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# Configuration coordinate energy level diagrams of intervalence and metal-to-metal charge transfer states of dopant pairs in solids.

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Configuration coordinate diagrams, which are normally used in a qualitative manner for the energy levels of active centers in phosphors, are quantitatively obtained here for intervalence charge transfer (IVCT) states of mixed valence pairs and metal-to-metal charge transfer (MMCT) states of heteronuclear pairs, in solid hosts. The procedure relies on vibrational frequencies and excitation energies of single-ion active centers, and on differences between ion-ligand distances of donor and acceptor, which are attainable empirically or in *ab initio* calculations. The configuration coordinate diagrams of the Yb<sup>2+</sup>/Yb<sup>3+</sup> mixed-valence pair in Yb-doped YAG and the Ce<sup>3+</sup>/Yb<sup>3+</sup> heteronuclear pair in Ce,Yb-codoped YAG, are obtained and described. They are drawn from empirical data of the single-ions and their usefulness is discussed. The first diagram suggests that IVCT states of Yb<sup>2+</sup>/Yb<sup>3+</sup> pairs may play an important role in the quenching of the Yb<sup>3+</sup> emission and it provides the details of the quenching mechanism. The second diagram supports the interpretation recently given for the energy transfer from Ce<sup>3+</sup> to Yb<sup>3+</sup> in Ce,Yb-codoped YAG via a MMCT Ce<sup>4+</sup>-Yb<sup>2+</sup> state and it provides the details. The analyses of the two diagrams suggest the formation of Yb<sup>2+</sup>/Yb<sup>3+</sup> pairs after the Ce<sup>3+</sup>-to-Yb<sup>3+</sup> MMCT as responsible for the temperature quenching of the Yb<sup>3+</sup> emission excited with Ce<sup>3+</sup>(4*f* → 5*d*) absorption in Ce,Yb-codoped YAG.

Keywords: MMCT, IVCT, Ce, Yb, YAG, concentration quenching, temperature quenching, energy transfer

## I. INTRODUCTION

Electron transfer between two metal ions involved in a redox process is an important phenomenon in biology, chemistry, and material science. It is often known as metal-to-metal charge transfer (MMCT). When the two metal ions differ only in oxidation state, the conventional name of this process is intervalence charge transfer (IVCT).<sup>1</sup> [The term IVCT is used sometimes for MMCT between non-equivalent metal ions and also for electron transfer processes not involving metals; here we will adopt the conventional meaning and we will call IVCT only to the homonuclear, symmetric MMCT.]

MMCT states are considered to have an important role in energy transfer processes between optically active centers of doped solids and in non-radiative decays which can drastically change the optical behavior of materials.<sup>2</sup> The blue to near-infrared conversion in Ce<sup>3+</sup>,Yb<sup>3+</sup>-codoped YAG (Ref. 3) and the ultraviolet to greenish-blue or to red conversions in Pr<sup>3+</sup>-doped CaTiO<sub>3</sub> and CaZrO<sub>3</sub> (Ref. 4) are respective examples.

IVCT absorptions, which have been found in a large number of mixed-valence molecular compounds,<sup>5,6</sup> have also been reported in lanthanide activated phosphors like Ce<sup>3+</sup>-doped LaPO<sub>4</sub> (Ref. 7), and lanthanide mixed-valence solids like Na<sub>5</sub>Eu<sub>7</sub>Cl<sub>22</sub> (Ref. 8). And IVCT luminescence has recently been reported to exist as well, and to be responsible for the anomalous emissions of Ce<sup>3+</sup> in elpasolites<sup>9</sup> and Yb<sup>2+</sup> in fluorites.<sup>10</sup> In fact, the IVCT states of mixed-valence pairs in doped solids have a high potential for changing the absorptions, emissions, and non-radiative decays of the ma-

terials when the pairs are formed, because these states are intercalated between the regular states of single-ion active centers.<sup>9,10</sup> Hence the importance of a detailed knowledge of the IVCT states in materials where such mixed-valence pairs are likely to form. Among others, this could be the case of solids activated with Ce<sup>3+</sup>, Pr<sup>3+</sup>, Eu<sup>2+</sup>, or Yb<sup>2+</sup> lanthanide dopants, in which preventing the coexistence of several valence states is difficult.<sup>11,12</sup>

It is common to address the participation of MMCT and IVCT states in energy transfer, non-radiative decay, and radiative processes with the help of schematic configuration coordinate energy level diagrams<sup>2-4,7</sup> (cf. Fig. 4 in Ref. 7, for instance). Here, we present an alternative to make quantitative IVCT and MMCT configuration coordinate diagrams using structural and energetic data of the single-ion active centers, which are attainable empirically or in *ab initio* calculations. We discuss their meaning and use as interpretative tools for the issues mentioned above.

As bases for the elaboration of the IVCT and MMCT configuration coordinate diagrams, we take the vibronic model for the IVCT absorption of the two-state problem of a mixed-valence pair of Piepho *et al.*<sup>13</sup> and its extension to excited states, which was used to analyze state-of-the-art *ab initio* calculations of the diabatic potential energy surfaces of Ce<sup>3+</sup>/Ce<sup>4+</sup> and Yb<sup>2+</sup>/Yb<sup>3+</sup> pairs of dopant ions in solids.<sup>9,10</sup> The latter revealed that, even though adiabatic potential energy surfaces calculated with a full consideration of electronic couplings between electronic states of donor and acceptor centres are necessary for IVCT absorption and emission intensities and non-radiative decay rates, a great deal of quantitative spectroscopic information on the pairs can be attained at the diabatic level, i.e. without explicit consid-

eration of donor-acceptor electronic couplings. Hence, we focus here on the diabatic approximation to configuration coordinate diagrams.

We discuss the definition of the configuration coordinate of a single-ion active center and its corresponding energy level diagram in Sec. II. This serves as a basis for the definition of the IVCT configuration coordinate of a mixed-valence pair in Sec. III, where the corresponding energy level diagram is discussed. The same is done for the MMCT states of a heteronuclear metallic pair in Sec. IV. Detailed discussions of IVCT and MMCT sample cases are given:  $\text{Yb}^{2+}/\text{Yb}^{3+}$  mixed-valence pair in YAG, Sec. III B, and  $\text{Ce}^{3+}/\text{Yb}^{3+}$  heteronuclear pair in YAG, Sec. IV B.

## II. CONFIGURATION COORDINATE OF A SINGLE ION ACTIVE CENTER

Let us briefly discuss on the configuration coordinate of a single ion active center and its corresponding configuration coordinate diagram. This diagram is a very useful simplified representation of the variation in energy of the electronic levels of the active center with the nuclear displacements. In this simplified description, only one vibrational coordinate is used, which is called the configuration coordinate, and the diagram is aimed at providing gross details of the crossings and relative positions of the electronic levels. Fine details, such as Jahn-Teller distortions, demand the use of several vibrational coordinates.

Let us consider the energies of the electronic levels of an optically active center with a single ion in the absence of (Jahn-Teller) vibronic couplings between degenerate states. The potential energy surface of the active center in its ground state can be written as a function of its normal vibrational coordinates as:

$$E_0(Q_1, Q_2, \dots) = \frac{1}{2} \sum_{\mu}^{N_{TS}} k_{\mu 0} Q_{\mu}^2 + \frac{1}{2} \sum_{\nu}^{N_{NTS}} k_{\nu 0} Q_{\nu}^2 + \dots \quad (1)$$

In this expression, the ground state electronic energy at equilibrium is taken as a reference for the energy scale and the equilibrium structure is the reference for the vibrational coordinates. In Eq. 1, the leading terms are quadratic and diagonal, and, for convenience, we have divided the normal modes into  $N_{TS}$  totally symmetric,  $\{Q_{\mu}\}$ , and  $N_{NTS}$  non-totally symmetric,  $\{Q_{\nu}\}$ . Then, the energy of an excited electronic state,  $E_i$ , can be written as:

$$E_i(Q_1, Q_2, \dots) = E_{i,e} + \frac{1}{2} \sum_{\mu}^{N_{TS}} k_{\mu i} (Q_{\mu} - Q_{\mu i})^2 + \frac{1}{2} \sum_{\nu}^{N_{NTS}} k_{\nu i} Q_{\nu}^2 + \dots \quad (2)$$

Here,  $E_{i,e}$  is the the minimum-to-minimum excitation energy from the ground state (vertical offset),  $Q_{\mu i}$  is the vibrational offset of the  $Q_{\mu}$  totally symmetric normal mode with the ground state (horizontal offset), and it is clear that non-totally symmetric normal modes have null offsets (because

symmetry makes all linear  $Q_{\nu}$  terms of the electronic energy to vanish). Eq. 2 implies that non-totally symmetric normal modes only contribute to electronic energy difference if the respective force constants are different, e.g.  $k_{\nu 0} \neq k_{\nu i}$ . Usually, such contributions are much smaller than those due to vibrational equilibrium offsets. So, the leading term of  $E_i - E_0$  is due to the totally symmetric vibrational modes,

$$E_i(Q_1, Q_2, \dots) - E_0(Q_1, Q_2, \dots) = E_{i,e} + \frac{1}{2} \sum_{\mu}^{N_{TS}} k_{\mu i} (Q_{\mu} - Q_{\mu i})^2 - \frac{1}{2} \sum_{\mu}^{N_{TS}} k_{\mu 0} Q_{\mu}^2 + \dots \quad (3)$$

In some highly symmetric active centers, like  $O_h$  octahedral  $\text{ML}_6$  sites and cubic  $\text{ML}_8$  sites, only the breathing modes are totally symmetric. In these cases, if the vibration of the first coordination shell is the only relevant, one single breathing mode contributes in Eq. 3,

$$Q_{\text{breath}} = \frac{1}{\sqrt{n}} (\delta_{L_1} + \delta_{L_2} + \dots + \delta_{L_n}), \quad (4)$$

$n$  being the number of breathing ligands and  $\delta_{L_i}$  the increment of the  $\text{M-L}_i$  distance with respect to its value in the reference structure.

