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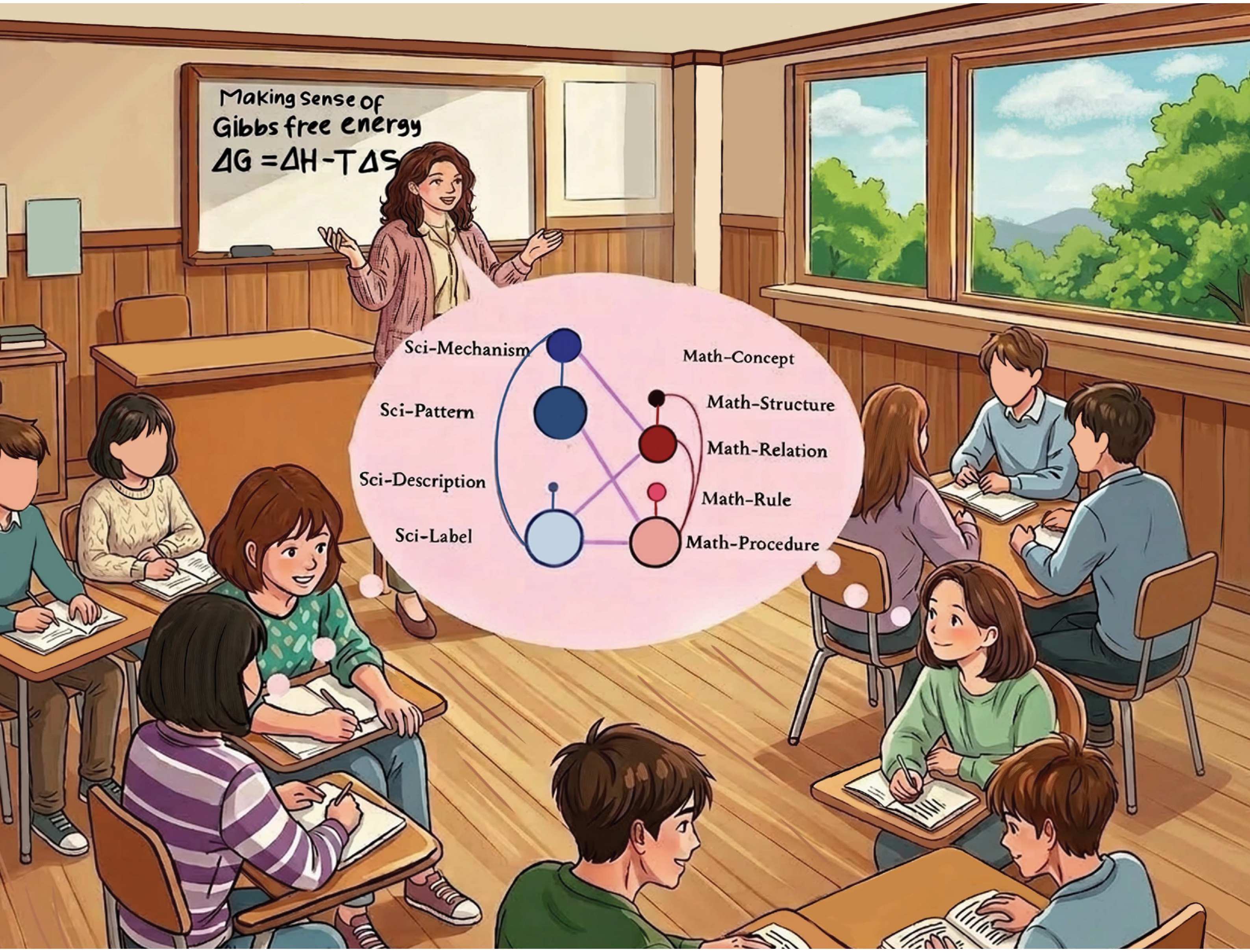

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Sensemaking opportunities provided by college chemistry instructors

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Studies have shown that students often struggle to solve quantitative science problems because they fail to make connections between mathematical equations and associated scientific phenomena. This struggle has been attributed to instructors providing limited sensemaking opportunities that connect science and mathematics sensemaking. Our prior research on college instructors teaching population growth shows that the types of sensemaking elicited varied and were organized in different ways. This case study extends the research by exploring sensemaking opportunities about mathematical equations provided by three instructors teaching Gibbs free energy. This study also examines different levels of connection between sensemaking types within and across the science and mathematics dimensions and analyzes factors (*i.e.*, pedagogical approaches, equation types) that might shape the types of sensemaking that were provided. The Sci-Math Sensemaking Framework was used to identify sensemaking opportunities provided by three instructors during the lessons and a comparative case study approach was employed. Findings showed that while each instructor provided both science and mathematics sensemaking opportunities, they had distinct ways of sequencing their sensemaking even when teaching the same scientific phenomenon using mathematical equations. Mathematics sensemaking was mostly presented separately from science sensemaking, with only a few instances of connected sensemaking occurring in instructors' lessons, albeit at varying levels. When instructors expose students to connected science and mathematics sensemaking, they model for students how to use resources from two different disciplines to understand scientific concepts or solve quantitative problems. We provide evidence to show that levels of connection between sensemaking types within science and mathematics dimensions reflect distinct approaches to developing more robust scientific explanations or to working with equations. Based on these case studies, we suggest that instructors' equation choices and pedagogical approaches support specific types of sensemaking and levels of connection between sensemaking opportunities.

Introduction

Teaching science not only involves attention to scientific content, but also the mathematical representations (*i.e.*, symbols, equations, operators) that represent relevant scientific phenomena. Policy documents advocate undergraduate students should utilize quantitative reasoning to interpret data, and mathematical modeling to generate solutions to science problems (American Association for the Advancement of Science (AAAS), 2009). However, studies of students' use of mathematics in science show that students often struggle to solve quantitative science problems because they do not make connections between equations and associated scientific phenomena (Bing and Redish, 2009; Taasoobshirazi and Glynn, 2009; Becker and Towns, 2012). Failure to make these connections can result in difficulties that include a lack of understanding of the science concepts represented in the equations, as well as the ability to use the equations to solve novel or complex quantitative problems (Becker and Towns, 2012; Schuchardt and Schunn, 2016; Eichenlaub and Redish, 2018). These struggles have been attributed to a lack of sensemaking opportunities related to mathematics provided by instructors during science instruction involving mathematical equations (Schuchardt and Schunn, 2016; Zhao *et al.*, 2021).

Zhao *et al.* (2021) explored different opportunities for sensemaking of mathematical equations in a biology classroom using the Sci-Math Sensemaking framework (Zhao and Schuchardt, 2021). They noted that types of sensemaking used by instructors were varied and organized in different ways. However, this study was limited to sensemaking opportunities provided by instructors teaching a single phenomenon in biology. This study extends

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this prior work to other scientific disciplines by exploring the teaching of a chemical phenomenon (Gibbs free energy). This study also investigated varying levels of connection between sensemaking types within and across dimensions, and analyzed factors (*e.g.*, pedagogical approaches, equation types) that may influence the types of sensemaking opportunities.

The overarching research questions guiding this study are:

1. What sensemaking opportunities about mathematical equations do instructors provide when teaching Gibbs free energy?

2. How do instructors make connections between sensemaking opportunities when teaching Gibbs free energy?

3. How do pedagogical approaches and types of equations support particular sensemaking types?

Literature review

Mathematical equations are often used in science to represent patterns, model, predict, and explain scientific phenomena (Bialek and Botstein, 2004; Hestenes, 2010; Becker and Towns, 2012; Rodriguez *et al.*, 2018; Lazenby and Becker, 2019). However, students struggle with making sense of conceptual knowledge embodied in mathematical equations. They have difficulty making connections between their mathematical understanding of the equation and their scientific knowledge of the phenomenon represented by the equation (Bing and Redish, 2009; Taasoobshirazi and Glynn, 2009; Becker and Towns, 2012). Thus, students tend to rely on algorithmic procedures to solve quantitative problems rather than using reasoning about mathematical or science content (Lythcott, 1990; Tuminaro and Redish, 2007; Bing and Redish, 2009; Kuo *et al.*, 2013; Becker *et al.*, 2017), possibly due to a lack of sensemaking opportunities during instruction (Schuchardt and Schunn, 2016; Zhao *et al.*, 2021). Indeed, when students were taught in a manner likely to promote sensemaking, they showed greater conceptual understanding and were better able to solve quantitative problems (Schuchardt and Schunn, 2016).

Sensemaking

Sensemaking is broadly defined as the process of making meaning of new situations using knowledge and skills acquired from past experiences (Martin and Kasmer, 2009; Kapon, 2016; Odden and Russ, 2019). Sensemaking has been conceptualized differently in science and mathematics education communities. Sensemaking in science education has been described as the construction of an explanation for the purpose of understanding a given scientific phenomenon (Odden and Russ, 2019). Whereas mathematics sensemaking has been defined as seeking coherence between conceptual and procedural knowledge of mathematics (Rittle-Johnson and Schneider, 2015; Dreyfus *et al.*, 2017; Kuo *et al.*, 2020). Students in science classes could make sense of equations by drawing on either scientific explanations or mathematical (conceptual or procedural) knowledge or blending both cognitive resources (Fauconnier and Turner, 1998; Bain *et al.*, 2019; Zhao *et al.*, 2021; Kaldaras and Wieman, 2023).

When instructors explicitly make connections between mathematics and science sensemaking during instruction, students are given the opportunity to move beyond the idea of mathematics and science as fragmented and disconnected disciplines (Li and Schoenfeld, 2019; Zhao *et al.*, 2021). Various instructional practices could provide students with the opportunity to make these connections and thus foster student engagement in sensemaking. Such practices include mathematical modeling (*e.g.*, Malone, 2008; Schuchardt and Schunn, 2016; Becker *et al.*, 2017; Li and Schoenfeld, 2019; Schuchardt and Roehrig, 2024) and making explicit connections between mathematics and scientific ideas (*e.g.*, Zhao *et al.*, 2021). Passive modes of engagement where learners are oriented towards receiving information (*e.g.*, listening to a lecture) may also offer students limited opportunities for sensemaking. When listening to a lecture, students may appear passively engaged, yet they may be covertly processing the information deeply (*e.g.*, by mentally connecting concepts or self-explaining the material) (Chi and Wylie, 2014). Therefore, in this article, we use the term "sensemaking opportunity" to refer to the situation created by instructor-scaffolded instruction or tasks that supports students in processing and making sense of the material, whether they lead to passive or active engagement.

Sci-Math Sensemaking framework

Instructors can utilize different types of sensemaking and connect them in different ways during instruction of mathematical equations in science classrooms (Zhao *et al.*, 2021). Based on a systematic literature review, Zhao and Schuchardt (2021) proposed nine categories of sensemaking in their Sci-Math Sensemaking framework, four in the science dimension: Sci-Label, Sci-Description, Sci-Pattern, Sci-Mechanism, and five in the mathematics dimension: Math-Procedure, Math-Rule, Math-Structure, Math-Relation, Math-Concept (Table 1). A smaller subset of sensemaking from the framework tends to be used by instructors in traditional science instruction (Zhao *et al.*, 2021). They often rely heavily on Math-Procedure, Sci-Label, and Sci-Description when demonstrating algorithmic problem-solving on the board.

The order of types of sensemaking within the dimensions are theorized to represent increasing levels of understanding (Zhao and Schuchardt, 2021). For example, logically, identifying components within a phenomenon (Sci-Label) and input/output associations (Sci-Pattern) may help generate an explanation of the mechanism which results in the observed patterns (Sci-Mechanism).

Sensemaking types can be organized as blended, coordinated, and adjacent

In addition to characterizing types of sensemaking, Zhao *et al.* (2021) also identified three different ways sensemaking was organized during instruction of mathematical equations in science classrooms: blended, coordinated, and adjacent sensemaking (Table 2).

