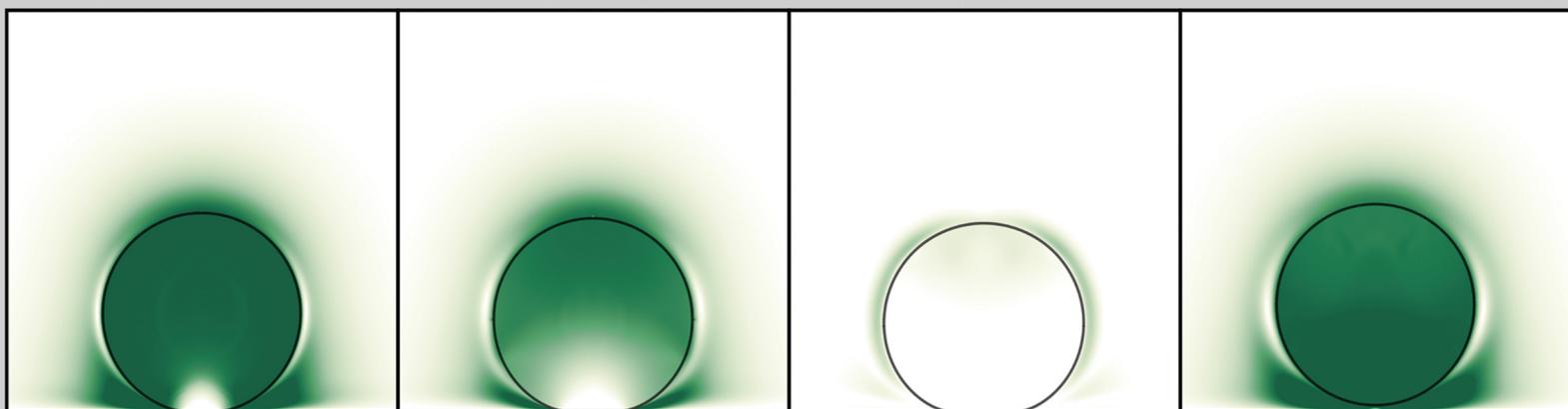
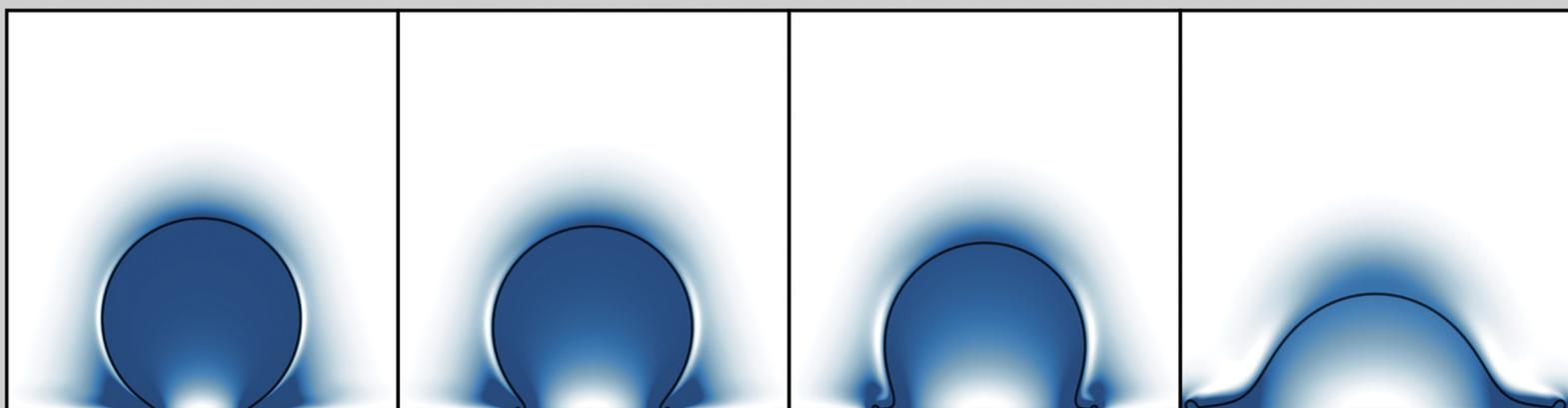


# Soft Matter

rsc.li/soft-matter-journal



ISSN 1744-6848



Cite this: *Soft Matter*, 2026, **22**, 2226

## Impacting spheres: from liquid drops to elastic beads

Saumili Jana, \*<sup>a</sup> John Kolinski, <sup>b</sup> Detlef Lohse <sup>ac</sup> and Vatsal Sanjay \*<sup>d</sup>

A liquid drop impacting a non-wetting rigid substrate spreads laterally, then retracts, and finally jumps off again. An elastic solid, by contrast, undergoes a slight deformation, contacts briefly, and bounces. The impact force on the substrate – crucial for engineering and natural processes – is classically described by Wagner’s (liquids) and Hertz’s (solids) theories. This work bridges these limits by considering a generic viscoelastic medium. Using direct numerical simulations, we study a viscoelastic sphere impacting a rigid, non-contacting surface and quantify how the elasticity number (El, dimensionless elastic modulus) and the Weissenberg number (Wi, dimensionless relaxation time) dictate the impact force. We recover the Newtonian liquid response as either  $El \rightarrow 0$  or  $Wi \rightarrow 0$ , and obtain elastic-solid behavior in the limit  $Wi \rightarrow \infty$  and  $El \gg 1$ . In this elastic-memory limit, three regimes emerge – capillary-dominated, Wagner scaling, and Hertz scaling – with a smooth transition from the Wagner to the Hertz regime. Sweeping  $Wi$  from 0 to  $\infty$  reveals a continuous shift from materials with no memory to materials with permanent memory of deformation, providing an alternate, controlled route from liquid drops to elastic beads. The study unifies liquid and solid impact processes and offers a general framework for the liquid-to-elastic transition relevant across systems and applications.

Received 27th October 2025,  
Accepted 3rd February 2026

DOI: 10.1039/d5sm01078k

rsc.li/soft-matter-journal

## 1 Introduction

Impacts of spherical bodies on rigid substrates span two classical limits that have long been treated separately: liquid drops<sup>1–5</sup> and elastic solids.<sup>6–9</sup> Both occur widely in nature and technology, where the normal force on the substrate is often the quantity of interest because it can damage engineered surfaces.<sup>10,11</sup> Drop impact governs fields ranging from inkjet printing<sup>12</sup> and spray cooling/coating<sup>13,14</sup> to forensics,<sup>15</sup> pesticide deposition,<sup>16</sup> and soil erosion.<sup>17</sup> Impacts of elastic solids arise in hardness testing,<sup>18</sup> granular media and suspensions,<sup>19</sup> sports,<sup>20</sup> and everyday bouncing of soft rubber balls. Despite this breadth, a unifying framework for the impact force across liquid and solid limits has remained elusive.

A falling liquid drop, after impact on a rigid surface, deforms and spreads laterally until it reaches its maximum extent. A pronounced peak in the temporal evolution of force

occurs at the instance of drop touchdown on the surface due to the inertia of the impact, whereas during droplet spreading this force is much smaller.<sup>5</sup> For perfectly wetting surfaces, the liquid sticks to them. However, for non-wetting surfaces, the drop retracts from its maximum spread and generates a Worthington jet which coincides with a second peak in the temporal evolution of force.<sup>3,4</sup> In the inertial regime, Wagner’s theory predicts that the impact force scales as

$$F \sim \rho_1 V_0^2 R_0^2, \quad (1)$$

where  $\rho_1$  is the density of the media,  $V_0$  is the impact velocity and  $R_0$  is the radius of the falling drop.<sup>11</sup>

By contrast, the elastic solids undergo slight deformation on impact with the substrate and bounce off following a brief contact with the substrate due to the exerted normal reaction. In such cases, the temporal evolution of force is characterized by a single maximum. Assuming the contact area to be small in comparison to the bead’s size, and considering a non-adhesive contact with small strains within the elastic limit, the situation can be treated as a Hertzian contact problem.<sup>9</sup> Thus, Hertz’s theory describes the scaling laws for the impact force in the case of the impact of an elastic bead on a rigid substrate,

$$F \sim (GR_0^2)^{2/5} (\rho V_0^2 R_0^2)^{3/5} \quad (2)$$

where  $G$  is the modulus of rigidity,  $\rho$  is the density of the solid medium, and  $R_0$  and  $V_0$  as mentioned before are the radius and impact velocity of the solid elastic bead.

<sup>a</sup> *Physics of Fluids Department, Max Planck Center Twente for Complex Fluid Dynamics, and J. M. Burgers Center for Fluid Dynamics, University of Twente, P.O. Box 217, 7500AE Enschede, The Netherlands. E-mail: s.jana@utwente.nl, d.lohse@utwente.nl*

<sup>b</sup> *Institute of Mechanical Engineering, School of Engineering, EPFL, Lausanne, Switzerland. E-mail: john.kolinski@epfl.ch*

<sup>c</sup> *Max Planck Institute for Dynamics and Self-Organisation, Am Fassberg 17, 37077 Göttingen, Germany*

<sup>d</sup> *CoMPhy Lab, Department of Physics, Durham University, Science Laboratories, South Road, Durham DH1 3LE, UK. E-mail: vatsal.sanjay@comphy-lab.org*



Viscoelastic media – here, soft elastic gels – bridge liquids and solids: when deformed they support both viscous flow and recoverable elastic stress.<sup>21</sup> Compared with Newtonian liquids, their rate-dependent rheology can markedly alter spreading, pinch-off, and rebound on impact.<sup>22,23</sup> Such soft media are relevant to inkjet printing,<sup>24</sup> drop deposition,<sup>25</sup> and spray atomization.<sup>26</sup> Soft solids such as hydrogels, comprising cross-linked networks with tunable elasticity, are widely used as biocompatible materials in rapid prototyping<sup>27</sup> and drug delivery;<sup>28</sup> see Chen *et al.*<sup>29</sup> for background.