In a more general case, it is convenient to define a configuration coordinate. In effect, we can make a rotation (unitary transformation) of the  $N_{TS}$  totally symmetric vibrational coordinates into a new set  $\{Q_{\mu'}\}$ , such that one of the transformed coordinates connects directly the minima of the ground and excited states. Let us call this coordinate  $Q_{\text{eff}}$ . (We represent in Fig. 1 the original and transformed coordinates in the case of two totally symmetric coordinates  $Q_1$  and  $Q_2$ .) Then,  $E_i$  and  $E_0$  will have a horizontal offset in this coordinate  $Q_{\text{eff}}$ , but their offset in all other vibrational coordinates will be zero. Accordingly, only the  $Q_{\text{eff}}$  vibrational mode contributes to the leading term of the  $E_i - E_0$  energy difference,

$$E_i(Q_1, Q_2, \dots) - E_0(Q_1, Q_2, \dots) = E_{i,e} + \frac{1}{2} k_{\text{eff},i} (Q_{\text{eff}} - Q_{\text{eff},i})^2 - \frac{1}{2} k_{\text{eff},0} Q_{\text{eff}}^2 + \dots \quad (5)$$

$Q_{\text{eff}}$  is, then, *the configuration coordinate* of the single ion active center, because it is the only one with horizontal offset and the only one that contributes to the leading term of the electronic energy differences.

[We may remark that this configuration coordinate for the potential energy surfaces of two states (ground and excited) is totally equivalent to the reaction coordinate defined between two minima of the potential energy surface of a chemical reaction (reactants and products). Accordingly, for higher-order approximations to the state energies in Eq. 3, the configuration coordinate is not exactly the straight line connecting the minima, but the curve that connects the lowest energy crossing point between the two surfaces with the two minima, with maximum descendent slope. However, we will assume the quadratic approximation is valid for our purposes.]

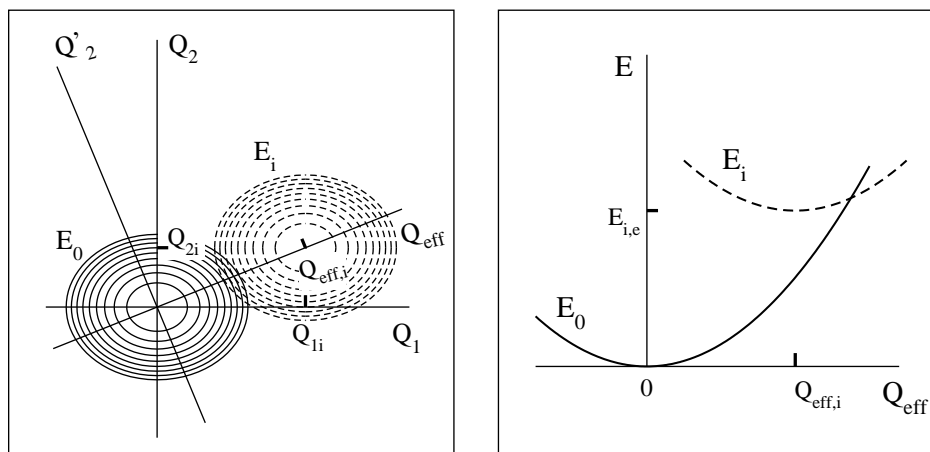


FIG. 1: Left: Definition of the configuration coordinate  $Q_{\text{eff}}$  of a single ion active center with two totally symmetric normal vibrational modes  $Q_1$  and  $Q_2$ . The ground state  $E_0$  and one excited state  $E_i$  potential energy surfaces are shown. Right: Corresponding configuration coordinate energy diagram.

We must bear in mind that, in general, the transformed vibrational coordinates are not normal modes, i.e. there are  $Q_{\text{eff}}Q_{\mu'}$  off-diagonal quadratic terms contributing to the electronic energies. If these terms are negligible, Eq. 5 holds and  $Q_{\text{eff}}$  is a real vibrational coordinate that connects the minima of  $E_0$  and  $E_i$ . Also, if all excited states have their minima aligned with  $E_0$  and  $E_i$  in the vibrational space,  $Q_{\text{eff}}$  is uniquely defined for all states. On the contrary, if  $Q_{\text{eff}}Q_{\mu'}$  off-diagonal terms are important or the electronic energy minima of all states are far from being aligned, then, for Eq. 5 to hold for all excited states,  $Q_{\text{eff}}$  must be some sort of not well defined, effective vibrational coordinate. For simplicity,  $Q_{\text{eff}}$  is often abbreviated by  $Q$ . The representation of the energies of the electronic states of the single ion active center along  $Q$  is called *the configuration coordinate diagram*.

### III. IVCT CONFIGURATION COORDINATE DIAGRAM OF A MIXED VALENCE PAIR

In this Section we describe the configuration coordinate diagram of the intervalence charge transfer (IVCT) states of donor-acceptor dopant pairs in a solid host. We will take an  $\text{Yb}^{2+}/\text{Yb}^{3+}$  mixed valence pair in Yb-doped YAG ( $\text{Y}_3\text{Al}_5\text{O}_{12}$ ) as a working example. We will assume both Yb ions substitute for Y in a  $D_2$  symmetry site with 8-fold Oxygen coordination (with long-range charge compensation in the case of  $\text{Yb}^{2+}$ ) and we will focus on the  $4f^N$  manifold. Consequently, we are interested in the energy levels resulting from the interplay between the  $4f^{14}$  electronic configuration of  $\text{Yb}^{2+}$  and the  $4f^{13}$  electronic configuration of  $\text{Yb}^{3+}$ . The  $4f^{14}$  closed-shell configuration of  $\text{Yb}^{2+}$  has a single totally symmetric A state. As it is shown in Table I, the  $4f^{13}$  open-shell configuration of  $\text{Yb}^{3+}$  has seven  $\Gamma_5$  Kramers dou-

TABLE I: Level energies relative to their respective ground states ( $E_{i,e}$ , in  $\text{cm}^{-1}$ ) and increments of the average Yb-O and Ce-O distances with respect to their values in the ground states of YAG: $\text{Yb}^{3+}$  and YAG: $\text{Ce}^{3+}$ , respectively ( $\Delta d$ , in Å).

Level	$E_{i,e}^a$	$\Delta d^e$	Level	$E_{i,e}^b$	$\Delta d^e$
YAG: $\text{Yb}^{3+}$			YAG: $\text{Ce}^{3+}$		
4f <sup>13</sup> levels			4f <sup>1</sup> levels		
1 $\Gamma_5(^2F_{7/2})$	0	0	1 $\Gamma_5(^2F_{5/2})$	0	0
2 $\Gamma_5(^2F_{7/2})$	611	0.00	2 $\Gamma_5(^2F_{5/2})$	289	0.00
3 $\Gamma_5(^2F_{7/2})$	696	0.00	3 $\Gamma_5(^2F_{5/2})$	770 <sup>c</sup>	0.00
4 $\Gamma_5(^2F_{7/2})$	782	0.00	4 $\Gamma_5(^2F_{7/2})$	2112	0.00
5 $\Gamma_5(^2F_{5/2})$	10321	0.00	5 $\Gamma_5(^2F_{7/2})$	2342	0.00
6 $\Gamma_5(^2F_{5/2})$	10620	0.00	6 $\Gamma_5(^2F_{7/2})$	2466	0.00
7 $\Gamma_5(^2F_{5/2})$	10674	0.00	7 $\Gamma_5(^2F_{7/2})$	3830	0.00
			Lowest 5d <sup>1</sup> level		
			8 $\Gamma_5(5d)$	20450 <sup>d</sup>	-0.02
YAG: $\text{Yb}^{2+}$			YAG: $\text{Ce}^{4+}$		
A	0	0.14	A	0	-0.16

<sup>a</sup>Reference 14.

<sup>b</sup>Reference 15.

<sup>c</sup>Reference 16.

<sup>d</sup>Reference 17.

<sup>e</sup>See text.

plets which are grouped into four  $^2F_{7/2}$  related levels spanning about  $800 \text{ cm}^{-1}$  and three  $^2F_{5/2}$  related levels spanning about  $500 \text{ cm}^{-1}$  and lying at about  $10000 \text{ cm}^{-1}$  above.<sup>14</sup>

Let us label the two distinguishable Yb dopant atoms of an  $\text{Yb}^{2+}/\text{Yb}^{3+}$  mixed valence pair as  $\text{Yb}_L$  and  $\text{Yb}_R$ . Starting with an  $\text{Yb}_L^{2+}-\text{Yb}_R^{3+}$  ionic configuration of the pair, we have seven levels with the same relative energies of the  $\text{Yb}^{3+}$  levels. Besides these states, we have those that result from electron transfer from  $\text{Yb}^{2+}$  to  $\text{Yb}^{3+}$ . Such electron transfer produces an  $\text{Yb}_L^{3+}-\text{Yb}_R^{2+}$  ionic configuration of the pair,



which also has seven levels with the same relative energies of the  $\text{Yb}^{3+}$  levels. These are called the IVCT states (of the original, reference  $\text{Yb}_L^{2+}\text{-Yb}_R^{3+}$  pair; obviously, the two sets of seven states are each others' IVCT states.) Altogether, the  $4f^N$  manifold of an  $\text{Yb}^{2+}/\text{Yb}^{3+}$  mixed valence pair in YAG is made of 14 energy levels. The energies of the seven levels of the  $\text{Yb}_L^{2+}\text{-Yb}_R^{3+}$  ionic configuration vary with the displacements of the oxygens around  $\text{Yb}_L$  and  $\text{Yb}_R$ , but their differences are very insensitive to these displacements because of the  $5s^25p^6$  shielding of the  $4f$  shells. The same is true for the seven levels of the  $\text{Yb}_L^{3+}\text{-Yb}_R^{2+}$  ionic configuration. On the contrary, the energy differences *between* the two sets of levels are very much dependent on the positions of the oxygens because they involve an ionization in one site and an electron attachment in the complementary site. Here we describe this dependence.

