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PAPER

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DOI: 10.1039/D5NJ04298DFrom Atomic Structure to Functional Properties: Orthorhombic Disphenoidal NaAlBr₄ as a Solid-State ElectrolyteReceived 00th January 20xx,
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Yohandys A. Zulueta,^a Narciso Antonio Villar Goris,^b J. R. Fernandez-Gamboa,^c Duc Toan Truong,^{d,e} Chi M. Phan^f and Minh Tho Nguyen^{d,e,*}

This paper presents a comprehensive computational investigation of the orthorhombic $P2_12_12_1$ phase of NaAlBr₄ as a halide-based solid-state electrolyte for sodium-ion batteries. Symmetry lowering from the conventional $Pnma$ phase generates one-dimensional Na⁺ conduction ribbons along the b-axis, enabling highly directional transport. Thermodynamic stability, confirmed through computed decomposition energies, global instability index values, and convex-hull analysis, demonstrates its strong bonding, resistance to elemental breakdown and intrinsic synthesizability. Mechanical analysis reveals moderate elastic anisotropy and high compressibility, supporting interface compatibility. Electronic structure calculations show a wide band gap (4.3–4.7 eV) which corresponds to an electrochemical stability window of 0–4.5 V, ensuring an insulating behaviour and a compatibility with Na metal anodes and high-voltage cathodes. Defect energetics identify NaBr Schottky and Na⁺ Frenkel defects as the most favourable intrinsic configurations, while Li⁺ substitution and divalent doping with Zn²⁺ and Mg²⁺ at Na⁺ sites further promote vacancy formation. Transport analysis, evaluated by bond valence computations, yields exceptionally low migration barriers (~0.10 eV) and room-temperature conductivities up to 0.11 S/cm. Although the orthorhombic $P2_12_12_1$ phase of NaAlBr₄ has not been observed experimentally yet, these results predict the $P2_12_12_1$ phase of NaAlBr₄ as a symmetry-enabled, defect-tunable halide electrolyte, advancing design strategies for next-generation sodium-ion batteries.

1. Introduction

The accelerating demand for safer, high energy density batteries implemented in electric vehicles, grid storage and portable electronics has intensified the search for advanced solid-state electrolytes (SSEs).^{1–10} Conventional liquid electrolytes, while being widely adopted, suffer from flammability, leakage and dendrite formation, posing serious safety risks and limiting long-term stability. Solid-state batteries (SSBs) address these challenges by replacing flammable liquids with inorganic SSEs, thereby enabling the use of alkali metal anodes and high-voltage cathodes.^{1–10} Of the leading SSE families—oxides, sulfides, polymers, and halide-based electrolytes have recently emerged as a compelling class due to their wide electrochemical stability windows, intrinsic compatibility with high-voltage cathodes, and moderate mechanical softness that facilitates defect

formation and ion migration.^{3–14} Bromide-based SSEs in particular offer enhanced polarizability and lattice flexibility as compared to their chloride analogues that can lower migration barriers and improve Na⁺ conductivity.^{14–18} These attributes position bromide derivatives as effective candidates for next generation metal ion batteries, especially when paired with structural tuning and defect engineering strategies.

Of bromide-based SSEs, the NaAlBr₄ (space group $Pnma$) has recently garnered attention due to its moderate Na⁺ ion conductivity (~1.2 × 10^{−5} S/cm at 30°C) and low activation energy (~0.43 eV), despite its exceptionally soft mechanical profile.¹⁶ Na₃YBr₆, another bromide-based SSE showing a low activation energy for Na⁺ migration (~0.15 eV) is attributed to the vibrational flexibility of Br[−] anions that expand migration channels.¹⁴ However, its mechanical softness still necessitates mitigation strategies such as grain boundary engineering and surface coatings to prevent fracture and enhance interfacial contact. These design principles are increasingly relevant for halide SSEs intended for separator roles in all-solid-state configurations.

Li₃InCl₆, a halide SSE used for lithium-ion systems, shares comparable outstanding properties.¹⁹ Despite its fragility, it was successfully integrated into composite cathodes using slurry-coating techniques and polymer binders, which improve mechanical cohesion and electrochemical performance. Its stability in ambient air and compatibility with high-voltage cathodes further demonstrate that mechanically soft halides can play as viable

^a Departamento de Física, Facultad de Ciencias Naturales y Exactas, Universidad de Oriente, CP 90500, Santiago de Cuba, Cuba

^b Autonomous University of Santo Domingo (UASD), Dominican Republic

^c Laboratório de processamento de Materiais tecnológicos (LPMaT). Universidade Federal do Amazonas (UFAM), Manaus, AM, Brazil.

^d Laboratory for Chemical Computation and Modeling, Institute for Computational Science and Artificial Intelligence, Van Lang University, Ho Chi Minh City, 70000 Vietnam. Email: minhtho.nguyen@vlu.edu.vn

^e Faculty of Applied Technology, Van Lang School of Technology, Van Lang University, Ho Chi Minh City, 70000 Vietnam

^f Discipline of Chemical Engineering, WASM MECE, Curtin University, Perth WA 6045, Australia

SSEs when supported by appropriate processing and interface design.²⁰⁻²⁴

Orthorhombic NaAlCl₄ (space group $P2_12_12_1$) recently emerged as a promising halide-based SSEs for sodium-ion battery applications, owing to its favourable electrochemical and structural characteristics.^{23,24} While NaAlBr₄ is conventionally reported to crystallize in the centrosymmetric orthorhombic $Pnma$ space group,¹⁶ its structural topology, comprising corner- and edge-sharing [NaBr₆] pentagonal pyramids and [AlBr₄] tetrahedral units, suggests latent flexibility that could accommodate symmetry lowering. Transitioning to the non-centrosymmetric $P2_12_12_1$ space group offers a compelling opportunity to unlock directional ionic conductivity and defect asymmetry, features often suppressed in centrosymmetric frameworks. Analogous behaviour has been observed in Na₃PS₄, where symmetry reduction from cubic to tetragonal enhances Na⁺ migration pathways and reveals anisotropic conduction channels.²⁵⁻²⁷

In this work, we consider the orthorhombic $P2_12_12_1$ phase of NaAlBr₄ as the central structure of investigation. By integrating density functional theory, defect energetics, and bond valence site energy analysis, we systematically explore its structural, electronic, thermodynamic, mechanical, and transport properties. Although this non-centrosymmetric phase has not been experimentally synthesized yet, our computed results predict that a symmetry lowering from $Pnma$ to $P2_12_12_1$ not only stabilizes the lattice but also creates one-dimensional Na⁺ conduction ribbons along the *b*-axis, enabling low migration barriers and high ionic conductivity. This symmetry-enabled conduction mechanism, combined with defect engineering strategies, positions the $P2_12_12_1$ phase of NaAlBr₄ as a promising halide-based SSE for sodium-ion battery applications.

2. Computational Details

The CASTEP code is employed to investigate the ground-state properties of NaAlBr₄ in the orthorhombic $P2_12_12_1$ space group, including lattice parameters, electronic structure, and total energy.²⁸ The computational setup follows our previous work on halide-based SSEs.²⁷ In line with the recommendations to employ a range of classical exchange-correlation functionals for predicting ground-state properties of emerging materials, density functional theory (DFT) calculations are conducted using the generalized gradient approximation (GGA). Specifically, the functionals considered include Perdew-Burke-Ernzerhof optimized for solids (PBESOL), revised Perdew-Burke-Ernzerhof (RPBE), Wu-Cohen (WC), and Perdew-Wang (PW91).²⁸ In addition, norm-conserving pseudopotentials are used to model the electronic configurations of the constituent species in reciprocal space.²⁹ A plane-wave energy cut-off of 700 eV is adopted to ensure the energy convergence. Geometry optimizations are carried out using convergence thresholds of 5×10^{-4} Å for atomic displacements, 5×10^{-6} eV/atom for total energy change, and maximum force and stress tolerances of 10^{-2} eV/Å and 2×10^{-2} GPa, respectively. A $4 \times 4 \times 2$ Monkhorst-Pack *k*-point grid is used to sample the Brillouin zone during structural relaxation.³⁰ These settings

ensure an accurate self-consistency convergence and provide us with reliable structural and electronic configurations for the $P2_12_12_1$ phase of NaAlBr₄.
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To investigate the optimal lattice parameters, mechanical properties, and intrinsic defect formation in NaAlBr₄, the general utility lattice program (GULP) is employed.³¹ In the Electronic Supplementary Data (ESI) file the interatomic potential parameters (FF), are included. Interatomic potential parameters are adopted from prior studies on halide-based systems,³²⁻³⁴ and adapted to reflect the bonding environment of Na⁺, Al³⁺, and Br⁻ ions. Short-range interactions are modelled using the Buckingham potential approximation, while long-range Coulombic forces are described by the formal charges and interatomic distances between species. To account for ionic polarization effects, the Dick and Overhauser model is adopted,³⁵ wherein each polarizable ion is treated as a positively charged core and a negatively charged shell connected by a harmonic spring. The sum of the core and shell charges yields the formal oxidation state of the ion, and the spring constant (*k*) governs the polarizability.

