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# Ion Density Deviations in Semipermeable Ionic Microcapsules

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By implementing the nonlinear Poisson-Boltzmann theory in a cell model, we theoretically investigate the influence of polyelectrolyte gel permeability on ion densities and pH deviations inside the cavities of ionic microcapsules. Our calculations show that variations in permeability of a charged capsule shell cause a redistribution of ion densities within the capsule, which ultimately affects the pH deviation and Donnan potential induced by the electric field of the shell. We find that semipermeable capsules can induce larger pH deviations inside their cavities than permeable capsules. Furthermore, with increasing capsule charge, the influence of permeability on pH deviations progressively increases. Our theory, while providing a self-consistent method for modeling the influence of permeability on fundamental properties of ionic microgels, makes predictions of practical significance for the design of microcapsules loaded with fluorescent dyes, which can serve as biosensors for diagnostic purposes.

## 1 Introduction

Polyelectrolyte (PE) microcapsules – ionic (charged) colloidal particles with hollow cavities<sup>1,2</sup> – have attracted great attention in the past decade due to their novel fundamental properties and their potential applications as biosensors to monitor local ion concentrations (such as pH) in cellular environments.<sup>3–14</sup> Charged PE shells, which may encapsulate pH-sensitive fluorescent dyes, generate electric fields that can cause deviations in local ion distributions and limit practical applications. Previous experiments<sup>15,16</sup> and theories<sup>17,18</sup> demonstrated that the pH and ion densities near a charged flat surface can deviate from their bulk values. For example, Bostrom *et al.*<sup>15</sup> demonstrated that ion and pH gradients emerge near biological flat membranes. Zhang *et al.*<sup>16</sup> experimentally confirmed that local ion concentrations and sensor read-outs can be attributed to surface charges. Janata<sup>17,18</sup> showed theoretically that pH shifts measured by optical sensors depend on bulk-surface interactions. The charged shell of a spherical microcapsule can induce variations in ion densities near the strongly curved surface of the capsule. Furthermore, because of the asymmetry between the inside and outside environments, the measured ion concentrations in microcapsule cavities can deviate greatly from those in bulk. Understanding such ion deviations is significant for biomedical applications of ionic microcapsules as biosensors, e.g., to avoid misdiagnosis of diseases, such as early-stage cancer.<sup>19–21</sup> Ion distributions can also affect the release properties of pressurized capsules for drug delivery.<sup>22</sup>

Ion density deviations inside of microcapsule cavities have been studied previously by experiments,<sup>23–26</sup> theories,<sup>25,27</sup> and simulations.<sup>28</sup> For example, Sukhorukov *et al.*<sup>23</sup> and Gao *et al.*<sup>24</sup> demonstrated that a pH difference between the inside and outside of PE capsules emerges when the capsules are permeable to small ions, but exclude poly(styrenesulfonate) ions of a particular molecular weight. Another approach, based on entrapping polyanions within PE microcapsules, also reported a redistribution of H<sup>+</sup> ions across a semipermeable microcapsule wall.<sup>25</sup> These ion density differences were attributed to the capsule semipermeability and explained by a macroscopic theoretical model of Donnan equilibrium.<sup>25,27</sup> Vinogradova *et al.*<sup>29–31</sup> applied Poisson-Boltzmann (PB) theory to study the osmotic pressure acting on semipermeable shells in polyanion solutions. Stukan *et al.*<sup>28</sup> used molecular dynamics simulation to study ion distributions near nanocapsules that are permeable to solvent and counterions, but impermeable to polyelectrolyte coils, which were modeled as soft colloids. Semipermeability to salt ions has also been linked to osmotic shock of rigid protein shells, such as viral capsids.<sup>32</sup>

Recently, we demonstrated, using non-linear PB theory, that even fully permeable microcapsules can induce ion density deviations inside of charged capsule cavities.<sup>33</sup> These deviations depend on the degree of dissociation of the PE making up the shells, rather than on the permeability of the capsule wall. In practical applications of PE microcapsules in cellular environments, however, the capsule walls usually exclude polyions, such as charged DNA or amino acids. In this case, the ion density deviations induced by the microcapsule should also depend on the permeability of the charged wall. The influence of permeability on deviations of local ion densities and pH induced by ionic microcapsules is still poorly understood.

