Energy & Environmental Science

Accepted Manuscript

ROYAL SOCIETY
OF CHEMISTRY

www.rsc.org/ees

Energy &
Environmental Science

RSCPublishing

OPINION

Comment on "\$ per W metrics for thermoelectric power generation: beyond *ZT*" by S. K. Yee, S. LeBlanc, K. E. Goodson and C. Dames, *Energy Environ. Sci.*, 2013, 6, 2561

Cite this: DOI: 10.1039/x0xx00000x

Received 00th January 2012,
Accepted 00th January 2012

DOI: 10.1039/x0xx00000x

www.rsc.org/

G. Nunes, Jr.[a]

A recent paper by Yee, LeBlanc, Goodson, and Dames[1] provides a powerful approach to the design of a low cost thermoelectric generation system, but makes an unjustified approximation. Avoiding that approximation is straightforward, and in no way undermines the validity of the original approach. It does, however, shift the optimal design of a thermoelectric generator and makes that design material dependent. The difference in cost between a generator designed using the results of the original paper, and one that uses the modification given here, could be as much as a factor of two.

In a recent article,[1] Yee *et al.* present a creative and powerful cost model for analysing and optimizing the economic feasibility of thermoelectric power generation. In this comment, we point out an unjustified approximation made in that paper, an approximation that could lead to a thermoelectric installation costing twice as much as it would if it were correctly optimized. It should be emphasized that the issues raised here do not detract in any way from the overall validity and utility of the original approach of Yee *et al.*, they only shift the location of what might be termed the "low cost valley" identified in the original paper (hereafter referred as Yee).

Consider the circuit model of a thermoelectric device shown in Fig. 1. This is a simplified version of Fig. 1b in Yee, in which, following the original analysis, we are taking the hot and cold heat exchangers to have identical performance (*i.e.*, $K_H = K_C \equiv K_X$), and are assuming any path for heat conduction in parallel with the thermoelectric device to be negligible. As given in Yee, Eqs. 4 and 5, energy balance at the hot and cold junctions of the thermoelectric lead to the relations

$$Q_H = K_X(T_H - T_1) = K_T(T_1 - T_2) + S_{pn}IT_1 - \frac{1}{2}I^2R, \qquad (1)$$

and

$$Q_C = K_X(T_2 - T_C) = K_T(T_1 - T_2) + S_{pn}IT_2 + \frac{1}{2}I^2R. \qquad (2)$$

($S_{pn}$is the combined Seebeck coefficient for the device, R is its total electrical resistance, and I is the electrical current through the device. The reader should refer to the original paper for a full explanation of the notation.)

In Eq. (12) of their paper, Yee *et al* make the approximation that

$$K_X(T_H - T_1\,) \approx K_T(T_1 - T_2) \approx K_X(T_2 - T_C), \qquad (3)$$

which is only correct for devices of such poor performance as to be uninteresting for energy generation. The problem does not lie with the approximate equivalence of the heat flow out of the hot reservoir and into the cold reservoir. Thermoelectricity is a second order effect, with the sad consequence that $Q_H \approx Q_C$ is a safe approximation to make.

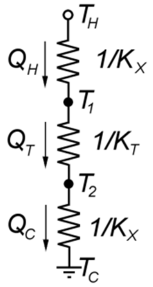

Fig. 1 Equivalent thermal circuit for a thermoelectric device, after Yee *et al.* (Ref. 1). The simplifying assumption that the hot and cold heat exchangers are identical has already been made.

To assume that $K_T(T_1 - T_2)$ is of the same order is to neglect *e.g.*, $S_{pn}IT_1$ in Eq. 1. That is, it neglects the Peltier heat flow. It is, of course, precisely that fact that the Peltier heat flow is not small that make a material useful as a thermoelectric. We compare the Peltier heat flow to the "ordinary" heat flow by taking the ratio

$$\frac{S_{pn}IT_1}{K_T(T_1 - T_2)}. \qquad (4)$$

Using equations 6, 9, and 10 in Yee, and under the assumption (discussed in detail in the original paper) that the electrical load has been optimized for maximum power generation, we find

$$\frac{S_{pn}IT_1}{K_T(T_1 - T_2)} = \frac{Z_{pn}T_1}{8}, \qquad (5)$$

where we have adopted the useful notational shorthand

$$Z_{pn} = \frac{S_{pn}^2\sigma}{k}. \tag{6}$$

σ and k are the electrical and thermal conductivities of the thermoelectric materials, respectively, as given by Yee. (Care should be taken to distinguish Z_{pn}, which uses the combined Seebeck coefficient $S_{pn} = S_p - S_n$, from the traditional figure of merit for a thermoelectric material,[2] which is calculated from S_p or S_n alone.) Using the values given by Yee in Fig. 2 of their paper, we find $Z_{pn}T_1/8 \approx 0.4$, which is not a particularly small number.