Defect energetics are evaluated using the multi-region strategy (Mott-Littleton),³⁶ which partitions the crystal lattice into two concentric spherical regions with radius *R*₁ and *R*₂ (*R*₁ < *R*₂). The defect, or defect cluster, is placed within the inner region *R*₁ where strong local interactions are explicitly treated. The outer region *R*₂ is approximated as a quasi-continuum, allowing for efficient treatment of long-range elastic and electrostatic responses.^{36,37} To ensure computational accuracy, *R*₁ is set at the value of 13 Å and *R*₂ at 27 Å, exceeding the maximum short-range cut-off of the interatomic potential and satisfying the condition *R*₂ - *R*₁ > cut-off. Geometry optimization and defect relaxation are performed using the Broyden-Fletcher-Goldfarb-Shanno (BFGS) algorithm, which iteratively updates atomic positions and lattice parameters.^{30,37} All defects are treated in the dilute limit to avoid spurious defect-defect interactions. An example of defect energetics computation is included in the Electronic Supplementary Data.

To complement the defect analysis, the bond valence site energy (BVSE) method is employed to identify diffusion pathways and estimate activation energies for Na⁺ migration.³⁸⁻⁴¹ This approach locates regions of minimal BVSE, which correspond to energetically favourable sites for mobile sodium ions. Bond valence contributions between Na⁺ and Br⁻ are calculated using tabulated empirical parameters (*L*_{Na-Br}) and (*b*_{Na-Br}), via equation (1):

$$S_{\text{Na-Br}} = \exp[(L_{0,\text{Na-Br}} - L_{\text{Na-Br}})/b_{\text{Na-Br}}] \quad (1)$$

The BVSE of Na⁺ is then computed analogously to a Morse-type potential, augmented by a Coulombic repulsion term (*E*_c) arising from interactions between mobile Na⁺-ions and the surrounding electrostatic field. This formulation captures both the short-range bonding preferences and long-range repulsive interactions that govern ion mobility.

Specifically, the bond valence energy of Na⁺-ion, $E_{BV}(\text{Na})$, is expressed in equation (2):

$$E_{BV}(\text{Na}) = \sum_{\text{Br}} H_0 \left[\sum_{i=1}^N \left(\frac{s_{\text{Na-Br}}}{s_{\text{min,Na-Br}}} \right)^2 - 2 \frac{s_{\text{Na-Br}}}{s_{\text{min,Na-Br}}} \right] + E_c \quad (2)$$

where H_0 is the bond dissociation energy constant and $s_{\text{min,Na-Br}}$ the optimal bond valence corresponding to the shortest energetically favourable Na-Br separation.³⁸⁻⁴¹ The summation over Br⁻ accounts for all coordinating halide ions within the local environment of the migrating Na⁺. The Coulombic term introduces a penalty for proximity to other mobile Na⁺-ions, effectively modelling repulsion and preventing artificial clustering in the energy landscape. This energy expression defines the potential surface over which Na⁺ migration is evaluated. Local minima correspond to energetically favourable sites, while first-order saddle points represent transition states. The resulting BVSE map enables rapid identification of conduction pathways and estimation of migration barriers.³⁸⁻⁴¹

The softBV-GUI tool is used to analyse sodium mobility within the NaAlBr₄ structure.⁴² All parameters from the BVSE formalism are seamlessly integrated into the code. Coulombic repulsions between mobile Na⁺ and immobile Al³⁺-ions are explicitly considered, while attractive Na⁺-Br⁻ interactions are inherently captured within the Morse-type framework. Migration pathways are mapped by identifying regions of low BVSE across mesh grids with a resolution of $\pm 0.1 \text{ \AA}^3$. For a comprehensive treatment of the bond valence methodology and its application to halide frameworks, the readers are referred to foundational and recent relevant literature.³⁸⁻⁴² The combined use of GULP and BVSE in this study has proven effective for modelling defect formation energies and estimating transport properties in halide-based SSEs materials.^{34,43,44}

3. Results and Discussion

3.1 Lattice and electronic properties

Figure 1 displays the unit cell of NaAlBr₄ structure. By assuming that NaAlCl₄ as a parent structure of the Br-compound, the crystal structure of NaAlBr₄ is orthorhombic and belongs to the $P2_12_12_1$ space group, with four formula units per unit cell ($Z = 4$). These compounds adopt a three-dimensional environment where Al³⁺ ion is coordinated by chloride ions. The Al³⁺ cation is tetrahedrally coordinated by four Br⁻ ions. Table 1 presents the lattice parameters and energy gap (Eg) of NaAlBr₄ derived from DFT calculations using various exchange-correlation functionals, including the lattice parameters obtained from classical FF computations, benchmarked against the reference structure of NaAlCl₄. Note that because the $Pnma$ and $P2_12_12_1$ phases of NaAlBr₄ belong to distinct space groups, their properties should be assessed independently.

In the ESI file, an example of geometry optimization of NaAlBr₄ in the $Pnma$ space group is included. After optimization, no imaginary frequencies are found in both phases, indicating transferability of the FF.

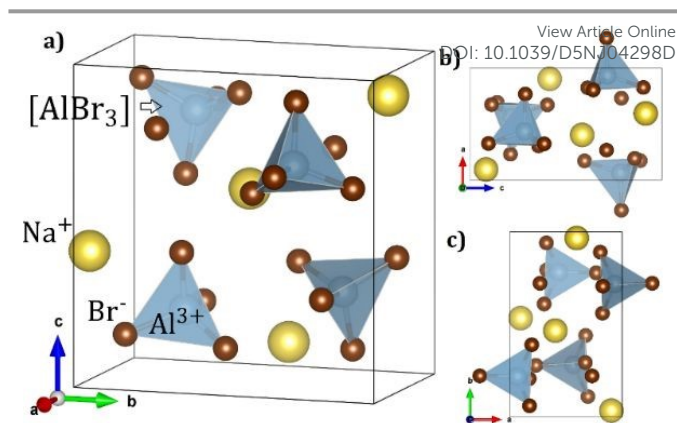


Fig. 1 Unit cell of NaAlBr₄ (space group $P2_12_12_1$) viewed along **a)** perspective, **b)** [010], and **c)** [001] directions, highlighting structural features and symmetry.

The DFT-optimized cells exhibit modest deviations (Δ) with respect to the lattice parameters of the isostructural NaAlCl₄ structure, with PBESOL showing a slight contraction along the a -axis (+0.17 \AA) and moderate expansions along b - and c -axis (-0.96 and -1.00 \AA , respectively), indicative of an anisotropic response to halide substitution. Calculated results using the WC, PW91 and RPBE approximation follow similar trends, even though PW91 slightly overestimates the a -axis (-0.06 \AA) while amplifying b - and c -axes expansions (-1.76 and -1.65 \AA). The RPBE functional overestimates slightly the a -axis (-0.78 \AA) while a large expansion along the b - and c -axes is observed (-2.12 and -2.50 \AA). As in the case of RPBE, the FF-derived parameters overestimate all three lattice parameters, reflecting the tendency of classical models to overextend the lattice due to simplified interatomic potentials. These trends align with known computational behaviour: DFT methods typically yield more compact geometries due to their treatment of electron correlation, whereas FF approaches often overestimate cell dimensions. Accordingly, the reported values serve as bounding estimates-DFT offering a lower bound and FF an upper bound-bracketing the plausible experimental geometry of NaAlBr₄. This framework is particularly valuable for guiding synthesis strategies and validating structural predictions in halide-based solid-state systems.