In this paper, by modeling a PE microcapsule as a uniformly

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charged shell that is permeable to some ionic species (counterions and salt ions), but only semipermeable to another species (polyions), we extend our previous work to analyze the impact of permeability of charged capsule shells on ion density deviations in aqueous solutions. By employing nonlinear PB theory, we systematically calculate deviations of local ion densities inside microcapsule cavities induced by the permeability of capsule shells. Unlike macroscopic models of Donnan equilibrium for neutral capsules,<sup>25,27</sup> here the redistribution of ion densities depends not only on the capsule wall's permeability, but also on its degree of dissociation (charge density). Our results demonstrate that capsule permeability can also significantly influence the properties of ionic microcapsules and their performance as biosensors.

The remainder of the paper is organized as follows. In Sec. 2, we define our model of semipermeable ionic microcapsules and describe our implementation of PB theory. In Sec. 3, we present numerical results from our calculations for the influence of capsule permeability on ion densities, Donnan potentials, and pH distributions. Finally, in Sec. 4, we summarize our findings and emphasize implications for practical applications.

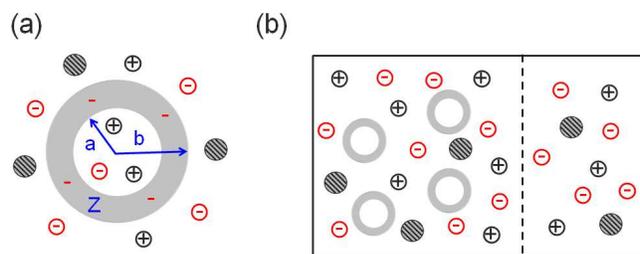
## 2 Models and Methods

### 2.1 Uniform-Shell Model of Ionic Microcapsules

We consider a bulk dispersion of ionic capsules, microions, and polyions dispersed in water. Within the primitive model of polyelectrolytes,<sup>34</sup> the solvent is idealized as a dielectric continuum with uniform dielectric constant  $\epsilon$ . We model the capsules as spherical shells of inner radius  $a$ , outer radius  $b$ , and valence  $Z$ . In a polar solvent, a capsule becomes charged when counterions dissociate from the PE chains that form the cross-linked network (hydrogel) making up its shell. Similarly, the polyions become charged by dissociation. In line with many experiments, we presume the capsules to be negatively charged. Approximating the distribution of ionized sites in the hydrogel as uniform within the volume of the shell, we represent the number density of fixed charge by a simple radial profile,

$$n_f(r) = \begin{cases} 0, & r > b, \\ \frac{3Z}{4\pi(b^3 - a^3)}, & a < r < b, \\ 0, & 0 < r < a, \end{cases} \quad (1)$$

where  $r$  is radial distance from the center of the shell [Fig. 1(a)]. Although neglecting charge discreteness and ion-specific effects, this coarse-grained model is consistent with the primitive model and valid on length scales longer than the typical spacing between ionized groups on the PE backbones.



**Fig. 1** (a) Model of a spherical microcapsule of inner shell radius  $a$ , outer shell radius  $b$ , and valence  $Z$  in water. The capsule is permeable to water and small ions (smaller spheres labeled as  $+$  and  $-$ ), but only partially permeable to polyions (larger, shaded spheres). (b) Bulk dispersion of microcapsules in Donnan equilibrium with a reservoir.

The microions comprise dissociated counterions and salt ions (e.g.,  $\text{Na}^+$  and  $\text{Cl}^-$ ). The polyion charge can be of either sign, depending on the species and the pH level of the solution. For simplicity, we model microions and polyions as point charges. For reasons made clear below, we furthermore consider only monovalent ions.

The capsules are permeable to water and microions, but only partially permeable to polyions. To quantify the degree of penetration of the capsule shell by polyions, we introduce a permeability factor  $\alpha$ , which ranges from 0 (no penetration) to 1 (complete penetration). Within the cavity ( $r < a$ ), we take the dielectric constant to be the same as in bulk, while in the shell, we assume a lower dielectric constant ( $\epsilon_{\text{shell}} < \epsilon$ ), as suggested by experiments on PE microgels in water.<sup>35,36</sup> In Donnan equilibrium, the capsules are confined to a fixed volume, while the solvent, microions, and polyions can freely exchange with a reservoir, with fixed number densities of salt ion pairs,  $n_0$ , and of polyions,  $n_{p0}$  [see Fig. 1(b)].