### A. IVCT model

In order to facilitate the extension to other mixed valence pairs and to simplify the notation, we will call  $\text{Yb}^{2+}$  the donor  $D$  and  $\text{Yb}^{3+}$  the acceptor  $A$ . More precisely,  $D$  and  $A$  will not refer to the single Yb ions, but to the defect centers they create in the solid; usually these are atomic moieties containing the Yb ions and their first coordination shells at least. Then the  $\text{Yb}_L^{2+}\text{-Yb}_R^{3+}$  ionic configuration of the  $\text{Yb}^{2+}/\text{Yb}^{3+}$  mixed valence pair can be called  $D_LA_R$ , or simply  $DA$ , and the  $\text{Yb}_L^{3+}\text{-Yb}_R^{2+}$  ionic configuration  $A_LD_R$ , or simply  $AD$ .

Let us describe first the energy of the electronic ground state of the  $\text{Yb}^{2+}/\text{Yb}^{3+}$  mixed valence pair as a function of two normal modes  $Q_L$  and  $Q_R$ , which describe vibrations around  $\text{Yb}_L$  and  $\text{Yb}_R$  respectively. We will comment later on the excited states. We may think of  $Q_L$  and  $Q_R$  as the configuration coordinates (Sec. II) of the  $\text{Yb}_L$  and  $\text{Yb}_R$  active centers. In general, they can be any totally symmetric normal vibrational coordinate of the centers.

The adiabatic ground state energy of the  $\text{Yb}^{2+}/\text{Yb}^{3+}$  pair at any value of  $Q_L$  and  $Q_R$  can be seen as the result of the diagonalization of a  $2 \times 2$  Hermitean matrix whose diagonal elements are the energies of the  $\text{Yb}_L^{2+}\text{-Yb}_R^{3+}$  and  $\text{Yb}_L^{3+}\text{-Yb}_R^{2+}$  ionic configurations of the pair (diabatic energies  $H_{11} = E_{D_0A_0}$  and  $H_{22} = E_{A_0D_0}$ ), and with the electronic coupling between the two ionic configurations as the off-diagonal element  $H_{12}$ . This diagonalization also gives the adiabatic energy of a second, excited state at each  $Q_L$  and  $Q_R$ . The consideration of the  $H_{12}$  electronic coupling is central for the energy barrier of the thermal electron transfer reaction  $\text{Yb}_L^{2+}\text{-Yb}_R^{3+} \rightarrow \text{Yb}_L^{3+}\text{-Yb}_R^{2+}$  and for the radiative and non-radiative transition probabilities. However, it is of minor importance for the energy of the corresponding optical electron transfer, which takes place at a fixed equilibrium structure of  $\text{Yb}_L^{2+}\text{-Yb}_R^{3+}$  with a relatively long Yb-Yb distance, and for the values of the vibrational coordinates at which crossings between potential energy surfaces occur.<sup>9,10</sup> In consequence, we will discuss the diabatic energies of  $\text{Yb}_L^{2+}\text{-Yb}_R^{3+}$  and  $\text{Yb}_L^{3+}\text{-Yb}_R^{2+}$ , which correspond to null elec-

tronic coupling between the oxidized and reduced members of the pair.

The diabatic energy of the ground state of the  $DA$  ionic configuration, as a function of the positions of the ligands that can be described with the coordinates  $Q_L$  and  $Q_R$ , can be written as

$$E_{D_0A_0}(Q_L, Q_R) = \frac{1}{2}k(Q_L - Q_{D_0})^2 + \frac{1}{2}k(Q_R - Q_{A_0})^2, \quad (6)$$

in a quadratic approximation with a common force constant for  $D$  and  $A$ ,  $k = \mu\omega^2$ ,  $\mu$  being the reduced mass of the vibration and  $\omega$  the vibrational frequency common to both vibrational coordinates. This equation establishes the minimum of  $E_{D_0A_0}$  as the reference energy.  $Q_{D_0}$  and  $Q_{A_0}$  are the values of the vibrational coordinates at the equilibrium structures of  $D$  and  $A$  respectively. Equivalently, if the donor and acceptor sites are identical, the diabatic energy of the ground state of the  $AD$  ionic configuration is

$$E_{A_0D_0}(Q_L, Q_R) = \frac{1}{2}k(Q_L - Q_{A_0})^2 + \frac{1}{2}k(Q_R - Q_{D_0})^2. \quad (7)$$

The diabatic potential energy surface  $E_{A_0D_0}(Q_L, Q_R)$  is identical to  $E_{D_0A_0}(Q_L, Q_R)$  but shifted  $-(Q_{D_0} - Q_{A_0})$  in the  $Q_L$  axis and  $+(Q_{D_0} - Q_{A_0})$  in the  $Q_R$  axis. They are represented in Fig. 2.

The crossing point between  $E_{D_0A_0}$  and  $E_{A_0D_0}$  with lowest energy is the activated complex (*ac*) of the thermal  $DA \rightarrow AD$  electron transfer reaction at the diabatic level. Within the adopted approximations of identical  $D$  and  $A$  sites and a common force constant for both, this activated complex is at the midpoint between their respective minima:  $Q_{L,ac} = Q_{R,ac} = Q_{ac} \equiv (Q_{D_0} + Q_{A_0})/2$ . The electron transfer reaction coordinate is the straight line in the  $(Q_L, Q_R)$  plane that connects the  $E_{D_0A_0}$  and  $E_{A_0D_0}$  minima and passes through the activated complex,

$$\frac{Q_L - Q_{D_0}}{Q_R - Q_{A_0}} = -1. \quad (8)$$

Then, a normal electron transfer reaction coordinate  $Q_{et}$  can be defined,

$$Q_{et} = \frac{1}{\sqrt{2}}(Q_R - Q_L), \quad (9)$$

which is null for the activated complex,  $Q_{et,ac} = 0$ , and takes opposite values for the  $DA$  and  $AD$  ionic configurations at the equilibrium,

$$-Q_{et,D_0A_0} = Q_{et,A_0D_0} = \frac{1}{\sqrt{2}}(Q_{D_0} - Q_{A_0}), \quad (10)$$

which gives

$$Q_{et,A_0D_0} - Q_{et,D_0A_0} = \sqrt{2}(Q_{D_0} - Q_{A_0}). \quad (11)$$

Using the  $Q_{et}$  defined in Eq. 9, the parametric form of the reaction coordinate is

$$\begin{cases} Q_L - Q_{ac} = -\frac{1}{\sqrt{2}}Q_{et}, \\ Q_R - Q_{ac} = +\frac{1}{\sqrt{2}}Q_{et}, \end{cases} \quad (12)$$

which makes it clear that an increment of  $Q_{et}$  means a simultaneous decrement of  $Q_L$  and increment of  $Q_R$ . This reflects the fact that a thermal electron transfer from  $D_L$  to

$A_R$ , which converts  $D_L A_R$  into  $A_L D_R$ , is accompanied by a contraction of the coordination shells around the donor  $D_L$  and a simultaneous expansion around the acceptor  $A_R$ .

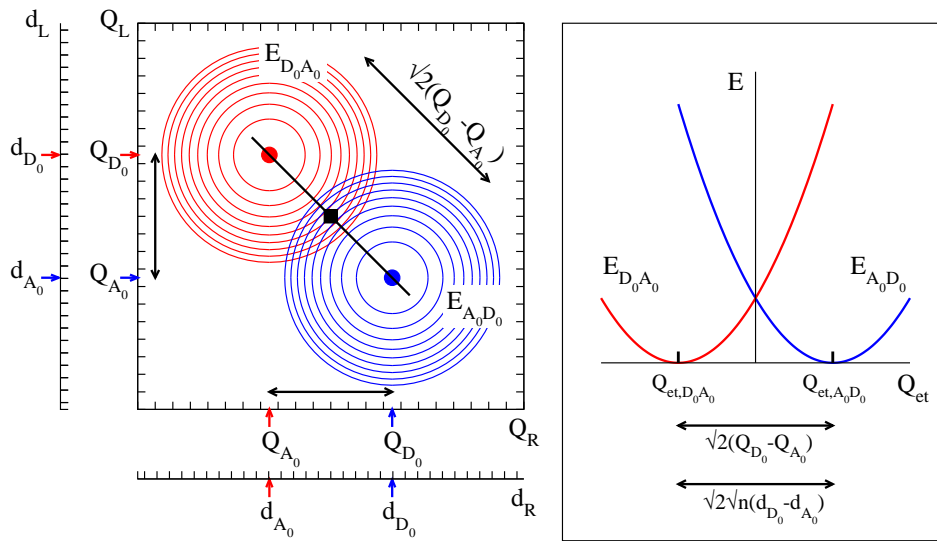


FIG. 2: Left: Ground state diabatic energy surfaces of the DA (red) and AD (blue) ionic configurations of the mixed valence pair. The black line that connects the DA minimum (red dot) with the AD minimum (blue dot) and passes through the activated complex (black square) is the electron transfer reaction coordinate, which is also the IVCT configuration coordinate. Right: IVCT configuration coordinate diagram with the ground state energies of DA (red) and AD (blue).