In this work, we parameterize the gel's elastic response using the elastic modulus  $G$  which is the proportionality constant between strain and elastic stresses, and the relaxation time  $\lambda$  that sets the decay timescale of those elastic stresses. Upon non-dimensionalizing the governing equations (Section 2.1), two material control parameters emerge. The elastocapillary number

$$Ec = \frac{GR_0}{\gamma} \quad (3)$$

compares the elastic modulus to the Laplace pressure, while the Deborah number

$$De = \frac{\lambda}{\sqrt{\rho_l R_0^3 / \gamma}} \quad (4)$$

compares the elastic-stress relaxation timescale to the inertio-capillary process time. Here  $\gamma$  represents the coefficient of surface tension.

The impact inertia is expressed by the Weber number

$$We = \frac{\rho_l V_0^2 R_0}{\gamma}, \quad (5)$$

which compares the inertial and capillary forces. Another important dimensionless control parameter of the system is the Ohnesorge number

$$Oh = \frac{\eta_l}{\sqrt{\rho_l \gamma R_0}} \quad (6)$$

which is the ratio of inertio-capillary and the visco-capillary timescales. Here  $\eta_l$  is the dynamic viscosity of the sphere.

Two combinations of these numbers will be central in what follows. The elasticity number

$$El = \frac{Ec}{We} = \frac{G}{\rho_l V_0^2}, \quad (7)$$

compares elastic to inertial stresses. The Weissenberg number

$$Wi = De \sqrt{We} = \frac{\lambda}{R_0 / V_0}, \quad (8)$$

compares the elastic relaxation time  $\lambda$  to the impact time  $R_0/V_0$  (see Section 2.1).

In this study, we simulate impacts of soft gel spheres on a rigid, non-contacting substrate using a volume-of-fluid, finite-volume framework. By varying the elastocapillary number ( $Ec$ ) and the Deborah number ( $De$ ), we traverse smoothly from liquid-like to solid-like response and compare the resulting force scalings with Wagner's (liquids) and Hertz's (elastic

solids) theories. We develop an expression for the peak force that transcends the two regimes, using a function of the elasticity parameter to compare the shear modulus with the impact stress. Consistent with these limits, we recover Newtonian-liquid behavior for  $De = 0$  or  $Ec = 0$ , while for  $De \rightarrow \infty$  at sufficiently large  $Ec$  the dynamics converge to those of an elastic solid.

## 2 Numerical framework

### 2.1 Problem description and governing equations

We consider an axisymmetric sphere of radius  $R_0$  approaching a rigid substrate with initial velocity  $V_0$ . For liquid drops the substrate is non-wetting; for viscoelastic and elastic beads it is non-contacting. The sphere is a viscoelastic medium of density  $\rho_l$ , dynamic viscosity  $\eta_l$ , elastic modulus  $G$ , relaxation time  $\lambda$ , and surface tension coefficient  $\gamma$ . The surrounding gas has density  $\rho_g$  and viscosity  $\eta_g$  (Fig. 1).

Lengths are scaled by  $R_0$  and time by the inertio-capillary timescale  $\tau_\gamma = \sqrt{\rho_l R_0^3 / \gamma}$ . The corresponding velocity and pressure/stress scales are  $u_\gamma = \sqrt{\gamma / (\rho_l R_0)}$  and  $\sigma_\gamma = \gamma / R_0$ , respectively. Throughout the manuscript, all variables with a tilde are non-dimensionalized using the above mentioned scales. The incompressible mass and momentum balances in the viscoelastic phase read

$$\nabla \cdot \mathbf{u} = 0 \quad (9)$$

and

$$\frac{\partial(\rho \mathbf{u})}{\partial t} + \nabla \cdot (\rho \mathbf{u} \mathbf{u}) = -\nabla p + \nabla \cdot (\boldsymbol{\sigma}_v + \boldsymbol{\sigma}_e) + \mathbf{f}_\gamma \quad (10)$$

where  $\mathbf{f}_\gamma$  is the non-dimensional capillary force density acting at the interface. The Newtonian (viscous) stress is

$$\tilde{\boldsymbol{\sigma}}_v = 2Oh \tilde{\mathcal{D}} \quad (11)$$

with  $\tilde{\mathcal{D}} = (\tilde{\nabla} \tilde{\mathbf{u}} + \tilde{\nabla} \tilde{\mathbf{u}}^T) / 2$  denoting the symmetric part of the velocity gradient tensor. Furthermore, the normal force on the substrate is obtained using the rate of change of sphere's momentum

$$\mathbf{F}(t) = \frac{4}{3} \pi R_0^3 \rho_l \frac{dV_{cm}}{dt}, \quad (12)$$

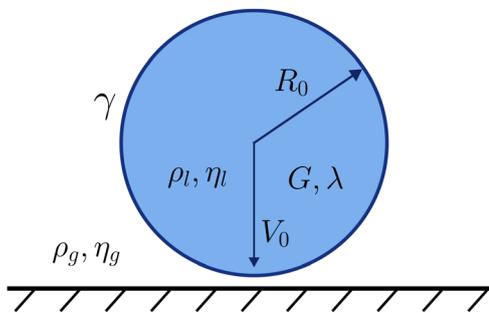


Fig. 1 Schematic: a viscoelastic sphere (radius  $R_0$ ) impacts a non-contacting rigid surface with velocity  $V_0$ . Material properties are  $\rho_l$ ,  $\eta_l$ ,  $G$ ,  $\lambda$ , and  $\gamma$  for the sphere;  $\rho_g$  and  $\eta_g$  for the gas.



where  $V_{\text{cm}}$  denotes the velocity of the centre of mass of the drop at any instant.

The elastic stresses arise from deformation of the microstructure quantified by the conformation tensor  $\mathcal{A}$ .<sup>21</sup> Using the Oldroyd-B constitutive model,<sup>30</sup>

$$\tilde{\sigma}_e = \text{Ec}(\mathcal{A} - \mathcal{I}) \quad (13)$$

with the elastocapillary number  $\text{Ec} = GR_0/\gamma$  (eqn (3)). The conformation tensor relaxes to  $\mathcal{I}$  on the Deborah timescale  $\text{De} = \lambda/\tau_\gamma$  (eqn (4)) *via*

$$\overset{\nabla}{\mathcal{A}} = -\frac{1}{\text{De}}(\mathcal{A} - \mathcal{I}), \quad (14)$$

where the upper-convected derivative is

$$\overset{\nabla}{\mathcal{A}} \equiv \frac{\partial \mathcal{A}}{\partial t} + (\tilde{\mathbf{u}} \cdot \tilde{\nabla})\mathcal{A} - 2\text{Sym}(\mathcal{A} \cdot (\tilde{\nabla}\tilde{\mathbf{u}})). \quad (15)$$

The Deborah number  $\text{De}$  quantifies material memory:  $\text{De} = 0$  recovers a Newtonian liquid characterized by Oh;  $\text{De} \rightarrow \infty$  yields an elastic solid limit with

$$\overset{\nabla}{\mathcal{A}} = 0 \quad (16)$$

and Oldroyd-B equivalent to a neo-Hookean solid.<sup>21</sup>

Despite being widespread due to simplicity, the Oldroyd-B model suffers from certain limitations.<sup>21,31</sup> It fails to capture the shear-thinning behaviour in viscoelastic fluids completely, and erroneously predicts unbounded stress growth in strong extensional flows.<sup>32</sup> This limitation can be addressed by incorporating the finite polymer extension (FENE-P model).<sup>33</sup> Also various other extensions of the Oldroyd-B model have been developed<sup>34</sup> like the Phan–Thien–Tanner (PTT)<sup>35</sup> model to account for such non-linearities. However, since we are not dealing with strong extensional flows, we restrict ourselves to the Oldroyd-B model in this study, as it is sufficient to describe our case.

## 2.2 Numerical methods and simulations

We solve the above equations with Basilisk C,<sup>36</sup> using a one-fluid formulation with surface tension as a singular interfacial force.<sup>37,38</sup> The liquid–gas interface is tracked by a volume-of-fluid (VoF) color function  $\psi$  advected by

$$\frac{\partial \psi}{\partial t} + \nabla \cdot (\psi \mathbf{u}) = 0, \quad (17)$$

with  $\psi = 1$  in liquid,  $\psi = 0$  in gas, and  $0 < \psi < 1$  at the interface. Mixture properties are

$$\rho = \psi \rho_l + (1 - \psi) \rho_g, \quad (18)$$

$$\eta = \psi \eta_l + (1 - \psi) \eta_g, \quad (19)$$

with fixed ratios  $\rho_r = \rho_g/\rho_l = 10^{-3}$  and  $\eta_r = \eta_g/\eta_l = 10^{-2}$  and a small sphere Ohnesorge number,  $\text{Oh} = 10^{-2}$ . A geometric VoF reconstruction applies capillary forces as

$$\mathbf{f}_\gamma \approx \gamma \kappa \nabla \psi, \quad (20)$$

where the curvature  $\kappa$  is computed *via* height functions.<sup>39</sup> Explicit surface-tension forcing imposes the standard capillary time-step constraint;<sup>40</sup> the explicit update of  $\sigma_e$  adds a typically milder constraint.

