At all times, one has  $\theta(t) < \theta_{sep}$ . Conversely, for  $A_T > 0.017$ , one has  $\theta_{b0} + 2A_T > \theta_{c2}$ , so that there are times when the steady state corresponds to a high temperature larger than  $\theta_{sep}$ . In this case, different dynamical behaviors of the thermochemical system are observed, depending on the frequency f of the ambient temperature oscillations. Time series of the system temperature  $\theta(t)$  for a fixed amplitude  $A_T = 0.03$  and different frequencies f are shown in Fig. 2.



Figure 2: Temperature  $\theta(t)$  of the system as a function of time t for a constant amplitude  $A_T = 0.03$  and selected values of the frequency f of the ambient temperature oscillations. The oscillation type is indicated in each case. The frequencies at which transitions between different oscillation types occur are indicated on the f-axis.

For sufficiently small frequencies,  $f \leq 0.16$ , the changes in the ambient temperature are slow enough for the thermochemical system to adjust to these changes. As a result, the oscillations of the system temperature may extend over the interval [0.18, 1]. If a long time interval I is considered, then  $\min_{t \in I} \theta(t) < \theta_{sep}$  and  $\max_{t \in I} \theta(t) > \theta_{sep}$ . In the following, we refer to such large-amplitude oscillations as type-I oscillations. For a high frequency, the sinusoidal term time-averages to zero and the effective ambient temperature is  $\theta_b \approx \theta_{b0} + A_T < \theta_{c2}$ . Therefore, the time evolution initiated from  $(\alpha_s, \theta_s)$ converges to small oscillations on the lower temperature branch of the  $\alpha_{\alpha}(\theta)$ nullcline. In this case, one has  $\theta(t) < \theta_{sep}$  at every time  $t \in I$ . Such a behavior is shown in Fig. 2 for  $f \geq 0.3049$  and further referred to as type-III oscillations. For intermediate frequencies, the system exhibits oscillations on the higher temperature branch obeying  $\theta(t) > \theta_{sep}$ , as illustrated for f = 0.028 and f = 0.3048 in Fig. 2. Oscillations on the higher temperature branch are of type II.

The transition between type-I and type-II oscillations occurs when  $\min_{t \in I} \theta(t)$ exceeds  $\theta_{sep}$ . For the amplitudes  $A_T \geq 0.0276$  the function  $\min_{t \in I} \theta(t)$  continuously increases with frequency, therefore this transition is a smooth one. Similarly the transition between type-I and type-III oscillations, that occurs when  $\max_{t \in I} \theta(t) < \theta_{sep}$  is a smooth one because for the amplitudes  $A_T < 0.0276$  the function  $\max_{t \in I} \theta(t)$  is continuous, decreasing function of frequency. The values of parameters  $(f, A_T)$  at which the transitions mentioned above occur are marked by thick dashed lines on Fig.3. A precise measurement of temperature is required to detect these transitions in a real system. On the other hand the transition between type-II and type-III oscillations, indicated by a thick solid line in Fig.3 is very sharp (cf. Fig. 2 and oscillations for f = 0.3048 and f = 0.3049). This transition is easy to detect because large changes in both  $\max_{t \in I} \theta(t)$  and  $\min_{t \in I} \theta(t)$  are involved. Therefore it can be used for discrimination oriented tasks.

In order to classify the oscillation types obtained for different frequencies f and amplitudes  $A_T$ , the set of Eqs. (8,9) is numerically solved with the initial condition ( $\alpha_s = 0.9951$ ,  $\theta_s = 0.1946$ ). The values of  $\min_{t \in I} \theta(t)$  and  $\max_{t \in I} \theta(t)$  are calculated within the time interval I = [500, 2000] time units. Figure 3 shows the domains associated with the different oscillation types in the parameter space  $(f, A_T)$ .

