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We have recently reported on the discovery that an infinite class of triaxial magnetic fields is capable of producing rotational flows in magnetic particle suspensions. These triaxial fields are created by applying a dc field orthogonally to a *rational* biaxial field, comprised of orthogonal components whose frequencies form a rational ratio. The vorticity axis can be parallel to any of the three field components and can be predicted by a careful consideration of the symmetry of the dynamic field. In this paper we not only test the field-symmetry predictions, but also quantify fluid vorticity as a function of the field parameters (strength, frequency ratio, phase angle and relative dc field strength) and particle shape. These measurements validate the symmetry predictions and demonstrate that rational fields are as effective as vortex fields for producing strong fluid mixing, yet have the advantage that small changes in the frequency of one of the field components can change the vorticity axis. This approach extends the possibilities for noncontact control of fluid flows and should be useful in areas such as microfluidics, and the manipulation and mixing of microdroplets.

1 Introduction

Methods of inducing vigorous noncontact fluid flow are of interest for heat and mass transfer and fluid mixing. Of particular interest are methods that do not create flow patterns with a characteristic macroscopic correlation length (such as that exhibited by natural convection) and are therefore adaptable to the nanoscale. We have recently shown that an infinite class of spatially-uniform applied magnetic or electric fields, called *symmetry-breaking rational fields*, can create strong fluid vorticity when applied to suspensions of magnetic or dielectric particles, respectively[1]. In fact, such fields create a *vortex fluid* capable of crawling up the sides of a container (active wetting) and exhibiting a negative viscosity. The purpose of this paper is not only to validate the symmetry-based theoretical predictions previously made, but to determine how the *magnitude* of the fluid torque density depends on a variety of experimental parameters, including those pertaining to the applied field and those pertaining to the particle suspension. In the course of these studies two new phenomena have been discovered: flow reversal and surface flow reversal, and these are presented as well.

Symmetry-breaking rational fields are comprised of three orthogonal components. Two of these are ac fields having a rational frequency ratio that can be written as an irreducible ratio n:m, where n and m are integers. One field component is higher in frequency than the other, so without loss of generality $n>m$. The third field is dc and serves to break the symmetry of the rational field, thus creating *deterministic* fluid vorticity. This vorticity can occur around the high frequency (H), low frequency (L), or dc (D) field axis.

This fluid vorticity is quite unexpected, because in general the field vector is *noncirculating* (*i.e.*, zero net rotation of the field vector over one field cycle) and thus there is no immediately obvious field parity of the type that something like a simple rotating field possesses. The strangeness of this manner of generating vorticity can be illustrated by the following example: When the increasing frequency ratio sequence 5:4, 6:4, 7:4,… is successively applied to a particle suspension, the repeating vorticity axis sequence H, H, H, L, H, H, H, D,… occurs, but only if the dc field is applied. Longer or shorter axis sequences occur for other denominators, and only for certain frequency ratios does reversing the dc field reverse the vorticity or does reversing the leads on the high frequency coil reverse the vorticity.

A consideration of the symmetry of the dynamic field leads to predictions of all of these effects[1]. These predictions are remarkably simple: 1) For *even*:*odd* or *odd*:*even* fields the vorticity axis is the odd axis and reversing the dc field reverses the vorticity; 2) For *odd*:*odd* fields the dc field is the vorticity axis and reversing the dc field *does not* reverse the vorticity; 3) A change in the phase of the high frequency field by 180°/m reverses the vorticity. These symmetry predictions are consistent with experimental observations, but symmetry alone cannot predict the dependence of the magnitude of the fluid torque density on the frequency ratio of the ac components or the phase angle between these components. Nor can it address how the torque density depends on ac field strength, dc field strength or particle shape. Yet a quantitative understanding of these issues is essential to developing applications that require vigorous vorticity.

In this paper we quantify the vorticity produced by three primary rational fields, 2:1 (*even*:*odd*), 3:2 (*odd*:*even*) and 3:1 (*odd*:*odd*). These fields produce vorticity around the low frequency, high frequency, and dc field axes, respectively. Using these fields we explore the dependence of the torque density on frequency, phase angle, field strength, and particle shape.

Three particle shapes are studied—spheres, platelets, and rods. In general we find that the various dependencies observed for platelets and rods are similar, whereas spheres are quite distinct. This difference is likely a reflection of the fact that an applied field can create a geometry-driven torque on an isolated platelet or rod, but cannot exert such a torque on an isolated magnetically soft sphere. Only if spheres form anisometric clusters, such as particle chains, can such a torque be generated, so the dynamic field must be able to create such structures without the formation of competing structures, such as particle sheets. We find that symmetry-breaking rational fields are generally as effective as vortex magnetic fields at creating strong mixing within magnetic particle suspensions, despite the fact that these fields are generally noncirculating.