Blended sensemaking occurs when science sensemaking is used to support or explain mathematical sensemaking or *vice*

Table 1 Categories of Sci-Math Sensemaking framework adapted to Gibbs free energy (Zhao and Schuchardt, 2021)

Dimension	Code	Description	Representative quote
Science	Sci-Label	Name variables or operations in mathematical equations with relevant aspects or processes of scientific phenomenon.	"...That G is free energy... H is enthalpy... T is temperature and S is entropy."
	Sci-Description	Use a mathematical equation to provide a quantifiable measure of a parameter or aspect of scientific phenomenon that cannot be directly measured.	"...the delta G is just the difference between reactants and products."
	Sci-Pattern	Describe the trend or pattern among aspects of scientific phenomenon represented as variables in a mathematical equation or as data points on a graph.	"The product[s] of the reaction have less heat content than the reactants."
	Sci-Mechanism	Connect to a mechanism that explains how or why relationships or aspects of the scientific phenomenon represented in the mathematical equation occur.	"That reaction will only proceed with continuous input of energy. It won't proceed on its own."
Mathematics	Math-Procedure	Explain or carry out step-by-step calculation or algorithmic procedures for obtaining a numerical answer. Assign values to variables.	"There are two electrons being passed, so our N is 2, the number of electrons... The RT at 298 over Faraday is 0.026 volts. We just divided that by two and it comes down to 0.013 volts."
	Math-Rule	Refer to generalizable statements that guide calculation across multiple problem types.	"They $\left[\frac{\text{units of [products]}}{\text{units of [reactants]}}\right]$ cancel out during calculation, so K_{eq} technically is dimensionless, it doesn't have units."
	Math-Structure	Discuss the form of the equation, the numbers and arrangement of variables and operations.	"Just know, on top is [the concentration of] whatever is accepting and getting reduced and the concentration of whatever takes on the electron is the donor, that's on the bottom $\left[\frac{\text{[electron acceptor]}}{\text{[electron donor]}}\right]$."
	Math-Relation	Focus on quantitative relationships (has numbers) between variables in the mathematical equation.	"The delta G value's going to be a negative number... But the delta H for this reaction is positive six kilojoules per mole."
	Math-Concept	Refers to a network of knowledge that enables explanation of what, how, and why of a mathematical idea. [*e.g.*, Expresses boundaries or limitations of mathematical equation (works/doesn't work)]	"You can say it [the equation] only applies when T is bigger than 12. Putting limits on when equations apply works."

Table 2 Three different ways to organize sensemaking adapted to Gibbs free energy (Zhao *et al.*, 2021)

Structure	Description	Example
Blended	Two types of sensemaking *across* dimensions (mathematics and science) co-occur and one type of sensemaking is used to support or explain the other type of sensemaking.	"And if we look at the position of delta S in the equation up here, delta S is sitting behind a minus sign. And so, a positive change in entropy contributes, because of this minus sign, to a negative delta G. (Sci-Pattern, Math-Structure)"
Coordinated	Two types of sensemaking within the *same* dimension co-occur and one type of sensemaking is used to support or explain the other type of sensemaking	If a reaction is in a situation where the reactants have lower free energy and the products have higher free energy, then that's a positive delta G (Sci-Pattern). And that reaction will only proceed with continuous input of energy. It won't proceed on its own (Sci-Mechanism).
Adjacent	Two types of sensemaking, either within the same dimension or across dimensions, are presented sequentially within the same activity but are not explicitly connected.	"A is the electron acceptor here, but the other one is called the 'electron donor', and that is B (Sci-Label)... Just know, on top is [the concentration of] whatever is accepting and getting reduced and the concentration of whatever takes on the electron is the donor, that's on the bottom (Math-Structure)."

Note. Within quotes, a specific type of sensemaking is highlighted with a specific color: dark blue for Sci-Label, blue for Sci-Pattern, light blue for Sci-Mechanism, and maroon for Math-Structure.

versa. Students who are provided with the opportunity to engage in blended sensemaking display increased scientific understanding and problem-solving skills (Kuo *et al.*, 2013; Schuchardt and Schunn, 2016; Bain *et al.*, 2018). Kaldaras and Wieman (2023) extended prior work on blended sensemaking by using the Sci-Math Sensemaking framework to describe different combinations and levels of blended sensemaking during interviews with students as they engaged with computer simulations. They introduced four blended categories–algorithmic, qualitative, quantitative, and conceptual. Each derived from combining a specific type of mathematics sensemaking (*i.e.*, Math-Procedure, Math-Structure, Math-Relation, Math-Concept) with any of the science sensemaking types (*i.e.*, Sci-Description (combines Sci-Label and Sci-Description), Sci-Pattern, Sci-Mechanism). These combinations form a hierarchical continuum, where the lowest level emerges from pairing Math-Procedure with any science sensemaking type (algorithmic), and the highest level arises from blending Math-Concept with any science sensemaking type (conceptual). Within the algorithmic and conceptual categories of blended sensemaking, there is a second hierarchy where connections to Sci-Pattern are considered lower level than connections to Sci-Mechanism. Additional evidence is needed to determine whether instructors engage in these combinations and levels of blended sensemaking when discussing mathematical equations in chemistry instruction.

In coordinated sensemaking, two types of sensemaking that are from the same dimension are connected and one is used to support the other. Coordinated sensemaking shares similarities with definitions of sensemaking in science which focus on combining different scientific ideas to construct explanations about phenomena (Odden and Russ, 2019; Odden, 2021) and mathematics which focus on generating coherence between formal and conceptual understanding of mathematics (Rittle-Johnson and Schneider, 2015; Dreyfus *et al.*, 2017; Li and Schoenfeld, 2019; Kuo *et al.*, 2020). Adjacent sensemaking takes place when two types of sensemaking occur sequentially but are not explicitly connected. The lack of an explicit connection means that students are forced to make those connections themselves and not all may do so (Zhao *et al.*, 2021). Explicit connections offer opportunities for sensemaking to all students, while implicit connections tend to limit these opportunities to only those students who choose to make those connections.

Zhao *et al.* (2021) used the Sci-Math Sensemaking framework to describe how undergraduate instructors represent mathematical equations in their instruction. They noted that types of sensemaking opportunities were varied, but the application of this framework was limited to the context of instructors teaching a single biological phenomenon. This study expands the application of the sensemaking framework to instruction of a chemistry topic. This study also explores varying levels of blended and coordinated sensemaking opportunities modeled by chemistry instructors. Moreover, this study contributes to the literature by examining how pedagogical strategies may provide affordances or constraints for specific types of sensemaking.

Additionally, the topics and equations addressed in population growth and Gibbs free energy are fundamentally different. Gibbs free energy equations express ideas about the relationship between enthalpy and entropy and Gibbs free energy. Population growth equations express ideas about how the mechanism for reproduction in an organism affects the number of organisms that are produced. Thus, one might expect instruction on Gibbs free energy to focus on sensemaking around mathematical relationships and scientific patterns while in the instruction on population growth characterized by Zhao *et al.* (2021), the focus was on sensemaking that explains how growth occurred (scientific mechanisms). Connecting scientific patterns to mathematical sensemaking is generally considered a lower level than connecting scientific mechanisms to mathematical sensemaking. Therefore, it will be insightful to examine whether the opportunities for sensemaking provided by instructors when teaching Gibbs free energy equations are indeed of a different type and level compared to the prior study on instruction of population growth equations.

Methods

Research design

The goal of this study was to identify the different types of sensemaking opportunities about mathematical equations provided by instructors teaching Gibbs free energy and how these sensemaking opportunities are connected. The Gibbs free energy concept is essential in chemistry and biochemistry research. It aids in understanding system characteristics and processes by predicting reaction spontaneity, direction, as well as the stability of molecular structures under the constraints of constant temperature and pressure. These constraints apply to all biological and chemical processes. A change in Gibbs free energy (ΔG) is not a measurable thing. It is defined by the equation that serves more as an expression of ideas. Therefore, understanding the equation is essential to understanding Gibbs free energy. A change in Gibbs free energy (ΔG) accounts for both the enthalpy (heat content, H) and entropy (disorder, S) changes in a system and is represented as $\Delta G = \Delta H - T\Delta S$. Interpretations and calculations of a change in Gibbs free energy are generally assumed to be under conditions of constant pressure and temperature. Thus, a change in Gibbs free energy (ΔG) represents the net balance between changes in enthalpy (ΔH) and entropy (ΔS).

Participants and instructional context

This study employed a case study design of three instructors from research universities. Each case was defined as a single instructor teaching mathematical equations related to Gibbs free energy. Three instructors (two female and one male) participated in this study (Table 3). At the time of the study, they had all been teaching for more than five years. Gary and Tessa are from the same Midwestern university and taught Gibbs free energy in introductory biology for majors (class size 100–150 students). Maria taught Gibbs free energy in a biochemistry class at a Southwestern university (class size 150–200 students).

Table 3 Participants and instructional context for each instructor

Instructor	Background	University	Class size	Course level
Tessa	Biology	R1 Midwestern university	100–150	Introductory Biology
Maria	Chemistry	R1 Southwestern university	150–200	Biochemistry
Gary	Biology	R1 Midwestern university	100–150	Introductory Biology

Note. Names are pseudonyms.

Data collection and analysis

Data collection included audio recordings of each instructor teaching Gibbs free energy, as well as slides used during the lesson. The primary data sources for the study were the transcripts from these lessons. Slides served as a secondary data source to provide additional details about the nature of class activities.

The purpose of the data analysis was to understand how the instructor sets up and affords sensemaking opportunities to the class as a whole rather than providing an exhaustive list of all instances of sensemaking. The data were analyzed in three steps. First, a rich description of the instruction of mathematical equations related to Gibbs free energy was generated based on the transcripts and slides. These rich descriptions focused on instructors' statements to the whole class and included a summary of the specific equations used by the instructor, the amount of instructional time spent teaching these equations, and the chronology of instructional events around mathematical equations. The first author constructed rich descriptions. The last two authors independently read the descriptions along with the transcripts and available slides and noted areas where the rich description did not reflect the lesson. These areas of disagreement were resolved with discussion and iterative revision of the rich descriptions.

Second, the Sci-Math Sensemaking framework was utilized to identify sensemaking opportunities around mathematical equations within the rich case descriptions. One researcher assigned the types of sensemaking elicited by the instructor using the Sci-Math Sensemaking framework (Zhao and Schuchardt, 2021, Table 1). Two independent researchers reviewed the assigned types of sensemaking and noted areas of disagreement that were resolved with discussion. Third, the ways instructors organized their sensemaking opportunities were identified by one researcher as blended and coordinated sensemaking (Zhao *et al.*, 2021, Table 2). Two independent researchers reviewed the identified organizational structure of sensemaking and noted areas of disagreement that were resolved with discussion.

Results and discussion

First, the rich descriptions of each instructor's implemented lesson on equations about Gibbs free energy are presented. This is followed by a descriptive summary of the sensemaking opportunities provided by each instructor. Within quotes, a specific type of sensemaking is bracketed by a specific colored symbol: ♣ for Sci-Label, ♦ for Sci-Description, ♥ for Sci-Pattern, ♠ for Sci-Mechanism, ∩ for Math-Procedure, ⊃ for Math-Rule, ⊗ for Math-Structure, and ∅ for Math-Relation.

Description of Tessa's instruction

Tessa's instruction of Gibbs free energy lasted for 50 minutes. Tessa began her Gibbs free energy lesson by introducing the mathematical equation, $\Delta G = \Delta H - T\Delta S$. She exposed students to Sci-Label as she defined each symbol or variable in the equation, "So, we're going to go through each of these symbols… ♣Delta means 'change in'. That G, I've already mentioned, is free energy… H is enthalpy… T is temperature and S is entropy♣ (Sci-Label♣)."

Tessa first defined the variable "S" as *entropy* (Sci-Label) followed by explaining the relationship between changes in the organizational state of matter and change in entropy (Sci-Pattern). Next, she related entropy to ΔG (Sci-Pattern) and pointed out the position of entropy in the equation, "behind a minus sign", (Math-Structure) to support her description of the relationship between a change in entropy and free energy, connecting two types of sensemaking across two dimensions (blended sensemaking).

> ♥When molecular arrangement becomes more random, then delta S is positive♥. And if we look at ⊗the position of delta S in the equation up here, delta S is sitting behind a minus sign⊗. And so, ♥a positive change in entropy contributes♥, because of ⊗this minus sign⊗, ♥to a negative delta G. So, increasing the entropy of a system is going to be one way of contributing to a negative delta G♥ (Sci-Pattern♥, Math-Structure⊗).

Using the same equation, Tessa continued by explaining the effect of temperature on molecular movement and how molecular movements may affect the likelihood that the reaction will occur (Sci-Mechanism), "♠At low temperature, molecules are moving slowly… if you speed up molecules by raising the temperature, reactions are more likely to occur. And at higher temperatures, reactions are likely to occur♠ (Sci-Mechanism♠)."

Tessa defined the next variable in the equation, "H", as *enthalpy* (Sci-Label), and made connections among the stability of molecules, enthalpy, and free energy (Sci-Pattern). Tessa ended her instruction of the equation $\Delta G = \Delta H - T\Delta S$ by describing the relationship among variables (Sci-Pattern).