Table 1. Lattice parameters (in \AA) and energy gap (Eg) of optimized NaAlBr₄ in $P2_12_12_1$ space group derived from DFT (using different functional) and FF computations. The Δ -values represent the deviations of lattice parameters with respect to those reported for NaAlCl₄ in Ref. 24.

		a	b	c	Eg (eV)
DFT (functional)	PBESOL	5.989	10.765	11.273	4.29
	Δ	0.17	-0.96	-1.00	
	WC	5.962	11.050	11.537	4.38
	Δ	0.20	-1.25	-1.26	
	PW91	6.214	11.560	11.926	4.53
	Δ	-0.06	-1.76	-1.65	
	RPBE	6.940	11.919	12.779	4.72
	Δ	-0.78	-2.12	-2.50	
FF		7.277	11.234	12.903	
	Δ	-1.12	-1.43	-2.63	
Ref. 24		6.157	9.800	10.274	

In Figure 2 displays the electronic properties of NaAlBr₄ under the PBESOL approximation. The band structure plots derived from PW91, RPBE and WC approximations are included in the ESI file (Fig. S2). Along different functional, the *E_g* values lies between 4.29-4.72 eV (yielding a mean value of *E_g* = 4.48 eV), revealing insulating behavior of NaAlBr₄, comparable with other SSEs.^{14–18,27,34,43,44} It is well-known that PBESOL is the traditional PBE approximation optimized for solids.²⁷ Insulating SSEs are desirable to avoid high electronic conductivity, which may affect the transport properties between the SSE-electrode interphase and, consequently, the battery performance.^{1–5} For now on, all properties derived from DFT calculations are computed using PBESOL approximation for further comparison with our previous works.^{27,34,43,44} Using the same PBESOL setup, we explore the ground-state properties of NaAlBr₄ in the *Pnma* phase. As shown in Table S1 (SI file), lattice parameters are deviated by less than 0.1 Å, and the band gap by 0.37 eV from reference values. This agreement further supports the choice of PBESOL to study the structural and electronic properties of *P2₁2₁2₁* phase.

In particular, the band structure (Figure 2a) reveals an indirect band gap of approximately 4.29 eV, confirming its insulating nature. Based on the DFT-derived band gap, the estimated electrochemical stability window of NaAlBr₄ in the *P2₁2₁2₁* phase spans approximately 0-4.48 V, suggesting intrinsic compatibility with both Na metal anodes and high-voltage cathodes under inert electrode conditions.

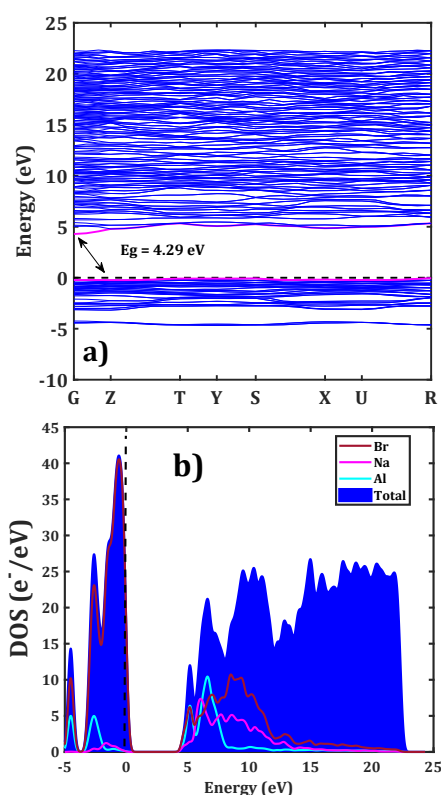


Fig. 2: a) Band structure and b) density of states of NaAlBr₄ computed using the PBESOL functional. Dash lines represent the Fermi level.

The valence band maximum and conduction band minimum occur at different special points (Z to G), indicating an indirect electronic transition. This characteristic implies limited optical absorption which can be beneficial for applications requiring electronic insulation without photoconductivity. The density of states (DOS Figure 2b) supports this interpretation, showing that the valence band is predominantly composed of Br *p*-states, while the conduction band is mainly influenced by Al³⁺ and Na⁺ orbitals.

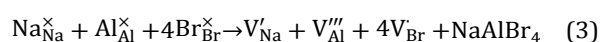
3.2. Defect formation and divalent doping strategies in NaAlBr₄

The concentrations of Na⁺ ions and their associated vacancies play a pivotal role in governing the transport properties of halide-based SSEs.^{1–10} These species directly influence the ionic conductivity by modulating the availability and mobility of charge carriers within the lattice. A well-balanced Na⁺/Na⁺ vacancy ratio not only facilitates efficient migration pathways but also affects defect formation energies, local lattice distortions, and the overall thermodynamic stability of the material.

In particular, the presence of Na⁺ vacancies can enhance conductivity by enabling vacancy-mediated diffusion mechanisms, provided that their distribution and interaction with dopants or intrinsic defects are energetically favourable. In this context, a precise control over the Na⁺ concentration and defect engineering are of essential importance for optimizing electrochemical performance in next-generation energy storage applications.^{1–15} Guided by these principles, we focus on intrinsic defects such as the NaBr Schottky and Na⁺ Frenkel pairs which represent the lowest-energy native configurations and directly govern vacancy-mediated transports.

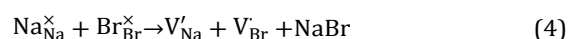
For extrinsic modifications, we select the following divalent cations, Mg²⁺, Zn²⁺, Ca²⁺, Sr²⁺ and Ba²⁺, to systematically probe the role of ionic size and charge compensation in promoting Na⁺ vacancy formation, whereas a Li⁺ substitution is considered due to its isovalent nature with respect to Na⁺ and its relevance to alkali-ion systems (Li⁺ remains the most employed metal ion in batteries). This combined strategy ensures that both the most probable intrinsic defects and the most experimentally relevant extrinsic dopants are captured, providing us with a comprehensive framework for defect engineering in NaAlBr₄.

Three Schottky defect mechanisms are considered. The first involves the full NaAlBr₄ Schottky scheme, written in the Kröger-Vink notation,⁴⁵ as described by equation (3):

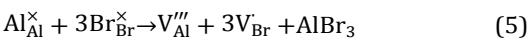


where Al_{Al}[×], Na_{Na}[×] and Br_{Br}[×] represent the constituent ions occupying their respective atomic positions at the NaAlBr₄ lattice structure, while V_{Na}['], V_{Al}^{'''} and V_{Br} symbolize Na⁺, Al³⁺ and Br[−] vacancies, respectively.

Formation of a NaBr Schottky defect is described by equation (4):



leading the stoichiometric $\text{Na}_{1-x}\text{AlBr}_{4-x}$ composition. The third basic scheme describes the formation of AlBr_3 Schottky defect as in equation (5):



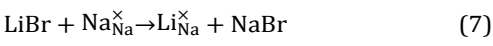
which leads to the stoichiometric $\text{NaAl}_{1-x}\text{Br}_{4-3x}$ formula.

In addition, the Na^+ Frenkel type mechanism explains the excess of Na^+ increasing the Na concentration as described by equation (6):



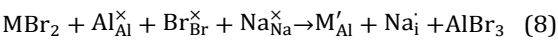
where for each Na^+ vacancy an extra Na^+ -ion occupying an interstitial site (Na_i) is required for charge compensation.

Another interesting scheme is to consider the Li^+ substitution at Na^+ -site in NaAlBr_4 , as in equation (7):



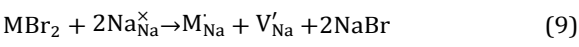
resulting in the stoichiometric $\text{Na}_{1-x}\text{Li}_x\text{AlBr}_4$ where no additional defect is required for charge compensation.

A divalent doping (M^{2+}) occupying an Al^{3+} -site in NaAlBr_4 lattice structure is given by equation (8):



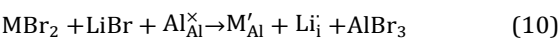
resulting in a stoichiometric $\text{Na}_{1+x}\text{Al}_{1-x}\text{M}_x\text{Br}_4$. From equation (8), for each M^{2+} replacing an Al^{3+} -site, a Na^+ interstitial is generated for charge compensation with the presence of $\text{M}_{\text{Al}}' - \text{Na}_i$ dimers.