2 Background

Because there are many instances in which *noncontact* methods of manipulating and agitating fluids would be useful (*e.g.*, cooling high-performance microprocessors, fluid handling in bioassays, microfluidic applications…), the search for effective, practical methods of inducing fluid flow has been longstanding. Magnetic fluids and suspensions naturally lend themselves to noncontact control, since an applied magnetic field easily penetrates most containers and can thereby be used to manipulate the magnetic fluid. Spatially uniform fields, such as we use, do not create a body force on a fluid, so most other methods of manipulation rely on field gradients. Gradients create a force on each volume element of the fluid that is described by the Kelvin force, $\mathbf{F} = -\mathbf{m} \cdot \nabla\mathbf{H}$, where $\mathbf{H}$ is the magnetic field and $\mathbf{m}$ is the fluid volume magnetic moment. Ferrofluids have been the focus of much of this research[2], because they are thermodynamically stable suspensions of particles that are not subject to sedimentation. Three principal approaches to inducing flow have been realized for ferrofluids, and each is specific to a particular type of ferrofluid flow phenomenon, which we now briefly discuss.

The first approach, thermomagnetic convection[3–5], is a passive mechanism by which *toroidal* convective motion in a ferrofluid is initiated by applying a substantial magnetic field gradient parallel to an imposed thermal gradient. This technique exploits the temperature-dependent susceptibility of ferrofluids (*i.e.*, a negative pyromagnetic coefficient) to generate a magnetic Kelvin force *gradient*. This force gradient has exactly the same effect as the gravitational force gradient that arises because of the thermal expansion of a fluid subjected to a temperature gradient. The magnetic field gradient can therefore produce convective flow patterns if it is sufficiently large and properly aligned relative to the thermal gradient. A field gradient can be produced by a permanent magnet, so the source of the energy driving the fluid motion is the heat transfer that maintains the thermal gradient, just as is the case with natural convection. Limitations of this approach are that a thermal gradient is required, in addition to an extremely large magnetic field gradient (of the order of 1–10 $T \cdot m^{-1}$), which makes scaling up this approach problematic. Because of this limitation thermomagnetic convection is usually applied to very thin fluid layers (1–10 mm) where it is difficult to induce natural convection because the required destabilizing thermal gradient scales as the *inverse* fourth power of the fluid thickness.

Ferrohydrodynamic pumping is another means of moving a ferrofluid, and occurs by applying a travelling solenoidal field to regions of a pipe, which produces *linear* flow of the ferrofluid down the pipe[6–8]. This effect is due to the field gradient producing a Kelvin force on the fluid and is the magnetic field equivalent of the peristalsis that occurs in intestines. In recent work by Mao *et al.*, a maximum volumetric flow rate of 0.69 $mL \cdot s^{-1}$ was achieved by applying a modest non-uniform propagating/travelling solenoidal field to regions of the pipe[8]. Finally, weak *rotational* flows within ferrofluids have also been studied, and occur when rotating magnetic fields are applied[9–11]. This effect is due to a lag between the magnetic moment of the particle and the rotating field vector.

Our own previous work focused on creating fluid vorticity in suspensions of micron-sized particles using a "vortex" magnetic field[12], wherein the field vector exhibits a precession-like motion that describes the surface of a cone. When such a field is applied to a suspension of magnetic particles, pronounced fluid vorticity develops throughout the entire fluid volume. Torsion fiber measurements have shown that the resultant torque has surprising dependencies on experimental parameters, in comparison to conventional stir bar mixing[12]. Within certain limits the torque is quadratic in the field (for spherical particles the specific torque density is roughly four times the energy density of the field), and independent of the field frequency and fluid viscosity. Theory has shown that these dependencies are due to the field-induced formation of volatile chain-like agglomerates that whirl around in pursuit of the dynamic field vector[13]. Vortex field mixing with suspensions of anisometric particles (platelets and nanorods) show similar trends in the mixing behavior[14]. However, when the dc field is removed—to create a simple rotating field—strong differences emerge: the spherical particles form static sheets, the rods continue to mix, but the platelet suspensions perform a stunning and unexpected trick—the formation of advection lattices[15].