> ♥The change in free energy of a reaction is going to have to do with the change in the chemical potential energy. It's going to have to do with the temperature at which the reaction is occurring. And it's going to have to do with the change in entropy of the reaction, as well♥ (Sci-Pattern♥).

Tessa then shifted to another equation, $\Delta G = G_{\text{products}} - G_{\text{reactants}}$. She related free energy of reactants and products to both entropy and chemical potential energy (Sci-Pattern). Tessa then used the difference between the two free energy values to provide a quantifiable measure of ΔG (Sci-Description), connecting two types of sensemaking within science dimension (coordinated science sensemaking).

> ♥The free energy of the reactants is something that takes into account both the entropy and the chemical potential energy. The free energy of the products is something that takes into account the chemical potential energy and entropy♥. And what we're thinking about when we calculate ♦the delta G is the difference between those two [the free energy of products and the free energy of reactants]♦ (Sci-Pattern♥, Sci-Description♦).

Tessa continued with a statement that compared free energy of reactants and product to conclude that delta G will be positive (Sci-Pattern) and explained how a reaction with a positive ΔG can proceed by connecting to a driving mechanism for reactions (continuous energy inputs) (Sci-Mechanism). This is an example of coordinated sensemaking, connecting two types of sensemaking within the same dimension.

> ♥If a reaction is in a situation where the reactants have lower free energy and the products have higher free energy, then that's a positive delta G♥. And ♠that reaction will only proceed with continuous input of energy. It won't proceed on its own♠ (Sci-Pattern♥, Sci-Mechanism♠).

Tessa referred back to the previous equation, $\Delta G = \Delta H - T\Delta S$, and provided students with graphs (Fig. 1) representing reactions with a positive and negative ΔG. She asked students in their small groups to discuss how the graphs and Gibbs free energy equation, $\Delta G = \Delta H - T\Delta S$, are relevant to transport across a membrane.

After allowing time for group discussions, Tessa brought the students back together and asked Team 4 and Team 6 to share their ideas with the class. Tessa created sensemaking opportunities for Sci-Mechanism when she asked several questions prompting Team 6 to make connections between the movement of glucose in the vesicle during transport across a membrane and variable T. For example, Tessa asked, "♠And so in order for glucose molecules to move across the membrane, does there have to be a change in temperature?♠ (Sci-Mechanism♠)"

Tessa asked students to return to their small groups and discuss, "♥how are each of these different letters—G, H, T, and S—[in the Gibbs free energy equation, $\Delta G = \Delta H - T\Delta S$] related to this example of something moving across the gradient?♥ (Sci-Pattern♥)" Tessa brought the students back together and asked Team 1 to share their discussion results with the class. Tessa focused on Sci-Pattern sensemaking as she asked Team 1 to describe how gradient concentration corresponds to variables in the equation ($\Delta G = \Delta H - T\Delta S$).

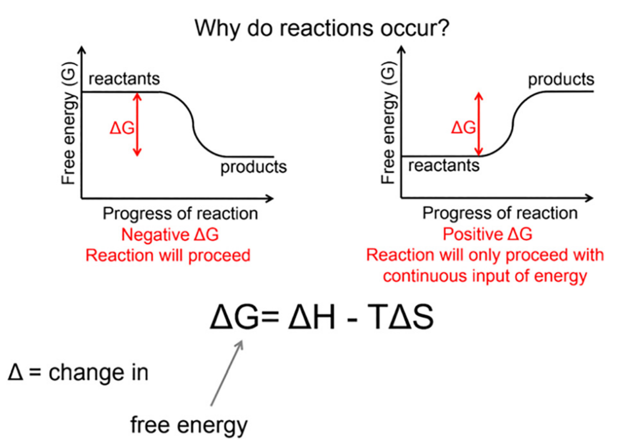

Fig. 1 Artifact of Tessa's instruction: the relationship between free energy and progress of reaction. *Note.* The red labels were absent when students were asked to analyze the graphs.

> ♥So, when you have a strong gradient with really different concentrations of glucose on either side of that membrane, that's very ordered. It's not random. There's an organization to that. And so entropy is low. But when molecules move down the concentration gradient to even it out, what's that doing to entropy? I know you said it already. It's increasing the entropy♥ (Sci-Pattern♥).

Tessa also presented opportunities for blended sensemaking as she added Math-Structure sensemaking when she elaborated on Team 1's answer about the relationship between increased entropy and negative free energy by referring to how ΔG and ΔS are positioned and connected (by *minus* sign) in the equation ($\Delta G = \Delta H - T\Delta S$), "So, it's because ♥there's a change in entropy when molecules move down a gradient and because that change in entropy is to increase entropy♥, ⊗with that minus sign in there⊗, ♥that leads to a negative delta G♥ (Sci-Pattern♥, Math-Structure⊗)." Tessa ended this whole class discussion by creating Sci-Label, Sci-Mechanism, and Sci-Pattern sensemaking opportunities respectively when she defined enthalpy and explained the effect of the scientific process (mechanism) of "*forming or breaking bonds*", on enthalpy, followed by summarizing the relationship among ΔG, the concentration gradient, and ΔS in the case of transport across the membrane.

Tessa shifted to discuss another equation, $\Delta G_3 = \Delta G_1 + \Delta G_2$, and used Math-Procedure sensemaking as she transitioned students from thinking about the Gibbs free energy equation to considering how Gibbs free energy can be summed across a series of equations to make predictions. To illustrate this she described in detail how that would happen in the example of ATP hydrolysis coupled with glutamine biosynthesis. Tessa showed students how to determine the ΔG values for glutamate (+3.4 kcal mol^{-1}) and ATP (−7.3 kcal mol^{-1}) using a step-by-step procedure which was displayed on her slide. She showed students that by summing the two ΔG values (as shown on her slide), she could predict whether glutamine biosynthesis would proceed.

> If we look simply from an energy, math perspective, I told you that ∩ATP releases 7.3 kilo Cals per mole∩. So, energetically, that's enough to drive this reaction [glutamine biosynthesis] forward if ∩it only requires 3.4 kilocalories per mole to be put in∩ (Math-Procedure∩).

Tessa focused on science sensemaking

Multiple opportunities for different types of science and mathematics sensemaking were provided during Tessa's instruction of three equations related to Gibbs free energy. Tessa focused on science sensemaking for most of the lesson, with occasional introduction of Math-Structure, and ended the lesson with Math-Procedure sensemaking (Fig. 2). Students were frequently exposed to Sci-Label, Sci-Pattern, and Sci-Mechanism sensemaking.

When Tessa showed students how to determine whether a reaction would proceed using the Gibbs free energy equation, $\Delta G = \Delta H - T\Delta S$, she started in the science sensemaking space, then moved to combine science and mathematics sensemaking, and ended in the science sensemaking (*i.e.* Sci-Label and

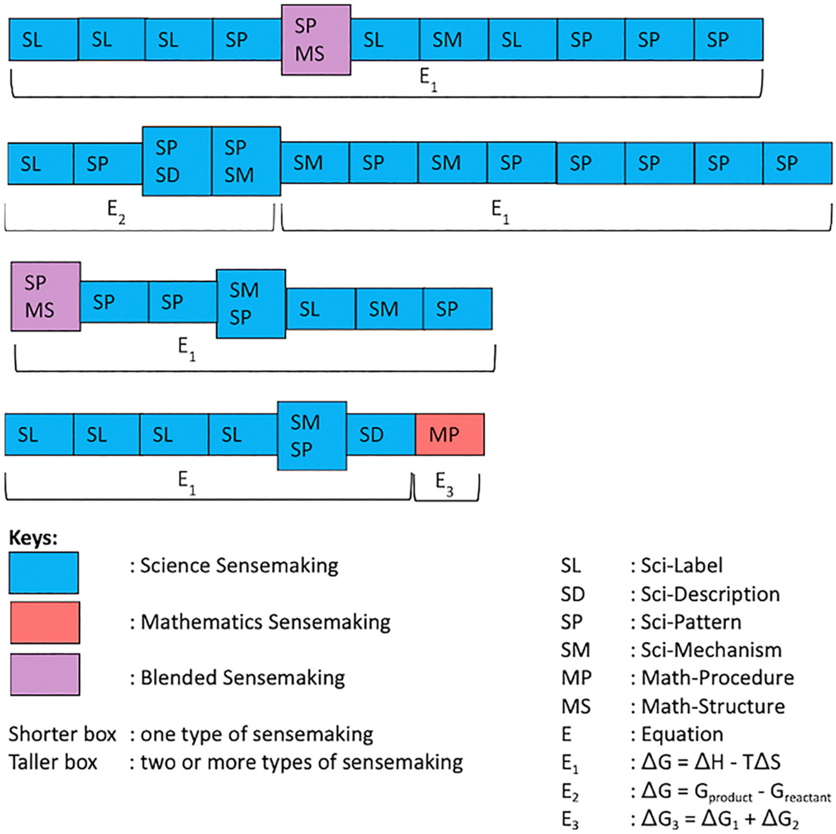

Fig. 2 The organization of sensemaking opportunities identified during Tessa's lesson on equations about Gibbs free energy (50 minutes).

Sci-Pattern to Sci-Pattern/Math-Structure to Sci-Label, Sci-Mechanism, and Sci-Pattern). Whereas with $\Delta G = G_{\text{products}} - G_{\text{reactants}}$, Tessa stayed in the science sensemaking space (*i.e.* Sci-Label and Sci-Pattern to Sci-Pattern/Sci-Description to Sci-Pattern/Sci-Mechanism). When the class discussed how the Gibbs free energy equation, $\Delta G = \Delta H - T\Delta S$, is relevant to transport across a membrane, Tessa again started in the science sensemaking space, then moved to combine science and mathematics sensemaking, and ended in the science sensemaking (*i.e.* Sci-Mechanism and Sci-Pattern to Sci-Pattern/Math-Structure to Sci-Pattern, Sci-Mechanism, Sci-Label, and Sci-Description). In contrast, Tessa only worked in the mathematics sensemaking space (*i.e.* Math-Procedure) during the instruction of $\Delta G_3 = \Delta G_1 + \Delta G_2$ to predict what would happen in the coupling reaction.

During her instruction, Tessa made connections between science sensemaking (*i.e.*, coordinated science sensemaking) and between science and mathematics sensemaking (*i.e.*, blended sensemaking). Two types of science sensemaking were connected in the following combinations: Sci-Description and Sci-Pattern; and between Sci-Pattern and Sci-Mechanism. Additionally, science sensemaking and mathematics sensemaking were connected twice (both Sci-Pattern/Math-Structure).

Description of Maria's instruction

Maria's instruction of mathematical equations related to Gibbs free energy lasted for 62 minutes over two days. During the first day of Gibbs free energy instruction, Maria presented the mathematical equation $\Delta G = \Delta H - T\Delta S$. Maria began with defining variables in the equation (Sci-Label). She then asked students to generate the characteristics of spontaneous reactions. After allowing students to share their ideas with the class, Maria paraphrased student-generated characteristics of spontaneous reactions by connecting them to a driving mechanism for reactions (continuous energy inputs) (Sci-Mechanism) and the relationship between ΔG and the direction of reactions (Sci-Pattern), "♠Spontaneous reactions occur without net input of additional energy ♠ … ♥[and go] in one direction if it's thermodynamically favorable… if this number [ΔG] is large enough, you can't go in the reverse direction♥ (Sci-Mechanism♠, Sci-Pattern♥)."

Maria went through each of the variables in the equation starting with enthalpy (H), naming reactions that had a positive ΔH as endothermic and a negative ΔH as exothermic (Sci-Label). She used the transfer of energy between system (reaction) and surroundings, which she had characterized as the driving mechanism for reactions (Sci-Mechanism), to provide an explanation for the labels (a positive ΔH as endothermic and a negative ΔH as exothermic), explicitly connecting two types of science sensemaking (coordinated science sensemaking, Sci-Label and Sci-Mechanism). Maria continued to provide coordinated science sensemaking as she compared these energy changes in a reaction (Sci-Mechanism) to the relative heat content of products and reactants (Sci-Pattern).

> ... ♣If you have a positive delta H, then the reaction is called endothermic♣. So, that means that ♠the reacting system takes up heat from the surroundings♠. ♣If a reaction has a negative delta H, then the reaction is exothermic♣. So, that means ♠the reaction happens and it releases heat♠. So, ♥the product[s] of the reaction have less heat content than the reactants♥ (Sci-Label♣, Sci-Mechanism♠, Sci-Pattern♥).

