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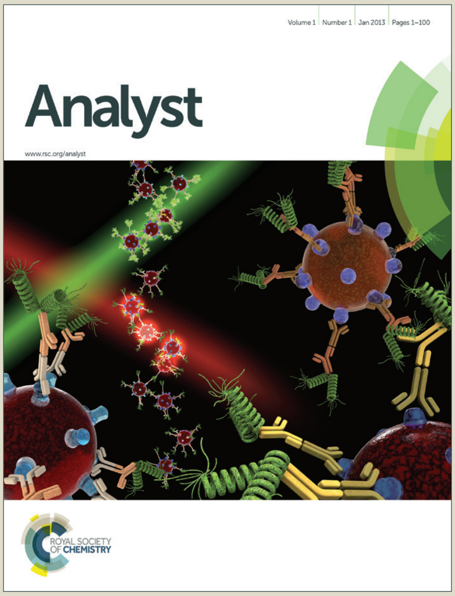

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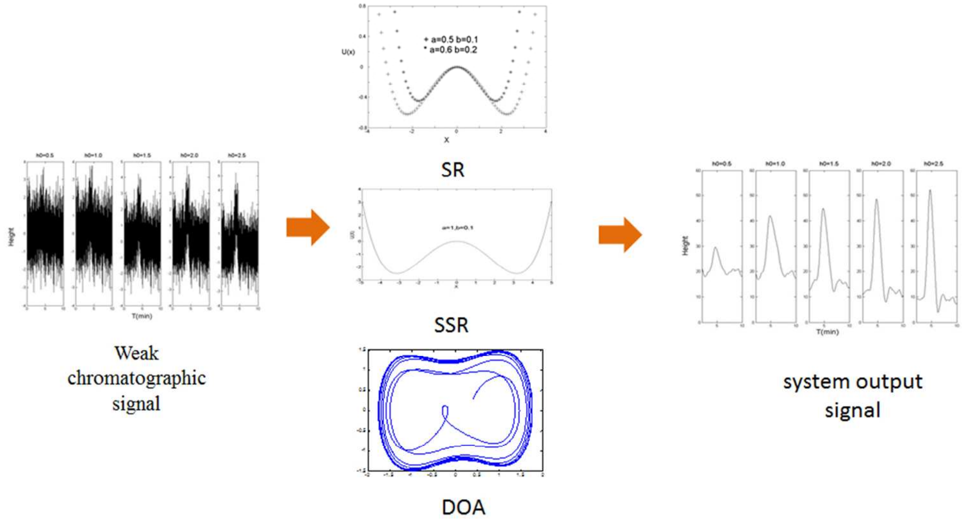

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Improving the detection sensitivity of chromatography by stochastic resonance

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Abstract

Improving the detection sensitivity of analytical instruments has been a challenging task for chemometricians since undetectability has been almost unavoidable in trace analysis, even under optimized experimental conditions and with use of modern instruments. Various chemometrics methods have been developed which attempt to address this detection problem with limited success (e.g., fast Fourier transform and wavelet transform). However the application of stochastic resonance (SR) creates an entirely new and effective methodology. Stochastic resonance is a phenomenon which manifested in non-linear systems where a weak signal can be amplified and optimized with the assistance of noise.

In this review, we summarize the use of basic SR, optimization of parameters and its modifications, including. periodic modulation stochastic resonance (PSRA), linear modulation stochastic resonance (LSRA), Single -well potential stochastic resonance (SSR) and Duffing oscillator algorithm (DOA) for amplifying subthreshold small signals. We also review the advantages and the disadvantages of various SR procedures.

Keywords: Stochastic resonance (SR); Periodic modulation stochastic resonance (PSRA); linear modulation stochastic resonance (LSRA);Single -well potential stochastic resonance (SSR); Duffing oscillator algorithm (DOA); Weak chromatographic signal

1. Introduction

With the development of analytical instruments, especially those hyphenated [e.g., high-performance liquid chromatography with ultraviolet detector (HPLC-UV), high-performance liquid chromatography with mass spectrometric (HPLC-MS) and gas chromatography with mass spectrometry (GC-MS)], the quantitation of compounds and its major metabolites can be produced conveniently[1-3]. Enhancing the detection limit, however, is still the key to obtaining information concerning trace components in the sample.