0.040

0.036

0.032

0.028

0.024

0.020

0.0

amplitude



Figure 3: Phase diagram illustrating the different oscillation types of the thermochemical system in the parameter space  $(f, A_T)$ . The thick dashed and solid lines divide the parameter space into regions where a given oscillation type is observed. The thick solid line separating the oscillations of type II and type III can be used for sensing the parameters describing periodic changes in the ambient temperature. The thin dashed line shows  $A_T = 0.03$  and refers to results from Fig. 2.

## 3 Master equation associated with the thermochemical system

The description of the internal fluctuations in isothermal chemical systems is well known. Reactions are considered as Markovian processes associated with discrete jumps for the number of particles and the master equation that describes these processes can be easily written [13, 14, 15] and numerically simulated [16]. The major difficulty with the stochastic analysis of a thermochemical model, if compared to a system involving thermoneutral chemical processes associated with discrete particle number jumps, is related to the continuous, unbound temperature jumps associated with Newtonian exchange of energy. We used the kinetic theory to determine the transition probabilities associated with exothermic reactive processes and exchanges with a thermostat and already studied the effects of the temperature fluctuations in different thermochemical systems [21, 25, 27].

In the case of the thermochemical system of interest, the evolution is described by the probability  $P(\theta, N_A, t)$  of finding the system at the temperature  $\theta$  with  $N_A$  particles of reactant A at time t. The total number of particles  $N = N_A + N_B + N_C$  controls the strength of the internal fluctuations. The master equation has the following integro-differential form [21, 25, 27]:

$$\frac{\partial}{\partial t} P(\theta, N_A, t) = \int_{\Delta \theta < \theta} d(\Delta \theta) P(\theta - \Delta \theta, N_A - \Delta N_A, t) w(\theta - \Delta \theta, N_A - \Delta N_A \to \theta, N_A) 
- P(\theta, N_A, t) \int_{\Delta \theta > -\theta} d(\Delta \theta) w(\theta, N_A \to \theta + \Delta \theta, N_A + \Delta N_A)$$
(16)

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in which the transition probability w is composed of three terms:

$$w(\theta, N_A \to \theta + \Delta \theta, N_A + \Delta N_A) = w_0(\theta, N_A \to \theta + \Delta \theta, N_A)$$
$$+w_2(\theta, N_A \to \theta + \Delta \theta, N_A + 1)$$
$$+w_1(\theta, N_A \to \theta + \Delta \theta, N_A - 1) (17)$$

The first term,  $w_0$ , is related to the Newtonian exchange of energy without reaction and the two last ones,  $w_2$  and  $w_1$ , are connected with reactions (2) and (1), respectively. The transition probability  $w_0$  for exclusive energy exchange is a continuous function of the temperature jump  $\Delta \theta$  and does not involve any chemical change. The explicit expression for  $w_0$  has been derived from the collision rates under the assumption that the velocity distribution of particles remains Maxwellian [21, 27]. The transition probability  $w_0$  can be cast into the following form:

$$w_0(\theta, N_A \to \theta + \Delta\theta, N_A) = \left(\frac{1}{2}NQK_0 - N_BK_2\right)\sqrt{\theta}\,\omega(\theta, \Delta\theta) \qquad (18)$$

where  $\omega(\theta, \Delta \theta)$  is the probability distribution of temperature jumps  $\Delta \theta$  at temperature  $\theta$ :

$$\omega(\theta, \Delta \theta) = \frac{\theta \theta_b}{(\theta + \theta_b)^3} \left( 2 + \frac{(\theta + \theta_b)(\frac{3}{2}N)|\Delta \theta|}{\theta \theta_b} \right) \\
\times \frac{3}{2}N \begin{cases} \exp\left(-\frac{3}{2}N\frac{|\Delta \theta|}{\theta}\right) & \text{for } \Delta \theta < 0 \\ \exp\left(-\frac{3}{2}N\frac{\Delta \theta}{\theta_b}\right) & \text{for } \Delta \theta > 0 \end{cases}$$
(19)