These advection lattices not only form in rotating fields but in a broader class of *biaxial* magnetic fields. Advection lattices are generally in the form of a lattice of antiparallel flow columns that extend orthogonal to the plane of the biaxial field. The appearance and flow characteristics of advection lattices, as they depend on various magnetic field parameters and fluid properties, have been previously reported[16]. Applying a third field component in the form of a dc field orthogonally to the biaxial field plane (parallel to the flow columns) alters the dynamics of advection lattices in numerous ways[17]. One such alteration is the creation of vigorous rotational flow, the subject of this paper. Such rotational flow has already been shown to have the potential for significantly enhancing heat transfer[18].

In the following section we describe the materials we use, the method of generating the dynamic fields, and the torsion fiber apparatus used to quantify the fluid torque density.

3 Experimental

The suspensions were prepared by dispersing 1.5 vol.% magnetic particles into isopropyl alcohol, and were contained in 1.8 mL vials. Figure 1 shows SEM images for the different magnetic particles that were studied. These included 4–7 μm spherical carbonyl iron powder (ISP Technologies Inc.); ~50 μm-wide by 0.4 μm-thick molybdenum Permalloy platelets (Novamet Corp.); and 8–10 μm-long, 300 nm-diameter cobalt

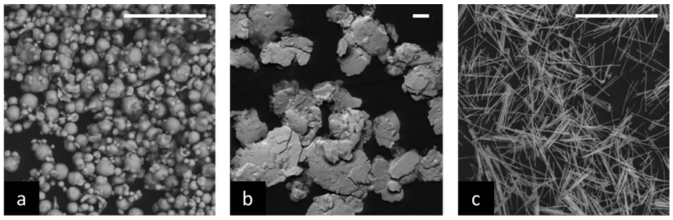

Fig. 1. SEM images of the magnetic particles. (**a**) 4–7 μm spherical agglomerate carbonyl iron powder, (**b**) Permalloy platelets roughly 50 μm across by 0.4 μm thick, and (**c**) cobalt nanorods 8–10 μm long by 300 nm diameter. The white scale bar in each image represents 20 μm.

nanorods (Richard Bell, Pennsylvania State University, Altoona College). (All of these materials are magnetically soft and so have little remanence.) The sample vial was mounted to a Macor fixture at the end of the torsion fiber and suspended in the central cavity of three orthogonally-nested Helmholtz coils, two of which are operated in series resonance with computer-controlled fractal capacitor banks[19] to generate uniform fields up to 200 G (0.02 T) in the low audio frequency range (~100–1000 Hz), while the third coil was used to produce a uniform dc field of up to 200 G. The specific torque density (torque per unit volume of magnetic particles) of the suspension was computed from measured angular displacements on a torsion balance employing a 96.0 cm-long, 0.75 mm-diameter nylon fiber with a torsion constant of ~13 μN·m rad^{-1}. A photograph of the triaxial Helmholtz magnet assembly and torsion balance is shown in Fig. 2.

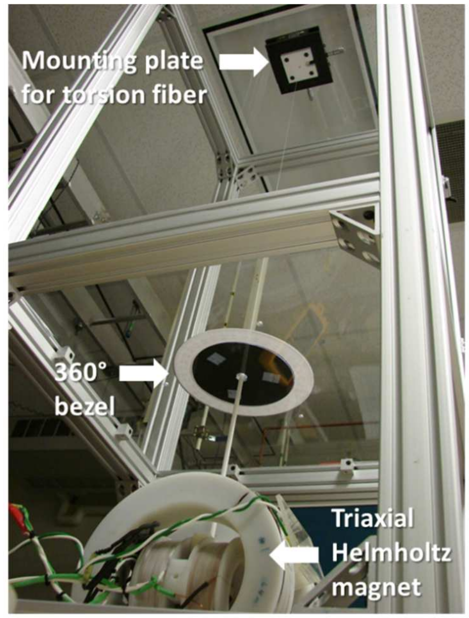

Fig. 2. Torsion balance and Helmholtz coils. An oblique upward view (three-point perspective) of the experimental apparatus, showing the three nested Helmholtz coils and the torsion balance (supported by the aluminum frame). Suspended in the central cavity of the coils is a vial containing the sample, which is attached to the end of the torsion fiber via the white rod-shaped fixture. The vorticity axis of the rotational flows must be oriented parallel to the torsion fiber axis to measure a body torque in the fluid.