At the substrate we impose no-penetration and no-slip, and a zero normal pressure gradient. To enforce a non-contacting (superhydrophobic) condition we set  $\psi = 0$  at the wall, maintaining a thin air cushion.<sup>41,42</sup> For liquid drops, enforcing this air cushion results in a non-wetting (superhydrophobic) substrate, while for elastic spheres this results in a non-contacting substrate. We stress that contact initiation in soft-solid impacts is generically air-mediated and can proceed annularly or patchily with a non-monotonic initial contact radius; see Zheng *et al.* (2021)<sup>43</sup> for direct observations of air-mediated contact in compliant-hemisphere impacts. Top and lateral boundaries use outflow (ambient pressure, zero tangential stress, and zero normal velocity gradient). Boundaries are positioned far enough to avoid spurious confinement effects. The axisymmetric domain size is  $8R_0 \times 8R_0$ . We employ quadtree adaptive mesh refinement (AMR)<sup>40,44</sup> with maximal refinement at the interface and in regions of large velocity gradients. Wavelet-based error control uses tolerances  $10^{-3}$  for  $\mathbf{u}$ ,  $\psi$ ,  $\kappa$ , and  $\mathcal{A}$ . Grid-independence tests confirm convergence. Unless stated otherwise, the minimum cell size is  $\Delta = R_0/512$  (*i.e.* 512 cells per radius on a uniform equivalent grid), increased to  $\Delta = R_0/2048$  when required. Further numerical details can be found in studies of Sanjay,<sup>42</sup> Popinet<sup>44</sup> and Dixit *et al.*<sup>31</sup>

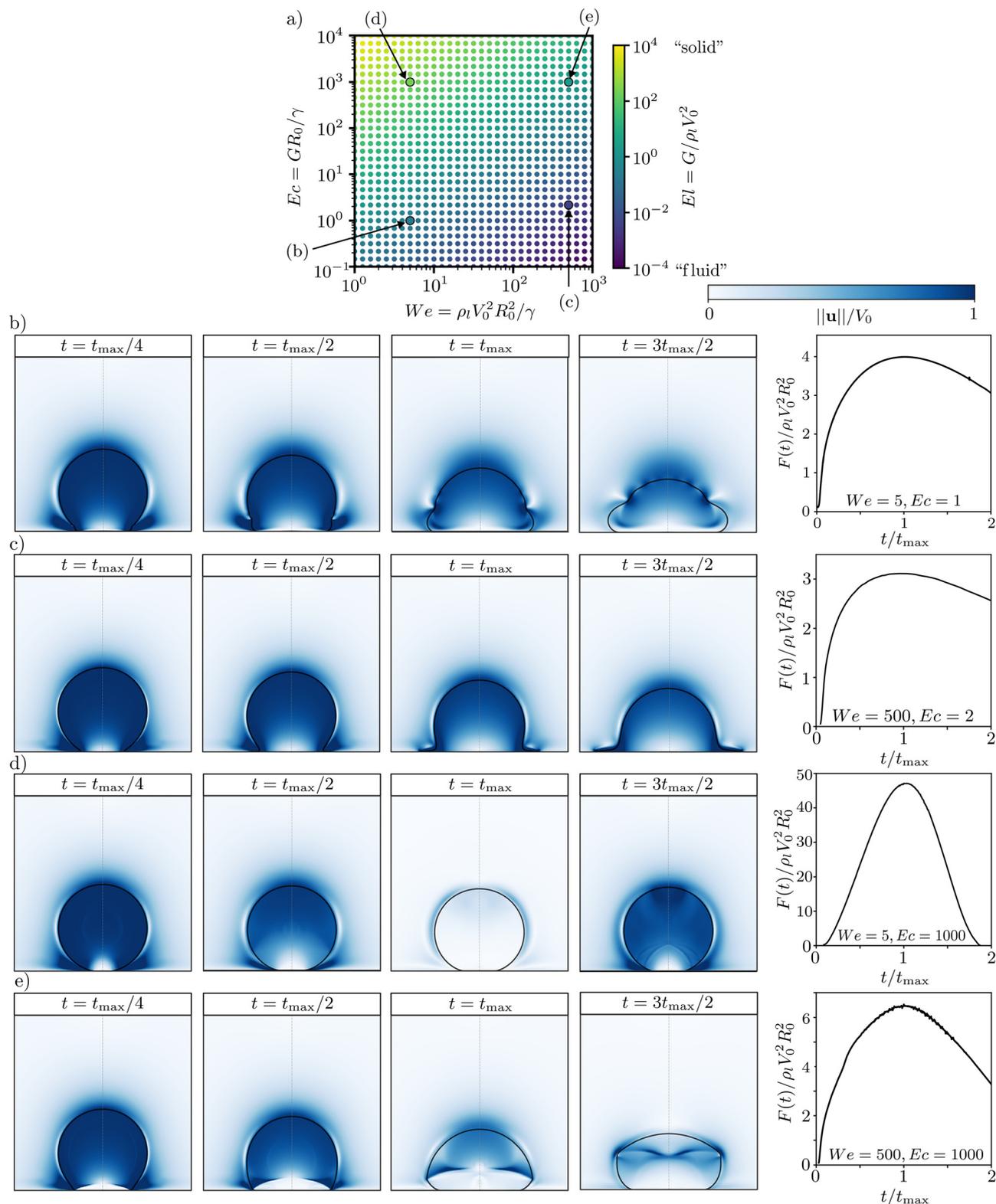
## 3 Wagner versus Hertz: permanent-memory impacts

In this section, we quantify the solid-impact limit by taking  $\text{De} \rightarrow \infty$ , so the material retains its deformation memory over the process time. In the numerics, we keep a small background viscosity, so the spheres are Kelvin–Voigt solids rather than perfectly elastic; this facilitates comparison with inertial liquid impacts at finite Oh and avoids the numerical breakdown of the inviscid (Euler)–elastic limit. We therefore approach the purely elastic response by letting  $\text{Oh} \rightarrow 0$ .

We sweep the  $(\text{Ec}, \text{We})$  space over  $\text{We} \in [1, 10^3]$  and  $\text{Ec} \in [10^{-1}, 10^4]$ . The normal reaction on the substrate is computed from the drop's momentum balance (eqn (12)). For liquid drops on non-wetting substrates,  $F(t)$  exhibits two peaks: an inertial peak at touchdown and a later peak associated with the formation of a Worthington jet.<sup>3</sup> The second, jetting peak occurs at a time  $t_2 \gg t_{\text{max}}$ , with the ratio  $t_2/t_{\text{max}} \sim \sqrt{\text{We}}$ ,<sup>4</sup> reflecting the phase difference between recoil and jet formation. In contrast, for elastic spheres the loading and unloading remain nearly in phase: the entire contact–rebound cycle fits within  $t \lesssim 2t_{\text{max}}$  and  $F(t)$  displays only a single, almost symmetric peak in this interval. To compare liquids and solids and to track the transition, we therefore focus on the first (inertial) peak  $F_{\text{max}}$ , non-dimensionalized as  $F_{\text{max}}/(\rho_l V_0^2 R_0^2)$ ; the corresponding time is  $t_{\text{max}}$ .

Fig. 2 shows representative cases across the parameter space. At low  $\text{Ec}$  the sphere flows and behaves liquid-like





**Fig. 2** (a) Phase space in the  $Ec$ – $We$  plane illustrating the range of simulations conducted in this work colored according to the elasticity number  $El = Ec/We$ . The four highlighted symbols locate typical cases representing the range of parameters explored. We chose  $(We, Ec) =$  (b) (5, 1), (c) (500, 2), (d) (5, 1000), (e) (500, 1000). In each case, the color scheme of each snapshot represents the magnitude of the velocity normalized by the impact velocity, alongside the corresponding force history  $F(t)/(\rho_l V_0^2 R_0^2)$  plotted versus  $t/t_{max}$  (right). The force traces are plotted up to  $t/t_{max} = 2$ : for the liquid-drop reference, the second peak associated with the Worthington jet<sup>3,4</sup> occurs at later times  $t_2 \gg t_{max}$  (with  $t_2/t_{max} \sim \sqrt{We}$ ) and is therefore outside the plotted window.



(Fig. 2b and c;  $Ec = 1, 2$ ). At low  $Ec$  and low  $We$ , capillarity is significant (Fig. 2b). At high  $Ec$  the sphere deforms slightly and rebounds after a short contact (Fig. 2d;  $Ec = 1000$ ). Increasing  $We$  at fixed  $Ec$  effectively softens the response (reduces  $El$ ) and increases the contact duration (Fig. 2d). The force traces reflect this evolution: for large  $El$  (high  $Ec$ , low  $We$ )  $F(t)$  is nearly symmetric, as in elastic impacts, while decreasing  $El$  (e.g. by increasing  $We$ ) skews  $F(t)$  in the manner typical of liquid impacts.<sup>3,4</sup> The peak magnitude also varies appreciably across cases. Since force and time are related as  $[F_{\max}/(\rho_l V_0^2 R_0^2)] \cdot [t_{\max}/(R_0/V_0)] \sim 1$  for any impacting sphere in general,  $t_{\max}$  reduces accordingly with an increase in  $F_{\max}$ .