### 2.2 Poisson-Boltzmann Theory of Bulk Dispersions

We model a bulk dispersion of ionic capsules via Poisson-Boltzmann theory.<sup>37</sup> In its density-functional formulation,<sup>38,39</sup> PB theory focuses on the grand potential functional,  $\Omega[n_{\pm}(\mathbf{r}), n_p(\mathbf{r})]$ , which is a unique functional of the number densities of positive and negative microions,  $n_{\pm}(\mathbf{r})$ , and of polyions,  $n_p(\mathbf{r})$ . We arbitrarily assume positively charged polyions of valence  $z_p$ . In reduced form, the PB approximation for the grand potential functional can be expressed as

$$\begin{aligned} \beta\Omega[n_{\pm}(\mathbf{r}), n_p(\mathbf{r})] = & \int d\mathbf{r} \left\{ n_+(\mathbf{r}) \left[ \ln \left( \frac{n_+(\mathbf{r})}{n_0} \right) - 1 \right] \right. \\ & \left. + n_-(\mathbf{r}) \left[ \ln \left( \frac{n_-(\mathbf{r})}{n_0} \right) - 1 \right] + n_p(\mathbf{r}) \left[ \ln \left( \frac{n_p(\mathbf{r})}{n_{p0}} \right) - 1 \right] \right\} \\ & + \frac{1}{2} \int d\mathbf{r} [n_+(\mathbf{r}) - n_-(\mathbf{r}) + z_p n_p(\mathbf{r}) - n_f(\mathbf{r})] \psi(\mathbf{r}), \quad (2) \end{aligned}$$

where the first integral accounts for the ideal-gas free energy and the second integral for the electrostatic potential energy. Here,  $\beta \equiv 1/(k_B T)$  and  $\psi(\mathbf{r}) \equiv \beta e \phi(\mathbf{r})$  is a dimensionless form of the electrostatic potential  $\phi(\mathbf{r})$ ,

$$e\phi(\mathbf{r}) = \int d\mathbf{r}' [n_+(\mathbf{r}') - n_-(\mathbf{r}') + z_p n_p(\mathbf{r}') - n_f(\mathbf{r}')] v(|\mathbf{r} - \mathbf{r}'|), \quad (3)$$

generated by Coulomb pair interactions,  $v(r) = e^2/(\epsilon r)$ , between elementary charges  $e$  on mobile microions dispersed throughout the system and fixed ions localized within the shells. The neglect of correlations among ions inherent in the mean-field approximation for  $\Omega$  is valid for weakly-correlated monovalent ions, but questionable for more strongly correlated multivalent microions.<sup>29–31</sup> For this reason, we restrict our considerations to monovalent ions.

In thermodynamic equilibrium, the grand potential functional is a minimum with respect to the microion and polyion densities. Minimizing Eq. (2) with respect to  $n_{\pm}(\mathbf{r})$  and  $n_p(\mathbf{r})$  yields Boltzmann distributions for the equilibrium densities:

$$n_{\pm}(\mathbf{r}) = n_0 e^{\mp \psi(\mathbf{r})} \quad (4)$$

and

$$n_p(\mathbf{r}) = \begin{cases} n_{p0} e^{-z_p \psi(\mathbf{r})}, & r > b, \\ \alpha n_{p0} e^{-z_p \psi(\mathbf{r})}, & 0 < r < b, \end{cases} \quad (5)$$

where the factor  $\alpha$  dictates the permeability of the capsule to polyions, ranging from completely impermeable for  $\alpha = 0$  to completely permeable for  $\alpha = 1$ . Substituting Eqs. (4) and (5) into the Poisson equation,

$$\nabla^2 \phi(\mathbf{r}) = -\frac{4\pi}{\epsilon} \rho(\mathbf{r}), \quad (6)$$

where

$$\rho(\mathbf{r}) = e[n_+(\mathbf{r}) - n_-(\mathbf{r}) + z_p n_p(\mathbf{r}) - n_f(\mathbf{r})] \quad (7)$$

is the local charge density, yields the Poisson-Boltzmann equation:

$$\nabla^2 \psi(\mathbf{r}) = \kappa_0^2 \sinh \psi(\mathbf{r}) + 4\pi \lambda_B [n_f(\mathbf{r}) - z_p n_p(\mathbf{r})]. \quad (8)$$

Here,  $\lambda_B \equiv e^2/(\epsilon k_B T)$  is the Bjerrum length ( $\lambda_B = 0.714$  nm in water at temperature  $T \simeq 293$ ,  $\epsilon = 78$ );  $\kappa_0 \equiv \sqrt{8\pi \lambda_B n_0}$  is the inverse Debye screening length in a reference reservoir of pure salt solution (absent polyions); and  $\kappa_p \equiv \sqrt{4\pi \lambda_B n_{p0}}$  can be interpreted as an inverse Debye screening length of the polyions. By solving Eq. (8) with appropriate boundary conditions, we obtain the microion and polyion density profiles.