Equation (18) does not include the inelastic particle-surface collisions that are related to energy transfer and also to reaction (2). The transition probability  $w_2$  corresponding to the latter processes describes an increment of A population by  $\Delta N_A = 1$  combined with a temperature jump,  $\Delta \theta$ . According to Eq. (19),  $w_2$  can be written as

$$w_2(\theta, N_A \to \theta + \Delta\theta, N_A + 1) = N_B K_2 \sqrt{\theta} \,\omega(\theta, \Delta\theta) \tag{20}$$

In contrast, the transition probability related to reaction (1) only involves a fixed, temperature increase,  $\Delta \theta_1 = Q/(\frac{3}{2}N)$ , following after the heat release q. The decrement of  $N_A$  associated with reaction (1) is  $\Delta N_A = -1$ . The transition function  $w_1$  has the standard form [13] following from the frequency of collisions related to reaction (1):

$$w_1(\theta, N_A \to \theta + \Delta\theta, N_A - 1) = N_A \sqrt{\theta} \exp\left(-1/\theta\right) \delta(\Delta\theta - \Delta\theta_1)$$
(21)

The kinetic Monte Carlo method used to simulate a master equation for discrete jumps is well founded [16] and its appropriate modification for the integro-differential form given in Eq. (16) is presented in detail in references [21, 25]. We briefly recall the main steps of the procedure. An initial temperature  $\theta$  and an initial number of molecules  $N_A$  are chosen. The escape rate from the state  $(\theta, N_A)$ ,

$$W(\theta, N_A) = \int_{\Delta\theta > -\theta} w(\theta, N_A \to \theta + \Delta\theta, N_A + \Delta N_A) d(\Delta\theta), \qquad (22)$$

is computed to select the time  $t = -\ln(r)/W(\theta, N_A)$  at which a process takes place, where r is a uniformly distributed random number in the interval (0, 1]. The probabilities of the three possible processes, non-reactive heat exchange, surface reaction (2), and exothermic reaction (1), are considered with relative weights proportional to the corresponding terms in Eq. (17). The system state is then modified according to the selected process and the transition rates are updated. The procedure is repeated in order to generate a stochastic trajectory. 4

of molecules

# Sensor reliability as a function of number Kinetic Monte Carlo simulations of the master equation given in Eq. (16)

are performed for different parameter values in the three parameter domains shown in Fig. 3. Stochastic time series for the number  $N_A$  of A particles and temperature  $\theta$  are generated. In the following, we focus on the time evolution of the temperature  $\theta$  of the system and begin with the presentation of non trivial fluctuation effects observed for parameter values for which the deterministic analysis predicts small-amplitude oscillations on the low temperature branch (type III).



Figure 4: Stochastic time series for temperature  $\theta(t)$  obtained by simulations of the master equation for oscillations of the ambient temperature of amplitude  $A_T = 0.03$  and frequency f = 0.7, and different numbers of particles: (a) N = 100 (purple) and  $N = 70000^2$  (black), (b) N = 5000.



Figure 5: Probability distribution function of temperature obtained by simulations of the master equation for oscillations of the ambient temperature of amplitude  $A_T = 0.03$  and frequency f = 0.7, and different numbers N of particles (the axis represents the common logarithm of N).

For high frequencies and moderate amplitudes, for which the deterministic approach predicts type-III oscillations, but relatively close to the boundary with the parameter domain associated with oscillations of type II, exotic behaviors emerge from the stochastic description, as illustrated in Figs. 4 and 5. For very small systems with N < 2000, the distribution of temperature has a single peak centered on a low temperature and a long right tail. This last property accounts for the high spikes observed in the time series given in Fig. 4(a) for N = 100. As the number of particles increases, for  $2000 \leq N \leq 10000$ , the distribution of temperature becomes bimodal with one peak centered on a low temperature and one peak centered on a high temperature. In this case, the typical shape of the time series is given in Fig. 4(b) and presents a random succession of periods of forced oscillations around either a small or a high temperature. For  $N \simeq 15000$ , the distribution becomes unimodal again, but is now centered on a high temperature: Oscillations on the higher temperature branch (type II) are observed. For 20000 < N < 60000, the distribution is again bimodal with a shrinking right peak and a rising left peak as N increases. The waiting time separating the jumps between oscillations on the lower and the higher branch becomes longer and obtaining a satisfying statistics requires a considerable computational effort that we reserve for the next parameter choice. Finally, for sufficiently large values  $N \geq 70000$  of the number of particles, a narrow peak centered on a low temperature is observed, in agreement with the smallamplitude oscillations on the low temperature branch observed in Fig. 4(a)for N = 70000. Hence, as the size of the system increases, we observe a first transition from oscillations of type III to oscillations of type II and a second abrupt transition from oscillations of type II to oscillations of type III. The deterministic behavior is recovered for  $N\simeq 70000$ .