The scheme of inclusion of a M^{2+} dopant at a Na^+ -site in NaAlBr_4 lattice structure is given by equation (9):



in which for each divalent dopant occupying a Na^+ -site (M_{Na}') a Na^+ vacancy is created for charge balancing, favoring the formation of $\text{M}_{\text{Na}}' - \text{V}_{\text{Na}}'$ dimer.

Lastly, equation (10) describes the incorporation of divalent dopant at the Al^{3+} -site, but compensating the charge with Li^+ interstitial (Li_i) instead of Na^+ interstitial:



resulting in a $\text{NaLi}_x\text{Al}_{1-x}\text{M}_x\text{Br}_4$ stoichiometry.

The results of defect energetics computations are collected in Table 2. During the geometry optimization of LiBr , NaBr , AlBr_3 and MBr_2 structures, no imaginary frequencies are encountered, and the lattice energy align well with those reported in the literature.⁴⁴ These findings support the transferability of the FF reproducing these structures. As it is shown, the creation of a Na^+ and Br^- are favorable, (~ 4.8 eV/vacancy), while the Al^{3+} vacancy is far unfavorable (44.9 eV/vacancy). The inclusion of Na^+ and Li^+ interstitially is negative, which implies that the NaAlBr_4 structure accepts this kind of point defects within the lattice structure. The substitution energy of Li^+ occupying a Na^+ -site ($E_{\text{Na}}^{\text{Li}}$) is also favorable (-0.47 eV/dopant). The substitution energy of Li^+ at the Na^+ -site in NaAlBr_4 is favourable due to several synergistic factors rooted in ionic size compatibility, charge neutrality, and lattice flexibility. Both Li^+ and Na^+ are monovalent cations that allow for isovalent substitution

without introducing charge imbalance or requiring compensatory defects. Additionally, the smaller ionic radius of Li^+ (0.76 Å) as compared to that of Na^+ (1.02 Å) can lead to local lattice contraction, potentially lowering the formation energy by relieving steric strain in the surrounding bromide framework.

The substitution behaviour of divalent cations in NaAlBr_4 reveals a clear correlation between ionic radius and defect energetics. Of the dopants considered, both Zn^{2+} and Mg^{2+} , with relatively small ionic radii of 0.74 and 0.72 Å, respectively, exhibit the most favourable substitution energies at the Na^+ -site (-12.5 and -9.1 eV/dopant), suggesting strong thermodynamic driving forces for incorporation and minimal lattice strain. In contrast, larger dopants such as Ca^{2+} (0.99 Å), Sr^{2+} (1.18 Å), and Ba^{2+} (1.35 Å) show progressively less favourable substitution energies, reflecting increased steric mismatch and local distortion. Substitution at the Al^{3+} site is energetically prohibitive for all dopants, with positive formation energies exceeding 22 eV, underscoring the strong size and charge incompatibility. These trends highlight the importance of ionic size compatibility and lattice flexibility in guiding dopant selection for defect engineering, with Zn^{2+} and Mg^{2+} cations emerging as promising candidates for enhancing Na^+ vacancy concentrations and potentially improving ionic conductivity.

The most favourable intrinsic defect in NaAlBr_4 is identified as the NaBr Schottky-type, comprising paired vacancies of Na^+ and Br^- . This defect exhibits the lowest formation energy among the intrinsic schemes considered, suggesting it may form in appreciable concentrations under ambient conditions. Such behaviour is consistent with trends observed in analogous halide systems, where Schottky-type defects play a central role in enabling ionic transport.^{16,22,25,38}

In contrast, defect schemes involving AlBr_3 units and full Schottky-type configurations are energetically less favourable, exhibiting significantly higher formation energies and thus lower likelihood of spontaneous formation, which reveal structural stability of NaAlBr_4 $P2_12_12_1$ framework.

Table 2. Vacancy (in eV/vac), lattice energies (in eV/f.u), Schottky formation and interstitial energies (in eV/defect) of NaAlBr_4 structure.

Basic defects and Lattice Energies				
$E_{\text{vac}}^{\text{Al}}$	$E_{\text{vac}}^{\text{Na}}$	$E_{\text{vac}}^{\text{Br}}$	E_i^{Na}	E_i^{Li}
44.94	4.71	4.80	-3.04	-3.59
$E_{\text{Na}}^{\text{Li}}$	$E_{\text{LiBr}}^{\text{Li}}$	$E_{\text{LiBr}}^{\text{Na}}$	$E_{\text{LiBr}_3}^{\text{Al}}$	$E_{\text{LiBr}_4}^{\text{NaAl}}$
-0.47	-8.10	-7.59	-50.68	-58.55
Formation Energies (eV/defect)				
Eq. (3)	Eq. (4)	Eq. (5)	Eq. (6)	Eq. (7)
3.44	0.96	4.33	1.67	0.04
Ionic Radius in Å (CN VI)				
Divalent Dopant		$E_L^{\text{MBr}_2}$	$E_{\text{Subs,Na}}^{\text{M}}$	$E_{\text{Subs,Al}}^{\text{M}}$
Mg^{2+}	0.72	-21.38	-9.06	26.90
Zn^{2+}	0.74	-24.17	-12.48	22.71
Ca^{2+}	0.99	-20.16	-7.65	28.79
Sr^{2+}	1.18	-17.23	-5.15	32.17
Ba^{2+}	1.35	-20.07	-7.89	28.51

The second most favourable defect configuration corresponds to the substitution of Li^+ at the Na^+ -site, as described in equation (7). As we mentioned before, Li^+ and Na^+ have the same monovalent charge the substitution of Li^+ at a Na^+ -site does not require charge compensation. This substitution introduces a stress-strain effect due to the ionic radius mismatch between Li^+ and Na^+ cations, which can locally distort the lattice and influence defect energetics. This is followed by the Na^+ Frenkel-type defect (equation (6)), involving the formation of a Na^+ interstitial and a corresponding Na^+ vacancy which may contribute to transport under specific thermodynamic conditions. The Na^+ Frenkel scheme yields a defect formation energy of 1.67 eV/defect. Interestingly, a molecular dynamics investigation employing a pretrained neural network potential reported formation energies for NaAlBr_4 in the $Pnma$ phase of 0.88 eV/defect for the Na^+ vacancy mechanism and 0.73 eV/defect for the Na^+ interstitial mechanism.¹⁶ By assuming that the Frenkel energy corresponds to the sum of vacancy and interstitial formation energies, the reported values amount to 1.61 eV/defect, which is remarkably close to the value obtained for the $P2_12_12_1$ phase. Such an agreement across distinct structural symmetries underscores the transferability of the force fields employed in capturing defect energetics beyond the reference $Pnma$ phase.

The solution energy (E_s) represents the energetic cost of introducing an isolated or clustered point defect from infinity into a specific crystallographic site within the lattice. The binding energy (E_B) is defined as the difference between the total energy of the defect cluster and the sum of the energies of its isolated constituents. The final solution energy (E_F) is then obtained by subtracting the binding energy from the initial solution energy, reflecting the net thermodynamic favorability of defect incorporation.^{22,25,46}

Figure 3 presents a comparative assessment of the solution energy, binding energy, and final solution energy for five divalent dopants in NaAlBr_4 , revealing clear distinctions in thermodynamic favourability across the three doping schemes. The substitution at the Na^+ -site consistently yields the most negative E_s and E_F values, particularly for Zn^{2+} and Mg^{2+} , indicating strong incorporation potential and stable defect formation. These dopants also exhibit favourable binding energies, supporting the formation of robust dopant-vacancy complexes that are conducive to enhanced ionic transport.

In contrast, Al^{3+} -site substitution schemes, whether compensated by Na^+ or Li^+ interstitials, show significantly higher solution energies and weaker binding interactions. This trend is especially pronounced for larger dopants such as Sr^{2+} and Ba^{2+} whose incorporation leads to substantial lattice strain and energetically unfavourable defect configurations. Even for smaller dopants like Zn^{2+} and Mg^{2+} , the Al^{3+} -site pathways remain less favourable as compared to Na^+ -site substitution, as reflected in their elevated E_F values.