### 2.3 Poisson-Boltzmann Cell Model

In general, solving the PB equation [Eq. (8) with boundary conditions matching an arbitrary arrangement of capsules is numerically quite challenging, although feasible in some systems, such as charged colloids.<sup>40–42</sup> For computational efficiency, we instead use a cell model,<sup>43,44</sup> whose boundary conditions are relatively simple. The cell model is justified by the wide disparity in size and charge between capsules and microions and by our focus on solutions of relatively low salt concentration.<sup>42,45</sup>

For spherical capsules, the cell model represents a bulk dispersion by a spherical cell – centered on a single capsule – of radius  $R$  determined by the capsule volume fraction  $\eta = (b/R)^3$  [see Fig. 1(a)]. Along with the capsule, the cell contains counterions and salt ions, which may freely penetrate the capsule, and polyions, whose penetration of the capsule is limited by the shell's permeability. The condition of electroneutrality of the cell relates the ion numbers:  $Z = N_+ - N_- + z_p N_p$ .

In the spherical cell model, the PB equation simplifies to

$$\psi''(r) + \frac{2}{r} \psi'(r) = \begin{cases} \kappa_0^2 \sinh \psi(r) - z_p \kappa_p^2 e^{-z_p \psi(r)}, & b < r < R, \\ \frac{1}{\chi} \left( \kappa_0^2 \sinh \psi(r) - \alpha z_p \kappa_p^2 e^{-z_p \psi(r)} + \frac{3Z\lambda_B}{b^3 - a^3} \right), & a < r < b, \\ \kappa_0^2 \sinh \psi(r) - \alpha z_p \kappa_p^2 e^{-z_p \psi(r)}, & 0 < r < a, \end{cases} \quad (9)$$

where  $r$  is the radial distance from the center of the cell and  $\chi = \epsilon_{\text{shell}}/\epsilon < 1$  is the ratio of the dielectric constant in the capsule shell to that in the bulk solvent.

Boundary conditions on Eq. (9) impose continuity of the electrostatic potential at the inner and outer boundaries of the

capsule shell,

$$\psi_{\text{in}}(a) = \psi_{\text{shell}}(a), \quad \psi_{\text{shell}}(b) = \psi_{\text{out}}(b), \quad (10)$$

vanishing of the electric field at the center of the cell and on

the cell boundary,

$$\psi'_{\text{in}}(0) = 0, \quad \psi'_{\text{out}}(R) = 0, \quad (11)$$

as required by spherical symmetry and electroneutrality, and continuity of the electric displacement on the inner and outer shell boundaries,

$$\psi'_{\text{in}}(a) = \chi \psi'_{\text{shell}}(a), \quad \chi \psi'_{\text{shell}}(b) = \psi'_{\text{out}}(b), \quad (12)$$

where the solutions in the three regions are labelled as  $\psi_{\text{in}}(r)$  ( $0 < r < a$ ),  $\psi_{\text{shell}}(r)$  ( $a < r < b$ ), and  $\psi_{\text{out}}(r)$  ( $b < r < R$ ).

By numerically solving the PB equation [Eq. (9)], along with the boundary conditions [Eqs. (10)-(12)], in the three radial regions (inside the cavity, in the shell, and outside the capsule), we calculate the equilibrium microion and polyion density distributions within the spherical cell.