Several methods have been developed to attempt improvement in detection limit including various smoothing

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and filtering algorithms[4-7], fast Fourier transform [8-10](FFT) and wavelet transform (WT) [11-13], etc. In practice, however, they may lose potentially useful information due to improper truncations. SR, however, offers an entirely new way to solve the problem. SR is a phenomenon wherein signal, noise and nonlinear systems cooperate to enhance the signal-to-noise ratio (SNR) of the output signal[14-16]. The conception of SR originally brought forward by Benzi et al and successfully applied to the study of ancient ice age weather fluctuation, where it was proposed as a plausible mechanism for periodic occurrences (approximately every 100,000 years) of ice ages on Earth during the last 700,000 years[17]. The birth of SR as an experimentally controlled physical phenomenon occurred in 1988, after its first laboratory demonstration in Schmitt triggers [18]. Since then SR has grown into a rapidly developing, interdisciplinary field of research, with numerous applications in biological, laser, electronic, quantum and other systems[19-21]. Because of its generic nature that random noise plays a beneficial role, SR finds useful applications in enhancing the detection of various types of signals such as periodic signal, aperiodic signal, 1D signals and 2D signals[22-24]. Furthermore, certain biological systems may even use SR for optimizing function and behavior[25].

Shao et al firstly had applied the SR algorithm to improve the detection limit in the analytical field[26-29]. It was proved that the SNR of the analytical signal can be greatly enhanced by this method, and an excellent quantitative relationship between different concentrations and their responses can be obtained. Since then, SR has attracted considerable interest. For example, Wang has applied SR to analyze the weak laser-Raman spectrum of CCl4, Pan et al have improved the detectability of analytes in GC and Xiang et al have improved the detectability of analytes in HPLC/UV and HPLC/MS[30-41].

SR is a phenomenon that is manifest in nonlinear systems whereby generally feeble input signal can be amplified and optimized by the assistance of noise. Basically, SR requires an energetic activation barrier, a weak input (periodic or a periodic signal) and a source of noise that is inherent in the system or that adds to the coherent input. The symmetric double well potential is the simple and common form of threshold to induce SR. When the SR is used to improve the detection sensitivity, the input signal, noise and nonlinear system cooperate well and the signal extracts energy from the noise both intrinsic and external to surmount the energy barrier and hop from one potential well to another. Consequently, the strength of signals increases and that of noise decreases. The output signal of the system is obtained by a better SNR compared with the input one. Stochastic resonance is a powerful approach to amplify subthreshold small signals, while these kind of weak signals were rarely considered by analysts before. However, the SR method has its disadvantages. For example, the weak chromatographic peak maybe distorted due to the presence of noise. And the optimization of nonlinear system parameters is also a problem for the experimenter without the knowledge of chemometrics. There have therefore been many efforts made to extract the correct information on the components in SR studies. For example, Deng et al determined three alkaloids (colchicine, scopolamine and ephedrine) in human plasma by Ultra Performance Liquid Chromatography/TOF-MS and used the re-scaling frequency stochastic resonance (IRSR) to process the weak signal. IRSR could extend the SNR form 2.5, 5.6 and 6.1 to 10.7, 13.2 and 13.6 for ephedrine, scopolamine and colchicine respectively. At the same time, the traditional chemometrics methods such as Savitzky-Golay filter, Whittaker-Eilers smoother and matched filtration just improved the SNR to 3.1-4.3, 5.6-8.7 and 6.3-10.2 for ephedrine, scopolamine and colchicine respectively. Obviously, the SR algorithm was powerful approach to amplify subthreshold small signals [42]. In order to simplify procedure of optimization, Zhang et al developed a new single -well potential stochastic resonance algorithm (SSR) to detect the weak signal. Although the detectability of the signal submerged by noise is not stronger as bistable system, SSR is enough to be applied in chromatographic field and successfully applied for extention of limit of quantification of Sudan IV[43, 44]. Xie et al conceived an experimental formula for an optimizing system based on a mass of experimental results in order to direct attention to both the SNR and the shape of the output chromatographic peak. With the help of this new criterion, stochastic resonance algorithms could not only improve the detection limit and quantification limit of weak signal but also promote the profile quality of output signal. Using this methodology, the weak signal of amiloride (30-100ng/ml) could be amplified obviously within the undistorted shape [34]. Cai et al studied the two-layer stochastic resonance algorithm and Stochastic Resonance Algorithm Based on Periodic Modulation to detect the weak chemical signal[35, 45]. After two layer linear modulation-based stochastic resonance, the SNR of dimethyl sulfide increased from 3 to 10[35]. Some applications of SR were list in Table 1 [46-49].It can be seen that SR has successful applications in different instrument including GC, HPLC/MS, HPLC/MS and Raman spectrometer, etc. And SR is a promising chemometrics method for trace analysis.