Figure 6: Stochastic time series for temperature  $\theta$  obtained by simulations of the master equation for oscillations of the ambient temperature of amplitude  $A_T = 0.027$  and frequency f = 0.3, and different numbers of particles: (a) N = 100 (purple) and N = 100000 (B4ack), (b) N = 30000.



Figure 7: Probability distribution function of temperature obtained by simulations of the master equation, for oscillations of the ambient temperature of amplitude  $A_T = 0.027$  and frequency f = 0.3, and different numbers N of particles (the axis represents the common logarithm of N).



Figure 8: Probability of finding the temperature  $\theta$  of the system characterized by the probability distribution given in Fig. 7 in different intervals as a function of  $\log_{10} N$ . The crosses and the dotted line show the probability of finding the system with a low temperature  $\theta \in [0.0, 0.42]$ . The squares and the solid line give the probability of finding the system with a high temperature  $\theta > 0.42$ .

Figures 6-8 show the behavior observed in the same parameter domain as in Figs. 4 and 5 but for smaller frequency values. As the number of particles increases, the analogous succession of two reverse transitions is observed, from oscillations on the lower temperature branch to oscillations on the higher temperature branch and vice versa. In addition, a splitting of the peaks of the distribution is observed in Fig. 7, due to the longer time spent around the extrema of the forced oscillations. As shown in the time series of Fig. 6(b), the temperature oscillates between the averaged minimum value  $\langle \theta_m^l \rangle = 0.22$ and the averaged maximum value  $\langle \theta_M^l \rangle = 0.33$  on the lower temperature branch, which coincides with the abscissa of the local maxima of the left peak in Fig. 7. Similarly, temperature oscillates between  $\langle \theta_m^h \rangle = 0.63$  and  $\langle \theta_M^h \rangle = 0.83$  on the higher temperature branch in Fig. 6(b), which coincides with the temperatures associated with the local maxima of the right peak in Fig. 7.

The nonintuitive similarity of the behaviors observed for very small and high values of the number of particles N is clearly illustrated in Fig. 8, which presents the probability of finding the system in different intervals of temperature as the function of  $\log_{10} N$ . The probability of observing the system at a low temperature ( $\theta < 0.42$ ), reaches maximum values for both N = 100 and N = 100000. The transition from oscillations on the higher temperature branch to oscillations on the lower temperature branch that is observed around N = 65000 is abrupt. We conclude that a moderate internal noise tends to stabilize oscillations on the higher temperature branch in the parameter domain where oscillations on the lower temperature branch are predicted by the deterministic approach.



Figure 9: Stochastic time series for temperature  $\theta$  obtained by simulations of the master equation for oscillations of the ambient temperature of amplitude  $A_T = 0.025$  and frequency f = 0.1, and different numbers of particles, N =100 (purple) and N = 50000 (black).



Figure 10: Probability distribution function of temperature obtained by simulations of the master equation for oscillations of the ambient temperature of amplitude  $A_T = 0.025$  and frequency f = 0.1, and different numbers of particles.