Accordingly, the energies of the two ionic configurations of the  $\text{Yb}^{2+}/\text{Yb}^{3+}$  mixed valence pair along the reaction coordinate are:

$$\begin{aligned} E_{D_0 A_0}(Q_{et}) &= \frac{1}{2}k(Q_{et} - Q_{et, D_0 A_0})^2, \\ E_{A_0 D_0}(Q_{et}) &= \frac{1}{2}k(Q_{et} - Q_{et, A_0 D_0})^2, \end{aligned} \quad (13)$$

which are two identical parabolae with a horizontal offset  $\sqrt{2}(Q_{D_0} - Q_{A_0})$ ,

$$E_{D_0 A_0}(Q_{et} - \sqrt{2}(Q_{D_0} - Q_{A_0})) = E_{A_0 D_0}(Q_{et}). \quad (14)$$

Since the coordinate orthogonal to  $Q_{et}$  does not have a horizontal offset (Fig. 2), the reaction coordinate  $Q_{et}$  is the IVCT configuration coordinate. The horizontal offset and the force constant  $k$  are the only two degrees of freedom of this model.

If  $Q_L$  and  $Q_R$  are the respective breathing modes of the  $\text{Yb}_L$  and  $\text{Yb}_R$  substitutional defects in YAG, which normally produce the maximum energy changes, they can be written as

$$\begin{aligned} Q_L &= \frac{1}{\sqrt{8}}(\delta_{O_{L1}} + \delta_{O_{L2}} + \dots + \delta_{O_{L8}}), \\ Q_R &= \frac{1}{\sqrt{8}}(\delta_{O_{R1}} + \delta_{O_{R2}} + \dots + \delta_{O_{R8}}), \end{aligned} \quad (15)$$

$\delta_{O_{L1}}$  being the increment of the  $\text{Yb}_L\text{-O}_{L1}$  distance with respect to its value in a given reference structure, and equivalently for the displacements of the other oxygens. The electron transfer reaction coordinate  $Q_{et}$  defined in Eq. 9 that corresponds to these definitions of  $Q_L$  and  $Q_R$  are shown in Fig. 3. Since all the  $\text{Yb-O}$  distances change equally during the breathing, we can write

$$Q_L = \sqrt{8}(d_L - d_{\text{ref}}), \quad Q_R = \sqrt{8}(d_R - d_{\text{ref}}), \quad (16)$$

where  $d_{\text{ref}}$ ,  $d_L$ , and  $d_R$  are, respectively, the average  $\text{Yb-O}$  distances in a reference structure, and around  $\text{Yb}_L$  and  $\text{Yb}_R$  at any moment. Then, for the donor and acceptor equilibrium structures we have

$$Q_{D_0} = \sqrt{8}(d_{D_0} - d_{\text{ref}}), \quad Q_{A_0} = \sqrt{8}(d_{A_0} - d_{\text{ref}}), \quad (17)$$

$d_{D_0}$  and  $d_{A_0}$  being the average  $\text{Yb-O}$  distances around the donor  $\text{Yb}^{2+}$  and the acceptor  $\text{Yb}^{3+}$  at equilibrium. In a more general case with a coordination number  $n$  of equal ligands,

$$Q_{D_0} = \sqrt{n}(d_{D_0} - d_{\text{ref}}), \quad Q_{A_0} = \sqrt{n}(d_{A_0} - d_{\text{ref}}). \quad (18)$$

This leads to the centers of the two parabolae in the configuration coordinate diagram (Eq. 13),

$$-Q_{et, D_0 A_0} = Q_{et, A_0 D_0} = \sqrt{n/2}(d_{D_0} - d_{A_0}), \quad (19)$$

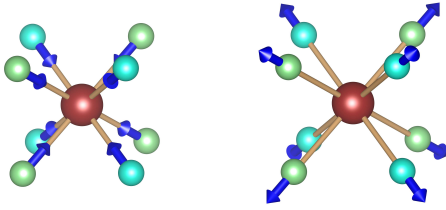


FIG. 3: Electron transfer reaction coordinate  $Q_{et}$  of an  $\text{Yb}^{2+}/\text{Yb}^{3+}$  pair in YAG.

and their offset

$$\sqrt{2}(Q_{D_0} - Q_{A_0}) = \sqrt{2n}(d_{D_0} - d_{A_0}). \quad (20)$$

Then, the horizontal offset (which is key to the model, together with the force constant) can be estimated from the difference between the ionic radii of the donor and acceptor ions,  $\text{Yb}^{2+}$  and  $\text{Yb}^{3+}$  in our working example.

The diabatic potential energy surfaces of other states of the  $4f^N$  manifold of the  $\text{Yb}^{2+}/\text{Yb}^{3+}$  mixed valence pair can be written as

$$\begin{aligned} E_{D_i A_j}(Q_L, Q_R) &= E_{D_i A_j, e} + \frac{1}{2}k(Q_L - Q_{D_0})^2 \\ &\quad + \frac{1}{2}k(Q_R - Q_{A_0})^2 \\ E_{A_j D_i}(Q_L, Q_R) &= E_{D_i A_j, e} + \frac{1}{2}k(Q_L - Q_{A_0})^2 \\ &\quad + \frac{1}{2}k(Q_R - Q_{D_0})^2, \end{aligned} \quad (21)$$

assuming they have the same equilibrium structures and vibrational frequencies as the ground state. The vertical offsets  $E_{D_i A_j, e}$  are the sum of the minimum-to-minimum excitation energies from the ground state to the  $i$  excited state of  $D$  and from the ground state to the  $j$  excited state of  $A$ :  $E_{D_i A_j, e} = (E_{D_i, e} - E_{D_0, e}) + (E_{A_j, e} - E_{A_0, e})$ . Those of the two complementary states  $D_i A_j$  and  $A_j D_i$  are identical: Then, their energies in the IVCT configuration coordinate diagram are

$$\begin{aligned} E_{D_i A_j}(Q_{et}) &= E_{D_i A_j, e} + \frac{1}{2}k(Q_{et} - Q_{et, D_0 A_0})^2, \\ E_{A_j D_i}(Q_{et}) &= E_{D_i A_j, e} + \frac{1}{2}k(Q_{et} - Q_{et, A_0 D_0})^2. \end{aligned} \quad (22)$$

In summary, Eq. 22, together with Eq. 19, or in a more general case with Eq. 10, constitute an IVCT configuration coordinate (diabatic) diagram of the D/A mixed valence pair.

The model of the potential energy surfaces can be improved by extending Eq. 21. Using different force constants for  $D$  and  $A$  leads to a different electron transfer reaction coordinate but a configuration coordinate diagram like Eq. 22 can still be used.<sup>9</sup> In this case, the minima of

$DA$  and  $AD$  are not aligned with the activated complex and the reaction coordinate is made of two straight segments, one between  $DA$  and the activated complex and another between the activated complex and  $AD$ . Also, using state-specific equilibrium structures, or state-specific force constants, or including anharmonic terms, will prevent from defining a unique reaction coordinate, because there is a different one for each  $D_i A_j - A_k D_l$  combination. In the case of excited states of a different configuration, like  $4f^{N-1}5d$ , using state-specific equilibrium structures, or at least configuration-specific equilibrium structures, might be necessary. If so, the one-dimension configuration coordinate diagram will break in a strict sense and working with the two-dimension energy surfaces is compulsory. However, the reaction coordinates of several  $D_i A_j - A_k D_l$  combinations can be similar enough so as to make the representation of all states of the mixed valence pair along one of these reaction coordinates meaningful.<sup>9</sup>

### B. IVCT configuration coordinate diagram of $\text{Yb}^{2+}/\text{Yb}^{3+}$ in YAG

In Fig. 4 we show the IVCT configuration coordinate diagram of the  $4f^N$  levels of an  $\text{Yb}^{2+}/\text{Yb}^{3+}$  mixed valence pair in YAG. It results from using Eqs. 19 and 22 with  $n=8$  and the following data: (1) The  $E_{D_i A_j, e}$  are the experimental  $4f \rightarrow 4f$  excitation energies of  $\text{Yb}^{3+}$ , which are shown in the second column of Table I (note that  $\text{Yb}^{2+}$  has only one state of the  $4f^{14}$  configuration). (2) For the  $d_{D_0} - d_{A_0}$  offset between donor and acceptor equilibrium distances, which is  $d_{\text{Yb}^{2+}-\text{O}} - d_{\text{Yb}^{3+}-\text{O}}$ , we have taken 90% of the difference between the ionic radii of  $\text{Yb}^{2+}$  and  $\text{Yb}^{3+}$  in coordination 8 (1.14 Å and 0.985 Å respectively, Ref. 18), which is  $d_{D_0} - d_{A_0} = 0.14$  Å. The reduction factor has been used to take into account the host effect, which has been found to make the equilibrium impurity-ligand distance in a doped host to lie between the cation-ligand distance in the undoped host and the distance that would correspond to the ionic radii mismatch between the host cation and the impurity.<sup>19</sup> (3) The force constant is  $k = \mu\omega^2$ , with  $\mu = m(\text{O}) = 15.999$  amu. We are not aware of direct experimental determinations of the breathing mode vibrational frequency of  $\text{Yb}^{3+}$  defects in YAG; we have taken a  $\bar{\nu} = \omega/(2\pi c) = 326$   $\text{cm}^{-1}$  mode reported by Lupei *et al.*<sup>20</sup> after an analysis of resonant vibronic effects in YAG: $\text{Yb}^{3+}$  and have scaled it 0.95% down to 310  $\text{cm}^{-1}$ . This corresponds to a 90% scaling of the force constant  $k$ , which is expected to be smaller in  $\text{Yb}^{2+}$  than in  $\text{Yb}^{3+}$ . Since

$$\frac{k}{\text{cm}^{-1} \text{Å}^{-2}} = 0.029660 \times \left( \frac{\mu}{\text{amu}} \right) \times \left( \frac{\bar{\nu}}{\text{cm}^{-1}} \right)^2,$$

we get  $k = 50434$   $\text{cm}^{-1} \text{Å}^{-2}$ .