Maria moved to the next variable in the equation, entropy (S), and provided the opportunity for Sci-Pattern sensemaking as she went on to compare entropy to the number of molecules in reaction and molecular movement during a phase change, "♥The more molecules that you have♥, the more players you have being disorganized, that means ♥you have more entropy. Another example is when you... have more freedom of [molecular] movement, and so you could have more disorganization♥ (Sci-Pattern♥)." Maria ended her first day of instruction of $\Delta G = \Delta H - T\Delta S$ with Sci-Pattern sensemaking by summarizing the correspondence between ΔG, ΔH, ΔS, and the likelihood of the reaction occurring.

Maria continued to use $\Delta G = \Delta H - T\Delta S$ on the second day of Gibbs free energy instruction. She asked students to address a true-false statement that related enthalpy to terms used to describe Gibbs free energy (*i.e.*, endergonic, non-spontaneous; Sci-Label). After allowing students to share and elaborate their answers, Maria summarized one student's response which connected higher temperatures to whether a reaction would be more exergonic (Sci-Pattern) and reminded the class about the terms used to describe ΔG and ΔH (Sci-Label).

Maria then used an example of melting ice to show a spontaneous situation with a negative ΔG and positive ΔH (Math-Relation) and equated the spontaneity of ice melting to a negative ΔG (Sci-Label), providing opportunity for blended sensemaking. Maria continued to provide blended sensemaking as she connected the positive ΔH to a mechanism of taking heat from surroundings during a phase change (Sci-Mechanism).

> ... it [ice melting] is going to be a spontaneous reaction. So, it's spontaneous, ∅the delta G value's going to be a negative number∅ indicating ♣that spontaneity♣. ∅But the delta H for this reaction is positive six kilojoules per mole∅. That indicates that ♠the system takes up heat from its surroundings to go from solid ice to liquid water♠ (Math-Relation∅, Sci-Label♣, Sci-Mechanism♠).

Maria then switched her focus to entropy (S), another variable in $\Delta G = \Delta H - T\Delta S$. She connected the possible molecular orientations to the increase in entropy (Sci-Pattern) and explained how the increase in entropy affects the spontaneity of reaction (Sci-Mechanism), providing an opportunity for coordinated science sensemaking.

> ♥They [water molecules] have a lot of different possible orientations. That's a lot more disorganization, that's a lot more freedom, so you have much more entropy♥. And that ♠increase in entropy drives the reaction to be spontaneous♠ (Sci-Pattern♥, Sci-Mechanism♠).

Maria provided Math-Procedure sensemaking during instruction of the equation $\Delta G_3 = \Delta G_1 + \Delta G_2$ as she transitioned students from thinking about the Gibbs free energy equation to considering how Gibbs free energy can be summed across a series of equations to get the final ΔG and make predictions about the spontaneity of reaction. She also equated the output of the procedure (a negative ΔG) to the spontaneity of reaction (Sci-Label), providing an opportunity for blended sensemaking.

> If these two [glucose phosphorylation and ATP hydrolysis] are coupled... you now have a reaction that incorporates both of those reactions into one, and ∩you have a new delta G now, which is the sum of those two individual delta Gs. So, delta Gs are additive, meaning you just add them up for reactions that are coupled... When these two reactions are coupled, you get a final delta G that's negative∩, ♣spontaneous♣ (Math-Procedure∩, Sci-Label♣).

Opportunities for different types of science and mathematics sensemaking emerged during instruction of the equation $K_{eq} = \frac{[\text{Products}]_{eq}}{[\text{Reactants}]_{eq}}$. Maria started by defining a quantifiable measure for K_{eq} (Sci-Description) for a reversible chemical reaction, A + B ⇋ C + D, as well as naming K_{eq} as the equilibrium constant (Sci-Label), providing an opportunity for coordinated sensemaking. She stated, "♦The concentration of the products, C and D at equilibrium over the concentration of the reactants at equilibrium gives us the constant K_{eq} $\left[K_{eq} = \frac{[\text{C}]_{eq}[\text{D}]_{eq}}{[\text{A}]_{eq}[\text{B}]_{eq}}\right]$♦, which ♣[$K_{eq}$] is the equilibrium constant♣ (Sci-Description♦, Sci-Label♣)."

Maria continued with Sci-Pattern sensemaking as she asked students about the correspondence between K_{eq}, and concentration of products and reactants, "♥if the K_{eq} is large, are there more products or more reactants at equilibrium?♥ (Sci-Pattern♥)" She agreed with the majority vote of the students that there are more products than reactants and connected Math-Structure to Math-Relation (coordinated mathematics sensemaking) when she expanded on this by pointing out the positions of products as the numerator and reactants as the denominator thereby referring to the structure of $K_{eq} = \frac{[\text{Products}]_{eq}}{[\text{Reactants}]_{eq}}$. She then used this position to determine which species (product or reactant) has the greater concentration for a given value K_{eq}.

> It's basically going back over here to the K_{eq} equation. ∅If you have a really big number [K_{eq}] here∅, that means that ⊗the numerator⊗ ∅is going to be bigger than∅ ⊗the denominator⊗, so that means ∅you're going to have more products at equilibrium∅ (Math-Relation∅, Math-Structure⊗).

Using the same equation, $K_{eq} = \frac{[\text{Products}]_{eq}}{[\text{Reactants}]_{eq}}$, Maria asked the class to work on a quantitative problem: determine whether the reaction of ATP hydrolyzed to ADP and P_i is at equilibrium in the cell for specific K_{eq} and concentrations. Maria reminded students of a mathematical rule, unit cancellation, to explain why K_{eq} does not have units.

> So, K_{eq} technically is dimensionless, ⊃it doesn't have units… because they $\left[\frac{\text{units of [products]}}{\text{units of [reactants]}}\right]$ cancel out during calculation… But sometimes in a calculation, units don't cancel… If they cancel mathematically, remove it, if they don't, stick it in there⊃ (Math-Rule⊃).

Maria brought students back together to go over the problem. She provided an opportunity for coordinated sensemaking between Math-Procedure and Math-Relation sensemaking as she showed students how to perform calculations to find the ratio of products and reactants (Math-Procedure) and related the result to the given value of K_{eq} quantitatively (Math-Relation). Maria then used this ratio to qualitatively characterize the correspondence between ATP and ADP (Sci-Pattern) to generate an answer to this problem, providing an opportunity of blended sensemaking.

> ∩You plug your numbers in, 0.5 millimolar, five millimolar, and five millimolar… And you do the math, and you get 0.5 millimolar, which you can convert to five times ten to the negative four [5×10^{-4}] molar∩ … So, for this is five times ten to the negative four [5×10^{-4}] is describing this reaction, ∅is this close to the K_{eq} value? It is actually very, very, very far from it. This up here is a huge number, this is a much smaller number, so this is far from equilibrium∅ … So, ♥ATP is far higher, and ADP is far lower in living cells than it would be at equilibrium♥ (Math-Procedure∩, Math-Relation∅, Sci-Pattern♥).

Maria then introduced another equation, $\Delta G = \Delta G'^{0} + RT\ln\frac{[\text{Products}]_{eq}}{[\text{Reactants}]_{eq}}$. During instruction of this equation, she only presented Sci-Label sensemaking when she named the variables ΔG as actual free energy change and $\Delta G'^{0}$ as the standard free energy change. In contrast, opportunities for four types of mathematics sensemaking (Math-Procedure, Math-Structure, Math-Relation, Math-Rule) were provided during instruction of the next equation ($\Delta G'^{0} = -RT\ln K_{eq}$). Maria first determined the value of ΔG at equilibrium (Math-Procedure) and used this value when she manipulated the structure of the equation ($\Delta G = \Delta G'^{0} + RT\ln K_{eq}$) to derive a new equation, $\Delta G'^{0} = -RT\ln K_{eq}$ (free energy equation at equilibrium; Math-Structure), providing an opportunity for coordinated mathematics sensemaking.

> At equilibrium, ∩your delta G equals zero. And if you were to plug in zero here [$\Delta G = \Delta G'^{0} + RT\ln K_{eq}$]∩ and ⊗move some constants around, you would derive a new equation, delta G prime naught equals minus RT, natural log of K_{eq} $\left[\Delta G'^{0} = -RT \ln K'_{eq}\right]$⊗ (Math-Procedure∩, Math-Structure⊗).

Maria then stated a quantitative relationship between ΔG and K_{eq}, "∅When K_{eq} is a lot bigger than one, delta G prime naught is large and negative. When K_{eq} is a lot smaller than one, delta G [prime naught] is large and positive∅ (Math-Relation∅)."

Using the same equation, $\Delta G'^{0} = -RT\ln K_{eq}$, Maria asked students to solve a quantitative science problem, "If you've got K_{eq} for the reaction ATP being hydrolyzed to ADP plus P_i, and it's 2.22 times 10 to the fifth [2.22×10^5], what is delta G prime naught for the synthesis of ATP from ADP and P_i at 25 degrees C?" After allowing students to work on this problem, Maria explained step-by-step how to calculate $\Delta G'^{0}$ for the breakdown of ATP (Math-Procedure). This was followed by introducing a mathematical rule (using the opposite sign for the same value of $\Delta G'^{0}$ for the breakdown of ATP) to find the value of $\Delta G'^{0}$ for the reverse reaction (the synthesis of ATP).

Maria focused on science sensemaking and then moved to mathematics sensemaking

Opportunities for different types of science and mathematics sensemaking were provided during Maria's instruction of five equations related to Gibbs free energy. Maria focused on science sensemaking for the first half of the lesson. She then moved to mathematics sensemaking with interspersed opportunities for Sci-Label and Sci-Pattern sensemaking and ended the lesson with mathematics sensemaking (Fig. 3).

Maria's instruction of $\Delta G = \Delta H - T\Delta S$ to determine whether a reaction will proceed was spent in the science sensemaking space (*i.e.*, Sci-Label, Sci-Mechanism, Sci-Pattern), then moved to blended sensemaking (*i.e.*, Math-Relation/Sci-Label, Math-Relation/Sci-Mechanism), followed by an instance of mathematics sensemaking (*i.e.*, Math-Procedure), and ended in the science sensemaking. Whereas with $\Delta G_3 = \Delta G_1 + \Delta G_2$, Maria only presented Math-Procedure and Sci-Label sensemaking. In contrast, when Maria led class discussion to determine whether a reaction is at equilibrium using eqn (4), $K_{eq} = \frac{[\text{Products}]_{eq}}{[\text{Reactants}]_{eq}}$, she started in the science sensemaking space, then moved to mathematics sensemaking, followed by an instance of blended sensemaking (Sci-Pattern/Math-Procedure) and ended with science sensemaking. Maria presented only Sci-Label sensemaking during the instruction of $\Delta G = \Delta G'^{0} + RT\ln\frac{[\text{Products}]_{eq}}{[\text{Reactants}]_{eq}}$, whereas during instruction of $\Delta G'^{0} = -RT\ln K_{eq}$, Maria provided only mathematics sensemaking (*i.e.*, Math-Structure, Math-Relation, Math-Procedure, Math-Rule).