The dependence of  $F_{\max}$  on  $We$  and  $Ec$  is summarized in Fig. 3. For  $Ec \lesssim 1$ ,  $F_{\max}$  follows the liquid-impact trend with the low- $We$  correction,<sup>3</sup>

$$\frac{F_{\max}}{\rho_l V_0^2 R_0^2} \approx \frac{3.2}{We} + 3.24. \quad (21)$$

As  $Ec$  increases,  $F_{\max}$  rises, most strongly at low  $We$ . At sufficiently large  $Ec$ ,  $F_{\max}/(\rho_l V_0^2 R_0^2)$  decreases with  $We$  with a log-log slope  $\simeq -2/5$ , indicating a transition from eqn (21) to  $F_{\max}/(\rho_l V_0^2 R_0^2) \sim We^{-2/5}$ . At fixed  $We$  (Fig. 3b),  $F_{\max}$  is nearly constant at small  $Ec$  and then increases steadily with  $Ec$ , with higher magnitudes at lower  $We$ .

Collapsing the data using  $El$  (Fig. 4a) reveals two regimes. For  $El \lesssim 1$  and sufficiently large  $We$ , the data lie near Wagner's constant level,  $F_{\max}/(\rho_l V_0^2 R_0^2) \approx 3.24$ . At small  $We$ , inertia competes with capillarity and the low- $We$  correction in eqn (21) is required.<sup>42</sup> For  $El \gtrsim 1$ , all points collapse onto a single master curve with slope  $2/5$ :

$$\frac{F_{\max}}{\rho_l V_0^2 R_0^2} \sim El^{2/5} \sim \left(\frac{G}{\rho_l V_0^2}\right)^{2/5}, \quad (22)$$

consistent with Hertz scaling for elastic impacts. Notably, surface tension does not enter this high- $El$  law, as expected for elastic solids. The transition from the Wagner (liquid) to the Hertz (elastic) regime is smooth. A contour map over  $(We, Ec)$  (Fig. 4b) visualizes the continuous variation of  $F_{\max}$  across the space.

## 4 Theory

In this section we identify the asymptotic limits of the peak impact force and develop a unified predictive model. First, we derive scaling expressions for  $F_{\max}$  in the two extreme regimes – an elastic contact limit *versus* a hydrodynamic impact limit – and then combine these results to propose a single predictive expression for the dimensionless maximum impact force.

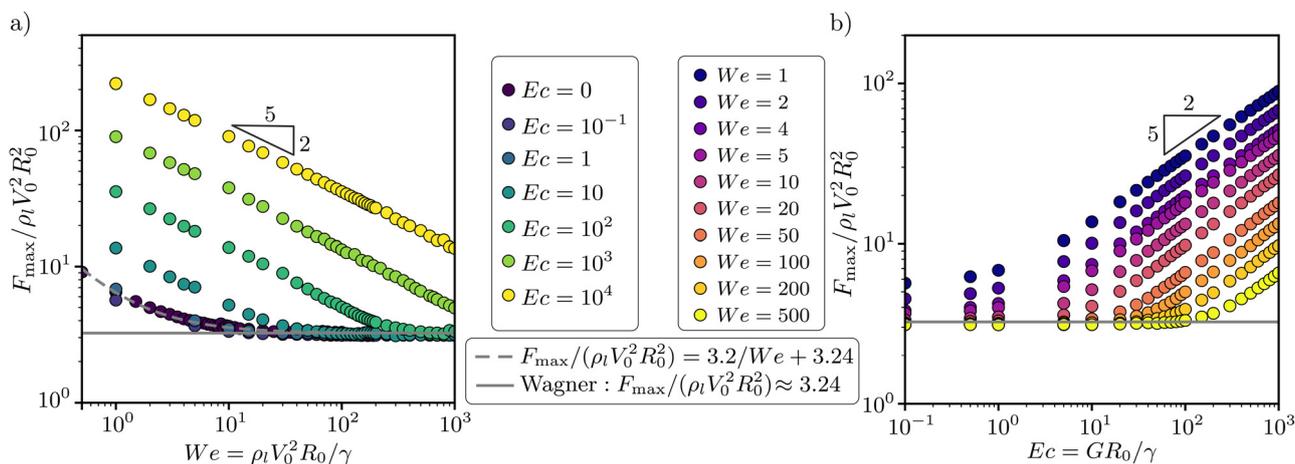
### 4.1 Purely elastic limit: Hertz contact theory

Consider a solid elastic sphere of radius  $R_0$ , mass  $m$ , and elastic modulus  $G$  (shear modulus, assuming an incompressible material) impacting a rigid flat surface with speed  $V_0$ . Upon contact, the sphere deforms and a normal force  $F$  develops according to Hertz's contact law. For a sphere indenting a half-space, the force-indentation relation is given by the  $3/2$ -power law of classical Hertz contact mechanics,<sup>6</sup>

$$F(\delta) = m\ddot{\delta} = -\frac{4}{3}E^* \sqrt{R_0} \delta^{3/2}, \quad (23)$$

where  $\delta(t)$  is the indentation depth and  $E^*$  is the effective Young's modulus of the contacting pair. For a sphere against a rigid flat,  $E^* = 2G/(1 - \nu)$ ; taking Poisson's ratio  $\nu \approx 0.5$  for an incompressible solid, we get  $E^* \approx 4G$ , so the prefactor  $(4/3)E^* \sqrt{R_0}$  in eqn (23) is about  $(16/3)G\sqrt{R_0}$ .

In the ideal elastic limit (no dissipation), the sphere will momentarily come to rest at maximum compression, converting



**Fig. 3** Peak force in the elastic-memory limit ( $De \rightarrow \infty$ ): (a) Variation of the normalized peak force,  $F_{\max}/(\rho_l V_0^2 R_0^2)$ , with the Weber number  $We = \rho_l V_0^2 R_0 / \gamma$  for different elastocapillary numbers  $Ec = GR_0 / \gamma$ . For  $Ec \lesssim 1$  the data follow the liquid-impact result: a high- $We$  Wagner plateau  $\approx 3.24$ , with the low- $We$  correction  $F_{\max}/(\rho_l V_0^2 R_0^2) \approx 3.2/We + 3.24$  (dashed line, eqn (21)). As  $Ec$  increases,  $F_{\max}$  rises, most clearly at low  $We$ , and for sufficiently large  $Ec$  the curves acquire a log-log slope  $-2/5$ , i.e.  $F_{\max}/(\rho_l V_0^2 R_0^2) \sim We^{-2/5}$  at fixed  $Ec$ , consistent with the approach to Hertzian elastic contact. (b) Dependence on  $Ec$  at fixed  $We$  (curves labelled by  $We$ ). At small  $Ec$  all series collapse to the liquid-like level ( $\approx 3.24$ ); above a  $We$ -dependent crossover,  $F_{\max}$  increases monotonically with  $Ec$ , following  $F_{\max}/(\rho_l V_0^2 R_0^2) \sim Ec^{2/5}$ , again, consistent with the approach to Hertzian elastic contact. Together, (a) and (b) show a continuous evolution from Wagner (liquid) to Hertz (elastic) behavior as  $Ec$  increases.



all its kinetic energy into elastic deformation energy. Using energy conservation between the moment of impact and the instant of maximum indentation  $\delta_{\max}$  (when  $\dot{\delta} = 0$ ), we have:

$$\frac{1}{2}mV_0^2 = \int_0^{\delta_{\max}} F(\delta)d\delta. \quad (24)$$

Substituting the Hertz law for  $F(\delta)$  and performing the integration yields the elastic energy stored at indentation  $\delta_{\max}$ :

$$\frac{1}{2}mV_0^2 = \int_0^{\delta_{\max}} \frac{16}{3}G\sqrt{R_0}\delta^{3/2}d\delta \quad (25)$$

$$= \frac{16}{3}G\sqrt{R_0} \cdot \frac{2}{5}\delta_{\max}^{5/2} \quad (26)$$

$$= \frac{32}{15}G\sqrt{R_0}\delta_{\max}^{5/2}. \quad (27)$$

Rearranging this result to solve for the peak indentation  $\delta_{\max}$  gives:

$$\delta_{\max} = \left(\frac{15mV_0^2}{64G\sqrt{R_0}}\right)^{2/5}. \quad (28)$$

The maximum force occurs at  $\delta = \delta_{\max}$ . Substituting the expression for  $\delta_{\max}$  in eqn (23), we get

$$F_{\max} = \frac{16}{3}G\sqrt{R_0}\left(\frac{15mV_0^2}{64G\sqrt{R_0}}\right)^{3/5}. \quad (29)$$