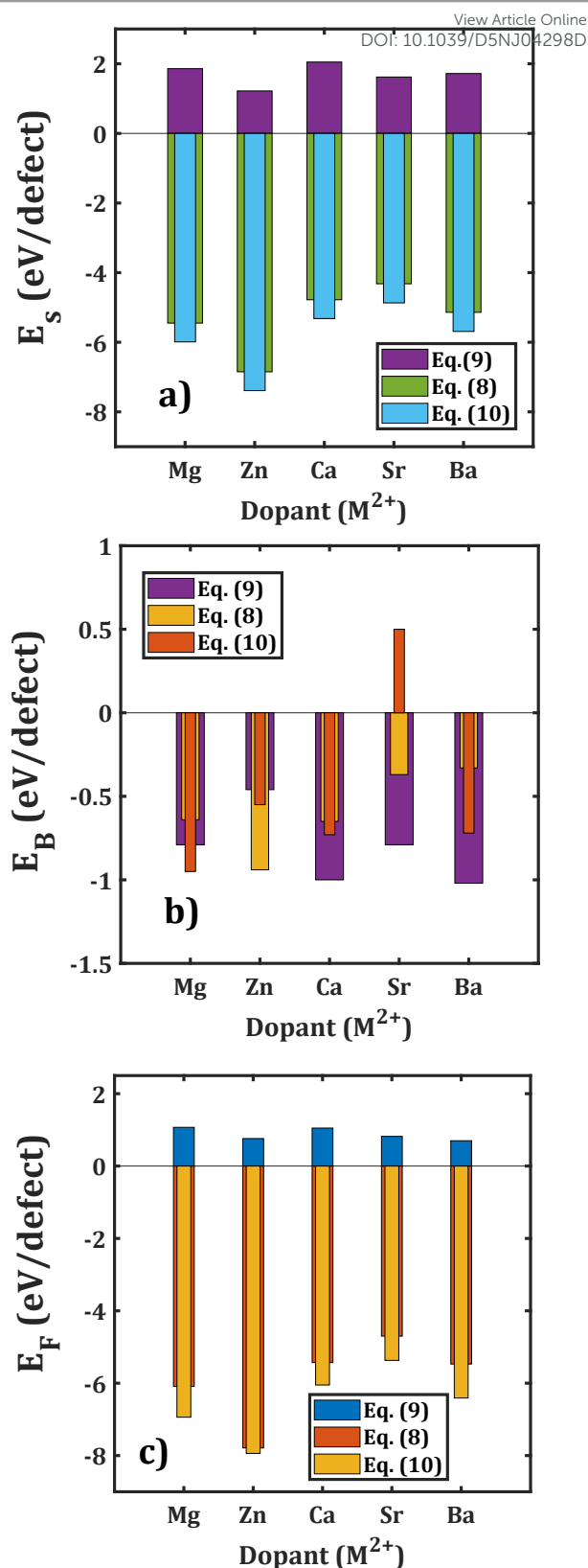


Fig. 3: a) Solution, b) binding and c) final solution energy of divalent doped NaAlBr_4 according to the schemes described in equations (8)–(10).

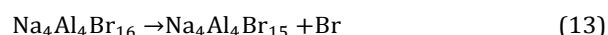
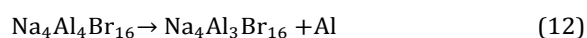
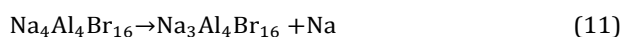
To date, defect energetics of NaAlBr₄ reveal a clear hierarchy of favourable configurations that align with principles of ionic size compatibility, charge neutrality, and lattice flexibility. Intrinsic defects such as the NaBr Schottky and Na⁺-Frenkel pairs exhibit low formation energies, indicating their potential to form under ambient conditions and contribute to ionic transport. Among extrinsic modifications, substitution at the Na⁺-site, particularly with Zn²⁺ and Mg²⁺, emerges as the most promising strategy, offering strong defect binding and minimal lattice strain. In contrast, Al³⁺-site substitutions are energetically prohibitive across all dopants, reinforcing the structural rigidity of the [AlBr₃] framework.

Collectively, these findings establish a robust foundation for targeted defect engineering in NaAlBr₄, guiding the rational design of halide-based SSEs with enhanced ionic conductivity and defect-tolerant architectures.

3.3 Thermodynamic and mechanical stability

Thermodynamic stability is a critical criterion in the design and optimization of SSEs, as it governs both the structural integrity and defect tolerance of the material under operating conditions. A thermodynamically stable SSE resists decomposition, phase transitions, and undesirable chemical reactions when interfaced with electrodes or exposed to varying temperatures and voltages. This stability ensures that the electrolyte maintains its ionic conductivity over time without forming secondary phases or degrading into electronically conductive species, which could compromise battery performance and safety. Moreover, stable defect configurations, such as low-energy vacancies or interstitials, can persist in sufficient concentrations to facilitate ion transport, making thermodynamic favorability a cornerstone for reliable and high-performance energy storage systems.

Thermodynamic properties of the NaAlBr₄ compound is evaluated by exploring the energetic cost for the inclusion-extraction processes of its individual constituent elements considering some hypothetical decomposition reactions in equations (11-13):



Note that the stoichiometry in equations (11)-(13) of NaAlBr₄ is augmented by 4 because the NaAlCl₄ framework is assumed to having the Z = 4 formula unit.⁴⁷⁻⁵¹ Consequently, a 2×2×1 supercell of NaAlBr₄ is used for the non-stoichiometric reactants to obtain their total energies by FF and DFT computations. The decomposition reaction energies are then in equation (14-16):

$$\Delta E(\text{Na}) = E(\text{Na}_4\text{Al}_4\text{Br}_{16}) - [E(\text{Na}) + E(\text{Na}_3\text{Al}_4\text{Br}_{16})] \quad (14)$$

$$\Delta E(\text{Al}) = E(\text{Na}_4\text{Al}_4\text{Br}_{16}) - [E(\text{Al}) + E(\text{Na}_4\text{Al}_3\text{Br}_{16})] \quad (15)$$

$$\Delta E(\text{Br}) = E(\text{Na}_4\text{Al}_4\text{Br}_{16}) - [E(\text{Br}) + E(\text{Na}_4\text{Al}_4\text{Br}_{15})] \quad (16)$$

for reactions (11) to (13), respectively. In equations (14) to (16), ΔE denotes the decomposition reaction energy, and E(R) the total energy of the compound R (Na, Al, Br, and the other defective NaAlBr₄). This procedure had been used in other battery materials.⁵² The computed decomposition energies are as follows: ΔE(Na) = 3.31 (3.39), ΔE(Al) = 52.71 (54.94) and ΔE(Br) = 2.49 (5.03) eV/atom derived from FF and DFT computations (in parentheses are the values from DFT calculations), confirming stability with respect to decomposition processes. The ΔE(Na) and ΔE(Br) values are both the lowest ones, in agreement with the results of defect energetics discussed in Section 3.2. This confirms that NaBr Schottky defect is responsible for the transport properties of NaAlBr₄.

In addition to the structural and dynamic stability criteria employed so far, the energy-above-hull metric provides a robust thermodynamic indicator of phase stability.⁵³ To evaluate this, we compared the total energy of NaAlBr₄ with those of its potential decomposition products, specifically NaBr and AlBr₃. All DFT energies were normalized per formula unit (Z = 4), and elemental reference energies for Na, Al, and Br were obtained from the Materials Project database.⁵⁴ The orthorhombic polymorph of NaAlBr₄ with space group *P*2₁2₁2₁ was evaluated using the same VASP computational setup employed by the Materials Project database.⁵⁴ It exhibits an energy above the convex hull of 5.56 meV/atom (0.0056 eV/atom), which is below of thermal energy at 25°C (k_BT=0.025eV) indicating near thermodynamic stability. Its decomposition pathway involves NaBr and AlBr₃, consistent with the expected binary phases in the Na-Al-Br system (Fig. S3).⁵³ In contrast, the *Pnma* polymorph lies directly on the convex hull (0.0 eV/atom), confirming its thermodynamic stability. While both structures decompose to the same binary phases, the *Pnma* variant is the ground-state configuration, and *P*2₁2₁2₁ represents a low-energy polymorph that may be accessible under kinetic control or non-equilibrium synthesis conditions.