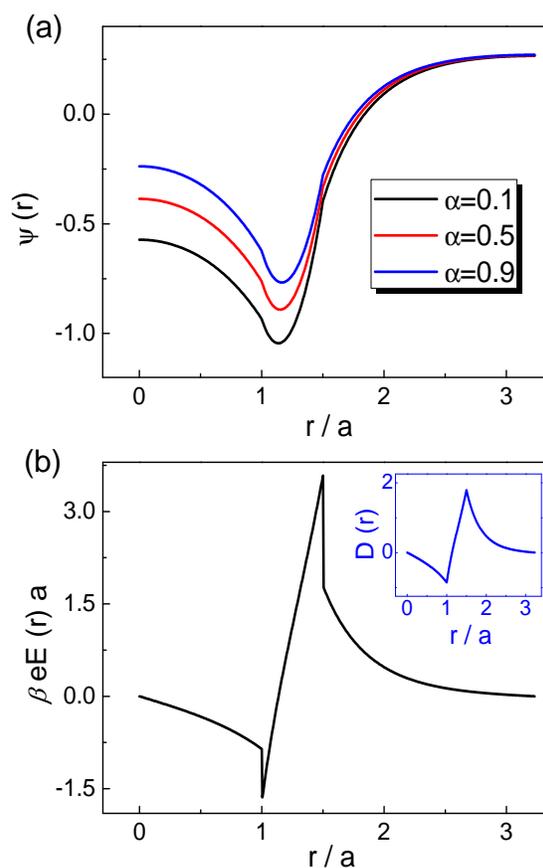
### 3 Results and Discussion

#### 3.1 Distribution of Electrostatic Potential and Field

To illustrate our theory, we present numerical results for system parameters typical of experiments on microcapsules. Specifically, we consider negatively charged microcapsule shells of inner radius  $a = 50$  nm, outer radius  $b = 75$  nm, and valence  $Z = 500$  dispersed in water at room temperature ( $\lambda_B = 0.72$  nm), in Donnan equilibrium with a salt reservoir of concentration  $n_0 = 0.1$  mM, at volume fraction  $\eta = 0.1$ , corresponding to a cell radius  $R = \eta^{-1/3}b \simeq 3.23 a$ . This concentration is sufficiently dilute to ensure independence of the ion distributions within neighboring cavities. We set the dielectric constant ratio between the microcapsule shell and the solution at  $\chi = 0.5$ , which is consistent with measured dielectric constants in hydrated ionic Poly(N-isopropylacrylamide) (PNI-PAM) microgels ranging from 63 at 15°C to 17 at 40°C.<sup>35,36</sup> In order to justify the mean-field PB approach, we consider here only weakly correlated monovalent polyions ( $z_p = 1$ ). Finally, we choose the polyion concentration,  $n_{p0} = 0.4n_0$ , to give a polyion screening length  $\kappa_p^{-1}$  somewhat longer than the screening length  $\kappa_0^{-1}$  in a reference reservoir of pure salt solution.

Figure 2(a) shows the distribution of electrostatic potential within the cell. Starting from the center, the potential decreases with increasing radial distance, reaching a minimum within the shell. The depth of this minimum increases with decreasing capsule permeability, reaching a value of  $\sim 1 k_B T$  (in potential energy) for  $\alpha = 0.1$ . The kinetic barrier to thermal diffusion of ions across the shell is thus sufficiently low to ensure equilibrium ion distributions. Outside the shell, the electrostatic potential increases toward the cell edge.

The corresponding electric field is shown in Fig. 2(b). Starting from zero at the center, the field decreases as  $r$  increases



**Fig. 2** (a) Distribution of reduced electrostatic potential  $\psi(r)$  vs. radial distance  $r$  from center of a microcapsule of inner shell radius  $a = 50$  nm, outer radius  $b = 75$  nm, valence  $Z = 500$ , and dielectric constant ratio  $\chi = 0.5$  in an aqueous solution of microcapsule volume fraction  $\eta = 0.1$ , reservoir salt concentration  $n_0 = 0.1$  mM, and polyion concentration  $n_p = 0.04$  mM. Results are shown for shell permeability  $\alpha = 0.1, 0.5,$  and  $0.9,$  (b) Electric field  $E(r)$  vs.  $r$  for permeability  $\alpha = 0.1.$  Inset shows distribution of displacement field  $D(r) = \epsilon E(r).$

inside the cavity, but then rises sharply within the shell. The discontinuities at the shell boundaries ( $r = a$  and  $1.5a$ ) originate from the difference in dielectric constant between the shell and the solution. The displacement field, rather than the electric field, is continuous at the boundaries [see inset to Fig. 2(b)]. Outside the shell, the field decreases as  $r$  increases, approaching zero at the edge of the cell to match the outer boundary condition. As a consistency check on our implementation of the theoretical model, we calculate the total number  $N_{tot}$  of microions and polyions in the cell,

$$N_{tot} = 4\pi \int_0^R dr r^2 [n_+(r) - n_-(r) + z_p n_p(r)], \quad (13)$$

and confirm global electroneutrality ( $N_{tot} = Z$ ) over a range of permeabilities.