In the IVCT configuration coordinate diagram (Fig. 4) we observe the energies of the seven levels of the  $\text{Yb}_L^{2+} - \text{Yb}_R^{3+}$  (in red) and the seven levels of the  $\text{Yb}_L^{3+} - \text{Yb}_R^{2+}$  (in blue) ionic configurations of the pair. They are grouped in two sets of

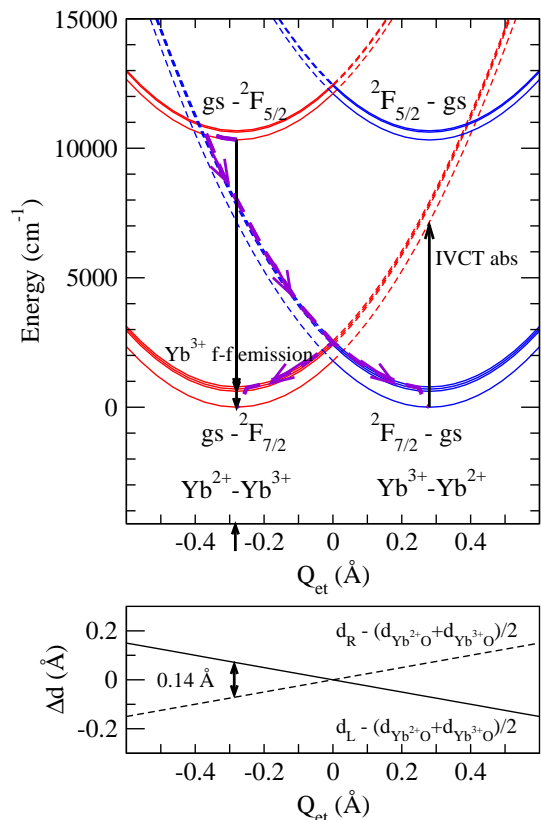


FIG. 4: Top: IVCT configuration coordinate diagram of a  $Yb^{2+}/Yb^{3+}$  mixed valence pair in YAG. Bottom: Changes of the Yb-O distance in the left Yb ( $d_L$ , full line) and right Yb ( $d_R$ , dashed line) with respect to their value at the activated complex,  $(d_{Yb^{2+}-O,e} + d_{Yb^{3+}-O,e})/2$ , along the configuration coordinate (or electron transfer reaction coordinate)  $Q_{et}$ .

four levels corresponding to  $Yb_R^{3+} (^2F_{7/2})$  and  $Yb_L^{3+} (^2F_{7/2})$ , and two sets of three levels about  $10000 \text{ cm}^{-1}$  above, corresponding to  $Yb_R^{3+} (^2F_{5/2})$  and  $Yb_L^{3+} (^2F_{5/2})$ . They have been represented with full lines for configuration coordinate values around their respective minima. We used dashed lines for their energies at configuration coordinate values corresponding to stressed structures. For instance, at  $Q_{et} = -0.28 \text{ \AA}$  we find  $Yb_L^{2+}-Yb_R^{3+}$  in equilibrium (full lines), with a long  $Yb_L$ -O distance and a short  $Yb_R$ -O distance; on the contrary, an  $Yb_L^{3+}-Yb_R^{2+}$  state is under strong stress in such structure (dashed lines) and it will relax towards a short  $Yb_L$ -O distance and a long  $Yb_R$ -O distance, increasing  $Q_{et}$  and releasing a large reorganization energy (about  $7500 \text{ cm}^{-1}$ ). It is clear that structurally stressed IVCT states intercalate between states of the pairs around their equilibrium. Vertical (Frank-Condon) transitions between full lines take place in  $Yb^{3+}$ . Vertical transitions between full and dashed lines are IVCT transitions; they end in structurally stressed states and take place between states with large horizontal offsets, which produces very wide bands. IVCT absorptions in mixed valence compounds are known long ago;<sup>5,6</sup> they take place without a corresponding emission. IVCT emissions from higher excited states have been recently reported in

mixed valence dopant pairs in solids. They have been found to be responsible for the anomalous emission of  $Ce^{3+}$ -doped elpasolites<sup>9</sup> and for the interplay between anomalous emissions and  $5d \rightarrow 4f$  emissions in Yb-doped fluorite hosts.<sup>10</sup> The IVCT emissions have large Stokes shifts associated with large reorganization energies.

It is interesting to observe in the IVCT configuration coordinate diagram that IVCT crossings between the excited  $Yb_L^{2+}-Yb_R^{3+} (^2F_{5/2})$  and the ground  $Yb_L^{3+} (^2F_{7/2})-Yb_R^{2+}$  manifolds can occur with low energy barriers. Obviously, the same is true for their symmetrical counterparts. With the present data we obtain a  $200 \text{ cm}^{-1}$  energy barrier from the minimum of the  $Yb_L^{2+}-Yb_R^{3+} (^2F_{5/2})$  state (which is the emitting state of the  $Yb^{3+} 4f \rightarrow 4f$  emission) to the crossing between the lowest  $Yb_L^{2+}-Yb_R^{3+} (^2F_{5/2})$  level and the highest  $Yb_L^{3+} (^2F_{7/2})-Yb_R^{2+}$  level. This result suggests the consideration of the nonradiative  $Yb^{2+} (4f_{7/2}) \rightarrow Yb^{3+} (4f_{5/2})$  electron transfer in  $Yb^{2+}/Yb^{3+}$  pairs as a possible mechanism responsible for quenchings of the  $Yb^{3+}$  emission, which is shown in violet in Fig. 4. This means that  $Yb^{2+}/Yb^{3+}$  pairs can be the active quenchers in the concentration quenching of the  $Yb^{3+}$  emission.<sup>21</sup> Also, they can explain the temperature quenching of the  $Yb^{3+}$  emission excited with  $Ce^{3+} 4f \rightarrow 5d$  absorption in Ce,Yb-codoped YAG.<sup>3</sup>

The uncertainties of the empirical data used to build the configuration coordinate diagram in Fig. 4 give only a qualitative or semi-quantitative meaning to the energy barrier just discussed. For instance, taking 95% or 85% of the ionic radii difference for  $d_{Yb^{2+}-O} - d_{Yb^{3+}-O}$  instead of 90%, leads to  $75 \text{ cm}^{-1}$  and  $400 \text{ cm}^{-1}$  energy barriers instead of the  $200 \text{ cm}^{-1}$  mentioned above. Similarly, using a higher/lower vibrational frequency would decrease/increase the barrier value in this case. Besides, the effective activation energies that are experimentally determined are smaller than the barrier energies calculated from configuration coordinate diagrams because of the effective overlap between vibrational wavefunctions below the crossing points.<sup>22</sup> Nevertheless, it is clear that a configuration coordinate diagram obtained with reasonable empirical data indicates that a crossing between a potentially emitting level  $Yb_L^{2+}-Yb_R^{3+} (^2F_{5/2})$  and a structurally stressed IVCT ground state level  $Yb_L^{3+} (^2F_{7/2})-Yb_R^{2+}$  is likely to occur and it would cause thermal quenching of the  $Yb^{3+} 4f \rightarrow 4f$  emission.

#### IV. MMCT CONFIGURATION COORDINATE DIAGRAM OF A METAL-METAL PAIR

In this Section, we describe the configuration coordinate diagram of the metal-to-metal charge transfer states of a pair of ions of different elements in a solid host. We will take a  $Ce^{3+}-Yb^{3+}$  pair in Ce,Yb-codoped YAG as a working example. We assume  $Ce^{3+}$  and  $Yb^{3+}$  substitute for Y in a  $D_2$  symmetry site with 8-fold Oxygen coordination. Regarding  $Ce^{3+}$ , we will focus on its states of the  $4f^1$  configuration and on its lowest  $5d^1$  state. As



it is shown in Table I, the seven  $4f^1$  states are grouped in three sets:<sup>15,16</sup> a first set with three levels spanning about  $800\text{ cm}^{-1}$ ,  $1-3\ \Gamma_5$ ; a second set starting at about  $2100\text{ cm}^{-1}$  above the ground state with three levels spanning  $350\text{ cm}^{-1}$ ,  $4-6\ \Gamma_5$ ; and a third set at about  $3800\text{ cm}^{-1}$  above the ground state with one level,  $7\ \Gamma_5$ . (This distribution of levels differs from the extended assumption of three  ${}^2F_{5/2}$  related levels and four  ${}^2F_{7/2}$  related levels separated by about  $2500\text{ cm}^{-1}$ ; a  $4f$  crystal field splitting of the same size of the  $4f$  spin-orbit coupling splitting is responsible for the partial break of such assumption.<sup>16</sup>) The lowest  $5d^1$  state is  $20450\text{ cm}^{-1}$  above the ground state.<sup>17</sup> These levels will combine with the previously described four  ${}^2F_{7/2}$  related levels and three  ${}^2F_{5/2}$  related levels of  $\text{Yb}^{3+}$  (Sec. III). The result of this combination is four main sets of levels. A first main set of 28 levels of  $\text{Ce}^{3+}(4f^1)\text{-Yb}^{3+}({}^2F_{7/2})$  character lies between 0 and  $4600\text{ cm}^{-1}$  and is divided in three subsets: 12  $\text{Ce}^{3+}(1-3\ \Gamma_5)\text{-Yb}^{3+}({}^2F_{7/2})$ , 12  $\text{Ce}^{3+}(4-6\ \Gamma_5)\text{-Yb}^{3+}({}^2F_{7/2})$ , and 4  $\text{Ce}^{3+}(7\ \Gamma_5)\text{-Yb}^{3+}({}^2F_{7/2})$ . A second main set of 21 levels of  $\text{Ce}^{3+}(4f^1)\text{-Yb}^{3+}({}^2F_{5/2})$  character lies between  $10300\text{ cm}^{-1}$  and  $14500\text{ cm}^{-1}$  and is divided in three subsets: 9  $\text{Ce}^{3+}(1-3\ \Gamma_5)\text{-Yb}^{3+}({}^2F_{5/2})$ , 9  $\text{Ce}^{3+}(4-6\ \Gamma_5)\text{-Yb}^{3+}({}^2F_{5/2})$ , and 3  $\text{Ce}^{3+}(7\ \Gamma_5)\text{-Yb}^{3+}({}^2F_{5/2})$ . A third main set of 4 levels of  $\text{Ce}^{3+}(5d_1^1)\text{-Yb}^{3+}({}^2F_{7/2})$  character lies between  $20450\text{ cm}^{-1}$  and  $21230\text{ cm}^{-1}$ . Finally a fourth main set of 4 levels of  $\text{Ce}^{3+}(5d_1^1)\text{-Yb}^{3+}({}^2F_{5/2})$  character lies between  $30770\text{ cm}^{-1}$  and  $31120\text{ cm}^{-1}$ . Between the third and the fourth set, states of  $\text{Ce}^{3+}(5d_2^1)\text{-Yb}^{3+}({}^2F_{7/2})$  character associated with the second  $5d^1$  excited state of  $\text{Ce}^{3+}$  may appear. Here we will only pay attention to the three first sets.