This article is intended to provide readers with an overview of various methodologies and approaches used to improve sensitivity thru SR methods, with special emphasis on applications to improve the detectability of analytes in chromatographic signals

2. Stochastic resonance

A model of a one-dimensional nonlinear system that exhibits stochastic resonance is the damped systems with the Langevin equation of motion [50, 51].

$$m\ddot{x} + \gamma\dot{x} + dU(x)/dx = H(t) \qquad (1)$$

Where the dot represents the time-derivative. This equation describes the motion of a particle of mass m moving in the presence of friction $\gamma\dot{x}$ in a potential $U(x)$ and with an additive stochastic force $H(t)$. When the system is heavily damped, the inertial term $m\ddot{x}$ can be neglected.

Rescaling Equation.1 with the damping term γ gives

$$\dot{x} = -dU(x)/dx + H(t) \quad (2)$$

$U(x) = -1/2ax^2 + 1/4bx^4$ is the simplest double-well potential with the constants a and b characterizing the system. Thus the Eq. (2) leads to

$$\dot{x} = ax - bx^3 + H(t) \quad (3)$$

The Fig.1 shows the simple double-well potential which has a local unstable maximum point at $x = 0$ and two stable minimum points at $x = \sqrt{a/b}$, respectively.

Where $H(t) = h(t) + \Gamma(t)$, $h(t)$ is the input signal and $\Gamma(t)$ is the noise to induce SR. The minima are located at $x = \pm\sqrt{a/b}$, where $x_{\min} = (a/b)^{1/2}$. These are separated by a potential barrier with the height given by $\Delta U = a^2/4b$. The barrier top is located at $x = 0$ (in figure 1). When the input signal, noise and nonlinear system cooperate well, the potential barrier will be lower and the signal which rests at one of the two minima of the potential may surmount the energy barrier hopping from one potential well to another. Thus, the intensity of signals will increase and that of the noise will be reduced. The output signal of the system will be obtained with a better SNR compared to the input signal. Supposing the input signal is a sinusoid, $h(t) = A\sin(\varpi_0 t)$, and D is the noise intensity. Where A is the intensity of input signal ϖ_0 is frequency of the input signal. Based on the adiabatic approximate theory [52], the SNR can be described as:

$$SNR = \sqrt{2}\mu^2 A^2 e^{-\mu^2/4D} / 4D^2 = \sqrt{2}\Delta U \left(A/D\right)^2 e^{-\Delta U/D} \quad (4)$$

The equation shows that the SNR of the output signal is decided by the potential barrier ΔU and the noise intensity D. The SNR of the Eq.4 has a maximum at $D_{\max} = \Delta U/2$. Based on the theory, SR can be observed by adding external noise, adjusting the input signal intensity, and modulating the nonlinear system simultaneously. The addition of external noise will damage the characteristic of the intrinsic noise and result in the serious distortion of the obtained signals. In order to do quantitative determination, only the parameters a and b of the system are modulated to match the input signal including real signal and intrinsic noise to achieve SR in the application of chromatography.

The output signal can be obtained by solving equation (3). Usually the discrete stochastic differential equation is solved by a fourth-order Runge–Kutta method in the application of chemical analysis, which has more precision than Euler scheme[53-55]. The algorithms can be described as follows:

$$\begin{aligned} &x_{n+1}=x_n+\frac{1}{6}(k_1+2k_2+2k_3+k_4), \quad n=0,1,\cdots,N-1 \\ &k_1=K(ax_n-bx_n^3+u_n) \\ &k_2=K[a(x_n+k_1/2)-b(x_n+k_1/2)^3+u_n] \\ &k_3=K[a(x_n+k_2/2)-b(x_n+k_2/2)^3+u_{n+1}] \\ &k_4=K[a(x_n+k_3/2)-b(x_n+k_3/2)^3+u_{n+1}] \end{aligned} \tag{5}$$

Where $u_n = h_n + \Gamma_n$ and K is length of step.

The input signal was first prepared by normalizing in [−1, 1] by means of the following equation：

$u_n = 2 \times (u_n - min\{u\})./(max\{u\} - min\{u\}) - 1$. Thus all samples in analyzed series will have the same strength in the nonlinear system. In order to kept quantitative linearity between concentration and peak strength, the normalization is essential for an accurate method. The final results can be obtained by inverse normalization of the output signals. Although the Runge–Kutta is the most common scheme to solve the discrete stochastic differential equation in the analytical field, there are alternative algorithms that may have greater accuracy in the particular situation[56-58].