Normalizing with the inertial force scale  $\rho_l V_0^2 R_0^2$ , we get

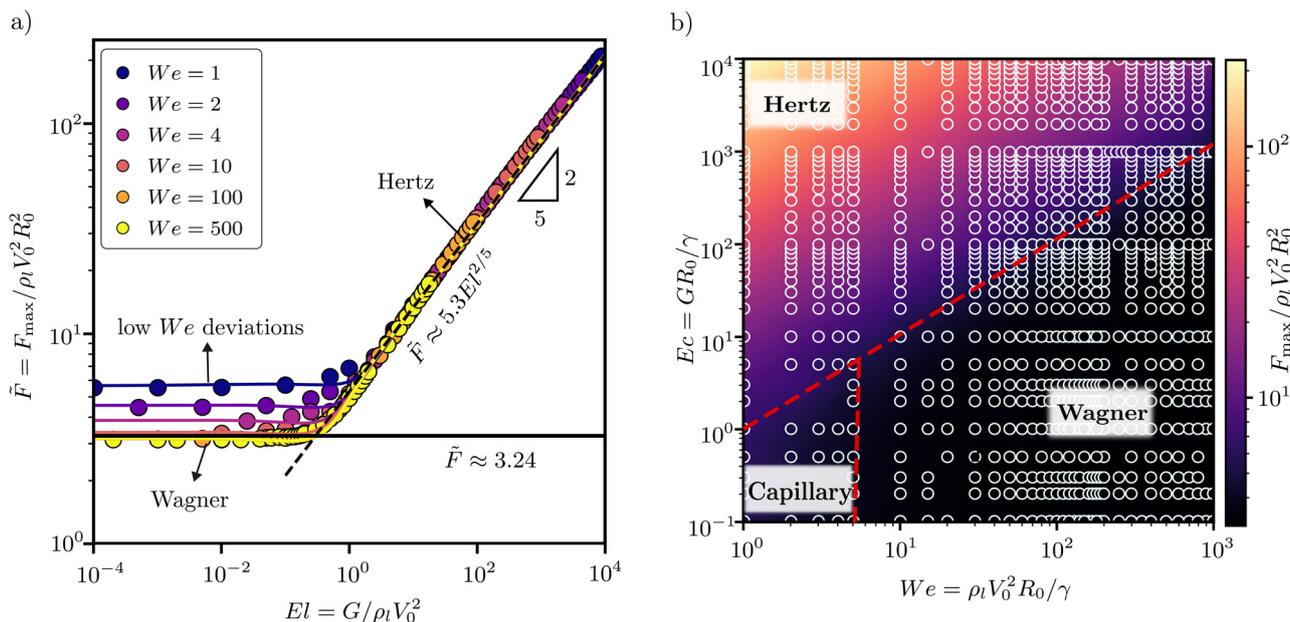
$$\frac{F_{\max}}{\rho_l V_0^2 R_0^2} = \frac{16}{3}\left(\frac{5\pi}{16}\right)^{3/5}\left(\frac{G}{\rho_l V_0^2}\right)^{2/5} \approx 5.3El^{2/5}, \quad (30)$$

which is consistent with our large El results, *cf.* Fig. 4a.

## 4.2 Purely liquid limit: Wagner impact theory

At the opposite extreme ( $Ec \rightarrow 0$ ) the sphere behaves as a liquid drop, and its impact dynamics are governed by inertia and capillarity in the classical Wagner limit.<sup>3,11</sup> Instead of an elastic compression, the drop undergoes rapid localized deformation at the moment of impact: the south pole flattens against the substrate while the remainder of the drop (including the north pole) is still moving downward at nearly the impact speed. The vertical momentum of the drop's center of mass is redirected into a radial outflow along the substrate, causing a small "wetted" area to grow outward from the impact point.<sup>4</sup> This Wagner-type mechanism – a thin spreading lamella initiated at the contact point<sup>45–48</sup> – contrasts sharply with the distributed Hertzian contact of an elastic solid. It produces a pronounced impulsive force at touchdown, as the drop's momentum is arrested over a short time and small area. The first force peak thus originates from pure inertial impingement of the liquid on the surface.<sup>5</sup> We have analyzed this case in past; for details see the references Zhang *et al.*,<sup>3</sup> Sanjay *et al.*,<sup>4</sup> Sanjay and Lohse.<sup>5</sup>

During the very early stage ( $t \sim \tau_\rho = D_0/V_0$ ), the normal force rises sharply to its first maximum  $F_{\max}$  as the drop's inertia is transferred to the substrate. At this moment the deformation is still localized: the contact radius has grown only to the order of



**Fig. 4** Unified scaling and regime map: (a) collapse of the normalized peak force versus the elasticity number  $El = Ec/We = G/(\rho_l V_0^2)$ . For  $El \lesssim 1$  and sufficiently large  $We$  the data sit on the Wagner plateau  $F_{\max}/(\rho_l V_0^2 R_0^2) \approx 3.24$ ; deviations at very small  $We$  reflect capillary corrections in eqn (21). For  $El \gtrsim 1$  all cases collapse onto a single power law with slope  $2/5$ ,  $F_{\max}/(\rho_l V_0^2 R_0^2) \sim El^{2/5}$  (eqn (22)), the hallmark of Hertz scaling. (b) Contours of  $F_{\max}/(\rho_l V_0^2 R_0^2)$  in the  $(We, Ec)$  plane (symbols: simulation points). The dashed guide  $El = 1$  marks the smooth crossover from the Wagner region (lower right) to the Hertz region (upper left); the low- $We$  corner is capillary-dominated and requires the correction in eqn (21). The map visualizes the continuous transition from liquid-like to solid-like impact forces as  $We$  and  $Ec$  are varied.



the drop's initial radius. In fact, experiments confirm that at the peak force time  $t_{\max}$ , the spread diameter  $D_f(t_{\max})$  is approximately equal to the initial drop diameter  $D_0$ , consistent with early-time self-similarity of the impact.<sup>4,5,49,50</sup> Wagner's inviscid theory predicts that the peak force scales with the inertial pressure on the drop's footprint. Non-dimensionalizing  $F_{\max}$  by  $\rho_1 V_0^2 R_0^2$  (with  $\rho_1$  the liquid density) yields a constant of order unity. Indeed, for large Weber numbers (negligible surface tension), simulations and experiments find  $F_{\max}/(\rho_1 V_0^2 R_0^2) \approx 3.24$ , cf. Fig. 3a of Zhang *et al.*<sup>3</sup> At lower  $We$ , surface tension enhances the impact, and the peak force increases with decreasing  $We$  (following a  $F_{\max}/(\rho_1 V_0^2 R_0^2) \sim We^{-1}$  correction in this regime). This initial peak is inertia-dominated and is relatively insensitive to liquid viscosity:  $F_{\max}$  remains nearly constant for drops with viscosity up to about  $100\times$  that of water.<sup>4</sup> Since our spheres have a very low background viscosity ( $Oh = 10^{-2}$ ), in the limit  $Ec \rightarrow 0$  our results for  $F_{\max}$  follow the same trends. Only for highly viscous drops (Ohnesorge number  $Oh \gtrsim 1$ ) does viscous dissipation significantly attenuate the first peak, reflecting the fact that most of the drop's momentum is redirected (and the force generated) before substantial viscous effects have time to act, cf. Sanjay and Lohse.<sup>5</sup>

### 4.3 Predictive interpolating model for maximum impact force

For intermediate conditions ( $El \sim \mathcal{O}(1)$ ), the sphere's deformation and the fluid's inertia both contribute, and the peak force deviates from either pure Hertz or Wagner scaling alone. We therefore express the dimensionless peak force as a weighted transition between the two asymptotic contributions following the approach of Sanjay & Lohse (2025),<sup>5</sup>

$$\frac{F_{\max}}{\rho_1 V_0^2 R_0^2} = 5.3f(El)El^{2/5} + (1 - f(El))\left(\frac{3.2}{We} + 3.24\right), \quad (31)$$

with a smooth transition function  $f(El)$  based on tanh function defined as

$$f(El) = \frac{1 + \tanh\left(\frac{El - a}{b}\right)}{2}, \quad (32)$$

where the  $We$ -dependent parameters  $a \approx 1.3$  for  $We \rightarrow 1$  and  $\approx 0.5$  for  $We \gtrsim 10$ , and  $b \approx 1.4$  for  $We \rightarrow 1$  and  $\approx 0.2$  for  $We \gtrsim 10$  (see ref. 51 for details of this fit). Here,  $a$  measures the critical  $El$  at transition from Wagner's to Hertz's scaling (Fig. 4). The width of this transition is indicated by  $b$ . The remaining coefficients in eqn (31) are fixed *a priori*: the Hertz prefactor 5.3 follows directly from the elastic analysis above (without fitting), while the constants 3.24 (Wagner plateau) and 3.2 (the  $We^{-1}$  capillary correction) are taken from the Newtonian impact model of Sanjay & Lohse (2025).<sup>5</sup> This construction ensures a continuous interpolation between the Hertz and Wagner limits. In spirit it follows the additive scaling approach of Sanjay & Lohse (2025)<sup>5</sup> for drop impacts, but unlike their model – which includes separate viscous regimes – here only the two primary regimes (elastic *vs.* inertial) are needed. The resulting formula smoothly bridges the two asymptotes and correctly reproduces

the peak-force scaling in both limits (this predictive curve is plotted in Fig. 4a for comparison).

## 5 Influence of elastic stress relaxation

The results in Section 3 established the two asymptotic force scalings for impacts with permanent memory ( $De \rightarrow \infty$ ): a Wagner plateau at a small elasticity number and a Hertz law at a large elasticity number (Fig. 4). These two limits also bound impacts when the material memory is finite: relaxing the memory shifts the response continuously from solid-like to liquid-like, with the transition controlled by the non-dimensional relaxation time. We quantify the memory effect with the Weissenberg number,  $Wi$  (eqn (8)), which compares the elastic relaxation time  $\lambda$  to the impact time  $R_0/V_0$  (see Section 2.1). While  $De$  compares  $\lambda$  to the inertio-capillary time,  $Wi$  is the more natural process-time measure here and increases with the degree to which elastic stresses persist during impact.<sup>52</sup> Thus  $Wi = 0$  ( $De = 0$ ) recovers a Newtonian liquid, whereas  $Wi \rightarrow \infty$  ( $De \rightarrow \infty$ ) yields an elastic-memory limit.