Opportunities for explicit connections between two or more types of sensemaking either within the same dimension or across two dimensions were provided during Maria's instruction. Two or more types of science sensemaking were connected (*i.e.*, coordinated science sensemaking) in the following combinations: Sci-Label and Sci-Description; Sci-Label and Sci-Mechanism; as well as Sci-Pattern and Sci-Mechanism. Also, two or more types of mathematics sensemaking were connected (*i.e.*, coordinated mathematics sensemaking) in the following combinations: Math-Procedure and Math-Rule; Math-Procedure and Math-Relation; Math-Procedure and Math-Structure; as well as Math-Structure and Math-Relation. Additionally, mathematics sensemaking and science sensemaking were connected (*i.e.*, blended sensemaking, Math-Procedure/Sci-Label,

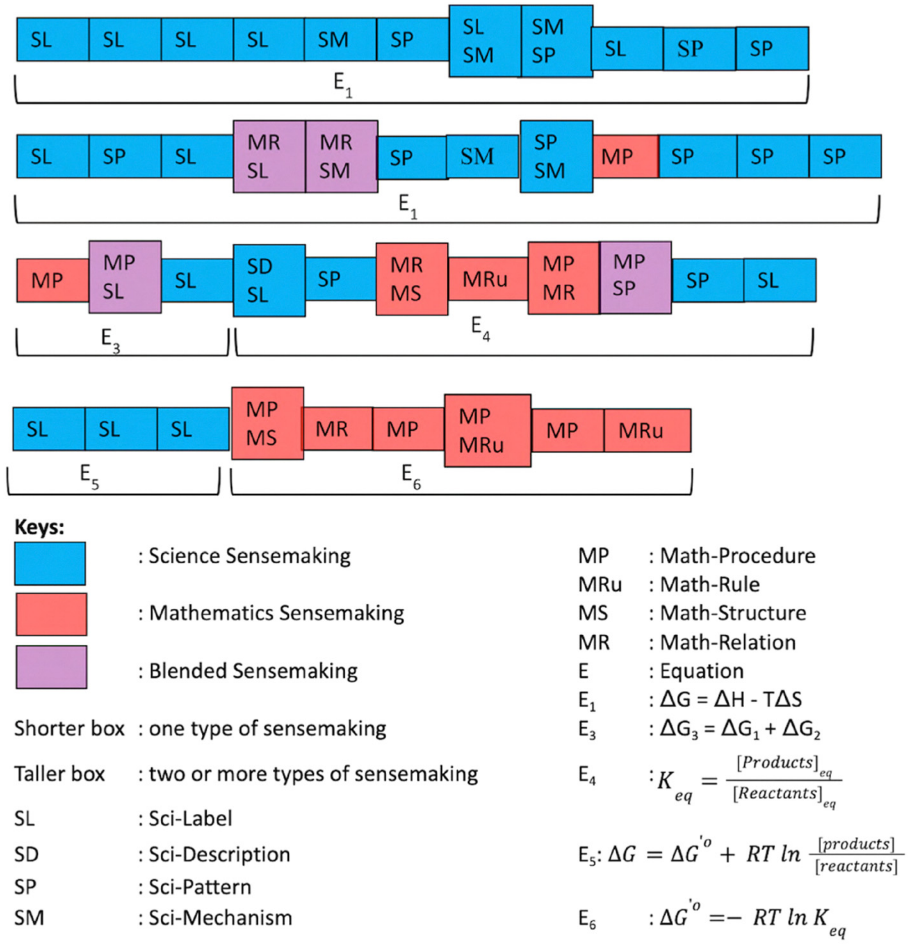

Fig. 3 The organization of sensemaking opportunities identified during Maria's lesson on equations about Gibbs free energy (62 minutes).

Math-Procedure/Sci-Pattern, Math-Relation/Sci-Label, Math-Relation/Sci-Mechanism).

Description of Gary's instruction

Gary's instruction of mathematical equations related to Gibbs free energy lasted for 20 minutes. Gary began by introducing the mathematical equations, $\Delta G'^0 = -nF\Delta E'^0$ and $\Delta G = -nF\Delta E$ to thermodynamically model what happens in a cell. He used $\Delta G'^0 = -nF\Delta E'^0$ to explain the relationship between the change in free energy under standard conditions ($\Delta G'^\circ$) and the variable "n" (Sci-Pattern) and named "n" as the number of electrons (Sci-Label), connecting two types of sensemaking within the science dimension (coordinated science sensemaking). He also labeled other variables in the equation, F as Faraday's constant and $\Delta E'^\circ$ as the change in standard reduction potential (Sci-Label).

> It turns out that ♥the change in free energy [under standard conditions, $\Delta G'^0$], with this movement of electrons, has this relationship. It is related to the negative n♥, where ♣n is the number of electrons in this redox reaction that are getting transferred… F is this thing called Faraday's constant and then, the change in the standard reduction potential♣ (Sci-Pattern♥, Sci-Label♣).

Gary shifted to another equation, $\Delta E = E_{\text{electron acceptor}} - E_{\text{electron donor}}$, and used Math-Procedure sensemaking to explain a step-by-step procedure for calculating the change in reduction potential (ΔE) under standard and nonstandard conditions. He also introduced a rule to find the value of standard reduction potential (E'^0) for the oxidation reaction (Math-Rule), providing an opportunity for coordinated mathematics sensemaking.

> And by convention, ∩when we calculate this delta [E] in the standard reduction potential… The way that we're going to do this is, we're going to take the reduction potential of whatever is accepting the electrons, whatever is getting reduced stays the same and we subtract the reduction potential for the half reaction of whatever is donating the electrons∩. ⊃We basically, effectively, turn around the sign [to get the value of $E'^0_{\text{electron donor}}$ or $E_{\text{electron donor}}$]⊃ (Math-Procedure∩, Math-Rule⊃).

Gary informed students that the calculation of ΔE involved the use of concentrations of compounds in the half reaction. Thus, he presented the Nernst equation, $E = E'^0 + \frac{RT}{nF}\ln\frac{[\text{electron acceptor}]^a}{[\text{electron donor}]^b}$ and exposed students to Math-Relation, Sci-Label, Math-Structure, and Math-Procedure sensemaking. Gary first presented Math-Relation sensemaking as he provided the ratio of concentration of reactant and product.

> … so, we're looking at a reduction half reaction of A going plus some number of electrons going to B. There could be stoichiometry; ∅it's some molar amount of A that gets reduced into some other molar amount [of B] that's not one to one [[A]:[B] ≠ 1:1]∅ (Math-Relation∅).

Next, Gary defined electron acceptor and electron donor (Sci-Label). He also pointed out the position of variables electron acceptor and electron donor in $E = E'^{0} + \frac{RT}{nF}\ln\frac{[\text{electron acceptor}]^{a}}{[\text{electron donor}]^{b}}$, for a chemical reaction with the format, $a\text{A} + ne^{-} \rightleftharpoons b\text{B}$.

> When you're looking at a reduction half reaction, ♣the acceptor is whatever is taking on the electron, to become reduced. A is the electron acceptor here, but the other one is called the 'electron donor', and that is B… it could give up that electron… and get oxidized♣. Just know, ⊗on top is [the concentration of] whatever is accepting and getting reduced and the concentration of whatever takes on the electron is the donor, that's on the bottom $\left[\frac{[\text{electron acceptor}]}{[\text{electron donor}]}\right]$⊗ (Sci-Label♣, Math-Structure⊗).

Gary continued to name other variables, F and T, in $E = E'^{0} + \frac{RT}{nF}\ln\frac{[\text{electron acceptor}]^{a}}{[\text{electron donor}]^{b}}$ (Sci-Label). At the end of the instruction of $E = E'^{0} + \frac{RT}{nF}\ln\frac{[\text{electron acceptor}]^{a}}{[\text{electron donor}]^{b}}$ Gary used Math-Procedure sensemaking to calculate the value of RT/F.

Next, Gary modeled algorithmic procedures to solve a quantitative problem: Calculate ΔG for the reaction, acetaldehyde + NADH + H^{+} $\leftrightharpoons$ ethanol + NAD^{+}, at pH 7 and 298 K. He first demonstrated how to calculate the E value of one-half reaction using $E = E'^{0} + \frac{RT}{nF}\ln\frac{[\text{electron acceptor}]^{a}}{[\text{electron donor}]^{b}}$ (Math-Procedure),

> Here's one of the half reactions that we're dealing with, going to ethanol. ∩If you look this up in the table, this would come to standard half reaction is negative 0.197 volts… And NAD^{+}—again, we're using the half reaction, here. This would come straight out of the standard reduction potential table. This is going to be 0.320 volts∩ (Math-Procedure∩).

Next, Gary determined the ratio of acetaldehyde and NAD^{+} concentrations (Math-Relation), "∅In this case, there is a one-to-one stoichiometry; this is one to one∅ (Math-Relation∅)." Gary continued exposing students to Math-Procedure sensemaking when he calculated the value of ΔE for the two half-reactions using the equation, $\Delta E = E_{\text{electron acceptor}} - E_{\text{electron donor}}$.

Gary then moved on to providing an opportunity for Sci-Pattern sensemaking, as he qualitatively connected the sign of ΔE to ΔG using $\Delta G = -nF\Delta E$.

> … ♥depending on the sign of delta E, you can tell what delta G sign is going to be here♥, because it's a negative N times the Faraday. ♥If delta E is positive, you already know delta G is negative. So, if that was the answer♥, is this spontaneous or not, you have your answer without even having to plug numbers in; ♥you'd know it's negative♥ (Sci-Pattern♥).

Using the same equation, $\Delta G = -nF\Delta E$, Gary named the variable n (Sci-Label) and demonstrated how to calculate ΔG in non-standard conditions (Math-Procedure). Gary ended his problem solving by describing the qualitative relationship between concentration and the directionality of electron flows in the redox reactions (Sci-Pattern).

Gary focused on Math-Procedure sensemaking

Opportunities for different types of science and mathematics sensemaking were provided during Gary's instructions of three equations related to Gibbs free energy. Gary focused on mathematics sensemaking, specifically Math-Procedure, with sporadic introduction of Sci-Label sensemaking and concluded the lesson with Sci-Pattern sensemaking (Fig. 4).

Gary started in the science sensemaking space for $\Delta G = -nF\Delta E$ (*i.e.* Sci-Label and Sci-Pattern), moved to mathematics sensemaking (*i.e.*, Math-Procedure), and ended in the science sensemaking (*i.e.*, Sci-Pattern). In contrast, Gary was in the mathematics sensemaking space (*i.e.* Math-Procedure and Math-Rule) during the instruction of $\Delta E = E_{\text{electron acceptor}} - E_{\text{electron donor}}$. Whereas with $E = E'^{0} + \frac{RT}{nF}\ln\frac{[\text{electron acceptor}]^{a}}{[\text{electron donor}]^{b}}$, Gary started in the mathematics sensemaking space, then moved to science sensemaking space, and ended in the mathematics sensemaking space without connecting any of these types of sensemaking to one another (*i.e.*, Math-Relation to Sci-Label or Math-Structure to Sci-Label, to Math-Procedure).

While no opportunities for blended sensemaking were provided during Gary's instruction, two opportunities for coordinated sensemaking were observed. Sci-Label was connected to Sci-Pattern in the same science sensemaking dimension and

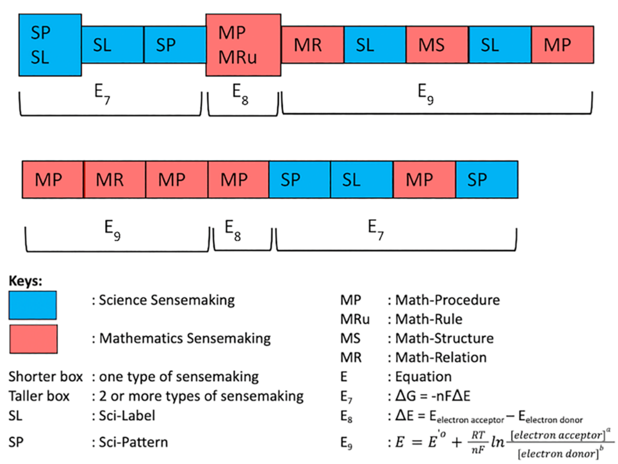

Fig. 4 The organization of sensemaking opportunities identified during Gary's lesson on equations about Gibbs free energy (20 minutes).

Math-Procedure was connected to Math-Rule in the same mathematics sensemaking space.

Sensemaking opportunities across instructors

When we compared the sensemaking opportunities across instructors, six salient features were observed across all instructors: (i) science and mathematics sensemaking opportunities were provided across instructors, (ii) sensemaking foci varied among instructors, (iii) science sensemaking tended to be presented separately from mathematics sensemaking, (iv) instructors occasionally organized their sensemaking opportunities as coordinated, (v) equation types afforded specific opportunities for sensemaking, and (vi) instructors opted for different pedagogical strategies. The following sections explore these key aspects.

Both science and mathematics sensemaking opportunities were provided

Based on the analysis of each instructor's sensemaking opportunities, both science and mathematics sensemaking opportunities were provided by all instructors when they taught mathematical equations associated with Gibbs free energy. Table 4 summarizes the types of sensemaking opportunities provided by each instructor.

Zhao and Schuchardt (2021) indicated that the types within the dimensions of the Sci-Math Sensemaking Framework are ordered theoretically to represent increasing levels of sensemaking from Sci-Label to Sci-Mechanism in the science sensemaking dimension and from Math-Procedure to Math-Concept in the mathematics sensemaking dimension. Drawing on this, we categorize Sci-Label and Sci-Description as lower-level science sensemaking (light blue, Table 4), while Sci-Pattern and Sci-Mechanism as upper-level science sensemaking (dark blue, Table 4). We also categorize Math-Procedure and Math-Rule as lower-level mathematics sensemaking (light orange, Table 4), while Math-Relation, Math-Structure and Math-Concept as upper-level mathematics sensemaking (dark orange, Table 4).