In conjunction with the full Schottky defect formation energy discussed in Section 3.2 and the decomposition energy, the global instability index (GII) provides a quantitative descriptor of structural stability in crystalline solids.⁵⁵ Within the BVSE, GII is calculated using the following equation (17):

$$\text{GII} = \left[\sum_{i=1}^N (\sum_j s_{ij} - q_i)^2 / N \right]^{1/2} \quad (17)$$

In this formulation, *s_{ij}* denotes the bond valence between atom *i* and its neighbouring atom *j*, *q_i* represents the formal oxidation state of atom *i*, and *N* is the total number of atoms in the unit cell. GII quantifies the degree of bond strain and valence mismatch within the lattice.⁵⁵ Lower GII values reflect closer alignment with ideal valence configurations, indicating an improved thermodynamic stability and a diminished likelihood of spontaneous structural degradation.⁵⁵ While GII does not account for electronic or vibrational contributions, it remains a practical and

insightful metric for screening metastable phases and guiding the design of structurally robust SSEs. The computed GII values for the optimized NaAlBr₄ structure are 0.045 and 0.052, as obtained from DFT and FF calculations, respectively. These low GII values reflect a high degree of valence matching and minimal bond strain within the lattice, indicating that NaAlBr₄ possesses a structurally coherent and thermodynamically stable framework. The close agreement between DFT and FF-derived values further reinforces the reliability of the lattice model and suggests that the material is resilient against spontaneous structural distortions. Such intrinsic stability is essential for SSE applications, as it supports defect tolerance, suppresses phase decomposition, and ensures consistent ionic transport under operational conditions.

To assess the mechanical stability and elastic behaviour of orthorhombic NaAlBr₄ the elastic stiffness tensor (C_{ij}) is computed and analysed to evaluate compliance with the Born stability criteria.⁵⁶

The C_{ij} matrix of the NaAlBr₄ compound, derived from FF and DFT computations, are given in equations (18-19):

$$C_{ij}^{FF} = \begin{pmatrix} 6.48 & 0.46 & -0.11 & 0 & 0 & 0 \\ 0.46 & 4.01 & 1.42 & 0 & 0 & 0 \\ -0.11 & 1.42 & 2.34 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1.56 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1.18 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1.39 \end{pmatrix} \quad (18)$$

$$C_{ij}^{DFT} = \begin{pmatrix} 31.20 & 12.95 & 9.29 & 0 & 0 & 0 \\ 12.95 & 17.13 & 6.74 & 0 & 0 & 0 \\ 9.29 & 6.74 & 15.01 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.91 & 0 & 0 \\ 0 & 0 & 0 & 0 & 6.10 & 0 \\ 0 & 0 & 0 & 0 & 0 & 2.17 \end{pmatrix} \quad (19)$$

The eigenvalues of the elastic stiffness tensor provide some insights into the principal modes of mechanical response within the crystal.^{56,57} Each eigenvalue corresponds to a distinct deformation pathway, representing the material's resistance to strain along a specific combination of stress directions.^{56,57} A strong mechanical stability criterion states that the C_{ij} -eigenvalues must be positive.^{55,56} The characteristic equation $\text{DET}(C_{ij} - \lambda_i I_{ij}) = 0$, where DET denotes the determinantal and I_{ij} the unity matrix of the same size as C_{ij} . The calculated eigenvalues are: $\lambda_1 = 1.18(0.91)$, $\lambda_2 = 1.39(2.17)$, $\lambda_3 = 1.50(6.10)$, $\lambda_4 = 1.56(8.83)$, $\lambda_5 = 4.75(11.02)$ and $\lambda_6 = 6.57(43.49)$ GPa for FF and DFT computations (in brackets). All the eigenvalues are positive which confirm the mechanical stability of the orthorhombic disphenoidal NaAlBr₄. The spread in λ -magnitude reflects a moderate degree of elastic anisotropy, with the lowest eigenvalues associated with shear or compliant deformation modes, and the highest values indicating stiffer responses to axial compression or tension. This distribution suggests that NaAlBr₄ exhibits direction-dependent mechanical behavior which may influence its structural integrity under external stress and its coupling to lattice dynamics.

For orthorhombic structures (such as in the case of space group $P2_12_12_1$), other necessary and sufficient conditions are required for mechanical stability ($C_{11} + C_{22}$

$-2C_{12} > 0$; $C_{11} + C_{33} - 2C_{13} > 0$; $C_{22} + C_{33} - 2C_{23} > 0$).^{56,57} These are clearly satisfied and thus confirm the mechanical stability of NaAlBr₄ and also reveal a moderate degree of elastic anisotropy. The negative value of C_{13} in equation (18) suggests a non-trivial coupling between normal strains along the x- and z- directions, potentially arising from anisotropic interatomic interactions or structural relaxation mechanisms. Such behavior may be relevant for understanding directional deformation responses or anisotropic thermal expansion in this structure.

From the C_{ij} components and the compliance matrix (C_{ij}^{-1}), other relevant properties are derived. To evaluate the macroscopic mechanical response of orthorhombic NaAlBr₄, elastic moduli and related indicators are computed using both FF and DFT with the PBESOL functional. Table 3 summarizes the bulk modulus (B), shear modulus (G), Young's modulus (E), and Pugh's ratio (B/G), derived via Reuss, Voigt, and Hill averaging schemes, along with additional mechanical descriptors.⁵⁸⁻⁶¹ The FF-derived Hill averages yield $B = 1.71$ GPa, $G = 1.43$ GPa, and $E = 1.82$ GPa, indicating a mechanically soft and compliant material. In contrast, DFT results show significantly higher stiffness, with $B = 12.41$ GPa, $G = 3.24$ GPa, and $E = 8.95$ GPa, reflecting stronger interatomic interactions and a more rigid lattice response. This discrepancy is expected, as FF models often tend to underestimate absolute stiffness due to their simplified potential forms.

The Pugh's ratio (B/G), as a descriptor of ductile or brittle material,⁶² further highlights the contrast: FF yields a borderline ductile value of 1.19, while DFT predicts 3.83, suggesting a more ductile and pressure-tolerant behavior under first-principles treatment. Similarly, the machinability index ($M=B/C_{44}$ Ref. 63) increases from 1.10 (FF) to 13.66 (DFT), reinforcing the enhanced mechanical resilience captured by DFT. Anisotropy indicators also diverge, while FF predicts low elastic anisotropy ($A_B = 0.07$, $A_G = 0.09$, $A^U = 1.15$),⁶⁴ DFT reveals more pronounced directional dependence. The compressibility (β) drops from 0.63 GPa⁻¹ (FF) to 0.09 GPa⁻¹ (DFT), and the Kleinman parameter (K) increases from 0.22 to 0.55, both consistent with a stiffer and less deformable lattice under DFT.⁶⁵

Table 3: Macroscopic mechanical properties of orthorhombic NaAlBr₄ (space group $P2_12_12_1$) derived from FF and DFT simulations. Bulk modulus (B), shear modulus (G), Young's modulus (E), and Pugh's ratio (B/G) are reported using Reuss, Voigt, and Hill averaging schemes. Additional indicators include the machinability index (M), anisotropy factors (A_B and A_G), compressibility (β), Kleinman parameter (K), and universal anisotropy index (A^U).

Computational Method	Force Field			Density Functional Theory		
	Reuss	Voigt	Hill	Reuss	Voigt	Hill
B (GPa)	1.59	1.82	1.71	11.34	13.48	12.41
G (GPa)	1.30	1.56	1.43	2.36	4.13	3.24
E (GPa)	6.38	3.10	1.82	6.62	11.23	8.95
B/G	1.23	1.16	1.19	4.81	3.27	3.83
M	1.03	1.17	1.10	12.48	14.84	13.66
A_B	0.07			0.09		
A_G	0.09			0.27		
β (GPa⁻¹)	0.63			0.09		
K	0.22			0.55		
Au	1.15			3.94		

While FF simulations capture the qualitative mechanical behavior and symmetry-consistent trends of NaAlBr₄, DFT provides a quantitatively more accurate and robust description of its elastic properties. These mechanical descriptors must be used as lower and upper bounds due to the lack of experimental verifications.