### 3.2 Donnan Potentials

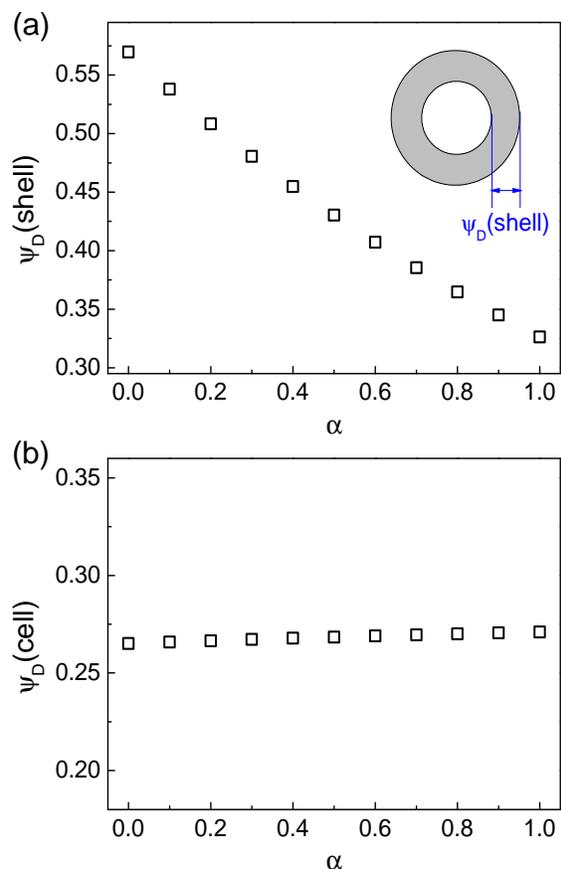
Chemical equilibrium between the inside and outside environments of ionic microcapsules can result in a significant difference in electrostatic potential between the two sides of the shell, as revealed in Fig. 2(a). We define this difference as the *shell Donnan potential*:

$$\psi_D(\text{shell}) \equiv \psi(b) - \psi(a), \quad (14)$$

which is analogous to the Donnan potential at the surface of a bulk polyelectrolyte gel.<sup>46</sup> The variation of the shell Donnan potential with capsule permeability  $\alpha$  is shown in Fig. 3(a). With increasing permeability, polyions increasingly penetrate the capsule shell, leading to a gradual decrease of  $\psi_D(\text{shell})$ . For comparison, Fig. 3(b) shows the Donnan potential at the cell edge,  $\psi_D(\text{cell}) \equiv \psi(R)$ , as a function of capsule permeability. Interestingly,  $\psi_D(\text{cell})$  is relatively insensitive to variation of  $\alpha$ , implying that capsule permeability also does not significantly influence the bulk osmotic pressure of the dispersion. On the other hand, the permeability *is* expected, to influence ion distributions inside and outside the capsules, with potential impact on practical applications of ionic microcapsules, such as in diagnosis of diseases. In the remainder of this section, we examine in detail ion density distributions induced by shell permeability, as well as corresponding pH deviations inside microcapsule cavities.

### 3.3 Influence of Permeability on Ion Densities

Next, we investigate the influence of capsule permeability on the ion density distributions within the cell. In practice, permeability may increase upon swelling, triggered by a change of solution pH or temperature. Figure 4(a) shows the distribution of polyion density over a range of permeabilities. As the capsule becomes more permeable ( $\alpha$  increases), polyions can more easily penetrate the capsule shell, thus increasing



**Fig. 3** Donnan potential  $\psi_D$  vs. capsule permeability  $\alpha$  at the edge of (a) the capsule shell ( $r = a$ ) and (b) the entire cell ( $r = R$ ). Other system parameters are as in Fig. 2.

the polyion density within the cavity. This redistribution of polyions results from entropy-driven thermal diffusion, which can overcome the electrostatic potential energy barrier [see Fig. 2(a)].

The rising concentration of polyions inside the capsule upon increasing permeability leads to an expulsion of positive microions and a corresponding decrease in  $n_+(r)$  for  $r < b$ , as shown in Fig. 4(b). Nevertheless, there is a net increase in the total concentration of positive ions inside the capsule. To maintain electroneutrality of the system, the density of negative microions also increases, as shown in Fig. 4(c). The redistributions of ion densities induced by changes in capsule permeability should be mirrored by corresponding changes in the local concentrations of  $H^+$  and  $OH^-$  ions, leading to deviations in local pH from bulk solution values.

### 3.4 Influence of Permeability on pH in Microcapsules

Next, we investigate the influence of microcapsule permeability on pH deviations inside the capsule cavities and explore the potential impact on applications to pH sensors in cellular environments. The polyions in our model are positively charged, corresponding to the real scenario of amino acids dispersed in an alkaline cellular environment. Under such conditions, the pH within the cell is determined by the concentration of hydroxyl ( $OH^-$ ) ions. Assuming the concentration of  $OH^-$  ions to be proportional to that of all negative microions, the local pH deviation induced by the charged shell can be approximated by