The parameters a and b in equation (3) not only define the height of the potential barrier, but also affect the profile of the potential well. When the input signal is fixed, the parameters a and b influence the quality of final output signal directly. The optimization of a and b must be selected for the various instruments or conditions in the experiment. The optimization method first reported was to fix one parameter among a and b, then to adjust another to optimize peaks with different heights. In such a way, some information obtained from the adjusting parameters of the target signal may be lost. Zhang et al presented a method to optimize a and b synchronously with the ratio between peak height to peak half-width as evaluating indicator[36]. The maximal SNR can be obtained using this method. However in the chromatographic experiments, the shape of the peak should also be taken into account. Xie et al devised an experimental formula for optimizing a system based on a mass of experimental results in order to address both the SNR and the shape of output chromatographic peak [34]. The experimental formula is as follow:

$$I = SNRi/SNR\max - \gamma\left|LA/RA - 1\right| \qquad (6)$$

$SNRi$ is the signal-to-noise ratio of system output signal that is defined as a ratio of the standard deviation of the peak area in the output signal to the baseline area, and $SNR\max$ is the maximal SNR with different parameters a and b. γ is the control coefficient of peak shape, LA and RA are the left sides and right side of the peak area, respectively. Using the ratio of left side and right side of peak area to evaluate the peak shape is an effective method to ensure undistorted chromatographic peak shape. If the chromatographic peak in output signal is a Gaussian peak, the value of $\left|LA/RA-1\right|$ is 0, while the value of $\left|LA/RA-1\right|$ for common chromatographic peak is within [0, 1]. The smaller value of $\left|LA/RA-1\right|$ indicates the undistorted peak shape. γ can control the proportion of peak shape in system output. Parameter γ can be increased to enhance the role of peak shape in the process of optimization if the chromatographic peak shape is poor in output signal. Otherwise Parameter γ could be decreased for the weak signal with better shape. The index I is a comprehensive criterion which contains both information regarding SNR and peak shape. Maximal I indicates greater SNR and better peak shape, and the corresponding a and b are the best system parameters. The quantitative analysis of amiloride in methanol solution showed that this method could not only improve the detection limit and quantification limit but retained an undistorted peak shape at the same time.

3. Periodic and linear modulation stochastic resonance

An algorithm based on SR can be an efficient approach to the detection of a weak signal buried in noise. However, when a chromatographic peak is extremely weak, the energy of the noise may be too high relative to that of useful signal and it may cause the noise to hop from one potential well to another together with the signal. As a result, the output signal will be distorted. Generally, distorted peaks are not suitable for chromatographic analysis. It has been found that the distorted signal can be corrected by introducing a periodic force into the nonlinear system, and this is named the periodic modulation-based stochastic resonance algorithm (PSRA)[59]. Pan et al have successfully employed PSRA to improve the Roman signal of CCl_4[60]. Subsequently the PSRA was also used to amplify and detect the weak liquid chromatography–mass spectrometry (LC–MS) signal of granisetron in plasma [45]. In the PSRA, a periodic modulation was introduced into the nonlinear system. Thus, the potential function can be developed into the following formula by adding such a periodic modulation:

$$\mathrm{U(x,t)} = -\frac{1}{2}ax^2 + \frac{1}{4}bx^4 + x\varepsilon\sin(\omega t) \qquad (7)$$

In the system described by PSRA, the potential barrier will vary with the periodic force (Fig. 2). When the values of ε and ω are suitale, the signal surmounts the potential barrier while the noise dose not. Thus, the peak shape of the output signal is improved. .The output signal can be also obtained by a fourth-order Runge–Kutta method. The scheme is similar with equation (5) except for $u_n = h_n + \Gamma_n - \varepsilon \sin(\omega t_n)$, where periodic force has been included into input signal.

However, it is often difficult to select appropriate parameters for this periodic force when periodic modulation is used to detect a weak experimental chromatographic peak. When run on a computer, the algorithm can generate errors if inappropriate parameters are used. Deng et al present a simple method called a linear modulation-based stochastic resonance algorithm (LSRA) [33, 35]. LSRA is used to simplify the selection of parameters as well as to avoid runtime errors. The theory of LSRA is similar to PSRA except that a linear force ($x(\alpha t + \beta)$) is introduced into the nonlinear system instead of a periodic force. When it is applied to detection of weak chromatographic peaks, linear modulation can produce the same effects as periodic modulation. Deng et al applied the LSRA to detect the weak chromatographic peaks of dimethyl sulfide (DMS) and the identification of chloramphenicol (CAP) residues in milk. The results from both of these tests had an undistorted peak shape and satisfactory SNR as well. The LSRA and PSRA can be selected if the peak was distorted by the basic SR. Since more parameters was introduced into the system, it is more difficult for experimentator without experience of chemometrics to optimize the parameters. Those modulating stochastic resonance algorithms require extensive selection of optimized parameters.