4 Results

In this section we present results of the measured specific torque densities for a variety of particle suspensions subjected to symmetry-breaking rational fields. We consider all three possible "parities" of frequency ratios—*even*:*odd*, *odd*:*even*, and *odd*:*odd*. These fields produce vorticity around the *low frequency*, *high frequency*, and *dc* field components, respectively. To measure a body torque within the suspension requires the vorticity axis of the rotational flows to be parallel to the torsion fiber, which is vertical. So for each of the three parity cases we simply applied the appropriate fields with the correct geometry to ensure that the vorticity axis was always collinear with the torsion fiber. (One could also rotate the entire triaxial magnet, but this is cumbersome and defeats the concept of being able to control the orientation of the vorticity axis with the applied field alone!) A study of the specific torque dependence on the phase angle between the biaxial field components was first performed for each rational field to identify the phase angle at which the largest torque values are produced. This optimal phase angle was then used during the subsequent study of the effect of the dc field amplitude. In all cases (except for the field strength study) the rms induction field of each ac *biaxial* field component was 150 G.

4.1 *Even:odd* biaxial fields

Phase dependence — The simplest rational field that produces vorticity around the low frequency axis is the *even*:*odd* 2:1 field (see Lissajous plots in Fig. 3). Figure 4a–c shows measured torque densities for each particle shape as a function of the phase angle between the ac biaxial field components. In all cases torque extrema occur at phase angles separated by 180°, which validates the symmetry prediction that flow maxima occur at intervals of 180°/*m* for all *n*:*m* rational fields[1]. The platelets and rods yield continuous, roughly sinusoidal torque curves, whose peak values shift slightly toward lower phase angles and increase modestly with the magnitude of the frequency. The curves for the platelets are roughly sinusoidal, while those for the rods are somewhat skewed.

In contrast, for spherical particles the torque density is especially sensitive to the phase angle: an adjustment of only 20° causes the torque to substantially drop from its maximum value at 0°. Although this decrease distorts the shape of the curve, the torque extrema are still separated by 180°. However, the torque minimum is strangely smaller in magnitude than the maximum. This occurs because the torques were measured by changing the phase angle while the field is applied. As the phase angle increases from 0° and the torque collapses it simply does not recover by simply adjusting the phase angle. This

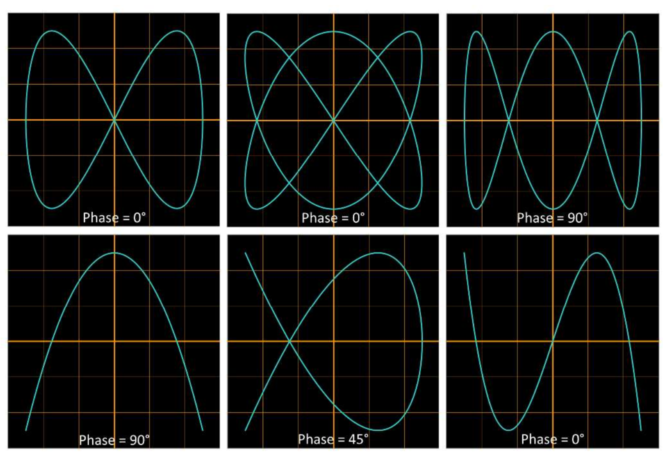

Fig. 3. Lissajous curves for three primary *n*:*m* rational fields. (**Top row**) Closed curves for 2:1, 3:2, and 3:1 fields. (**Bottom**) Corresponding open curves.

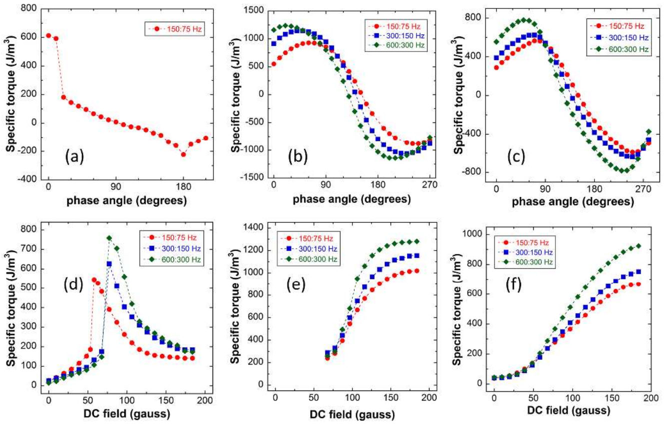

Fig. 4. Torque curves for 2:1 (*even:odd*) rational biaxial fields. Top row shows the torque density as a function of phase angle for (**a**) spheres, (**b**) platelets, and (**c**) rods. The bottom row shows the torque density as a function of the dc field amplitude.

suggests that at intermediate phases the particles form a relatively stable quasi-static structure that is not easily disrupted[20]. However, if the field is *entirely turned off* and re-applied with an initial phase angle of 180°, a torque magnitude comparable to that obtained at 0° is obtained, apparently because turning off the field allows the stable quiescent structure to collapse via sedimentation.