Electron transfer from  $\text{Ce}^{3+}$  to  $\text{Yb}^{3+}$  results into a  $\text{Ce}^{4+}\text{-Yb}^{2+}$  pair with a closed-shell ground state much more stable than all its excited states. This level may lie between the  $\text{Ce}^{3+}\text{-Yb}^{3+}$  states just discussed. Such a consideration has led to propose its involvement in the mechanism of energy transfer from  $\text{Ce}^{3+}$  to  $\text{Yb}^{3+}$  in Ce,Yb-codoped YAG, as well as in the quenching of the  $\text{Ce}^{3+}$ -excited  $\text{Yb}^{3+}$ -emission of this material.<sup>3</sup> The question we tackle here is how to represent the energies of the levels of the  $\text{Ce}^{3+}\text{-Yb}^{3+}$  and  $\text{Ce}^{4+}\text{-Yb}^{2+}$  pairs together in a simplified diagram, which is called the MMCT configuration coordinate diagram.

### A. MMCT model

In order to make the notation more general, we will call  $\text{Ce}^{3+}$  the donor  $D$  and  $\text{Yb}^{3+}$  the acceptor  $A$ . After the electron transfer they result into  $\text{Ce}^{4+}$  and  $\text{Yb}^{2+}$ , which will be respectively called  $D^+$  and  $A^-$ . As in the IVCT case,  $D$ ,  $A$ ,  $D^+$ , and  $A^-$  refer to the defect centers the respective ions create in the solid, which will be atomic moieties containing at least their first coordination shells, rather than to the single ions. Then, the pair  $\text{Ce}^{3+}\text{-Yb}^{3+}$  is  $DA$  and the pair  $\text{Ce}^{4+}\text{-Yb}^{2+}$  is  $D^+A^-$ .

We aim at describing the energies of the states of the  $\text{Ce}^{3+}\text{-Yb}^{3+}$  and  $\text{Ce}^{4+}\text{-Yb}^{2+}$  pairs ( $DA$  and  $D^+A^-$ ) as functions of two vibrational coordinates  $Q_D$  and  $Q_A$ , which describe vibrations of the moieties containing Ce ( $D$  and  $D^+$ ) and

Yb ( $A$  and  $A^-$ ) respectively. Note that in MMCT, contrary to IVCT,  $D$  and  $D^+$  refer to a different element than  $A$  and  $A^-$ , so that there is no need to use the *left* and *right* atoms to differentiate them; in other words, Ce is always the *left* atom and Yb the *right* atom. This is why we use  $Q_D$  and  $Q_A$  here instead of  $Q_L$  and  $Q_R$ . As in IVCT,  $Q_D$  and  $Q_A$  can be the configuration coordinate described in Sec. II of the Ce and Yb active centers, but they can also be other totally symmetric vibrational coordinates of these centers.

The diabatic energy of the ground state of the  $DA$  pair, as a function of the positions of the ligands that can be described with the coordinates  $Q_D$  and  $Q_A$ , can be written as

$$E_{D_0A_0}(Q_D, Q_A) = \frac{1}{2}k_D(Q_D - Q_{D_0})^2 + \frac{1}{2}k_A(Q_A - Q_{A_0})^2, \quad (23)$$

in a quadratic approximation.  $Q_{D_0}$  and  $Q_{A_0}$  are the values of the vibrational coordinates of the donor and acceptor moieties at the respective equilibrium structures of  $D$  and  $A$ .  $k_D = \mu_D\omega_D^2$  and  $k_A = \mu_A\omega_A^2$  are, respectively, the donor and acceptor force constants,  $\mu_D$  and  $\mu_A$  their vibrational masses, and  $\omega_D$  and  $\omega_A$  their vibrational frequencies. As in IVCT, with this equation we are establishing the ground state energy of the  $DA$  pair at its minimum as the reference energy.

The diabatic energy of the ground state of the  $D^+A^-$  pair that results after MMCT from  $DA$  can be written as

$$E_{D_0^+A_0^-}(Q_D, Q_A) = E_{D_0^+A_0^-,e} + \frac{1}{2}k_D(Q_D - Q_{D_0^+})^2 + \frac{1}{2}k_A(Q_A - Q_{A_0^-})^2. \quad (24)$$

Here, as in the IVCT model of Sec. III, we assume a common force constant  $k_D$  for  $D$  and  $D^+$  on one side, and  $k_A$  for  $A$  and  $A^-$  on the other;  $Q_{D_0^+}$  and  $Q_{A_0^-}$  are the values of the vibrational coordinates of the donor and acceptor moieties at the respective equilibrium structures of  $D^+$  and  $A^-$ .  $E_{D_0^+A_0^-,e}$  is the vertical offset between the minima of the  $D^+A^-$  and  $DA$  ground state potential energy surfaces. In an empirical approach to the problem, it can be considered an empirical parameter. Also, it might eventually be convenient to regard it as the sum of: 1) the adiabatic ionization potential of  $D$  in the host,  $IP_D$  (energy difference between the ground states of  $D^+$  and  $D$  at their relaxed structures), 2) the negative adiabatic electron affinity of  $A$  in the host,  $-EA_A$  (energy difference between the ground states of  $A^-$  and  $A$  at their relaxed structures), and 3) the interaction energy change due to the creation of a hole in  $D$  and an electron in  $A$ ; for long  $D$ - $A$  separations, this can be approximated by  $(q_A - q_D - 1)/d_{DA}$ ,  $d_{DA}$  being the distance between donor and acceptor, and  $q_D$  and  $q_A$  their respective charges:

$$E_{D_0^+A_0^-,e} = IP_D - EA_A + (q_A - q_D - 1)/d_{DA}. \quad (25)$$

The  $E_{D_0A_0}(Q_D, Q_A)$  and  $E_{D_0^+A_0^-}(Q_D, Q_A)$  potential energy surfaces are represented in the left hand side of Fig. 5.

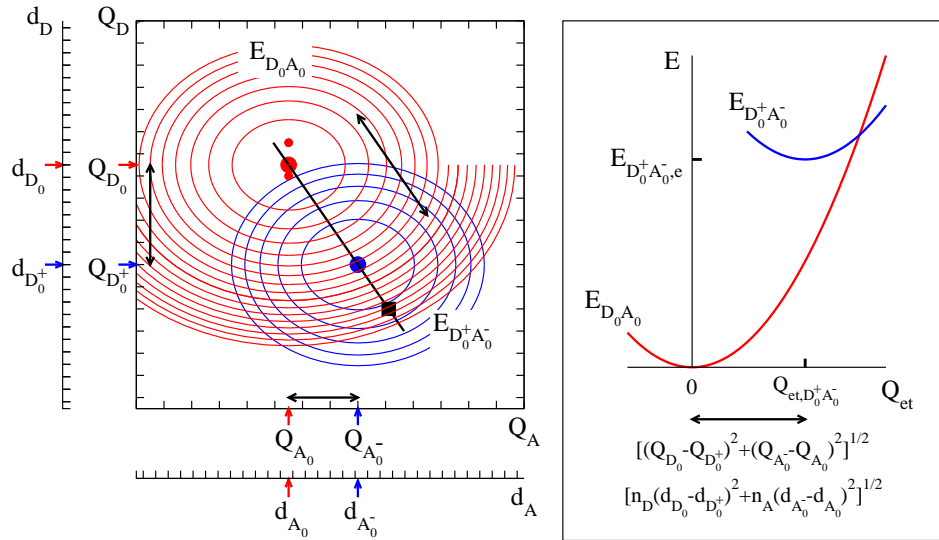


FIG. 5: Left: Ground state diabatic energy surfaces of  $DA$  (red) and  $D^+A^-$  (blue). The black line that connects the  $DA$  minimum (red dot) with the activated complex (black square) and returns to the  $D^+A^-$  minimum (blue dot) is the electron transfer reaction coordinate, which is also the MMCT configuration coordinate. The minima of other two  $DA$  energy surfaces (not represented here) are indicated with red dots. Right: MMCT configuration coordinate diagram with the ground state energies of  $DA$  (red) and  $D^+A^-$  (blue).