4. Single -well potential stochastic resonance (SSR)

It is difficult to change the optimized system parameters for stochastic resonance in detecting weak signals. Therefore attention is focused on the simple model of stochastic resonance, which has fewer system parameters. One of the characteristic features of stochastic resonance in systems without a threshold is bistability. Although it is not difficult to find a monostable system where a signal is enhanced by external noise, SNR generally has a tendency to decrease with a noise increase. Recently it was shown that the SNR can be enhanced by noise for underdamped single-well systems and for a special type of monostable system [61]. Ditzinger and his coworkers investigated a system that was not bistable and concluded that bistability was not a necessary condition for SR[62] . Grigorenko et al confirmed that stochastic resonance could be obtained in the simplest monostable system ($\mathrm{U(x)} = -\mathrm{a} + \frac{1}{2}bx^2$) [63].There is single-well potential which has no potential barrier. When SR takes place, the input signal xtracts energy from the noise to move along the brim of the single -well potential. Thus it can get the height, which the single input signal cannot reach. There may be the reasons that signal strength increases and that of noise decreases. Consequently, the weak signal will obtains a better SNR than the input signal. The output signal can be obtained by a fourth-order Runge–Kutta method in the application of chemical analysis. The derivative of single-well potential is$-bx$. Thus the algorithm of a single-well potential can be obtained by removing the cubic term in equation (5).

Zhang et al compared the single-well potential stochastic resonance algorithm with a traditional double-well potential stochastic resonance algorithm (DSR)[43]. From the simulated experiment, it can be demonstrated that DSR's detectabilty of the signal submerged by noise is much stronger than that of SSR. However the noise intensity is usually smaller in the chromatogram Thus the SSR algorithm is enough to be applied in the chromatographic analysis. Compared with Figs.3 and 4 presents the SNR of weak signal can be greatly enhanced by the SSR and DSR in the determination of dichloromethane using the gas chromatograph. SSR transduces the signal optimally when a certain amount of noise is presented. And the parameter in SSR method required only one. Thus it is easier to be selected compared with the traditional DSR. Using the signals enhanced by SSR, the method extended the limit of detection and the limit of quantification of Sudan IV from 0.03 and 0.1 μg/ml to 0.008 and 0.02 μg/ml respectively, and exhibited good linearity, accuracy, and precision, which ensure an excellent determination of the analyte [44]. In the future it is likely that the method could be combined with or embedded in chromatographic workstations to function online.

5. Duffing oscillator algorithm (DOA)

Chaotic motion can be characterized in a variety of ways. It is now widely agreed that a dynamics can be called chaotic only if it exhibits the property of sensitive dependence on initial conditions (SDIC). Although chaos is a sufficient condition for SDIC, it is not a necessary condition (a periodic motion may also exhibit SDIC, in particular when the system parameters are close to those for a bifurcation point). In a bifurcation process the system is structurally unstable and even a small perturbation may cause a qualitative change in the state of the system. The bifurcation behavior of the driven Duffing oscillator is used to detect a weak signal. The basic idea is that a weak periodic signal emerged in noise can be detected by the Duffing oscillator via a transition from chaotic motion to periodic motion. Chaos has potential application outlook in weak signal detection because of its properties, which are sensitive to certain signals and immune to noise at the same time[64, 65]. The SR phenomenon may be found in the bistable Duffing oscillator system[66]. Wu et al have applied Duffing equation to experimental weak signals of X-ray diffraction and Raman spectrum [26]. And Zhang et al reported a DOA to improve the SNR of chromatography. Using signal enhancement by DOA, this method extends the SNR of low concentrations of methylbenzene in the water from 2.662 to 29.90 [39].The Holmes Duffing equation was chosen to improve the SNR of chromatography system. It can be described by the following equation:

$$\mathrm{m}\ddot{x} + \gamma\dot{x} + \frac{dU(x)}{dx} = fH(t) \tag{8}$$

The $U(x)$ is the same double-well potential previously mentioned. Thus the Eq. (8) leads to

$$\mathrm{m}\ddot{x} + \gamma\dot{x} - ax + bx^3 = fH(t) \tag{9}$$

The output signal can be obtained by solving equation (9). In order to solve the duffing differential equation, the Eq.9 was changed as follows

$$\begin{aligned} \dot{x} &= y \\ m\dot{y} &= -\gamma y + ax - bx^3 + fH(t) \end{aligned} \tag{10}$$