3.4 Estimation of transport properties of NaAlBr₄

Evaluation of transport properties is essential for advancing solid-state batteries where the solid SSE plays a pivotal role in enabling safe and efficient ion conduction. SSEs must support the migration of Na⁺-ions through a rigid lattice, often with limited structural flexibility. Understanding key transport metrics such as diffusion and ionic conductivity, activation energy, and migration pathways allows us to identify bottlenecks and optimize SSEs with higher performance.¹⁻¹⁵

In order to explore transport properties, the BVSE method is now used to probe both migration pathways and activation energies for Na⁺ ions within the crystal lattice. To determine the Na⁺ migration pathways in NaAlBr₄, a 2×1×1 supercell is constructed, containing eight Na⁺ sites in total. Figure 4 presents energy profiles along BVSE-guided NEB migration paths in NaAlBr₄, computed using DFT (Figure 4a) and FF methods (Figure 4b). Both profiles reveal a three-segment migration mechanism with distinct energy barriers and intermediate minima. However Figure 4a shows a higher migration energy barrier, indicating a less favourable ion transport pathway in the DFT-derived profile. In contrast, Figure 4b specifically highlights the 1D ribbon conduction channel, characterized by a relatively symmetric energy landscape and a significantly lower central barrier of ~0.12 eV. This reduced activation energy arises from the structurally continuous and electrostatically favourable environment of the ribbon pathway which minimizes lattice distortion and repulsive interactions. As a result, Na⁺ ions can migrate more smoothly between adjacent sites, facilitating fast ionic conduction within the NaAlBr₄ lattice. These findings underscore the importance of accurate path selection and support the use of hybrid computational strategies to cross and validate ionic transport predictions in SSEs.

Figure 5 compares the energy landscapes of the optimized NaAlBr₄ structure derived from DFT (Figure 5a) and force field methods (Figure 5b). The blue isosurfaces represent regions of low site energy, indicating favourable positions for Na⁺ migration. In Figure 5a, the isosurfaces appear more localized and fragmented, suggesting a limited connectivity between low energy sites and a more constrained migration environment. In contrast, Figure 5b reveals a more continuous and extended distribution of isosurfaces, particularly along the crystallographic *c*-axis, which corresponds to the 1D ribbon conduction channel.

This enhanced connectivity reflects a structurally coherent and energetically favourable pathway for Na⁺-ions, consistent with the lower migration barrier observed in the BVSE-derived NEB profile.

Figure 6 presents the linearized Arrhenius plots for Na⁺ ion transport in the optimized NaAlBr₄ structure, comparing diffusion coefficients (Figure 6a) and ionic conductivities (Figure 6b) derived from DFT and FF computations. In both plots, FF data (red squares) exhibit significantly lower activation energies (~0.12 eV for diffusion and ~0.10 eV for conductivity) compared to the DFT-derived values of 0.53 and 0.50 eV, respectively. At ambient temperature (i.e.; 25°C), FF calculations yield markedly higher diffusion and conductivity ($D_0 = 4.48 \times 10^{-6} \text{ cm}^2/\text{s}$, $\sigma_0 = 0.11 \text{ S/cm}$) than those obtained from DFT ($D_0 = 6.02 \times 10^{-13} \text{ cm}^2/\text{s}$, $\sigma_0 = 2.02 \times 10^{-8} \text{ S/cm}$). The enhanced mobility captured by the BVSE model aligns with the presence of the 1D ribbon conduction channel identified in the energy landscape and NEB profile analyses. Previous studies on NaAlCl₄ (space group *P2₁2₁2₁*) and NaAlBr₄ (space group *Pnma*) showed a limited Na⁺ conductivity at room temperature, typically below $4 \times 10^{-6} \text{ S/cm}$ for NaAlCl₄ and around $1.2 \times 10^{-5} \text{ S/cm}$ for the *Pnma* phase of NaAlBr₄.^{2,16} In contrast, the present work investigates a distinct orthorhombic phase of NaAlBr₄ (space group *P2₁2₁2₁*), revealing markedly enhanced transport properties. BVSE-guided NEB computations and energy landscape analyses identify a 1D ribbon conduction channel unique to this phase, enabling smooth Na⁺ migration with a central barrier of 0.12 eV. This structural feature translates into significantly lower activation energies and higher transport properties at 25°C compared to both previously reported phases. These findings underscore the critical role of crystal symmetry and conduction topology in governing ionic mobility, positioning the *P2₁2₁2₁* phase of NaAlBr₄ as an emerging candidate for room-temperature SSE applications.

Table 4 presents a comparative analysis of halide and sulfide-based SSEs, highlighting their diverse transport and mechanical properties. The data underscore the critical influence of crystal symmetry, defect chemistry, and anion softness on ionic conductivity. Among the halide systems, NaAlBr₄ in the *P2₁2₁2₁* phase demonstrates notably high simulated conductivity and a low activation energy, attributed to symmetry-enabled one-dimensional Na⁺ migration channels and the favourable formation of NaBr Schottky defects. In contrast, the *Pnma* polymorph of NaAlBr₄ exhibits a substantially lower conductivity and a higher migration barriers illustrating the pronounced effect of lattice topology on the transport behaviour.¹⁶

A similar symmetry-driven enhancement is observed in NaAlCl₄, where the transition from *Pnma* to *P2₁2₁2₁* improves conductivity by nearly an order of magnitude and reduces activation energy from 0.45 eV to 0.35 eV.^{2,50} This improvement is attributed to increased Na⁺ sublattice occupancy and reduced bottleneck constraints, despite the fact that the chloride lattice is mechanically stiffer than its bromide analogue. These findings underscore the critical role of symmetry tuning and defect engineering in optimizing the performance of halide-based SSEs.

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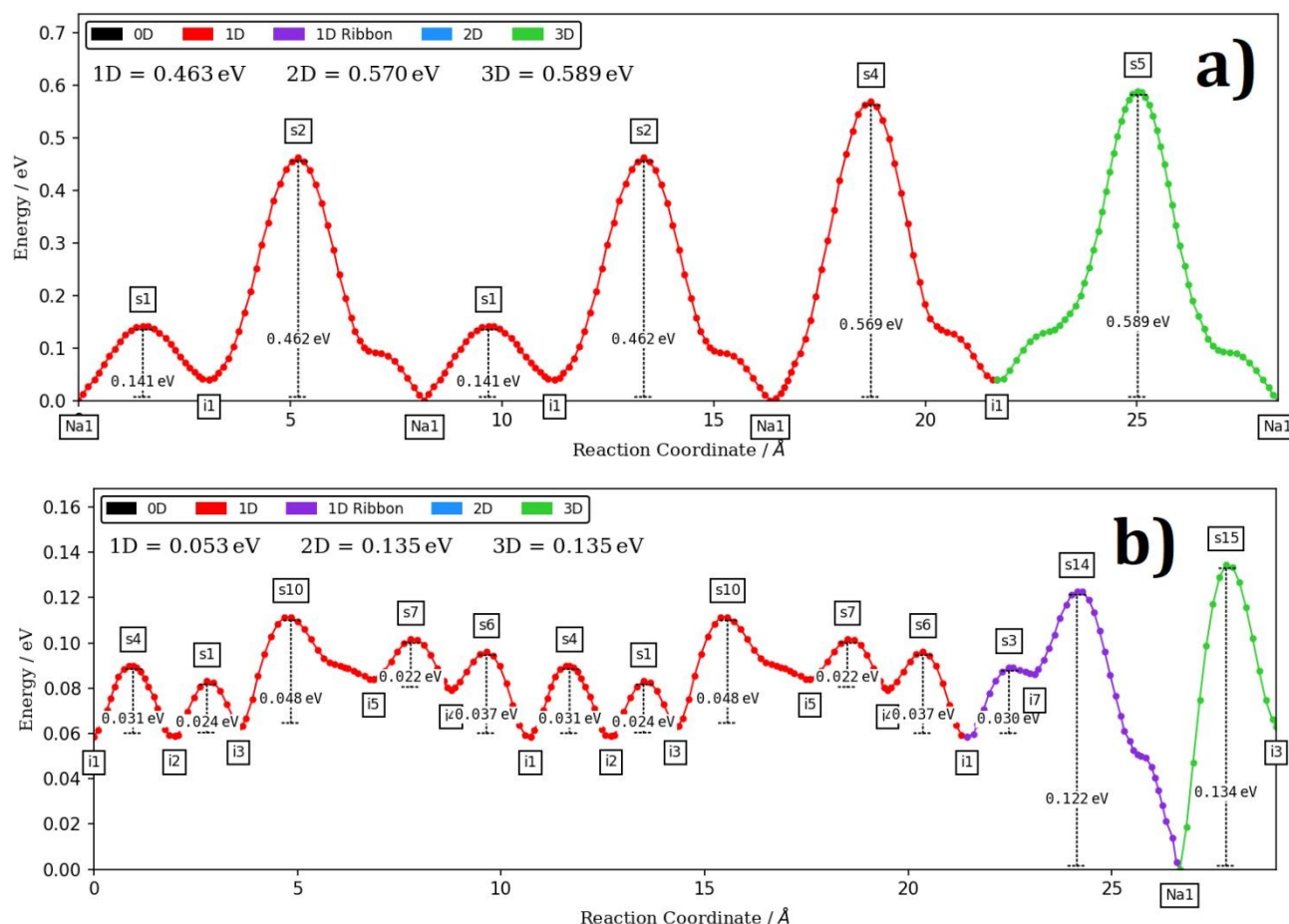


Fig. 4: Energy versus reaction coordinate profiles for NaAlBr_4 , optimized using a) DFT and b) FF computations.