Both nanorods and platelets give larger specific torques than spheres and do not develop this troublesome stable phase, so are better choices than spheres. Platelets have a higher packing density than rods and are the best choice for inducing vorticity in situations where congestion would be problematic.

dc field dependence — The dependence of the specific torque on the dc field amplitude was determined by varying the dc field at the phase angle that produced the maximum torque for each particle type. Trends for the anisometric particles (Fig. 4e–f) are similar: the torque increases monotonically with the dc field and saturates at higher fields. A balanced field gives nearly a maximum torque. In contrast, the spherical particles display peculiar torque curves characterized by a narrow maximum peak at a dc field substantially lower than balanced (~50–100 G), Fig. 4d. The discontinuity in the torque is again due to the formation of quasi-static structures that compete with the particle mixing, as was seen in the phase angle study (Fig. 4a). Again, platelets and rods show much more robust torque development.

Field strength and frequency — Finally, we investigated the dependence of the specific torque on the strength of the applied field for 2:1 biaxial fields. Plots of the specific torque versus the field squared (proportional to the energy density of the field) are shown in Fig. 5 for both types of anisometric particles. In both cases the torques increase monotonically with field strength, and the specific torque densities are *of the order of the energy density of the field*. These curves display similar trends to those observed for anisometric particles subject to vortex magnetic fields[14]. Moreover, the curves for the cobalt nanorods are largely independent of field frequency, whereas the Permalloy platelets show a significant frequency dependence at low fields. As the field frequency increases, the torque falls off more rapidly at lower field strengths, suggesting a Mason number effect[14] where the field-induced attractive forces have become insufficient to hold particle structures together at higher frequencies. For vortex fields such particle structures are responsible for torque production.

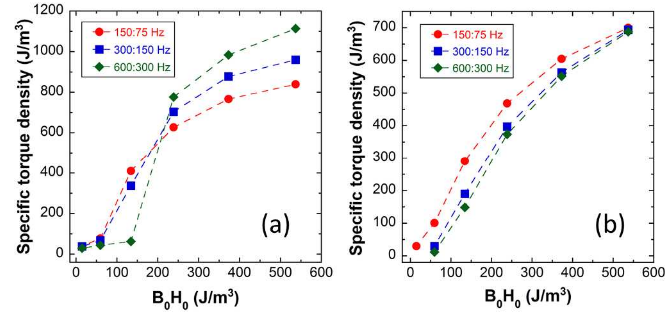

Fig. 5. Field strength study for 2:1 (*even:odd*) rational biaxial fields. Field squared dependence (twice the field energy density) of the specific torque for (**a**) Permalloy platelets and (**b**) Co nanorods. The sharp narrow peak in the torque curve for the spherical iron particles (Fig. 4d), which occurs at fields lower than a balanced field, made their field strength dependence too difficult to study.

4.2 *Odd:even* biaxial fields

Phase dependence — The simplest rational field that produces vorticity around the high frequency axis is the *odd*:*even* 3:2 field (see Lissajous plots in Fig. 3). This field also produces large specific torques, even though it is noncirculating[1]. The phase angle studies, Fig. 6a–c, show that for all particles the exrema in the specific torque are now separated by 90°, since here $m = 2$ and $\Delta\phi = 180°/m$. The platelets and rods produce symmetrical, nearly sinusoidal torque curves with maxima at similar phase angles. As with the 2:1 case, the spherical particles again exhibited strong sensitivity to the value of the phase angle, resulting in a discontinuous and asymmetric torque curve that is dependent on the suspension history.

dc field dependence — The dependence of the torque density on the dc field amplitude (Fig. 6d–f) is distinct for each particle shape. For spherical particles, there is only a narrow range of dc field amplitude that gives appreciable torque. In addition, there is a strange phenomenon that was not observed for the 2:1 field: *flow reversal*. As the dc field is progressively increased the torque increases to a maximum, decreases to zero and then reverses, as does the observed flow! Also, as with the 2:1 case, the peak torque obtains at slightly higher dc fields as the frequency of the biaxial field is increased.