Table 4 shows that all instructors provided lower-level sensemaking (*i.e.*, Sci-Label, Math-Procedure). Opportunities for the other two lower-level sensemaking types (*i.e.*, Sci-Description, Math-Rule) were also provided by certain instructors. Maria and Tessa offered Sci-Description, while Maria and Gary offered Math-Rule. The presence of Sci-Label in all instructors' Gibbs free energy lessons is to be expected since labeling the variables in the equation is the first step typically taken in teaching science involving mathematical equations (Hansson *et al.*, 2015; Schuchardt, 2016; Schuchardt and Schunn, 2016). Math-Procedure is also an important prerequisite to engaging in higher-level mathematics sensemaking (Zhao and Schuchardt, 2021). In this study, instructors tended to focus instruction on using mathematical procedures when performing calculations, which substantiates prior findings on the role of mathematics in science (*e.g.*, Hansson *et al.*, 2015). When mathematics is used solely as a calculational tool for learning science, procedural skills may take priority over conceptual understanding in both disciplines (National Research Council (NRC), 2001; Baldinger *et al.*, 2020).

All instructors also provided upper-level sensemaking (*i.e.*, Sci-Pattern, Math-Structure) in their Gibbs free energy lessons. Gibbs free energy equations inherently emphasizes identifying patterns within the equation, how variations in one variable affect others. Thus, one might expect instruction on Gibbs free energy to focus on sensemaking around scientific patterns and mathematical relationships. While Maria and Gary provided both Sci-Pattern and Math-Relation, Tessa only used Sci-Pattern that may relate to her learning objective for students to develop an understanding of a qualitative relationship between change in free energy and spontaneity of reactions. Additionally, all instructors tended to use Math-Structure when they pointed out the position of specific variables in the equation (the position of entropy, Tessa; the position of reactants and product, Maria; the position of electron donor and electron acceptor, Gary) to support their statement about: scientific pattern (the relationship between a change in entropy and free energy, Tessa) or mathematical relationship (concentration comparisons between reactants and products; Maria). An understanding of mathematical structures serves as a resource to arrange symbols and operations to represent a specific relationship in the phenomenon (Redish and Kuo, 2015; Pospiech, 2019). A larger sample size is needed to explore other contexts that lead to the use of Math-Structure sensemaking.

Opportunities for the highest level of science sensemaking (Sci-Mechanism) were provided by Maria and Tessa in their Gibbs free energy instruction. No instructor exposed students to Math-Concept sensemaking. By providing opportunities for multiple types of sensemaking, instructors exposed students to different resources from the two disciplines that might be important for students to develop their understanding of the scientific phenomenon and mathematical equations (Zhao *et al.*, 2021).

Table 4 Sensemaking opportunities identified in three instructors' lessons on equations in Gibbs free energy

	Science sensemaking				Mathematics sensemaking				
Instructor	Sci-Label	Sci-Description	Sci-Pattern	Sci-Mechanism	Math-Procedure	Math-Rule	Math-Relation	Math-Structure	Math-Concept
Tessa	X	X	X	X	X			X	
Maria	X	X	X	X	X	X	X	X	
Gary	X		X		X	X	X	X	

Keys: : Lower level science sensemaking. : Upper level science sensemaking. : Lower level mathematics sensemaking. : Upper level mathematics sensemaking.

This case study also demonstrates that the instructor provided opportunities for different types of science sensemaking in various sequences, rather than following a unidirectional progression (*e.g.*, from naming to patterning to mechanism). Logically, describing the relationship between aspects of the phenomenon (Sci-Pattern) or explaining scientific processes of why the phenomenon occurs (Sci-Mechanism) requires first being able to identify components within the phenomenon in question (Sci-Label). However, following the introduction of Sci-Label, instructors followed different sequences for science sensemaking, For example, Gary generally moved from a basic level, identifying objects (*e.g.*, naming n as the number of electron; Sci-Label), to the next level in science sensemaking, patterns (*e.g.*, the relationship between n and free energy; Sci-Pattern), and then moved back to Sci-Label. Tessa progressed from Sci-Label (*e.g.*, naming ΔS as a change in entropy), to Sci-Pattern (*e.g.*, the relationship between entropy and free energy), and then to the highest level of science sensemaking, scientific processes (*e.g.*, the effect of temperature on molecular movement; Sci-Mechanism). Whereas Maria transitioned from Sci-Label (*e.g.*, labeling a negative ΔH as exothermic) to scientific processes underpinning the phenomenon (*e.g.*, energy transfer between system and surroundings; Sci-Mechanism) and then moved back to patterns (*e.g.*, differences in heat content between products and reactants). These observations suggest that while instructors took different orders for science sensemaking, they made explicit transitions from lower- to upper-level science sensemaking. Making this transition explicit to students may provide opportunities for them to engage in similar shifts between levels. The effect on students' understanding of different sequences of science sensemaking warrant further investigation in future research.

Sensemaking foci varied among instructors

There were differences in the specific types of sensemaking opportunities that instructors focused on in their lessons. Tessa focused on science sensemaking for most of the lesson (*i.e.*, Sci-Label, Sci-Pattern, Sci-Mechanism) with intermittent use of Math-Structure, and concluded the lesson with Math-Procedure sensemaking (Fig. 2). Maria focused on science sensemaking for the first half of the lesson (*i.e.*, Sci-Label, Sci-Pattern, Sci-Mechanism) and then moved to mathematics sensemaking (*i.e.*, Math-Procedure, Math-Relation) with interspersed opportunities for Sci-Label and Sci-Pattern sensemaking (Fig. 3). Tessa and Maria both started with providing opportunities for science sensemaking before introducing mathematics sensemaking.

In contrast, although Gary also started the lesson with science sensemaking, he focused on mathematics sensemaking, specifically Math-Procedure for most of the lesson, with sporadic introduction of Sci-Label sensemaking and ended the lesson with Sci-Pattern sensemaking (Fig. 4). The difference in Gary's sensemaking focus, compared to that of Tessa and Maria, may be attributable to his specific learning objectives for students. If an instructor's goal is for students to solve problems primarily through calculation, they will theoretically tend to emphasize Sci-Label and Math-Procedure, as seen in Gary's case. When problem solving is confined to Math-Procedure, students tend to memorize the step-by-step procedures demonstrated by the instructor (Li and Schoenfeld, 2019). This approach can hinder their progress as they encounter more complex problems, where such strategy often proves insufficient (Tuminaro and Redish, 2007). Similarly, when instruction is limited to Sci-Label, students lack sufficient opportunities for the highest level of science sensemaking (Sci-Mechanism), which entails explaining the underlying mechanisms to understand how and why the phenomenon occurs (Zhao and Schuchardt, 2021). This mechanistic reasoning is essential for deep understanding of science ideas and more robust explanations (Russ *et al.*, 2008; Illari and Williamson, 2012).

Science and mathematics sensemaking were presented separately

Even though all instructors provided both science and mathematics sensemaking opportunities when teaching Gibbs free energy involving mathematical equations, science sensemaking was mostly presented separately from mathematics sensemaking (Fig. 2–4). Instructors' efforts to connect these two sensemaking dimensions were evident as some instructors provided opportunities for blended sensemaking during instruction (purple lines, Fig. 5).

Kaldaras and Wieman (2023) introduced a hierarchical continuum of blended sensemaking, where the lowest level emerges from pairing Math-Procedure with any science sensemaking type (algorithmic), and the highest level arises from blending Math-Concept with any science sensemaking type. However, our study was conducted in the context of instructors teaching the same chemical phenomenon. Our findings showed that Tessa relied on science sensemaking, Maria provided science and mathematics sensemaking equally, and Gary tended to use more mathematics sensemaking. Notably, only a few instances of blended sensemaking emerged in this study. Thus, we proposed an alternative hierarchical structure of blended sensemaking to better reflect our results, while still drawing on Kaldaras and Wieman's (2023) idea of hierarchy. We proposed three levels of blended sensemaking: lower, transitional, and upper. The lower-level blended sensemaking emerges from pairing Sci-Label or Sci-Description with any lower-level mathematics sensemaking (*i.e.*, Math-Procedure, Math-Rule; purple line, dotted gray box, Fig. 5b). Whereas the upper-level involves blending Sci-Pattern or Sci-Mechanism with any higher-level mathematics sensemaking (*i.e.*, Math-Relation, Math-Structure, Math-Concept; purple line, solid gray box, Fig. 5a and b). The transitional blended sensemaking occurs when cross-dimensional connections emerge between any lower-level and upper-level sensemaking types.

In this case study, Maria had one instance of lower-level blended sensemaking, two instances of transitional blended sensemaking, and one instance of upper-level blended sensemaking. The lowest level of blended sensemaking was exemplified by Maria's blending of Sci–Label with Math–Procedure (purple line, dotted gray box, Fig. 5b) as she equated the output

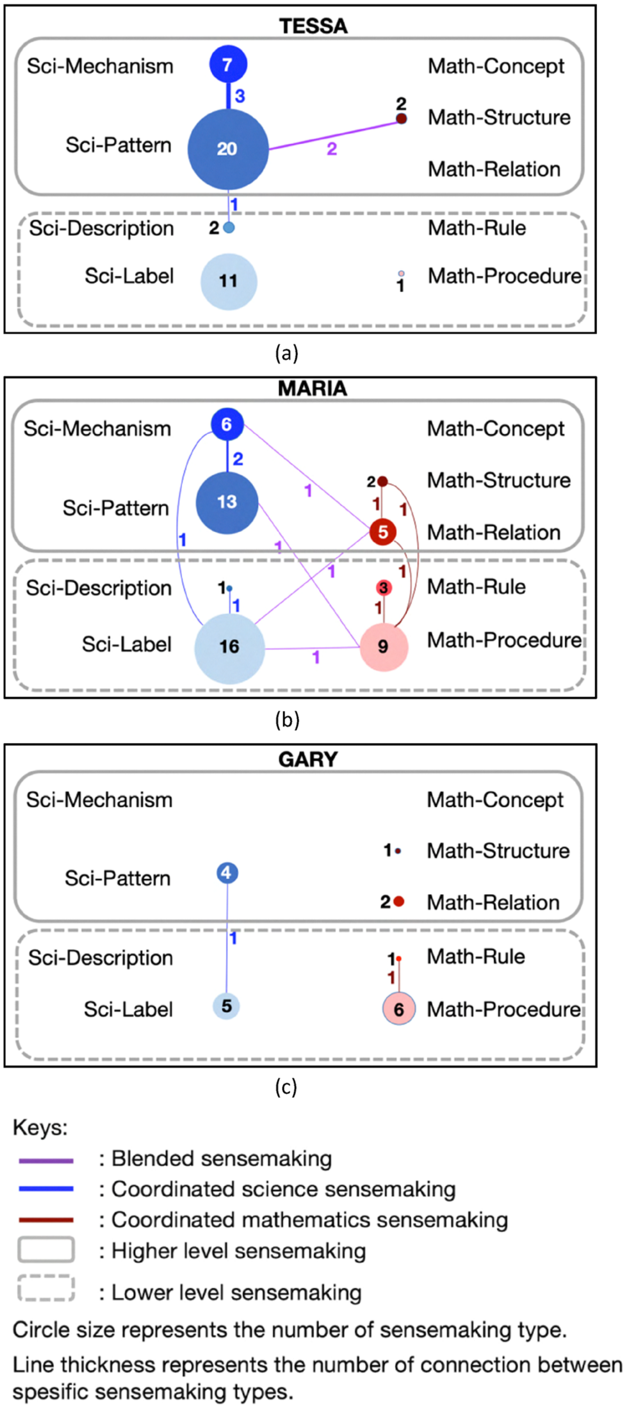

Fig. 5 Organizational structure of sensemaking types as blended and coordinated observed in: (a) Tessa's instruction, (b) Maria's instruction, and (c) Gary's instruction.

of the procedure (a negative ΔG) to the spontaneity of reaction (Sci-Label). This concrete evidence of the foundational level fills a gap in Kaldaras and Wieman's (2023) work, which had validated all other levels of blended sensemaking through student interviews involving PhET simulations, except the lowest level (*i.e.*, Sci–Description (combines Sci-Label and Sci–Description) with Math–Procedure (or Math–Rule)). Although the least sophisticated, this level is a critical prerequisite for developing more advanced blended sensemaking (Kaldaras and Wieman, 2023).