Fig. 5 (fixed  $We = 100$  and  $El = 40$ ) visualizes the progressive loss of elastic behavior as  $Wi$  decreases from  $Wi \rightarrow \infty$  to 0. In the large- $Wi$  limit, the bead contacts briefly and rebounds; the force trace is nearly symmetric with a large peak, characteristic of Hertz-like loading. Reducing  $Wi$  increases the contact time and skews  $F(t)$  towards a liquid-like evolution with a much smaller peak. At  $Wi = 0$  the material has no memory and behaves as a Newtonian liquid: the sphere spreads and flows, and  $F(t)$  exhibits the familiar asymmetric shape.

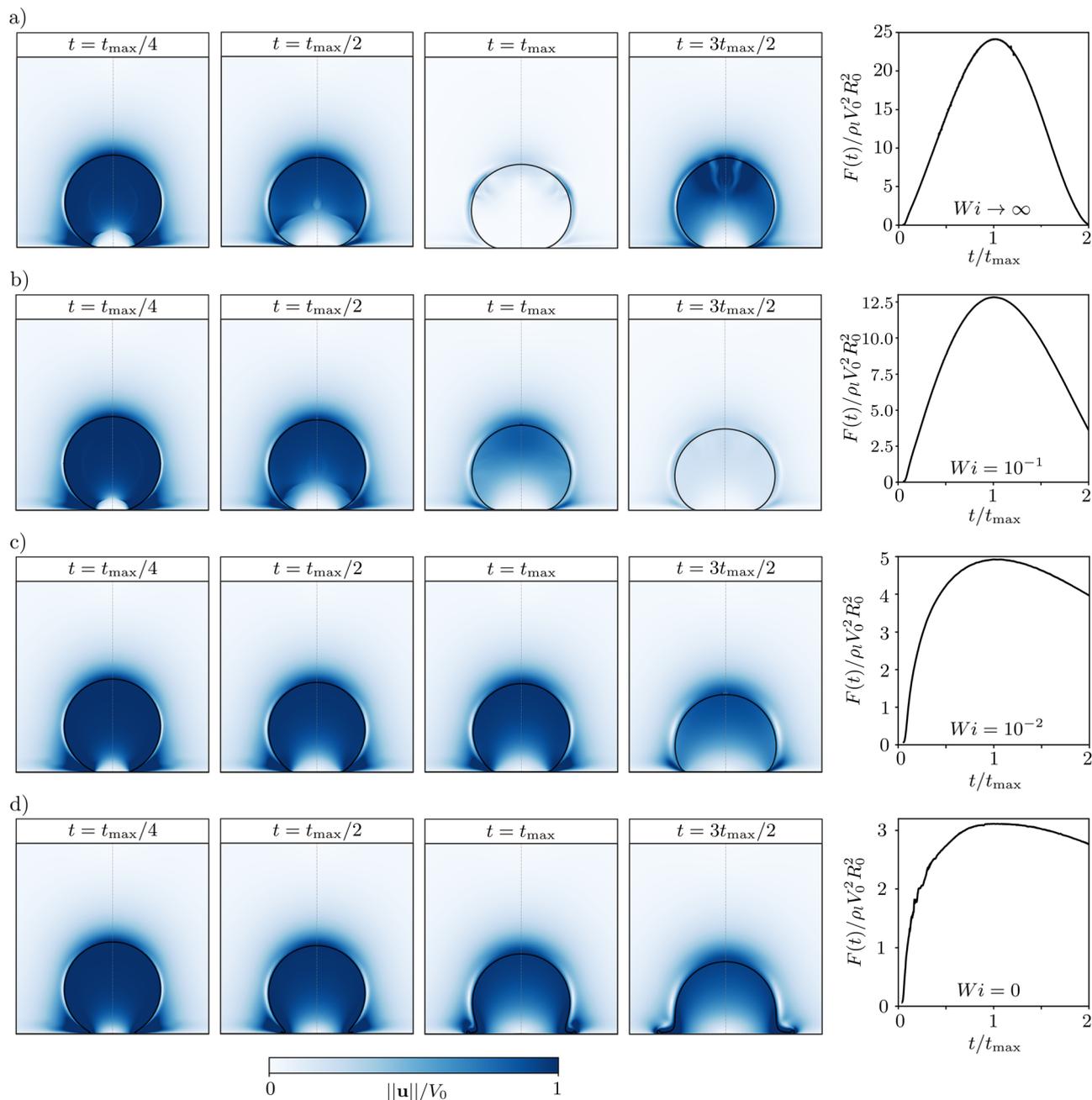
To quantify the role of memory, we plot  $F_{\max}/(\rho_1 V_0^2 R_0^2)$  versus  $El = Ec/We = G/(\rho_1 V_0^2)$  for several  $Wi$  values at  $We = 100$  in Fig. 6. At  $Wi = 0$ , the data follow the Newtonian liquid level established in Section 3. On the other hand, as  $Wi \rightarrow \infty$ , the points converge to the elastic-memory master curve of Section 3, namely the Hertz scaling  $F_{\max}/(\rho_1 V_0^2 R_0^2) \approx 5.3El^{2/5}$  at large  $El$ . Between these limits, weaker memory (smaller  $Wi$ ) delays the transition to Hertz scaling, while stronger memory (larger  $Wi$ ) makes the elastic response apparent already for softer spheres. Notably, even at  $Wi \rightarrow \infty$  the plateau persists for  $El \ll 1$ , because the modulus is introduced exclusively through  $El$ : vanishing  $G$  implies a liquid-like bound irrespective of memory.

Taken together with Section 3, these results show that both material stiffness (*via*  $El$ ) and material memory (*via*  $Wi$ ) govern the peak impact force: the Wagner and Hertz laws remain the bounding asymptotes, while  $Wi$  sets how rapidly the system transitions between them.

## Conclusions and outlook

In this work, we investigate the impact of a viscoelastic sphere on a non-contacting rigid surface and chart a continuous transition in impact dynamics from liquid-like to solid-like behavior by tuning the material parameters. The primary peak force  $F_{\max}$  (associated with the inertial impact) smoothly crosses over from Wagner's inertial-drop scaling to Hertz's





**Fig. 5** Relaxing material memory at fixed  $We$  and  $El$ . Evolution of shape (left) and force (right) when  $Wi$  decreases from  $\infty$  to 0 at  $We = 100$  and  $El = 40$ : (a)  $Wi \rightarrow \infty$  (elastic-memory limit); (b)  $Wi = 10^{-1}$ ; (c)  $Wi = 10^{-2}$ ; (d)  $Wi = 0$  (Newtonian). For each case, the color scheme of each snapshot represents the magnitude of the velocity normalized by the impact velocity, alongside the corresponding force history  $F(t)/(\rho_1 V_0^2 R_0^2)$  plotted versus  $t/t_{\max}$  (right). As  $Wi$  decreases, the contact time increases and  $F(t)$  becomes increasingly liquid-like with a reduced  $F_{\max}$ .

elastic-contact scaling as the elasticity number  $El = Ec/We$  increases. For small  $El$  (liquid-like response), we reproduce  $F_{\max}/(\rho_1 V_0^2 R_0^2) \rightarrow 3.24$  matching the constant plateau from Wagner's theory. In contrast, for large  $El$  (elastic-dominated regime),  $F_{\max}/(\rho_1 V_0^2 R_0^2)$  grows following a power-law  $\approx 5.3El^{2/5}$ , consistent with Hertz's prediction for elastic spheres. These two limiting behaviors bound the force response, and the transition between them is gradual rather than abrupt. The Weissenberg number  $Wi = De\sqrt{We}$ , which quantifies the

polymer's relaxation time relative to the impact time, governs this memory-driven crossover: as  $Wi$  increases from 0 (no elastic memory) to  $\infty$  (permanent memory), the peak-force scaling shifts continuously from the Wagner limit to the Hertz limit. Thus, by adjusting  $Wi$ , one can smoothly interpolate between liquid-drop and elastic-solid impact outcomes.

The modeling choices in this work were made to isolate the physics of the liquid-to-elastic transition and enable direct comparison to the classical limits. The viscoelastic sphere obeys



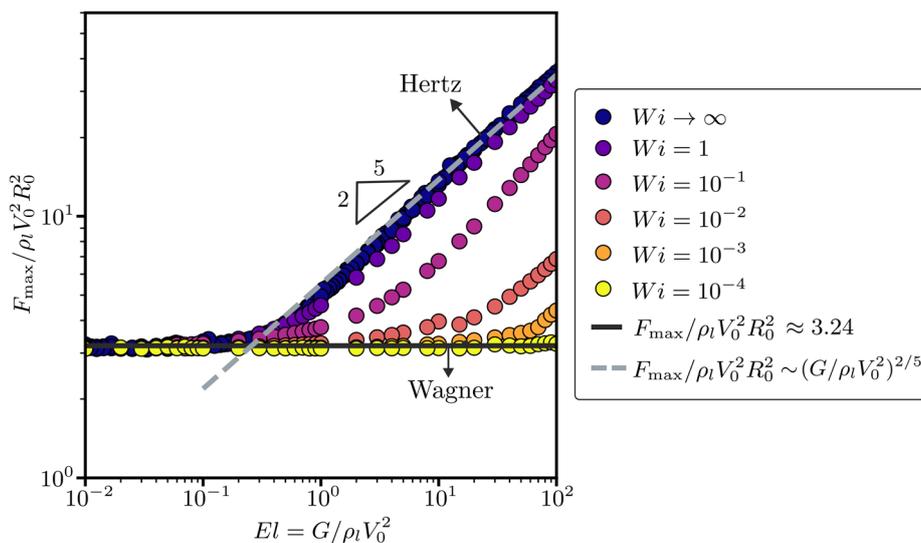


Fig. 6 Peak force versus elasticity number at different  $Wi$  (all at  $We = 100$ ). The black horizontal line indicates the Wagner plateau ( $\approx 3.24$ ); the dashed guide has slope  $2/5$  (Hertz). Increasing  $Wi$  shifts the departure from the plateau to lower  $El$  and drives the curves toward the Hertz master law; at  $Wi = 0$  all data sit on the Wagner level.