The torque curves for the platelets maximize near a balanced field and show a broad shoulder in this region, then abruptly fall off for fields below ~100 G. In contrast, a maximum is not seen for the nanorods, but this is probably

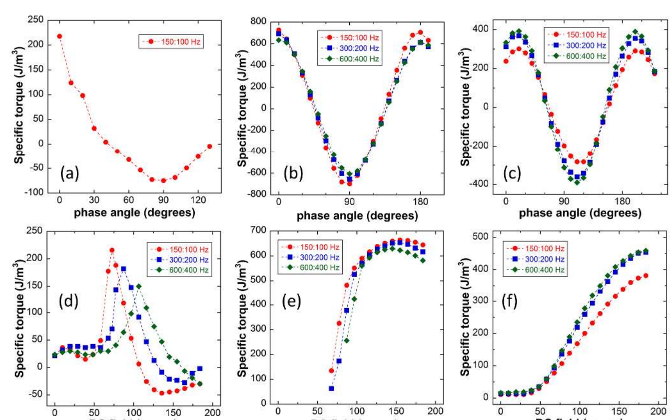

Fig. 6. Torque curves for 3:2 (*odd:even*) rational biaxial fields. Top row shows plots as a function of the phase angle for (**a**) spheres, (**b**) platelets, and (**c**) rods. The bottom row shows the corresponding torque curves as a function of the dc field amplitude.

simply due to the fact that we cannot apply a sufficiently large dc field with the current supply we have. The torque curves for the nanorods look remarkably similar to those for the 2:1 fields.

Frequency dependence — The anisometric particle data show the torque density to be fairly insensitive to field frequency, which is exactly the same result found for a vortex field. In the case of a vortex field this weak dependence could be attributed to the formation of volatile particle chains whose typical size simply adjusts to the field frequency. Higher frequencies lead to shorter chains, but because these are more numerous the torque density is unaltered. This suggests that for these rational fields torque production is due to the formation of particle clusters. For vortex fields there is also no predicted dependence on particle size, which might well be the case for these rational fields. Spherical particles have a stronger frequency dependence, which suggests that for these particles the Mason number is simply too large. The large Mason number is expected because the spherical shape produces large demagnetizing fields and thus low internal fields and low polarization.

4.3 *Odd:odd* biaxial fields

Phase dependence — The simplest rational field that produces vorticity around the dc field axis (other than a vortex field, which is 1:1) is the *odd*:*odd* 3:1 field (see Lissajous plots in Fig. 3). Currently, our largest Helmholtz coil is only capable of resonating at 50 Hz and 100 Hz, so this limits the number of 3:1 fields that we could investigate here to two: 150:50 Hz and 300:100 Hz. (For the latter case the frequencies were actually 300.6:100.2 Hz, since the resonant frequency of 100.2 Hz was dictated by the value of capacitance we were able to achieve given the capacitors we have.) As Fig. 7a–c shows, the torque maxima are 180° apart, as expected for this field. As with the previous two cases, the curves for the anisometric particles are very similar—nearly symmetric and sinusoidal, though the platelets give significantly greater torque. Although the torque curve for the spherical particles does not show the same sensitivity on the phase angle as in the two previous field cases, a peculiar flattening out is observed at ~120–180°, distorting the curve from a simple sinusoidal shape.

dc field dependence — The dc field study (Fig. 7d–f) shows that all three particles produce torque curves with a clear maximum. The torque curve for the spherical particles possesses a notably more jagged appearance, showing an abrupt jump near 100 G, whereas those produced with the anisometric particles are smooth and continuous. Nonetheless, in all three cases the torque maximizes for a dc field of ~100 G (or lower).

4.4 Surface flow reversal

Rotational flow in magnetic platelet suspensions subject to ac-ac-dc triaxial fields *other than* a vortex magnetic field was first observed in recently reported open-cell experiments[17]. In these experiments it is easy to observe the flow direction at the free fluid surface, especially if tracer particles are used. As mentioned earlier, a consideration of field symmetry cannot identify the flow direction, at least for noncirculating fields, but does show that the flow can be reversed by changing the phase angle between the ac biaxial field components by 180°/m. As mentioned above, for even, odd fields reversing the polarity of the dc field also reverses flow.