Within these approximations, the activated complex of the thermal  $DA \rightarrow D^+A^-$  reaction is in the straight line that connects their minima. Its exact position depends on the value of the energy offset  $E_{D_0^+A_0^-,e}$ . In any case, as in the IVCT case, the electron transfer reaction coordinate is the straight line in the  $(Q_D, Q_A)$  plane that passes through the  $E_{D_0A_0}$  and  $E_{D_0^+A_0^-}$  minima,

$$\frac{Q_D - Q_{D_0}}{Q_A - Q_{A_0}} = m \equiv \frac{Q_{D_0^+} - Q_{D_0}}{Q_{A_0^-} - Q_{A_0}}. \quad (26)$$

with a negative slope  $m$ . Then, a normal reaction coordinate could be defined like in Eq. 9, which would be null in the activated complex. However, since the position of the activated complex can be very different for different pairs of  $DA$  and  $D^+A^-$  states, it can be more convenient to define here the normal reaction coordinate so that it is null for the  $DA$  ground state at equilibrium:

$$Q_{et} = \frac{1}{\sqrt{1+m^2}} [(Q_A - Q_{A_0}) + m(Q_D - Q_{D_0})]. \quad (27)$$

Then, the parametric form of the reaction coordinate is

$$\begin{cases} Q_D - Q_{D_0} = \frac{m}{\sqrt{1+m^2}} Q_{et}, \\ Q_A - Q_{A_0} = \frac{1}{\sqrt{1+m^2}} Q_{et}. \end{cases} \quad (28)$$

which, with a negative  $m$ , indicates that an increment of  $Q_{et}$  means a simultaneous decrement of  $Q_D$  and increment of  $Q_A$ .

Being the only coordinate with horizontal offset,  $Q_{et}$  is the MMCT configuration coordinate and the energies of the ground states of the  $Ce^{3+} \cdot Yb^{3+}$  and  $Ce^{4+} \cdot Yb^{2+}$  pairs as functions of it constitute the configuration coordinate diagram for these states. Using Eqs. 23, 24, and 28, they are:

$$\begin{aligned} E_{D_0A_0}(Q_{et}) &= \frac{1}{2} k_{et} (Q_{et} - Q_{et,D_0A_0})^2 \\ E_{D_0^+A_0^-}(Q_{et}) &= E_{D_0^+A_0^-,e} + \frac{1}{2} k_{et} (Q_{et} - Q_{et,D_0^+A_0^-})^2, \end{aligned} \quad (29)$$

with

$$\begin{aligned} Q_{et,D_0A_0} &= 0 \\ Q_{et,D_0^+A_0^-} &= \sqrt{(Q_{D_0} - Q_{D_0^+})^2 + (Q_{A_0^-} - Q_{A_0})^2}, \end{aligned} \quad (30)$$

and

$$k_{et} = \frac{1}{1+m^2} (m^2 k_D + k_A). \quad (31)$$

The MMCT configuration coordinate diagram with  $E_{D_0A_0}(Q_{et})$  and  $E_{D_0^+A_0^-}(Q_{et})$  is represented in the right hand side of Fig. 5. The three degrees of freedom of this MMCT configuration coordinate diagram are the electron transfer reaction coordinate curvature,  $k_{et}$ , the MMCT horizontal offset  $\sqrt{(Q_{D_0} - Q_{D_0^+})^2 + (Q_{A_0^-} - Q_{A_0})^2}$ , and the vertical offset  $E_{D_0^+A_0^-,e}$ .

If  $Q_D$  and  $Q_A$  are the breathing modes of the donor and acceptor, with  $n_D$  and  $n_A$  equal ligands respectively, we have

$$\begin{aligned} Q_{D_0} - Q_{D_0^+} &= \sqrt{n_D} (d_{D_0} - d_{D_0^+}), \\ Q_{A_0^-} - Q_{A_0} &= \sqrt{n_A} (d_{A_0^-} - d_{A_0}), \end{aligned} \quad (32)$$

and a horizontal offset

$$Q_{et,D_0^+A_0^-} - Q_{et,D_0A_0} = Q_{et,D_0^+A_0^-} = \sqrt{n_D(d_{D_0} - d_{D_0^+})^2 + n_A(d_{A_0} - d_{A_0^-})^2}. \quad (33)$$

For excited states of  $DA$  and  $D^+A^-$ , the diabatic energy surfaces are

$$\begin{aligned} E_{D_iA_j}(Q_D, Q_A) &= E_{D_iA_j,e} + \frac{1}{2}k_D(Q_D - Q_{D_i})^2 \\ &\quad + \frac{1}{2}k_A(Q_A - Q_{A_j})^2, \\ E_{D_k^+A_\ell^-}(Q_D, Q_A) &= E_{D_k^+A_\ell^-,e} + \frac{1}{2}k_D(Q_D - Q_{D_k^+})^2 \\ &\quad + \frac{1}{2}k_A(Q_A - Q_{A_\ell^-})^2. \end{aligned} \quad (34)$$

The energy offsets of  $DA$  are the sums of the minimum-to-minimum excitation energies from the ground state to the  $i$  excited state of  $D$  and from the ground state to the  $j$  excited state of  $A$ :  $E_{D_iA_j,e} = (E_{D_i,e} - E_{D_0,e}) + (E_{A_j,e} - E_{A_0,e})$ . Correspondingly, the energy offsets of  $D^+A^-$  are the sums of the minimum-to-minimum excitation energies from the ground state to the  $k$  excited state of  $D^+$  and from the ground state to the  $\ell$  excited state of  $A^-$ , plus the ground state energy offset  $E_{D_0^+A_0^-,e}$ , which fulfils Eq. 25:  $E_{D_k^+A_\ell^-,e} = E_{D_0^+A_0^-,e} + (E_{D_k^+,e} - E_{D_0^+,e}) + (E_{A_\ell^-,e} - E_{A_0^-,e})$ . The  $D^+A^-$  ground state energy offset  $E_{D_0^+A_0^-,e}$  is the only parameter of the model that depends on the donor-acceptor distance  $d_{DA}$ . This means that, when two or more  $DA$  pairs made of the same elements coexist in the same host with different  $D$ - $A$  distances,  $d_{DA}$  and  $d'_{DA}$ , their MMCT manifolds  $E_{D_k^+A_\ell^-}$  are identical and shifted in energy with respect to one another. Eq. 25 gives  $1/d_{DA} - 1/d'_{DA}$  for the shift.

In general, there will be one reaction coordinate for each  $D_iA_j$ - $D_k^+A_\ell^-$  combination. However, if the horizontal offsets between different states of  $DA$  is much smaller than the MMCT horizontal offset, it is not a bad approximation to use the ground state reaction coordinate Eq. 27 for all the states. In this case, we have

$$\begin{aligned} E_{D_iA_j}(Q_{et}) &= E_{D_iA_j,e} + \frac{1}{2}k_{et}(Q_{et} - Q_{et,D_0A_0})^2 \\ E_{D_k^+A_\ell^-}(Q_{et}) &= E_{D_k^+A_\ell^-,e} + \frac{1}{2}k_{et}(Q_{et} - Q_{et,D_0^+A_0^-})^2. \end{aligned} \quad (35)$$

Summarizing, Eq. 35, together with Eq. 33, or in a more general case with Eq. 30, constitute the MMCT configuration coordinate (diabatic) diagram of the  $DA$  and  $D^+A^-$  metal-metal pairs in a host.

Alternatively, if the horizontal offsets between different states of either  $DA$  or  $D^+A^-$  are taken into account, then the MMCT configuration coordinate diagram results from evaluating Eq. 34 along the reaction coordinate of Eqs. 26, 27, and 32. If the reaction coordinate of a set of excited states  $D_iA_j$ - $D_k^+A_\ell^-$  is taken as the configuration coordinate, instead of the ground state combination  $D_0A_0$ - $D_0^+A_0^-$ , then, Eqs. 26 and 27 are still valid to define such a coordinate, as long as  $Q_{D_0}$ ,  $Q_{A_0}$ ,  $Q_{D_0^+}$ , and  $Q_{A_0^-}$ , are substituted by the corresponding  $Q_{D_i}$ ,  $Q_{A_j}$ ,  $Q_{D_k^+}$ , and  $Q_{A_\ell^-}$ .

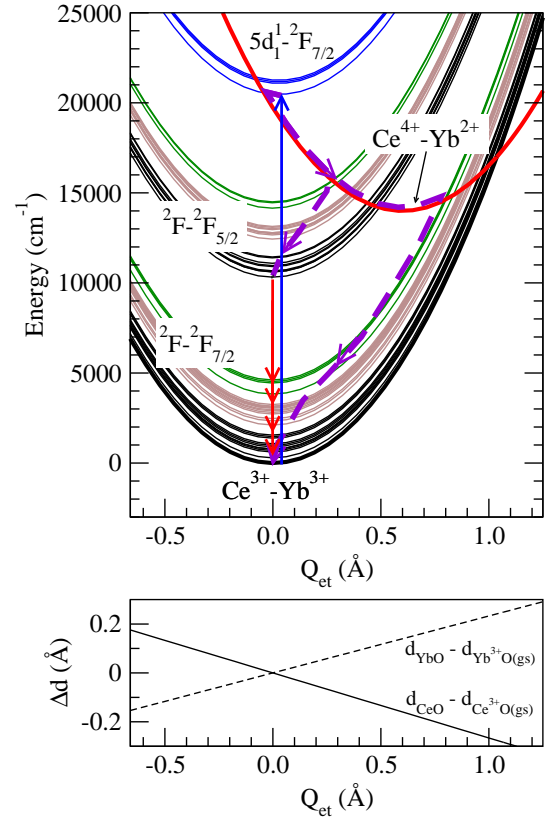


FIG. 6: MMCT configuration coordinate diagram of  $Ce^{3+}/Yb^{3+}$  pairs in YAG. The following processes are indicated:  $Ce^{3+}$  lowest  $4f \rightarrow 5d$  absorption (blue arrow); energy transfer to the  $^2F_{5/2}$  excited state of  $Yb^{3+}$  and non-radiative decay to the ground states of  $Ce^{3+}$  and  $Yb^{3+}$  through the  $Ce^{4+}\cdot Yb^{2+}$  MMCT state (dashed violet line); and  $Yb^{3+}$   $4f \rightarrow 4f$  emission (red arrows).