For convenience, the mass m of particle was set at 1. In the proposed DOA, the discrete duffing differential equation was solved by a fourth-order Runge–Kutta method. The algorithm can be described as follows:

$$\begin{aligned}
x_1 &= x(i), \qquad\qquad i = 0,1,\cdots,N-1 \\
y_1 &= y(i), \\
f_1 &= -\gamma\, y_1 + ax_1 - bx_1^3 + fH(i) \\
x_2 &= x(i) + y_1 \times \frac{h}{2} \\
y_2 &= y(i) + f_1 \times \frac{h}{2} \\
f_2 &= -\gamma\, y_2 + ax_2 - bx_2^3 + fH(i) \\
x_3 &= x(i) + y_2 \times \frac{h}{2} \\
y_3 &= y(i) + f_2 \times \frac{h}{2} \\
f_3 &= -\gamma\, y_3 + ax_3 - bx_3^3 + fH(i+1) \\
x_4 &= x(i) + y_3 \times h \\
y_4 &= y(i) + f_3 \times h \\
f_4 &= -\gamma\, y_4 + ax_4 - bx_4^3 + fH(i+1) \\
x(i+1) &= x(i) + \frac{h(y_1 + 2y_2 + 2y_3 + y_4)}{6} \\
y(i+1) &= y(i) + \frac{h(f_1 + 2f_2 + 2f_3 + f_4)}{6}
\end{aligned}$$

In here x is output signal. h is length of step. It starts with normalization of the input signal ($\mathrm{H}(\mathrm{t})$) to the interval [-1, 1]. By optimizing the parameters in equation (10), the best output can be obtained. Finally an inverse normalization is performed to obtain the output signal. In order to optimize the parameters in DOA a genetic algorithm was selected because Genetic algorithm is especially beneficial when the search space is complex with many local minima, so that conventional techniques fail to find global minima and a full search is not feasible. DOA was the original model of SR applying in the analytical field. Thus a study of DOA can help in understanding the mechanism of SR and finding more reasonable model. But its application is limited by more parameters at this time.

6. Conclusion

We here summarized the basic principles and the modifications of SR, emphasizing applications to improve the detection sensitivity of chromatographic instruments. Because the theory of SR is unlike that of more common methods (e.g., FFT and WT), SR offers new approaches to the resolution problem. Noise has always been considered to be a simply disorder and a nuisance to be avoided, but it shows its constructive properties in SR because of its contribution to signals.

In SR, the optimized parameters must be provided for individual systems since the algorithm can generate errors and distort the profile of a peak if inappropriate parameters are used. The optimization of parameters must consider the intensity, the profile of chromatographic peak and the noise. There were efforts made to modify the model of SR in resolving chemical signals. i.e.: (1) a new evaluating indicator was presented to consider the SNR and the profile of chromatographic peak at the same time; (2) modulation stochastic resonance algorithms were proposed to reduce distortion of the chromatographic peak; and (3) the simpler stochastic resonance model was introduced to simplify the procedure of optimization. However, these modifications have their limitations. The SNR and profile of chromatography can be considered at the same time in the new evaluating indicator and the modulation stochastic resonance algorithms. However, as more parameters were added to the system, the SSR was unable to modify the profile of the peak. So far, the efficacy of SR is not efficiency to meet the criteria of online analysis needed to support wider usage. Improving the detection sensitivity in complex samples, especially, is an unprecedented challenge, as it involves researching the interactions between target peak and interfered peak.

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Table 1 Application of stochastic resonance in in different instrument