In addition to these predicted reversal mechanisms we have now discovered that flow reversal occurs when increasing the

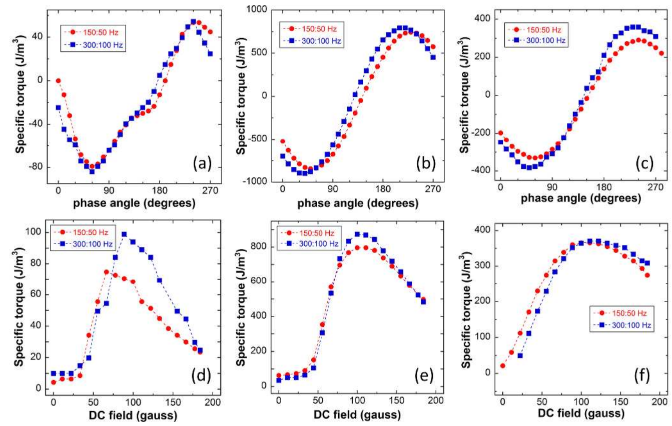

Fig. 7. Torque curves for 3:1 (*odd:odd*) rational biaxial fields. Top row shows plots as a function of the phase angle for (**a**) spheres, (**b**) platelets, and (**c**) rods. Bottom row shows the corresponding torque curves as a function of the dc field amplitude.

amplitude of the dc field component. When a dc field is applied orthogonal to the plane of a 2:1 rational field and progressively increased, rotational flow will develop in a dilute platelet suspension (a few volume percent). In an open cell, the fluid can be observed to move across the top surface, provided the axis of vorticity is not normal to this surface. At low dc fields, the fluid initially flows sluggishly, increasing in vigor as the dc field is increased. Near some critical dc field value, B^*_{dc}, the flow becomes vastly attenuated and sometimes appears frustrated, pulsating in one direction and then the opposite, or sometimes a nearly direct upwelling of fluid is observed that flows radially in variable directions at the surface. As the dc field is increased further, rotational flow resumes but in the *opposite direction* and becomes stronger as the dc field approaches the rms value of the rational field components (150 G). If the amplitude of the dc field is instead lowered to zero from its maximum value, a small but discernible hysteresis effect is observed. Essentially a reverse of the above-described events occurs with the subtle but consistent effect that flow reversal occurs at a slightly lower field (~2–3 G lower). Figure 8 shows that B^*_{dc} increases as a power law of the root frequency of the 2:1 rational field. For this experiment the rms values of the magnetic induction of the biaxial components were set to 150 G. The surface flow direction is labeled as either “north” or “south” simply to distinguish the direction. A study of 4:3 fields revealed the flow direction to remain constant over the entire range of dc field amplitude, as well as with 3:1 fields, so flow reversal is apparently not merely a function of field symmetry.

To determine if the small magnetic remanence of the multilayered, magnesium fluoride-coated, Ni core platelets (JDSU, Flex Products Group) could play a role in dc-field driven flow reversal we did some additional experiments with pure Permalloy platelets (Novamet). These platelets also displayed the reversal effect, so remanence is not a determining factor. Suspensions of carbonyl iron spheres also showed dc-field driven flow reversal but the Co nanorods did not, so particle shape does play a role.

We were surprised to find that the observed flow reversal did not lead to a measured torque reversal. In fact, the only instance of torque reversal was for the spherical particles subjected to 3:2 fields (Fig. 6d). These results indicate that the flow reversal observed at the free top surface does not occur throughout the bulk of the fluid. At this time it is not clear what the origin of this surface reversal effect is and why it only

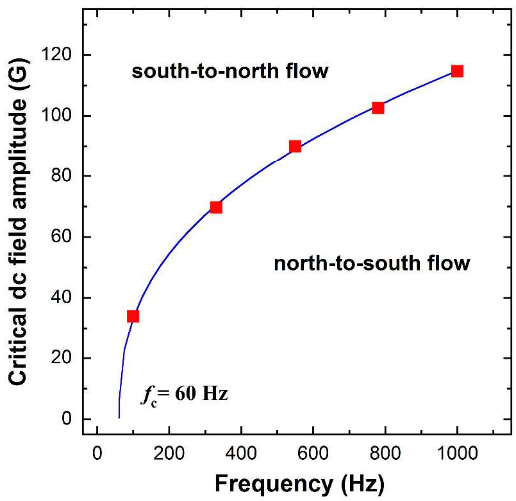

Fig. 8. Critical-field, flow reversal study. The critical dc field amplitude at which surface flow reversal occurs increases with the root component frequency magnitude as a power law: $B^*_{dc} = 7.95(f - f_c)^{0.39}$. These data were collected for the multilayer JDSU platelets subjected to ac-ac-dc triaxial fields whose ac biaxial component comprised a 2:1 field, 150 G_{rms}, phase = 0°.