Transitional blended sensemaking was observed when Maria established a connection between Math-Procedure and Sci-Pattern by using her calculational result—the ratio of products and reactants—to qualitatively compare two biochemical compounds, ATP and ADP (purple line, Fig. 5b). Another example of transitional blended sensemaking was illustrated by Maria's blending of Sci-Label and Math-Relation. Additionally, upper-level blended sensemaking between Sci-Mechanism and Math-Relation was observed as Maria translated quantitative relationships among variables (*e.g.*, a negative ΔG and positive ΔH) into explanations of the underlying mechanism (*e.g.*, the system absorbs heat from its surroundings; purple line, solid gray box, Fig. 5b). This blended sensemaking occurred during key moments of instruction to promote understanding of the scientific ideas represented in the equation.

In contrast, Tessa had two instances of upper-level blended sensemaking connecting (purple line, solid gray box, Fig. 5a). For example, Tessa made explicit connection between Sci–Pattern and Math-Structure that may help students recognize patterns (*e.g.*, an increase in entropy leads to a negative ΔG) by drawing attention to the equation's structure (*e.g.*, the placement of ΔS after the minus sign). Explicitly addressing the links between science and mathematics during instruction may make these connections available to all students and thus, provides an opportunity to improve students' ability to solve more complex quantitative problems in science (Schuchardt, 2016; Schuchardt and Schunn, 2016; Eichenlaub and Redish, 2018).

While our prior work also documented instances of blended sensemaking between Sci-Pattern and Math-Structure, and between Sci-Mechanism and Math-Relation (Zhao *et al.*, 2021), we did not investigate various levels of blended sensemaking. Thus, this case study extends this line of research by not only specifying the types of science and mathematics sensemaking being blended in the context of chemistry instruction, but also by describing the levels of blended sensemaking provided by instructors during Gibbs free energy lessons (purple lines, Fig. 5).

It is also worth noting that in this case study, no clear progression was observed along the hierarchy of blended sensemaking in either instructor's lesson. For example, lower-level blended sensemaking (*i.e.*, Sci–Label/Math–Procedure) occurred after Maria had already demonstrated a more advanced form (*i.e.*, Sci–Mechanism/Math–Relation). In another case, transitional and upper-level blended sensemaking (*i.e.*, Sci–Label/Math–Relation and Sci–Mechanism/Math–Relation, respectively) occurred within a single statement, bypassing Sci–Pattern/Math–Relation, again deviating from the intended sequence. These observations suggest that instructors moved fluidly back and forth between levels for specific purposes, highlighting the need for further research to understand the lack of a distinct progression.

Unlike previous work on instructors' sensemaking opportunities during population growth instruction (Zhao *et al.*, 2021), explicit connections between scientific processes (Sci-Mechanism) and mathematical structure (Math-Structure) were absent from these instructors' Gibbs free energy lesson. This absence may be due to the nature of the equations and the phenomenon. ΔG, while useful in discussing ideas in chemistry, is an intangible entity. The equation, $\Delta G = \Delta H - T\Delta S$, is not a model of scientific processes but a representation of the definition of ΔG as the net balance of entropy and enthalpy. Thus, ideas about mechanisms are not explicitly represented in the equation, yet both Tessa and Maria incorporated Sci-Mechanism into their instruction to explain factors affecting the spontaneity of reaction. In contrast, equations associated with population growth generally include mechanisms involved in population growth (*e.g.*, reproduction or death). This absence of linking Sci-Mechanism to Math-Structure could also be due to expert blindness where that connection is obvious to the instructors, but there is a missed opportunity to help the students make that connection (Nehm and Ridgway, 2011).

Instructors occasionally organized their sensemaking opportunities as coordinated sensemaking

While only two instructors provided blended sensemaking opportunities, all instructors presented two or more types of sensemaking within the same dimension, where one type of sensemaking used to support or explain the other type of sensemaking (coordinated sensemaking; Zhao *et al.*, 2021). These coordinated sensemaking opportunities occurred in various combinations and different levels (science, blue lines; mathematics, red lines; Fig. 5).

Coordinated science sensemaking. Sensemaking in science has been defined during studies of student discussions as generating an explanation about the phenomenon by connecting different scientific ideas (Odden and Russ, 2019; Odden, 2021). This idea has been extended to science sensemaking with the definition of coordinated science sensemaking as connections between different types of science sensemaking (Zhao *et al.*, 2021). While prior research found coordinated sensemaking primarily involving Sci-Label and Sci-Pattern (Zhao *et al.*, 2021), instructors in this study varied in the extent to which they provided coordinated science sensemaking, as well as in the types and levels of science sensemaking that were connected.

Similar to blended sensemaking, we propose three levels of coordinated science sensemaking: lower, transitional, and upper. The lower-level coordinated science sensemaking emerges from pairing Sci-Label with Sci-Description (blue line, dotted gray box, Fig. 5b). Whereas the upper-level involves connecting Sci-Pattern with Sci-Mechanism (blue lines, solid gray box, Fig. 5a and b). The transitional coordinated science sensemaking occurs when connections emerge between any lower-level and upper-level science sensemaking types.

These levels of coordinated science sensemaking distinctly reflect the development of a more comprehensive explanation of the phenomenon. In this case study, Maria had one instance of lower-level coordinated science sensemaking, one instance of transitional coordinated science sensemaking, and two instances of upper-level coordinated science sensemaking (blue lines, Fig. 5b). A lower-level coordinated science sensemaking was observed when Maria established a connection between Sci-Label and Sci-Description by providing a quantifiable measure for a variable or property of a scientific phenomenon (*e.g.*, K_{eq} described as the ratio between the amounts of products and reactants in a reversible chemical reaction at equilibrium; Sci-Description) while naming that variable or property (*e.g.*, K_{eq} as the equilibrium constant; Sci-Label; blue line, dotted gray box, Fig. 5b). Making the connection between Sci-Label and Sci-Description explicit may help students build a robust definition of the variable by naming it in terms of aspects or processes of a scientific phenomenon, as well as using an equation to provide a quantifiable measure. This coordinated science sensemaking may serve to make the definitions of the variables more meaningful for students, and thus more memorable. This lower level coordinated science sensemaking may also be important for instructors to scaffold or support higher levels.

Maria demonstrated transitional coordinated science sensemaking by linking Sci-Label (*i.e.*, a positive ΔH as endothermic) to Sci-Mechanism (*i.e.*, endothermic indicates that the system absorbs heat from the surroundings; blue line, Fig. 5b). When instructors connect Sci-Label to Sci-Mechanism, they may help their students understand what parts of the phenomenon are involved in a chemical process (mechanism). Additionally, upper-level coordinated science sensemaking between Sci-Pattern and Sci-Mechanism was observed in Maria's instruction. For example, she linked the increase in entropy to molecular orientations (Sci-Pattern) and explained how this increase affects reaction spontaneity (Sci-Mechanism; blue line, solid gray box, Fig. 5b).

In contrast, Gary had only one instance of transitional coordinated science sensemaking as he explained the relationship between variables in an equation (*i.e.*, the change in free energy relates to negative n; Sci-Pattern) while defining the variable (*i.e.*, n as the number of electrons; Sci-Label; blue line, Fig. 5c). When instructors connect Sci-Label to Sci-Pattern sensemaking, they may help their students understand which parts of the phenomenon are being compared. Sci-Label/Sci-Pattern connections may also be fundamental in helping students transition to higher level coordinated science sensemaking.

Tessa had one instance of transitional coordinated science sensemaking and three instances of upper-level coordinated science sensemaking (blue lines, Fig. 5a). Transitional coordinated science sensemaking was exemplified by Tessa's connection between Sci–Description and Sci-Pattern (blue line, Fig. 5a) as she related the free energy (G) of reactants and products to both entropy and chemical potential energy (Sci-Pattern) and used the difference between these two Gs values to provide a quantifiable measure of ΔG (Sci-Description).

Tessa also used upper-level of coordinated science sensemaking (*i.e.*, Sci-Pattern/Sci-Mechanism), However, this was absent in Gary's instruction. Omitting the connection between Sci-Pattern and Sci-Mechanism may limit students' ability to fully grasp the phenomenon. Whereas explicitly modeling this coordinated science sensemaking may help them connect the chemical process that is responsible for producing the observed pattern in the phenomenon. This level is likely results in more robust explanations (Odden and Russ, 2019; Odden, 2021), thereby deepening students' understanding of the phenomenon.

The differences in the instances and levels of coordinated science sensemaking among instructors may be due to the number of science sensemaking types they include in their Gibbs free energy instruction. While both Tessa and Maria incorporated all types of science sensemaking in their instruction (Table 4), Gary only sporadically introduced Sci-Label and Sci-Pattern sensemaking and never exposed students to the Sci-Mechanism sensemaking during his instruction. As a result, Maria and Tessa employed multiple combinations of coordinated science sensemaking (*i.e.*, Sci-Label/Sci-Description, Sci-Label/Sci-Mechanism, Sci-Description/Sci-Pattern, Sci-Pattern/Sci-Mechanism; blue lines, Fig. 5a and b), while Gary provided only one (*i.e.*, Sci-Label/Sci-Pattern; blue line, Fig. 5c). Additionally, this characterization of coordinated science sensemaking levels—from basic (Sci-Label/Sci-Description), through transitional, to advanced (Sci-Pattern/Sci-Mechanism)---extends prior work on instructors' sensemaking opportunities in population growth instruction (Zhao *et al.*, 2021). That work has solely shown that coordinated science sensemaking can occur in biology lessons involving mathematics (Zhao *et al.*, 2021).

Coordinated mathematics sensemaking. Similar to coordinated science sensemaking, we propose three levels of coordinated mathematics sensemaking: lower, transitional, and upper. The lower-level coordinated mathematics sensemaking emerges from pairing Math-Procedure with Math-Rule (red lines, dotted gray box, Fig. 5b and c). Whereas the upper level involves connections among higher-level mathematics sensemaking types (*i.e.*, Math-Relation, Math-Structure, Math-Concept; red lines, solid gray box, Fig. 5b). Transitional coordinated mathematics sensemaking occurs when connections emerge between any lower-level and upper-level mathematics sensemaking types.

These levels of coordinated mathematics sensemaking characterize distinct approaches to working with equations. In this case study, Gary had one instance of lower-level coordinated mathematics sensemaking. In contrast, Maria had one instance of lower-level coordinated mathematics sensemaking, two instances of transitional coordinated mathematics sensemaking, and one instance of upper-level coordinated mathematics sensemaking (red lines, Fig. 5b). Lower-level coordinated mathematics sensemaking was observed when Maria and Gary occasionally brought up mathematical rules when carrying out calculations. These rules (*e.g.*, using the opposite sign for the same $\Delta G'^0$ value in the breakdown reaction, Math-Rule) were used to support the execution of specific steps in the calculation procedures (*e.g.*, calculating $\Delta G'^0$ for the synthesis reaction). Modeling this coordinated mathematics sensemaking appears as one way to work with equations during problem solving—filling in values, applying mathematical rules, and calculating outputs—with the learning outcome likely being procedural fluency.

Maria demonstrated transitional coordinated mathematics sensemaking (*i.e.*, Math-Procedure/Math-Relation, Math-Procedure/Math-Structure; red lines, Fig. 5b). For example, she employed Math-Procedure (*e.g.*, filling the value of ΔG at equilibrium) to support her manipulation of the equation structure (*e.g.*, deriving $\Delta G'^0 = -RT\ln K_{eq}$, from $\Delta G = \Delta G'^0 + RT\ln K_{eq}$; Math-Structure). Modeling this combination, used in the context of deriving a new equation, could support students in engaging more flexibly with problems that require rearranging variables.

Maria also used upper-level coordinated mathematics sensemaking by combining Math-Relation and Math-Structure (red line, solid gray box, Fig. 5b), a connection not observed in prior research (Zhao *et al.*, 2021). For example, Maria used the positions of variables in the equation (*e.g.*, products as the numerator and reactants as the denominator in $K_{eq} = \frac{[\text{Products}]_{eq}}{[\text{Reactants}]_{eq}}$; Math-Structure) to make quantitative comparisons (*e.g.*, determining which species (product or reactant) has the greater concentration for a given value K_{eq}; Math-Relation). This coordinated mathematics sensemaking was used to solve a more complex problem that goes beyond straightforward substitution and calculation, potentially helping students progress from procedural fluency to conceptual understanding.

This case study extends prior work on coordinated mathematics sensemaking (Zhao *et al.*, 2021) by identifying varied levels of coordinated mathematics sensemaking that may reflect different ways of working with equations: filling in values and calculating an output, solving a more complex problem, or deriving a new equation. The findings also underscore the need for future research to examine whether similar combinations occur during problem-solving and equation derivation in other contexts, as well as to explore other roles of coordinated mathematics sensemaking in different science topics.