Analytes	Analytical platform	Parameters of SR-model	SR algorithm	Extended the limit of detection
CCL_4	Raman spectrometer	a=1;b=1, γ =1.7	SR in the Duffing system	display as diagrammatic form [19]
α-Al_2O_3	X-ray diffractometer	a=1;b=1, γ =1.6	SR in the Duffing system	display as diagrammatic form [19]
Erythrosin	Photoacoustic spectroscopy	a=1;μ=0.08	SR	display as diagrammatic form [20]
CO_2	GC- TCD	a=0.15;b=0.0187	SR	Improve intensity form 0.0172 to 0.0696[22]
dichloromethane, chloroform, 1,2-dichloroethane,benzene, ethylbenzene, and xylene.	GC-FID	a=0.032;b=$\times 10^{-6}$	SR	extent LOQ form 15.0,15.4,12.0,2.4,1.2and 1.3μg/L to 5.4,5.5,2.5,0.6,0.4 and 0.4μg/Lrespectively.[24]
dimethyl sulfide	GC- pulsed flame photometric detector	a=5×10^{-3};b=2×10^{-3}; α =0. 1; β =−0. 08	linear modulation-based stochastic resonance	Increase the SNR from 3 to 10[28]
roxithromycin	HPLC-UV	a=9.9×10^{-5};b=8.0×10^{-6}	SR	Improve LOQ from0.5 to 0.1ng/ml[29]
methylbenzene	HPLC-UV	a=0.0677;b=0.067,h=0.0835;f=1.6; γ =1.345	Duffing oscillator algorithm	extent intensity form 2.662 to 29.90[[illegible]]
glyburide	HPLC-UV	a=3.01×10-6,b=1.2×10-5	SR	Improve LOQ from15 to 1ng/ml[32]
three alkaloids	UPLC/TOF-MS	a=0.1;b=0.05;F=0.05times of iteration=4	re-scaling frequency stochastic resonance	extent SNR form 2.5,5.6 and 6.1 to 10.7,13.2 and 13.6 for ephedrine, scopolamine and colchicine respectively[35]
dichloromethane	GC-FID	b=0.02	single-well potential stochastic resonance	display as diagrammatic form [36]
granisetron	LC-MS	a=0.17;b=0.0045; ε =0.32; ω =0.31	PSRA	Improve LOQ and LOD from 0.2and 0.05ng/ml to 0.02 and 0.01ng/ml[38]
Sudan I	HPLC-UV	a=2.5×10^{-2};b=1×10^{-5}	SR	Improve LOQ and LOD from 0.1 and 0.03μg/ml to 0.02 and 0.006μg/ml[39]
triclocarban	HPLC-UV	a=6×10^{-3};b=1.0×10^{-6}	SR	Improve LOQ and LOD from 50and 10ng/ml to 5 and 1.0ng/ml[40]
Clenbuterol	HPLC-MS	b=0.0574	AdaptiveSingle-Well Stochastic Resonance	Improve LOQ and LOD from 0.3and 0.1ng/ml to 0.05 and 0.025ng/ml[41]