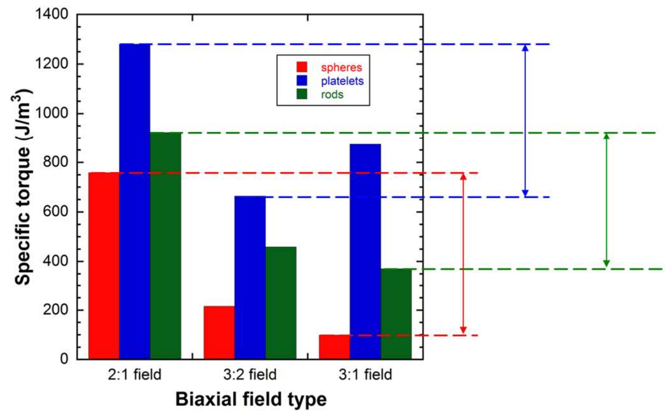

Fig. 9. Summary of specific torque densities. Comparison of the peak torque values produced by each particle shape for each type of biaxial field. The platelets produced the highest peak torque values for all fields.

manifests under certain conditions. However, we point out that a similar effect has been observed with ferrofluids subjected to rotating magnetic fields (the so-called spin-up flow case), in which the flow direction of the free top surface of the ferrofluid is observed to co-rotate or counter-rotate with the applied field depending on the field strength and/or frequency, whereas the bulk of the fluid is always found to co-rotate with the rotating field as verified by torque measurements[21] and the ultrasonic velocity profile method[22].

5 Discussion

A comparison of the absolute values of the peak specific torque densities we have achieved is in Fig. 9. For all particles tested, the 2:1 fields produced the highest torque values. The next highest values—for spheres and nanorods at least—were produced using 3:2 fields, followed by 3:1 fields. Moreover, under all field conditions tested, the anisometric particles produced higher torques than the spherical particles, and the platelets achieved higher torques than the nanorods. For the three different fields the Permalloy platelets gave maximum torques from ~630–1280 J·m^{-3}; the cobalt nanorods gave ~370–920 J·m^{-3}; and the carbonyl iron spheres ranged from ~100–760 J·m^{-3}. The disparity between the peak torques produced by the anisometric and spherical particles becomes progressively greater moving from the 2:1, to 3:2, and finally to the 3:1 fields. In fact, for 2:1 fields the spheres achieved ~59% of the peak torque produced by the platelets, but fell to only ~12% for the 3:1 field.

In a prior study[14] of mixing in *vortex* magnetic fields (in which case n:m = 1:1), cobalt nanorods were found to produce the highest specific torques (~1300–1500 J·m^{-3}), followed by platelets (~600–900 J·m^{-3}), and lastly spherical particles (~400–500 J·m^{-3}). However, the cobalt rods used in that study were considerably shorter (l = 2–4 µm) compared to those used here (l = 8–10 µm), which may account for the difference via reduced steric interactions with the shorter rods, thus enabling more effective dispersion and more efficient mixing. Nonetheless, comparing the ranges of values obtained for the different particles from these two studies demonstrates that rational biaxial fields can generate mixing as powerful as that produced by vortex fields, and that anisometric particles produce substantially greater torques than spherical particles. The JDSU platelets have the additional advantage of being coated with MgF_2, rendering them inert to a wide range of chemicals.

6 Conclusions

We have measured the torque densities of rotational flows produced by a new class of ac-ac-dc triaxial magnetic fields. These fields are comprised of a rational biaxial field—a field whose orthogonal components have frequencies that form a rational ratio—and an orthogonal dc field. Depending on the "parity" of the rational biaxial field (*i.e.*, *even*:*odd*; *odd*:*even*; or *odd*:*odd*), the vorticity axis of the rotational flow can be oriented along any of the three orthogonal field component axes. Moreover, the intensity of the rotational flow can be controlled by judicious selection of the phase angle between the biaxial field components or the amplitude of the dc field. Torque measurements demonstrate that these flows are as effective as those produced by vortex magnetic fields. This new class of rational triaxial fields in combination with vortex magnetic fields comprises a general class of triaxial magnetic fields capable of producing strong, field-controllable, rotational flows that should be useful for mixing and heat and mass transfer operations. Future work will focus on further quantifying the heat transfer efficacy of these rotational flows, and investigating the effect of a time-dependent third field component.

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Notes and references

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