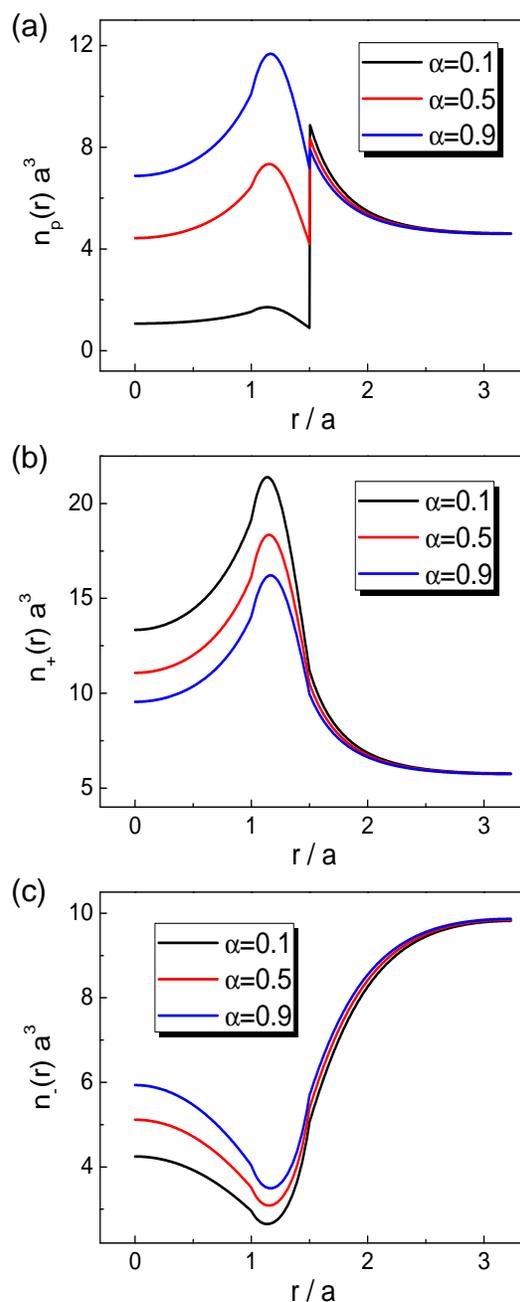
$$\Delta pH(r) = \log[n_-(r)/n_-(R)]. \quad (15)$$

The average pH deviation inside the capsule cavities is then

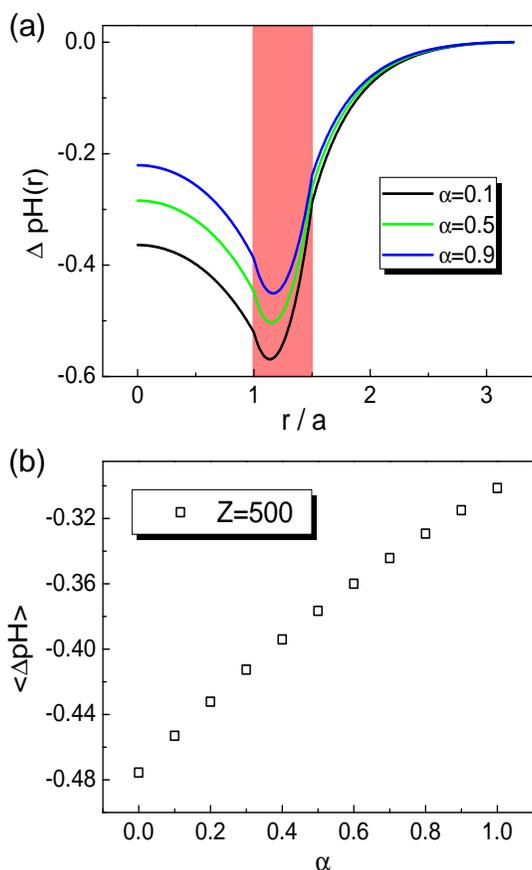
$$\langle \Delta pH \rangle \equiv \frac{3}{a^3} \int_0^a dr r^2 \Delta pH(r). \quad (16)$$

For a capsule of valence  $Z = 500$ , Fig. 5 shows (a) the local pH deviation profile and (b) the average pH deviation induced by the charged shell over a range of capsule permeabilities. With increasing  $\alpha$ , the local and average pH deviations inside the cavity decrease in magnitude, as a result of variations in ion concentrations discussed in Sec. 3.3. Thus, in alkaline environments, pH deviations inside cavities of ionic microcapsules induced by the charged shells are suppressed by increasing permeability to positive polyions. In other words, semipermeable charged capsules can induce larger pH deviations inside their cavities than permeable capsules.