Figures 9 and 10 reveal yet another type of non trivial stochastic behavior observed for low values of both the amplitude and the frequency of the forcing, in the parameter domain where the deterministic approach predicts large-amplitude oscillations of type I. For  $A_T = 0.025$  and f = 0.1, a single transition is observed as system size increases. For small values of the number of particles, N < 1000, the distribution of temperature given in Fig. 10 has a single peak and a long right tail. The time series obtained for N = 100 is shown in Fig. 9. It closely resembles the ones given in Figs. 4 and 6 and corresponds to oscillations on the lower temperature branch randomly perturbed by high spikes. We conclude that a very strong noise tends to stabilize the lower temperature branch, even in the parameter domain where the deterministic equations predicts large-amplitude oscillations. As N increases, the distribution becomes more flat in a well-defined interval of temperature and the long tail disappears: The large-amplitude oscillations predicted by the deterministic analysis are recovered for N = 50000. The longer time spent around the extrema of the oscillations shown in Fig. 9 explains the small maxima of the temperature distribution observed for  $\theta \simeq 0.3$  and  $\theta \simeq 0.9$  in Fig. 10.

As we have presented in details the effects of the internal fluctuations on the thermochemical system with oscillations of the ambient temperature, we are able to propose a simple criterion allowing us to use the thermochemical system as a discriminator. First of all, we omit very small values of N and consider  $N \ge 20000$ . Secondly, we introduce the probability  $P(\theta < \theta_{sep})$ that, during a given time interval (here for  $t \in [500, 5000]$ ) the temperature  $\theta$  along a stochastic trajectory remains smaller than the threshold  $\theta_{sep} = 0.5$ 

introduced in the deterministic approach. Finally, we differentiate 5 classes of behavior, depending on the values of the probability  $P(\theta < \theta_{sep})$ . Oscillations on the higher temperature branch of type II are attributed to the case where  $P(\theta < \theta_{sep}) \leq 10^{-4}$ . Intermediate behaviors between type-II and type-III oscillations, observed when the type-II oscillations randomly switch into less stable type-III oscillations (cf. Fig. 4b), are associated with the case where  $10^{-4} < P(\theta < \theta_{sep}) \leq 0.1$ . Large-amplitude oscillations of type I are associated with the case  $0.1 < P(\theta < \theta_{sep}) \leq 0.9$ . This case also describes systems for which the stabilities of low- and high- temperature oscillations are similar. Intermediate behaviors between type-II and type-III oscillations are associated with the case where  $0.9 < P(\theta < \theta_{sep}) \leq 0.9999$ . In this case the stability of low-temperature oscillations is larger than the one of high-temperature oscillations, but random switches between these low- and high-oscillations are still frequent. Small-amplitude oscillations on the lower temperature branch of type III correspond to the case where  $0.9999 \leq P(\theta < \theta_{sep})$ . It is to be noted that such an easy-to-implement discriminating protocol is less accurate than the computationally demanding determination of the probability distribution of temperature. In particular, the criterion does not discriminate between large-amplitude oscillations and the random succession of oscillations on the lower and the higher branches. The master equation was solved to generate a stochastic trajectory during a time interval of 5000 units and the results of the first 500 units were omitted in the calculation of the probability  $P(\theta < \theta_{sep})$ . Figure 11 summarizes the results obtained for  $20000 \le N \le 10^6$ . The domain of stability of the higher temperature steady state is less affected by the fluctuations which

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justifies that we did not present a detailed analysis for parameter values in domain II. For intermediate system sizes,  $N \simeq 20000$ , the stabilization of the higher temperature steady state in the domain of stability of the lower steady state is observed for high frequencies as already shown in Figs. 4-8, whereas the stabilization of the large-amplitude oscillations are obtained for small frequencies, as illustrated in Figs. 9, 10. The deterministic behavior is recovered for  $N = 10^6$  over the whole range of parameter values. In this case, even the boundary between oscillations of types II and III is detected by the discriminator (crosses in Fig. 11).

We conclude that the effects of the internal fluctuations in the thermochemical system with oscillations of the ambient temperature do not limit its use as a discriminator if the particle number is larger than  $N = 10^5$ .