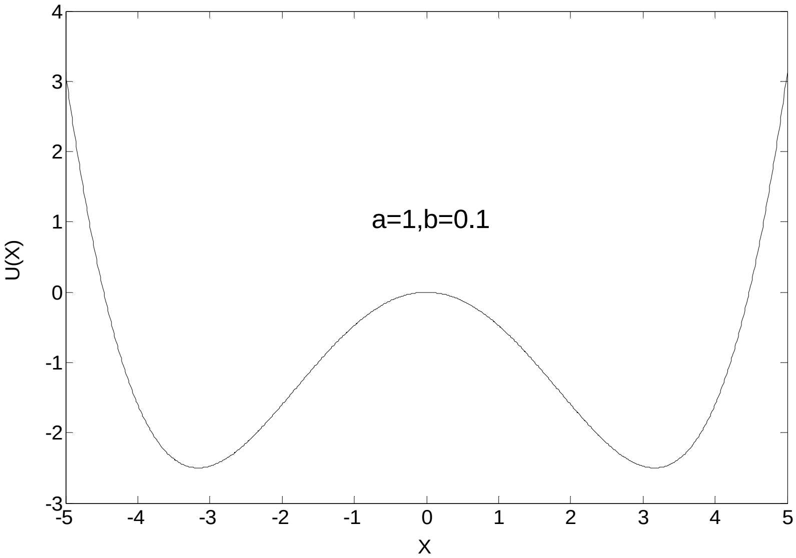


Fig.1 Double-well potentials with the a=1, b=0.1

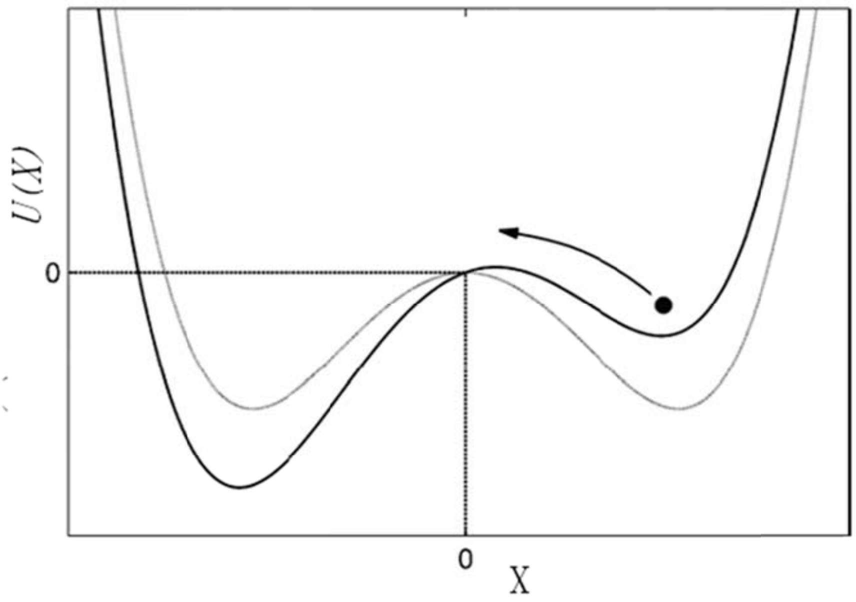


Fig. 2. The profile of double-well potential function of PSRA.

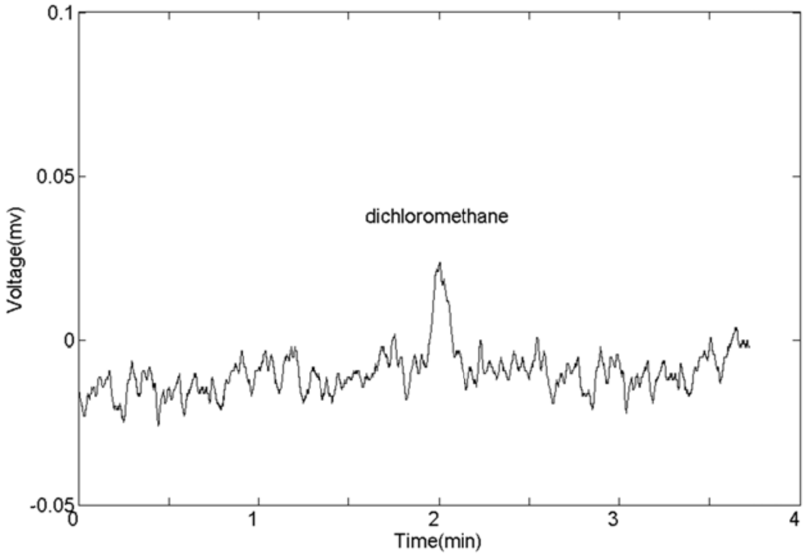
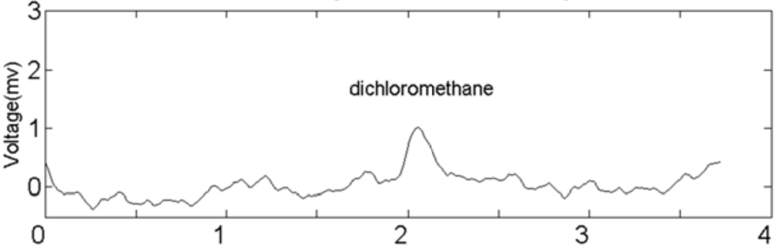


Fig. 3 Experimental chromatogram of dichloromethane at a concentration of 1.5µg/ml

a. Obtained signal of dichloromethane by SSR



b. Obtained signal of dichloromethane by DSR

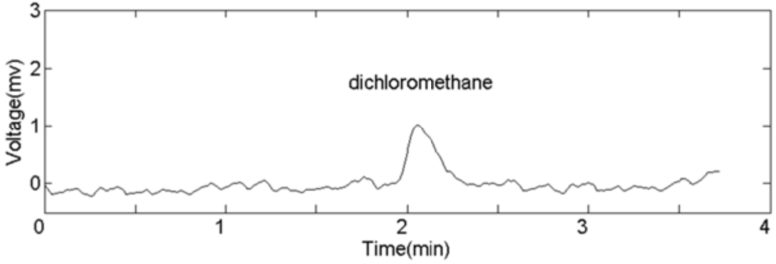


Fig. 4.a.Obtained chromatogram of dichloromethane by the single-well potential with b = 0.02

Fig. 4.b Obtained chromatogram of dichloromethane by the quartic bistable system with a = 0.002 and b = $4.01*10^{-5}$