Oldroyd-B, which neglects finite microstructure extensibility and shear-thinning; this is acceptable for our flow history but can be systematically relaxed with constitutive models that incorporate finite extensibility and rate-dependent viscosity.<sup>21,32,34,35</sup> The substrate is non-contacting, so the thin gas layer is present but not explicitly resolved with lubrication and wetting dynamics; prior work shows that air cushioning, skating on a gas film, and nanoscale first contact depend sensitively on slip, compressibility and rarefaction.<sup>53–64</sup> We stress that in the present work, the substrate is assumed to remain perfectly rigid. Small elastic deformations of the substrate can, however, modify the intervening gas layer,<sup>65</sup> which in turn may alter the impact dynamics and the measured force signature of the impacting sphere. Furthermore, the onset of contact will modify the shear stress at the interface – including effects such as adhesion – that can modify the peak force, and will alter the stresses upon rebound. Finally, a small background viscosity renders the ( $De \rightarrow \infty$ ) limit to be of the Kelvin–Voigt type rather than being ideally elastic; classical analyses quantify how viscoelastic dissipation and elastic waves perturb the Hertzian impact.<sup>7,8</sup> These assumptions do not alter the governing exponents (Wagner versus Hertz) but they may affect quantitative prefactors and very-short-time, near-contact details.

Several direct extensions can sharpen and generalize these results. Experiments with soft hydrogel/elastomer beads at moderate  $V_0$  can probe the crossover regime and test the full  $F(t)$  waveform (peak magnitude, rise time, and symmetry/skewness), leveraging recent studies on soft-solid and gel impacts and elastohydrodynamic bouncing,<sup>9,66–71</sup> and using established force-measurement protocols from liquid-drop impacts.<sup>3,4,72–74</sup> Furthermore, incorporating a thin-gas lubrication model with dynamic wetting (including slip and, if needed, rarefaction/compressibility) will resolve when and how contact initiates and how this feeds back on the very-early-time force.<sup>43,49,53,56–61,64</sup>

Linking impact memory with lubricated-impact and wetting transitions will place the near-contact force history on firmer ground.<sup>75–79</sup>

## Author contributions

V. S., J. K., and D. L. conceived the study. S. J. and V. S. planned the numerical simulations. S. J. performed the simulations and analyzed the data. V. S. and D. L. developed the theory. S. J. and V. S. designed the structure of the manuscript. S. J., V. S., D. L. and J. K. wrote the manuscript. V. S. and D. L. supervised the project. All authors discussed the results and approved the final manuscript.

## Conflicts of interest

There is no conflict of interest.

## Data availability

All codes used in this work are available as an open-source repository at <https://github.com/comphy-lab/Soft-Sphere-Impacts>.

Supplementary information (SI) contains simulation videos clearly demonstrating the striking contrast between liquid drop impact and elastic sphere impact. See DOI: <https://doi.org/10.1039/d5sm01078k>.

## Acknowledgements

We would like to thank Vincent Bertin, Pierre Chantelot, Mazyar Jalaal, Andrea Prosperetti, and Jacco Snoeijer for discussions. This work was carried out on the national e-infrastructure of SURFsara, a subsidiary of SURF cooperation,



the collaborative ICT organization for Dutch education and research. This work was sponsored by NWO – Domain Science for the use of supercomputer facilities. This work was supported by NWO-Canon grant FIP-II grant. V. S. acknowledges start-up funding from Durham University. Open Access funding provided by the Max Planck Society.

## Notes and references

- H. Wagner, Über Stoß- und Gleitvorgänge an der Oberfläche von Flüssigkeiten, *Z. Angew. Math. Mech.*, 1932, **12**, 193–215.
- J. Philippi, P.-Y. Lagrée and A. Antkowiak, Drop impact on a solid surface: short-time self-similarity, *J. Fluid Mech.*, 2016, **795**, 96–135.
- B. Zhang, V. Sanjay, S. Shi, Y. Zhao, C. Lv and D. Lohse, Impact forces of water drops falling on superhydrophobic surfaces, *Phys. Rev. Lett.*, 2022, **129**, 104501.
- V. Sanjay, B. Zhang, C. Lv and D. Lohse, The Role of Viscosity on Drop Impact Forces on Non-Wetting Surfaces, *J. Fluid Mech.*, 2025, **1004**, A6.
- V. Sanjay and D. Lohse, Unifying Theory of Scaling in Drop Impact: Forces and Maximum Spreading Diameter, *Phys. Rev. Lett.*, 2025, **134**, 104003.
- H. Hertz, Ueber die Berührung fester elastischer Körper, *J. Reine Angew. Math.*, 1882, **1882**, 156–171.
- Y.-H. Pao, Extension of the Hertz Theory of Impact to the Viscoelastic Case, *J. Appl. Phys.*, 1955, **26**, 1083–1088.
- S. C. Hunter, Energy Absorbed by Elastic Waves during Impact, *J. Mech. Phys. Solids*, 1957, **5**, 162–171.
- V. Bertin, Similarity solutions in elastohydrodynamic bouncing, *J. Fluid Mech.*, 2024, **986**, A13.
- M. Ahmad, M. Schatz and M. Casey, Experimental investigation of droplet size influence on low pressure steam turbine blade erosion, *Wear*, 2013, **303**, 83–86.
- X. Cheng, T.-P. Sun and L. Gordillo, Drop impact dynamics: impact force and stress distributions, *Annu. Rev. Fluid Mech.*, 2022, **54**, 57–81.
- D. Lohse, Fundamental fluid dynamics challenges in inkjet printing, *Annu. Rev. Fluid Mech.*, 2022, **54**, 349–382.
- S. Shiri and J. C. Bird, Heat exchange between a bouncing drop and a superhydrophobic substrate, *Proc. Natl. Acad. Sci. U. S. A.*, 2017, **114**, 6930–6935.
- J. Kim, Spray cooling heat transfer: The state of the art, *Int. J. Heat Fluid Flow*, 2007, **28**, 753–767.
- F. R. Smith, C. Nicloux and D. Brutin, Influence of the impact energy on the pattern of blood drip stains, *Phys. Rev. Fluids*, 2018, **3**, 013601.
- L. He, L. Ding, B. Li, W. Mu, P. Li and F. Liu, Optimization Strategy to Inhibit Droplets Rebound on Pathogen-Modified Hydrophobic Surfaces, *ACS Appl. Mater. Interfaces*, 2021, **13**, 38018–38028.
- M. A. Nearing, J. M. Bradford and R. D. Holtz, Measurement of force vs. time relations for waterdrop impact, *Soil Sci. Soc. Am. J.*, 1986, **50**, 1532–1536.
- Y. Tirupataiah and G. Sundararajan, A dynamic indentation technique for the characterization of the high strain rate plastic flow behaviour of ductile metals and alloys, *J. Mech. Phys. Solids*, 1991, **39**, 243–271.
- B. Andreotti, Y. Forterre and O. Pouliquen, *Granular Media: Between Fluid and Solid*, Cambridge University Press, 2013.
- R. Cross, The bounce of a ball, *Am. J. Phys.*, 1999, **67**, 222–227.
- J. H. Snoeijer, A. Pandey, M. A. Herrada and J. Eggers, The relationship between viscoelasticity and elasticity, *Proc. R. Soc. A*, 2020, **476**, 20200419.
- C. Clasen, J. Eggers, M. A. Fontelos, J. Li and G. H. McKinley, The beads-on-string structure of viscoelastic threads, *J. Fluid Mech.*, 2006, **556**, 283–308.
- S. Tamim and J. Bostwick, Oscillations of a soft viscoelastic drop, *npj Microgravity*, 2021, **7**, 42.
- P. Calvert, Inkjet Printing for Materials and Devices, *Chem. Mater.*, 2001, **13**, 3299–3305.
- V. B. Bergeron, D. B. Bonn, J. M. Martin and V. L. Louis, Controlling Droplet Deposition with Polymer Additives, *Nature*, 2000, **405**, 772–775.
- J. Thompson and J. Rothstein, The atomization of viscoelastic fluids in flat-fan and hollow-cone spray nozzles, *J. Non-Newtonian Fluid Mech.*, 2007, **147**, 11–22.
- B. Derby, Printing and Prototyping of Tissues and Scaffolds, *Science*, 2012, **338**, 921–926.
- J. Li and D. Mooney, Designing hydrogels for controlled drug delivery, *Nat. Rev. Mater.*, 2016, **1**, 16071.
- D. Chen, Q. Wen, P. Janmey, J. Crocker and A. Yodh, Rheology of Soft Materials, *Annu. Rev. Condens. Matter Phys.*, 2010, **1**, 301–322.
- J. G. Oldroyd and A. H. Wilson, On the formulation of rheological equations of state, *Proc. R. Soc. London, Ser. A*, 1950, **200**, 523–541.
- A. K. Dixit, A. Oratis, K. Zinelis, D. Lohse and V. Sanjay, Viscoelastic Worthington Jets and Droplets Produced by Bursting Bubbles, *J. Fluid Mech.*, 2025, **1010**, A2.
- G. H. McKinley and T. Sridhar, Filament-stretching rheometry of complex fluids, *Annu. Rev. Fluid Mech.*, 2002, **34**, 375–415.
- R. Bird, P. Dotson and N. Johnson, Polymer solution rheology based on a finitely extensible bead-spring chain model, *J. Non-Newtonian Fluid Mech.*, 1980, **7**, 213–235.
- R. Tanner, *Engineering Rheology*, OUP, Oxford, 2000.
- N. P. Thien and R. I. Tanner, A new constitutive equation derived from network theory, *J. Non-Newtonian Fluid Mech.*, 1977, **2**, 353–365.
- S. Popinet and collaborators, Basilisk C: volume of fluid method, <https://basilisk.fr> (Last accessed: August 23, 2023), 2013–2023.
- G. Tryggvason, R. Scardovelli and S. Zaleski, *Direct Numerical Simulations of Gas-Liquid Multiphase Flows*, Cambridge University Press, 2011.
- J. U. Brackbill, D. B. Kothe and C. Zemach, A continuum method for modeling surface tension, *J. Comput. Phys.*, 1992, **100**, 335–354.