Figure 11: The solid and dashed lines separate the domains associated with different oscillation types predicted by the deterministic approach in the parameter space  $(f, A_T)$ . The symbols give results deduced from simulations of the master equation over 4500 time units for a few selected values of the number N of particles. Open triangles correspond to  $P(\theta < \theta_{sep}) \leq 10^{-4}$  (type-II oscillations), solid triangles  $t^{35} 10^{-4} < P(\theta < \theta_{sep}) \leq 0.1$ , crosses to  $0.1 < P(\theta < \theta_{sep}) \leq 0.9$  (type-I oscillations), solid circles to  $0.9 < P(\theta < \theta_{sep}) \leq 0.9999$ , and open cycles to  $0.9999 \leq P(\theta < \theta_{sep})$  (type-III oscillations).

## 5 Conclusions

—par In this paper, we address the problem of size reduction for a chemical information processing device. We present a simple thermochemical system that can discriminate between different types of periodic changes in the ambient temperature. The discriminator input is analog and contains two real variables: the amplitude  $A_T$  and the frequency f of periodic changes. In biology such input variables can, for example, represent the daily changes in the temperature of environment. The output is discrete and classifies the type of periodic changes of system temperature. The discrimination is based on the observation that, according to the deterministic analysis (cf. Figs. 2,3), the time evolution of the system temperature depends on  $A_T$  and f. The proper selection of the model parameters  $(K_0, K_2, Q)$  makes the time evolution of the discussed system extremely sensitive to small changes of the amplitude and the frequency in the region around the thick solid line shown in Fig. 3. Figures 3 and 11 illustrate that the change in ambient oscillation amplitude by 0.0005 unit can be amplified by the reactions over 1000 times and resulting medium oscillations differ by 0.5 temperature unit.

We have analyzed the reliability of discriminator as the function of the number of molecules involved in the reactions. For the selected values of reaction parameters a system composed of  $10^6$  molecules functioned as predicted by the deterministic kinetic equations. The results for  $2 \cdot 10^5$  and  $8 \cdot 10^4$  molecules are qualitatively similar to the deterministic ones, however the line separating the domains in the parameter space  $(f, A_T)$  corresponding to different types of evolution is shifted towards lower amplitudes (cf. Fig.

11). In our opinion, discriminators operating with  $10^5$  molecules can be still functional, but their design and analysis should be based on equations that include internal fluctuations [28]. It can be also noticed that the boundary region between different oscillation types in which the discrimination gives a fuzzy answer (it is marked by symbols separating open triangles and open circles in Fig. 11) enlarges with decreasing number of molecules. For  $N = 10^6$ there is no more than a single separating symbol. It means that for a fixed amplitude the range of frequencies, where malfunction of discriminator is expected, reduces to  $\Delta f \sim 0.02$  frequency unit. For  $2 \cdot 10^4$  molecules and  $A_T \sim 0.03$  the boundary region extends to the whole frequency range.

Simulations performed for yet smaller numbers of molecules indicate that the system evolution is so strongly affected by internal fluctuations that the observation of time evolution does not give information about the ambient temperature. We conclude that the considered discriminator needs more than  $10^5$  molecules for a reliable operation.

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#### References

A Chemical and Biological Information Technology (C<sup>hem</sup>B<sup>io</sup>IT)
 Science and Technology Roadmap http://www.cobra-