Notably, none of the instructors in this case study or the prior case study made connections between Math-Procedure and Math-Concept (Zhao *et al.*, 2021). High school and undergraduate mathematics instruction generally focuses on procedural problem solving as opposed to conceptual understanding. Thus, many science instructors may not have the requisite background to make connections to mathematical concepts. Additionally, they are teaching science courses where the objectives focus on teaching science concepts. Given the context, connecting to mathematical concepts may be overlooked, particularly given the time constraints for covering the required material. However, providing opportunities of coordinated mathematics sensemaking between Math-Procedure and Math-Concept may help students understand not just how to perform calculations, but also why those calculations work. How to provide appropriate support for science instructors to foster connections between Math-Concept sensemaking and other types of mathematics sensemaking is thus an important area for future investigations.

Equation types afforded distinct opportunities for sensemaking

Tessa and Maria both taught the same equation (1 and 3; Table 5). Eqn (1), $\Delta G = \Delta H - T\Delta S$, reflects both mathematical and scientific ideas. This equation type affords multiple opportunities for sensemaking (*e.g.*, Sci-Label, Sci-Pattern, Sci-Mechanism), as well as facilitating connections between sensemaking types (*e.g.*, Sci-Mechanisms/Math-Relation, Sci-Pattern/Math-Structure, Sci-Pattern/Sci-Mechanism). When they switched to a more "basic" equation, $\Delta G_3 = \Delta G_1 + \Delta G_2$, their sensemaking levels also decreased (*i.e.*, Math-Procedure) because this equation has minimal conceptual underpinning compared to $\Delta G = \Delta H - T\Delta S$. The use of the same equations might explain why the sensemaking opportunities provided in the first half of Maria's lesson have more in common with Tessa's.

Maria included additional equations (Table 5). These equations (4 and 6, Table 5) had increased levels of complexity. Eqn (4) affords multiple sensemaking opportunities (3 science and 4 mathematics sensemaking types) and facilitates connections within each dimension (*i.e.*, Sci-Label/Sci-Description, Math-Procedure/Math-Relation, Math-Structure/Math-Relation). The types and levels of mathematical sensemaking opportunities Maria provided during instruction of eqn (6) increased, incorporating four types, including upper-level Math-Relation and Math-Structure. Theoretically, eqn (6) may invoke the mathematical concept of natural logarithm (ln). However, such conceptual engagement with the natural logarithm was not evident in Maria's instruction. The decision to exclude the mathematical concept during instruction of this equation may relate to the instructor's learning objectives, instructional focus, or adequacy of requisite background, which warrants further investigation.

In contrast, Gary used equations (7–9, Table 5) that were less connected to science ideas. Eqn (7) and (8) primarily promoted lower level sensemaking, while eqn (9) included greater mathematical complexity, similar to Maria's eqn (6). This might help explain Gary's greater focus on mathematics than science sensemaking. The similar form of eqn (6) and (9), along with their weaker connections to science ideas might explain why the sensemaking opportunities in the second half of Maria's lesson appeared more similar to Gary's overall pattern.

Instructors opted for different pedagogical strategies

We acknowledge that instructors employed different pedagogical strategies to teach the Gibbs free energy topic, including lectures, small-group discussions, whole-class discussions, individual problem solving, questioning, and making explicit connections between science and mathematics. These strategies shape what students do in the classroom (learning activities). Learning activities can promote different modes of engagement (*e.g.*, passive, active), which in turn may lead to varying levels of learning outcomes (*e.g.*, recall, application, transfer; Chi and Wylie, 2014). We assume that a similar pattern holds in the context of sensemaking: instructional strategies lead to different modes of engagement, which may support students in processing and making sense of new information or problems, and we refer to this as a "sensemaking opportunity."

Maria and Tessa provided more opportunities for active modes of engagement. They employed pedagogical strategies such as questioning, small-group discussions, whole-class discussions, and making explicit connections between science and mathematics, which in turn created a stronger opportunity for sensemaking. For example, Maria employed Socratic questioning (*i.e.*, asking individual students about the characteristics of spontaneous reactions), which in turn created more opportunities for science sensemaking (*i.e.*, Sci-Pattern, Sci-Mechanism). This was evident in the way she responded to students' contributions by paraphrasing their ideas, reflecting both scientific patterns (*e.g.*, the relationship between ΔG and reaction direction) and processes (*e.g.*, the need for continuous energy input). Tessa also fostered students' engagement in sensemaking by making explicit connections between science and mathematics while elaborating on the presenting group's response. By showing students how to link concepts across the two disciplines, she provided stronger opportunities for engagement in blended sensemaking, thereby supporting gains in students' basic understanding and problem solving skills (Schuchardt, 2016; Schuchardt and Schunn, 2016; Eichenlaub and Redish, 2018).

In contrast, Gary rarely invited students' contributions and promoted a passive mode of engagement, as students primarily observed how to solve problems algorithmically on the board. While passive engagement may still allow for sensemaking—if students are covertly processing the material (Chi and Wylie, 2014)—the opportunity is generally limited to Math-Procedure and Sci-Label, which may hinder students from developing a clear understanding of the phenomenon or their ability to solve a more complex problem (Tuminaro and Redish, 2007). Even when sensemaking opportunities were provided in Gary's lessons, they were passive and relied on the individual student to make connections and actively process the sensemaking.

Overall, instructors created instructional moments through their pedagogical strategies (*e.g.*, work on a problem individually or in groups, whole-class discussions, questioning, explicit connection across disciplines, lectures) that supported students in processing and making sense of information by drawing on knowledge and skills acquired from past experiences (*i.e.*, sensemaking). Such moments were considered as opportunities for sensemaking, and these opportunities may be strengthened through instructors' pedagogical choices.

Additionally, the way instructors structured their questions was observed to promote particular types of sensemaking. For

Table 5 Mathematical equations used by instructors

Equation label	Equation	Tessa	Maria	Gary
Eqn (1)	$\Delta G = \Delta H - T\Delta S$	X	X	
Eqn (2)	$\Delta G = G_{\text{products}} - G_{\text{reactants}}$	X		
Eqn (3)	$\Delta G_3 = \Delta G_1 + \Delta G_2$	X	X	
Eqn (4)	$K_{\text{eq}} = \frac{[\text{Products}]_{\text{eq}}}{[\text{Reactants}]_{\text{eq}}}$		X	
Eqn (5)	$\Delta G = \Delta G'^0 + RT\ln\frac{[\text{Products}]_{\text{eq}}}{[\text{Reactants}]_{\text{eq}}}$		X	
Eqn (6)	$\Delta G'^0 = -RT\ln K_{\text{eq}}$		X	
Eqn (7)	$\Delta G = -nF\Delta E$			X
Eqn (8)	$\Delta E = E_{\text{electron acceptor}} - E_{\text{electron donor}}$			X
Eqn (9)	$E = E'^0 + \frac{RT}{nF}\ln\frac{[\text{electron acceptor}]^a}{[\text{electron donor}]^b}$			X

Note. The equations used by each instructor are marked with an X.

example, "*how*" questions (*i.e.*, how gradient concentration corresponds to variables in the equation $\Delta G = \Delta H - T\Delta S$; Tessa's instruction) implicitly required students to reason about how a particular form of equation is applicable in a scientific context based on the observed patterns. Thus, "*how*' questions promoted Sci-Pattern and Math-Structure sensemaking.

Both Maria and Gary presented quantitative problems on the board, but the problems differed in nature, fostering distinct sensemaking opportunities. Maria asked more conceptual questions (*i.e.*, determine whether the reaction is at a specific condition in the cell), while Gary's were more procedural (*i.e.*, calculating the ΔG value for the reaction). Maria's questions promoted Math-Procedure, Math-Relation, and Sci-Pattern, whereas Gary's emphasized Math-Procedure and Math-Rule. These observations show that the structure of the prompts plays a crucial role in guiding student thinking and supporting sensemaking.

Conclusions and implications

This study shows different instructors expose students to different types of sensemaking and have distinct ways of sequencing their sensemaking even when teaching the same scientific phenomenon using mathematical equations. Describing the types of sensemaking included in a lesson and tracking the sequence of sensemaking that instructors provided can offer information about different ways to approach the Gibbs free energy topic and has the potential to make instructors aware of whether the instructional opportunities provided for sensemaking match their learning objectives for students.

Prior work has shown that blended and coordinated sensemaking could occur in a biology setting (Zhao *et al.*, 2021). This case study extends prior research by specifying the types and levels of science and mathematics sensemaking that are being blended or coordinated. Instructors presented few instances of blended sensemaking which has been associated with increased ability to solve novel or more complex quantitative problems and better understanding of the phenomenon (Kuo *et al.*, 2013; Schuchardt and Schunn, 2016; Bain *et al.*, 2018). When the instructors exposed students to blended sensemaking, these instructors showed students how to use resources from two different disciplines. Coordinated sensemaking across different levels, which has been hypothesized to lead to better understanding of the phenomenon (Odden, 2021), was also rare. Combined, these studies suggest that instructors may benefit from professional development or curriculum that facilitates presentation of blended or coordinated sensemaking. Model-based instruction may foster connections between science and mathematics (Hestenes, 2010; Schuchardt and Roehrig, 2024) and increase conceptual understanding and qualitative problem solving (Schuchardt and Schunn, 2016). Thus, a workshop that provides instructors the opportunity to engage with a mathematical modeling curriculum as students could serve as a meaningful form of professional development. Instructors will leave with hands-on experience in using and connecting their scientific and mathematical resources during mathematical modeling activities. They will also engage in in-depth discussion about features of the curriculum that promote modeling and how mathematical modeling could be adapted to their classrooms.

Similar to prior work on instructors' sensemaking opportunities (Zhao *et al.*, 2021), we also recognize that these instructors opted for different pedagogical strategies when teaching the same chemistry topic that incorporates mathematical equations. Analysis of these strategies, presented in detail in this paper, suggests that sensemaking opportunities may be strengthened through deliberate pedagogical choices. Examining both instructors' pedagogical choices and the types of sensemaking and connections they foster can provide deeper insights than focusing solely on how instructors teach. Extending such analyses across multiple chemistry topics and disciplines may help clarify why students struggle with mathematics in science and inform strategies to improve instruction. This case study also showed that instructors used various types of mathematical equations, which appeared to create opportunities for particular types of sensemaking.

Limitation and future directions

While this case study revealed differences in the types and sequences of sensemaking opportunities instructors chose to incorporate in their instructions, it only examined sensemaking opportunities provided by three instructors teaching the same chemical phenomenon. Expanding this research to a larger sample size and additional lessons on other mathematical equations in science at the college and K-12 levels will allow for general principles to be abstracted on what factors facilitate exposure to blended and coordinated sensemaking. This information will allow for development of resources that help instructors align the types of sensemaking they present to their students with both their learning objectives and the development of quantitative skills and conceptual understanding at both the K-12 and college levels where engage in mathematical thinking has been identified as a core skill (AAAS, 2009; NRC, 2012). In addition, this study only shows what sensemaking the instructors modeled, but it does not provide information about what the students gained from these lectures. Therefore, future research needs to examine the effect of sensemaking opportunities provided by the instructors on students' learning outcomes.

Additionally, while we acknowledge differences in the focus of sensemaking opportunities provided by our instructors, specifically that our male instructor emphasized mathematics sensemaking over science ones, we cannot attribute these differences to gender due to the small sample size. Future research with a larger sample is needed to explore the association between instructor characteristics and sensemaking opportunities, as well as how these factors influence student learning.

Author contributions

All authors contributed to the conceptualization and design of the study. AS conducted the data collection. DD led the data analysis and drafted the manuscript, with support from the other authors. GR and AS contributed to refining the research objectives and enriching the discussions. All authors contributed to the interpretation of the findings and critically reviewed the manuscript for accuracy and scientific rigor. All authors read and approved the final manuscript.

Conflicts of interest

All authors declare no competing interests. This study was conducted with approval from the Institutional Review Board (Protocol FWA00000312) of the University of Minnesota, and all instructors consented to participate in this study.

Data availability

All relevant data generated or analyzed during this case study are included in the published article. However, the raw data supporting the findings are not publicly available due to data privacy regulations protecting the personal information of participants. Anonymized data may be made available by the corresponding author upon reasonable request. Access will be granted only after signing a data access agreement that outlines permitted uses and ensures participant confidentiality.

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