Fortunately, it is relatively easy to avoid the approximation of Eq. 3 without adding any significant complexity to Yee's analysis. If we add Eq. 1 to Eq. 2, and use Yee, Eq. 6 to substitute for the current, we find

$$T_1 - T_2 = \frac{K_X}{K_X + 2K_T + \frac{\alpha^2(T_1+T_2)}{2R}}(T_H - T_C), \tag{7}$$

which is a modified version of Yee's Eq. 13. Using Yee's Eq. 10 to substitute for the device resistance, we have

$$T_1 - T_2 = \frac{1}{1 + \zeta\frac{K_T}{K_X}}(T_H - T_C), \tag{8}$$

where

$$\zeta = 2 + \frac{Z_{pn}T_m}{4}. \tag{9}$$

T_m is the average device temperature $(T_1 + T_2)/2$. The approximation made by Yee *et al.* amounts to $Z_{pn}T_m/4 \approx 0$, whereas, for the example numbers used in their paper, this term is approximately 3/4.

If one follows through the argument given by Yee from this point forward, one finds that optimizing the cost of a thermoelectric generator amounts to minimizing the factor (cf. Yee *et al.,* Eq. 25)

$$\left(\zeta F + \tilde{L}\right)^2\left(1 + \frac{\tilde{L}_C}{\tilde{L}} + \frac{\tilde{L}_{HX}}{\tilde{L}F}\right). \tag{10}$$

In Eq. 10, $\tilde{L}$ is a dimensionless form of the thermoelectric device leg length, and F is a dimensionless fill factor. $\tilde{L}_C$ and $\tilde{L}_{HX}$ are dimensionless "cost lengths." We do not replicate the motivation and definition of these terms here, but refer the reader to the original paper. We only point out that the utility of Yee's approach comes from the discovery that designing a low cost thermoelectric generator amounts to optimizing the choice of F and $\tilde{L}$ for fixed $\tilde{L}_C$ and $\tilde{L}_{HX}$.

This optimum occurs along the line

$$F = \tilde{L}/\zeta, \tag{11}$$

in the region defined by

$$F < \sqrt{\frac{\tilde{L}_{HX}}{2\zeta}}, \tag{12}$$

and

$$\tilde{L} < \sqrt{\frac{\zeta\tilde{L}_{HX}}{2}}. \tag{13}$$

Yee's original results can be recovered by taking the limit $\zeta \to 2$.

The impact of Yee's original approximation on the cost of a thermoelectric installation can be assessed by comparing Eq. 10 as written here to Eq. 10 with $\zeta = 2$. Such a comparison is made in Fig. 2, expressed as a per cent excess cost.

Part of the appeal of Yee's original result was that the optimization condition was independent of the performance of the thermoelectric material, and it is true that this independence is lost in the present analysis. On the other hand, ignoring the Peltier heat flow amounts to underestimating the performance available from the thermoelectric material, and as a consequence, over spending on the construction of a generator. For example, a waste heat recovery system using a material with a not unreasonable figure of merit[2] $ZT_m = 1.65$ would, in the case that $|S_n| \approx S_p$, have $Z_{pn}T_m = 6.6$. If it were designed without consideration of the Peltier heat flow, it would cost twice what it would need to. For a large installation, the extra expense could have a significant impact on the economic viability.

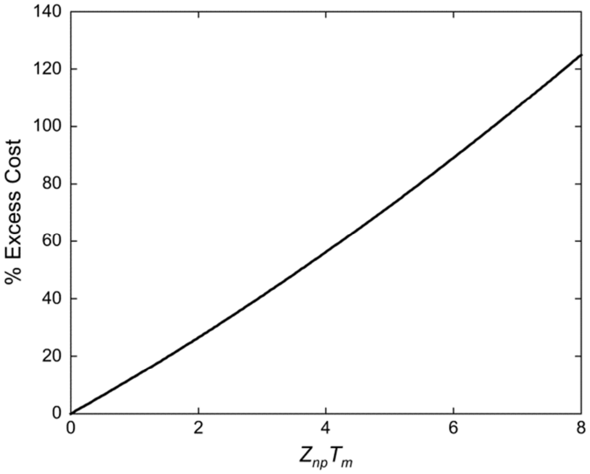

Fig. 2 Excess cost for a thermoelectric generator optimized under the approximation that $\zeta = 2$ instead of $\zeta = 2 + Z_{pn}T_m/4$.

Notes and references

[a] DuPont Central Research and Development, Experimental Station, Wilmington, DE 19880 USA.

1 S. K. Yee, S. LeBlanc, K. E. Goodson, C. Dames, *Energy Environ. Sci.,* 2013, **6**, 2561.
2 D. M. Rowe, *Thermoelectrics Handbook*, CRC Press, 2006, Section 1.3.