project.eu/uploads/2/0/8/9/20895162/chembioit\_roadmap\_draft\_280113.pdf

- [2] A. Henson, J. M. Parrilla Gutierrez, T. Hinkley, S. Tsuda, L. Cronin, Phil. Trans. R. Soc. A 373, 20140221 (2015); J. Gorecki, K. Gizynski, J. Guzowski, J. N. Gorecka, P. Garstecki, G. Gruenert, P. Dittrich, Phil. Trans. R. Soc. A 373, 20140219 (2015); C. L. Nehaniv, J. Rhodes, A. Egri-Nagy, P. Dini, E. Rothstein Morris, G. Horvth, F. Karimi, D. Schreckling, M. J. Schilstra, Phil. Trans. R. Soc. A 373, 20140223 (2015); A. Adamatzky, Phil. Trans. R. Soc. A 373 20140216 (2015).
- [3] J. Gorecki, J. N. Gorecka, K. Yoshikawa, Y. Igarashi, and H. Nagahara, Phys. Rev. E 72, 046201 (2005); K.Yoshikawa, H. Nagahara, T. Ichino, J. Gorecki, J. N. Gorecka and Y. Igarashi, Vol. 5, 5365 (2008).
- [4] M. Tanaka, H. Nagahara, H. Kitahata, V. Krinsky, K. Agladze, and K. Yoshikawa, Phys. Rev. E 76, 016205 (2007).
- [5] H. Ueno, T. Tsuruyama, B. Nowakowski, J. Gorecki, K. Yoshikawa, Chaos 25, 103115 (2015), early version available at http://arxiv.org/abs/1508.04600.
- [6] J.D. Murray, Mathematical Biology. I. An Introduction, 3rd ed, (Springer-Verlag, Berlin Heidelberg, 2002).
- M.A. Krasnoselsky, A.W. Pokrowsky, Sistemy s gisteresisom (Systems with Hysteresis, in Russian), (Nauka, Moscov, 1983).
- [8] A. L. Kawczynski and J. Gorecki, J. Phys. Chem. 1992, 96, 1060.
- [9] A. L. Kawczynski and J. Gorecki, J. Phys. Chem. 1993, 97, 10358.

- [10] J.Gorecki and A.L. Kawczynski, J. Phys. Chem. 1996, **100**, 19371-19376.
- [11] R. McFee, Am. J. Phys., 1973, **41**, 230234.
- [12] M. Falcioni, D. Villamaina, A. Vulpiani, A. Puglisi, and A. Sarracino, Am. J. Phys., 2011, 79, 777-785.
- [13] G. Nicolis and I. Prigogine, Self-Organization in Nonequilibrium Systems (Wiley, New York, 1977).
- [14] N. G. van Kampen, Stochastic Processes in Physics and Chemistry (North-Holland, Amsterdam, 1992).
- [15] C. W. Gardiner, Handbook of Stochastic Methods (Springer, Berlin, 2004).
- [16] D. T. Gillespie, Annu. Rev. Phys. Chem. 58, 35 (2007).
- [17] G. Nicolis, F. Baras, and M. Malek Mansour, in *Nonlinear Phenomena* in *Chemical Dynamics*, edited by A. Pacault (Springer, Berlin, 1981), p. 104.
- [18] G. Nicolis and M. Malek Mansour, Phys. Rev. A 29, 2845 (1984).
- [19] F. Baras, Ph.D. thesis, Université Libre de Bruxelles, 1985.
- [20] R. D. Present, *Kinetic Theory of Gases* (McGraw-Hill, New York, 1958).
- [21] B. Nowakowski and A. Lemarchand, Phys. Rev. E 64, 061108 (2001).
- [22] B. V. Volter and I. Y. Salnikov, Modeling and Optimization of Catalytic Processes (Nauka, Moscow, 1965) (in Russian).

- [23] A. L. Kawczynski and B. Nowakowski, Phys. Chem. Chem. Phys. 10, 289 (2008).
- [24] A. Lemarchand and B. Nowakowski, J. Phys.: Condens. Matter 19, 065130 (2007).
- [25] A. Kolbus, A. Lemarchand, A. Kawczynski, and B. Nowakowski, Phys. Chem. Chem. Phys. 12, 13224 (2010).
- [26] B. Nowakowski, A. Kawczynski, J. Phys. Chem. 109, 3134 (2005).
- [27] B. Nowakowski and A. Lemarchand Physica A **311**, 80 (2002).
- [28] H. Malchow, L. Schimansky-Geier, Noise and Diffusion in Bistable Nonequilibrium Systems Teubner Texte zur Physik 5, (Teubner Verlag, Leipzig 1986).