## B. MMCT configuration coordinate diagram of $Ce^{3+}/Yb^{3+}$ in YAG

In Fig. 6 we show a MMCT configuration coordinate diagram of  $Ce^{3+}/Yb^{3+}$  in YAG, which results from representing Eq. 34 along the ground state electron transfer reaction coordinate defined by Eqs. 26, 27, and 32. The representation along the  $Ce^{3+}(5d_1)-Yb^{3+}(^2F_{7/2}) \rightarrow Ce^{4+}\cdot Yb^{2+}$  reaction coordinate in the same scale is hard to distinguish at sight. We used the following data: (1) The  $E_{D_iA_j,e} = E_e(Ce_i^{3+}\cdot Yb_j^{3+})$  energy offsets were obtained from the experimental excitation energies of  $Ce^{3+}$  and  $Yb^{3+}$ , which are shown in Table I. Since we do not have empirical data on the minimum-to-minimum  $Ce^{3+}$ -to- $Yb^{3+}$  charge transfer excitation energy, we treat it here as an empirical parameter such that it provides a diagram consistent with the experiments; we used  $E_{D_0^+A_0^-,e} = E_e(Ce^{4+}\cdot Yb^{2+}) = 14000 \text{ cm}^{-1}$ . (2) We used  $d_{A_0} - d_{A_0} = d_{Yb^{2+}O} - d_{Yb^{3+}O} = 0.14 \text{ \AA}$ , as in Sec. III B, and  $d_{D_0} - d_{D_0} = d_{Ce^{3+}O} - d_{Ce^{4+}O} = 0.16 \text{ \AA}$ , using the same procedure as in  $Yb^{2+}/Yb^{3+}$  (90% of the difference between the ionic radii of  $Ce^{3+}$  and  $Ce^{4+}$  in coordination 8, 1.143 Å and 0.97 Å respectively.<sup>18</sup>). We used the same Ce-O

distance offset in all  $4f^1$  states of  $Ce^{3+}$  and a  $-0.02 \text{ \AA}$  offset for the lowest  $5d^1$  state (Table I), slightly larger than the  $-0.014 \text{ \AA}$  found in *ab initio* calculations in YAG:Ce<sup>3+</sup>, which has been considered to be underestimated.<sup>23</sup> (3) The donor and acceptor force constants are  $k_D = \mu \omega_D^2$  and  $k_A = \mu \omega_A^2$ , with  $\mu = m(O) = 15.999 \text{ amu}$ . We used  $\bar{\nu}_D = \omega_D / (2\pi c) = 210 \text{ cm}^{-1}$ , which is 5% larger than the  $200 \text{ cm}^{-1}$  vibrational sequence found in Ref. 17 for YAG:Ce<sup>3+</sup> (it corresponds to using a common force constant for Ce<sup>3+</sup> and Ce<sup>4+</sup> 10% larger than that of Ce<sup>3+</sup>; the force constant is expected to be larger in Ce<sup>4+</sup> than in Ce<sup>3+</sup>). And we used  $\bar{\nu}_A = \omega_A / (2\pi c) = 310 \text{ cm}^{-1}$ , which is the common vibrational frequency for Yb<sup>2+</sup> and Yb<sup>3+</sup> that we used in Sec. III B.

In the MMCT configuration coordinate diagram, we observe the states of the Ce<sup>3+</sup>-Yb<sup>3+</sup> pair, which have been discussed above, and, crossing them, the ground state of the Ce<sup>4+</sup>-Yb<sup>2+</sup> pair, with a horizontal offset of  $0.60 \text{ \AA}$  (Eq. 33). This diagram is basically consistent with the experiments and the interpretations given in Ref. 3. The lowest Ce<sup>3+</sup>( $5d_1^1$ )-Yb<sup>3+</sup>( $^2F_{7/2}$ ) state can decay nonradiatively to the Ce<sup>4+</sup>-Yb<sup>2+</sup> MMCT state with a small energy barrier. This barrier is  $73 \text{ cm}^{-1}$  and the crossing is produced on the left side of the Ce<sup>3+</sup>( $5d_1^1$ )-Yb<sup>3+</sup>( $^2F_{7/2}$ ) minimum with the above data; although the energy of the barrier can change with the parameters of the model, the basic interpretation is maintained for a relatively wide range around the present data. Next, the Ce<sup>4+</sup>-Yb<sup>2+</sup> MMCT state can decay directly to the Ce<sup>3+</sup>( $4f^1$ )-Yb<sup>3+</sup>( $^2F_{5/2}$ ) manifold, which can yield Yb<sup>3+</sup>  $^2F_{5/2} \rightarrow ^2F_{7/2}$  emission. This is consistent with the observations of energy transfer from Ce<sup>3+</sup> to Yb<sup>3+</sup> in YAG:Ce,Yb, with its temperature dependence, and with the Yb<sup>3+</sup> emission not showing rise time, so that it supports the given interpretation as due to a thermally activated decay through an intermediate Ce<sup>4+</sup>-Yb<sup>2+</sup> charge transfer state,<sup>3</sup> and complements it with a more detailed description. A value of  $E_e(\text{Ce}^{4+}\text{-Yb}^{2+})$  around  $2000 \text{ cm}^{-1}$  higher is also consistent with the experiments: e.g. a value of  $16000 \text{ cm}^{-1}$  gives a  $30 \text{ cm}^{-1}$  barrier and a crossing on the right side of the minimum. Note that, according to Eq. 25, several  $E_e(\text{Ce}^{4+}\text{-Yb}^{2+})$  values associated to different Ce-Yb distances can coexist in the material.

An additional interesting feature was observed in the experiments of Ref. 3: The intensity of the Yb<sup>3+</sup>  $4f \rightarrow 4f$  emission as excited with the Ce<sup>3+</sup>  $4f \rightarrow 5d$  absorption strongly decreases above 110 K, after the initial increase with temperature due to the thermally activated crossing to the Ce<sup>4+</sup>-Yb<sup>2+</sup> MMCT state. The crossing between the Ce<sup>4+</sup>-Yb<sup>2+</sup> MMCT state and the lowest Ce<sup>3+</sup>( $4f^1$ )-Yb<sup>3+</sup>( $^2F_{7/2}$ ) manifold was suggested as a possible explanation. The present MMCT configuration coordinate diagram does not rule out such an explanation because the crossing exists, although it has a relatively high barrier ( $590 \text{ cm}^{-1}$  with the present data). However, the consideration of both the MMCT diagram of Ce<sup>3+</sup>-Yb<sup>3+</sup> in YAG (Fig. 6) and the IVCT dia-

gram of Yb<sup>2+</sup>-Yb<sup>3+</sup> in YAG (Fig. 4), suggests an alternative explanation: The Ce<sup>3+</sup>-to-Yb<sup>3+</sup> MMCT produced after the Ce<sup>3+</sup>  $4f \rightarrow 5d$  absorption increases the probability of formation of Yb<sup>2+</sup>-Yb<sup>3+</sup> pairs, hence, of quenching the Yb<sup>3+</sup>  $4f \rightarrow 4f$  emission via IVCT non-radiative decay, as discussed in Sec. III B.

## V. CONCLUSIONS

Quantitative configuration coordinate diagrams for intervalence charge transfer states of mixed valence pairs and metal-to-metal charge transfer states of heteronuclear pairs of dopant ions in solid hosts, have been introduced and discussed in detail. They are obtained with the use of vibrational frequencies and excitation energies of single-ion active centers, together with differences between ion-ligand distances of the single-ion donor and acceptor centers. These data are attainable empirically, either from direct measurements or from estimations, e.g. based on ionic radii, and they can be calculated with *ab initio* methods. The IVCT configuration coordinate diagram of Yb<sup>2+</sup>/Yb<sup>3+</sup> mixed valence pair in Yb-doped YAG, and the MMCT configuration coordinate diagram of Ce<sup>3+</sup>/Yb<sup>3+</sup> heteronuclear pair in Ce,Yb-codoped YAG, have been obtained and discussed. Empirical data of the individual ions doped in YAG have been used for this purpose.

The analysis of the Yb<sup>2+</sup>/Yb<sup>3+</sup> IVCT diagram suggest that quenching of the Yb<sup>3+</sup>  $4f \rightarrow 4f$  emission takes place by means of IVCT non-radiative decay in Yb<sup>2+</sup>/Yb<sup>3+</sup> pairs.

The analysis of the Ce<sup>3+</sup>/Yb<sup>3+</sup> MMCT diagram supports a previous interpretation of energy transfer from Ce<sup>3+</sup> to Yb<sup>3+</sup> in Ce,Yb-codoped YAG, after Ce<sup>3+</sup>  $4f \rightarrow 5d$  excitation, via a Ce<sup>4+</sup>-Yb<sup>2+</sup> MMCT state. The diagram provides the details of this process. The energy of the structurally relaxed Ce<sup>4+</sup>-Yb<sup>2+</sup> pair is estimated to lie at either about  $14000 \text{ cm}^{-1}$  or  $16000 \text{ cm}^{-1}$  above the structurally relaxed Ce<sup>3+</sup>-Yb<sup>3+</sup> pair. According to the diagram, there is a higher probability of nonradiative decay from the intermediate Ce<sup>4+</sup>-Yb<sup>2+</sup> pair to the excited Ce<sup>3+</sup>( $4f^1$ )-Yb<sup>3+</sup>( $^2F_{5/2}$ ) manifold than to the ground Ce<sup>3+</sup>( $4f^1$ )-Yb<sup>3+</sup>( $^2F_{7/2}$ ) manifold of the Ce<sup>3+</sup>-Yb<sup>3+</sup> pair. Altogether, the two diagrams suggest that the temperature quenching of the Yb<sup>3+</sup>  $4f \rightarrow 4f$  emission excited with Ce<sup>3+</sup>  $4f \rightarrow 5d$  absorption is due to the formation of Yb<sup>2+</sup>-Yb<sup>3+</sup> pairs after MMCT from Ce<sup>3+</sup>-to-Yb<sup>3+</sup> in Ce<sup>3+</sup>-Yb<sup>3+</sup> pairs.

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