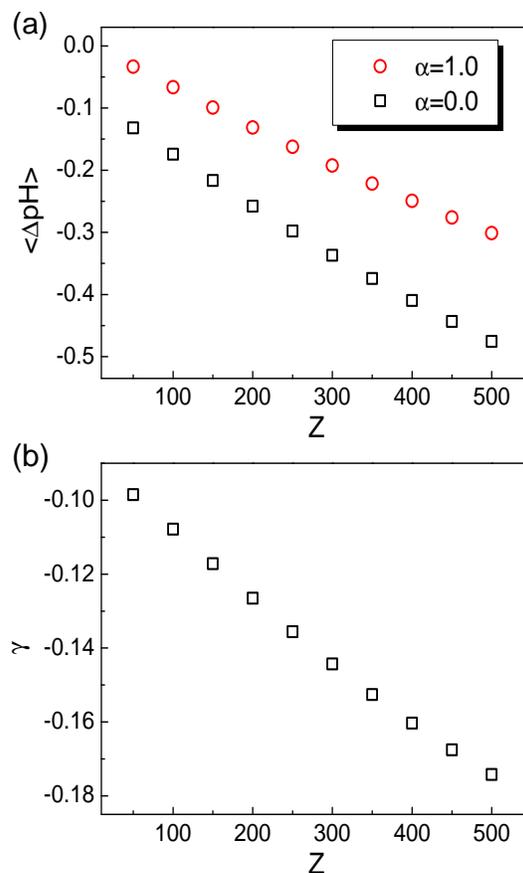
In a previous paper,<sup>33</sup> we demonstrated that the valence of ionic microcapsules dominates pH deviations inside the cavities. The larger the capsule valence, the greater the pH deviations. Here we find that the permeability of the capsule can also influence pH deviations. Finally, we investigate how the deviations induced by permeability vary with capsule valence.



**Fig. 4** Density distributions of (a) polyions, (b) positive microions, and (c) negative microions vs. radial distance  $r$  from center of microcapsule for permeability  $\alpha = 0.1, 0.5$ , and  $0.9$ . Other system parameters are as in Fig. 2.



**Fig. 5** (a) Deviation of local pH from bulk value vs. radial distance  $r$  from center of microcapsule [Eq. (15)] for valence  $Z = 500$  and permeability  $\alpha = 0.1, 0.5$ , and  $0.9$ . (b) Deviation of average pH inside cavity ( $r < a$ ) vs. permeability [Eq. (16)]. Other system parameters are as in Fig. 2.



**Fig. 6** (a) Average deviation of pH from bulk value inside microcapsule cavity vs. valence  $Z$  for permeability  $\alpha = 0$  and  $1$ . (b) Permeability-induced average pH deviation vs. valence [Eq. (17)]. Other system parameters are as in Fig. 2.

Figure 6(a) illustrates the variation with capsule valence of the average pH deviation inside the cavity for the two extremes of permeability,  $\alpha = 0$  and  $1$ . We find that, with increasing valence, the average pH deviation increases in magnitude. For capsules that are impermeable to polyions ( $\alpha = 0$ ), however, the increase is more rapid than for fully permeable capsules ( $\alpha = 1$ ). To quantify this effect, we define

$$\gamma = \langle \Delta \text{pH} \rangle_{\alpha=0} - \langle \Delta \text{pH} \rangle_{\alpha=1}, \quad (17)$$

which measures the average pH deviation inside the capsule cavity induced solely by permeability of the capsule shell. Figure 6(b) shows the permeability-induced average pH deviation inside the cavities as a function of capsule valence, illustrating that, as valence increases, the average pH deviation associated with capsule permeability increases in magnitude.

## 4 Conclusions

In summary, by applying a cell model implementation of the nonlinear Poisson-Boltzmann theory to solutions of ionic microcapsules that are semipermeable to polyions, we have analyzed the influence of capsule permeability on ion densities and pH deviations inside the capsule cavities. In real biological systems, variations in microcapsule permeability could result from variations in porosity of the polyelectrolyte networks making up the capsule shells associated with the size distribution of polyions (e.g., amino acids) or from network swelling/de-swelling induced by changes in temperature, pH, or salt concentration.

Our results show that, upon varying capsule permeability, the ion densities redistribute so as to fulfill the competing requirements of minimum free energy and global electroneutrality. Ultimately, increasing permeability suppresses deviations in microion density and pH induced by the charged shells. These findings have potential relevance for the design of microcapsules that encapsulate fluorescent dyes to serve as ionic biosensors for diagnostic purposes. Although we have focused, in this study, on solutions containing only monovalent ions, in order to justify our use of the mean-field PB theory, our approach could be extended to multivalent ion solutions by incorporating ion correlations into the PB theory<sup>47</sup> or by performing molecular simulations in the cell model.

## Acknowledgments

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