- 39 S. Popinet, Numerical models of surface tension, *Annu. Rev. Fluid Mech.*, 2018, **50**, 49–75.
- 40 S. Popinet, An accurate adaptive solver for surface-tension-driven interfacial flows, *J. Comput. Phys.*, 2009, **228**, 5838–5866.
- 41 J. M. Kolinski, L. Mahadevan and S. M. Rubinstein, Drops can bounce from perfectly hydrophilic surfaces, *Europhys. Lett.*, 2014, **108**, 24001.
- 42 V. Sanjay, PhD thesis – Research UT, graduation UT, University of Twente, Netherlands, 2022.
- 43 S. Zheng, S. Dillavou and J. M. Kolinski, Air mediates the impact of a compliant hemisphere on a rigid smooth surface, *Soft Matter*, 2021, **17**, 3813–3819.
- 44 S. Popinet, A quadtree-adaptive multigrid solver for the Serre-Green-Naghdi equations, *J. Comput. Phys.*, 2015, **302**, 336–358.
- 45 J. Eggers, M. A. Fontelos, C. Josserand and S. Zaleski, Drop dynamics after impact on a solid wall: theory and simulations, *Phys. Fluids*, 2010, **22**, 062101.
- 46 C. Josserand and S. T. Thoroddsen, Drop impact on a solid surface, *Annu. Rev. Fluid Mech.*, 2016, **48**, 365–391.
- 47 M. Negus, PhD thesis, University of Oxford, 2022.
- 48 M. J. Negus, M. R. Moore, J. M. Oliver and R. Cimpeanu, Droplet impact onto a spring-supported plate: analysis and simulations, *J. Eng. Math.*, 2021, **128**, 3.
- 49 S. Mandre, M. Mani and M. P. Brenner, Precursors to Splashing of Liquid Droplets on a Solid Surface, *Phys. Rev. Lett.*, 2009, **102**, 134502.
- 50 J. Philippi, P.-Y. Lagrée and A. Antkowiak, Drop impact on a solid surface: short-time self-similarity, *J. Fluid Mech.*, 2016, **795**, 96–135.
- 51 S. Jana and V. Sanjay, Code repository: Soft Sphere Impacts, <https://github.com/comphy-lab/Soft-Sphere-Impacts>, 2026.
- 52 R. J. Poole, The Deborah and Weissenberg numbers, *Rheol. Bull.*, 2012, **53**, 32–39.
- 53 F. T. Smith, L. Li and G. X. Wu, Air cushioning with a lubrication/inviscid balance, *J. Fluid Mech.*, 2003, **482**, 291–318.
- 54 S. T. Thoroddsen, T. G. Etoh and K. Takehara, Air entrapment under an impacting drop, *J. Fluid Mech.*, 2003, **478**, 125–134.
- 55 S. T. Thoroddsen, T. G. Etoh, K. Takehara, N. Ootsuka and Y. Hatsuki, The air bubble entrapped under a drop impacting on a solid surface, *J. Fluid Mech.*, 2005, **545**, 203–212.
- 56 J. M. Kolinski, S. M. Rubinstein, S. Mandre, M. P. Brenner, D. A. Weitz and L. Mahadevan, Skating on a film of air: drops impacting on a surface, *Phys. Rev. Lett.*, 2012, **108**, 074503.
- 57 R. C. A. van der Veen, T. Tran, D. Lohse and C. Sun, Direct measurements of air layer profiles under impacting droplets using high-speed color interferometry, *Phys. Rev. E: Stat., Nonlinear, Soft Matter Phys.*, 2012, **85**, 026315.
- 58 M. M. Driscoll and S. R. Nagel, Ultrafast interference imaging of air in splashing dynamics, *Phys. Rev. Lett.*, 2011, **107**, 154502.
- 59 E. Q. Li, I. U. Vakarelski and S. T. Thoroddsen, Probing the nanoscale: the first contact of an impacting drop, *J. Fluid Mech.*, 2015, **785**, R2.
- 60 M. V. Chubynsky, K. I. Belousov, D. A. Lockerby and J. E. Sprittles, Bouncing off the walls: the influence of gas-kinetic and van der Waals effects in drop impact, *Phys. Rev. Lett.*, 2020, **124**, 084501.
- 61 P. Garca-Geijo, G. Riboux and J. Gordillo, The skating of drops impacting over gas or vapour layers, *J. Fluid Mech.*, 2024, **980**, A35.
- 62 K. Langley, E. Q. Li and S. T. Thoroddsen, Impact of ultra-viscous drops: air-film gliding and extreme wetting, *J. Fluid Mech.*, 2017, **813**, 647–666.
- 63 L. Zhang, T. Soori, A. Rokoni, A. Kaminski and Y. Sun, Thin film instability driven dimple mode of air film failure during drop impact on smooth surfaces, *Phys. Rev. Fluids*, 2021, **6**, 044002.
- 64 J. E. Sprittles, Gas microfilms in droplet dynamics: When do drops bounce?, *Annu. Rev. Fluid Mech.*, 2024, **56**, 91–118.
- 65 K. R. Langley, A. A. Castrejón-Pita and S. T. Thoroddsen, Droplet impacts onto soft solids entrap more air, *Soft Matter*, 2020, **16**, 5702–5710.
- 66 S. Mitra, Q. Vo and T. Tran, Bouncing-to-wetting transition of water droplets impacting soft solids, *Soft Matter*, 2021, **17**, 5969–5977.
- 67 M. Jalaal, C. Seyfert, B. Stoeber and N. J. Balmforth, Gel-controlled droplet spreading, *J. Fluid Mech.*, 2018, **837**, 115–128.
- 68 L.-H. Luu and Y. Forterre, Drop impact of yield-stress fluids, *J. Fluid Mech.*, 2009, **632**, 301–327.
- 69 G. Martouzet, L. Jørgensen, Y. Pelet, A.-L. Biance and C. Barentin, Dynamic arrest during the spreading of a yield stress fluid drop, *Phys. Rev. Fluids*, 2021, **6**, 044006.
- 70 F. Chevy, A. Chepelianskii, D. Quéré and E. Raphaël, Liquid Hertz contact: softness of weakly deformed drops on non-wetting substrates, *Europhys. Lett.*, 2012, **100**, 54002.
- 71 E. A. Agüero, L. F. L. Alventosa, D. M. Harris and C. A. Galeano-Rios, Impact of a rigid sphere onto an elastic membrane, *Proc. R. Soc. London, Ser. A*, 2022, **478**, 20220340.
- 72 B. R. Mitchell, J. C. Klewicki, Y. P. Korkolis and B. L. Kinsey, The transient force profile of low-speed droplet impact: measurements and model, *J. Fluid Mech.*, 2019, **867**, 300–322.
- 73 J. Li, B. Zhang, P. Guo and Q. Lv, Impact force of a low speed water droplet colliding on a solid surface, *J. Appl. Phys.*, 2014, **116**, 214903.
- 74 B. Zhang, J. Li, P. Guo and Q. Lv, Experimental studies on the effect of Reynolds and Weber numbers on the impact forces of low-speed droplets colliding with a solid surface, *Exp. Fluids*, 2017, **58**, 1–12.
- 75 J. H. Snoeijer and B. Andreotti, Moving contact lines: scales, regimes, and dynamical transitions, *Annu. Rev. Fluid Mech.*, 2013, **45**, 269–292.
- 76 D. Richard and D. Quéré, Bouncing water drops, *Europhys. Lett.*, 2000, **50**, 769–775.
- 77 P. K. Sharma and H. N. Dixit, Regimes of wettability-dependent and wettability-independent bouncing of a drop on a solid surface, *J. Fluid Mech.*, 2021, **908**, A37.
- 78 V. Sanjay, P. Chantelot and D. Lohse, When does an impacting drop stop bouncing?, *J. Fluid Mech.*, 2023, **958**, A26.
- 79 P. Chantelot and D. Lohse, Drop impact on superheated surfaces: short-time dynamics and transition to contact, *J. Fluid Mech.*, 2021, **928**, A36.