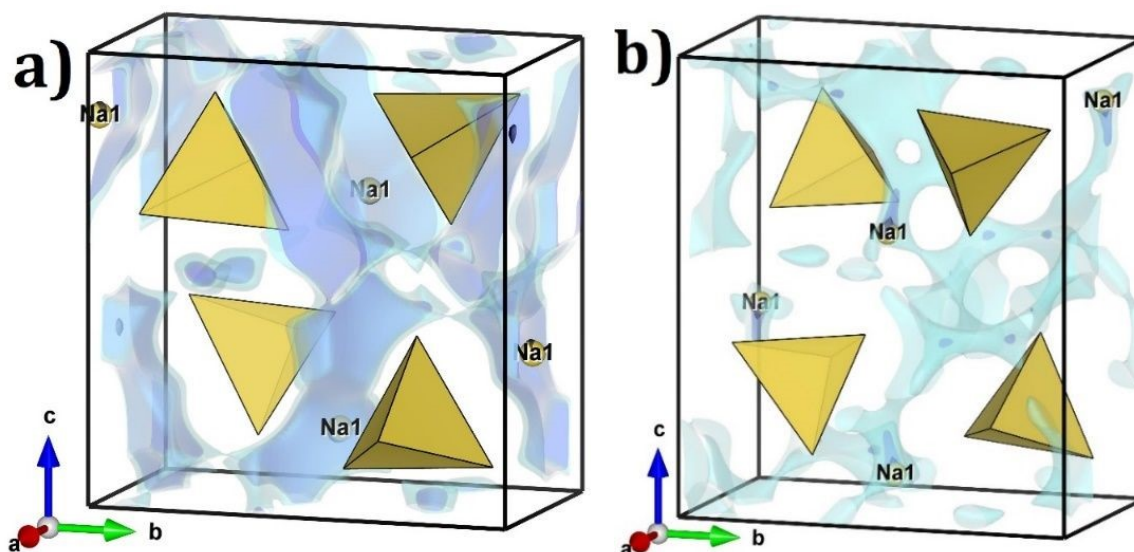


Fig. 5: Energy landscape of the optimized NaAlBr_4 structure (blue isosurfaces) derived from a) DFT and b) FF computations. Yellow polyhedral represent the AlBr_4 tetrahedral

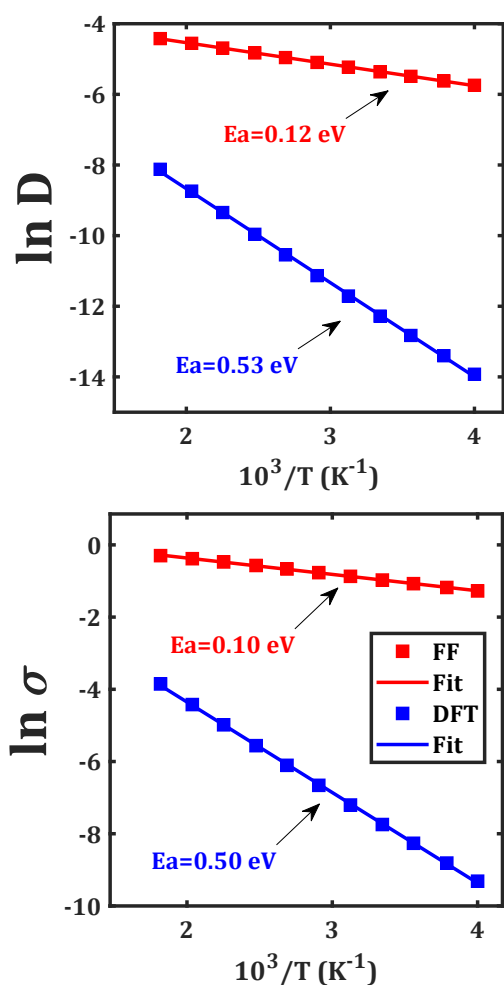


Fig. 6: Linearized Arrhenius dependence of Na⁺-ion: a) diffusion coefficient (D), and b) conductivity (σ) of the optimized NaAlBr₄ structure derived from DFT and FF computations.

In particular, systems characterized by inherently soft lattices exhibit heightened sensitivity to structural perturbations, making them especially responsive to targeted modifications in symmetry and defect landscapes. Na₃YBr₆ is structurally flexible, has a poor conductivity ($\sim 4.6 \times 10^{-8}$ S/cm) due to limited long-range Na⁺ connectivity, and a low activation energy (~ 0.15 eV).¹⁴ This suggests that vibrational freedom alone is insufficient without coherent migration pathways. Li₃InCl₆, a benchmark Li-ion halide SSE, offers higher conductivity ($\sim 1.5 \times 10^{-3}$ S/cm) and moderate mechanical robustness, making it attractive for slurry-based integration.⁶⁵ Its monoclinic symmetry supports defect-assisted transport and air stability, features that could inform analogous Na⁺ systems.

While the BVSE method offers rapid identification of conduction pathways and qualitative estimates of migration

barriers, it has inherent limitations.³⁸⁻⁴² The approach simplifies short-range repulsion and does not account for explicit lattice relaxation or temperature-dependent structural dynamics, which can lead to underestimated activation energies.

Table 4: Summary of selected halide and sulfide SSEs, showing space group, ionic conductivity, activation energy, mechanical stiffness, and stability descriptors.

Formula	Space Group	Mobile ion	σ_0 (Scm ⁻¹)	Ea^σ (eV)	Notes	Ref.
NaAlBr ₄	<i>Pnma</i> (No. 62)	Na ⁺	1.2×10^{-5}	0.43-0.53	Limited connectivity; higher migration barriers; less favorable defect topology	16
NaAlBr ₄	<i>P2₁2₁2₁</i> (No. 19)	Na ⁺	10^{-8} -0.11	0.10-0.5	1D ribbon-like Na channels; low Ea; enhanced by NaBr Schottky and Mg/Zn doping	<i>This work</i>
NaAlCl ₄	<i>P2₁2₁2₁</i> (No. 19)	Na ⁺	3.9×10^{-6}	~ 0.45	Fragile lattice; improved via ball milling and composite strategies	50
NaAlCl ₄	<i>P2₁2₁2₁</i> (No. 19)	Na ⁺	1.5×10^{-5}	~ 0.35	Improved Na ⁺ sublattice occupancy; symmetry-enabled conduction; soft lattice	2
Na ₃ YBr ₆	<i>P2₁/c</i> (No. 14)	Na ⁺	4.6×10^{-8}	~ 0.15	Br vibration expands bottlenecks; low Ea but poor mobility	14
Li ₃ InCl ₆	<i>C2</i> (No. 5)	Li ⁺	1.5×10^{-3}	~ 0.31 -0.35	Air-stable; defect-assisted transport; slurry-compatible	66
Na ₃ PS ₄	<i>P4₂1c</i> (No. 114)	Na ⁺	1×10^{-4}	~ 0.25	Grain boundary limited; improved via ball milling and vacancy engineering	67
Li ₁₀ GeP ₂ S ₁₂	<i>P4₂/nmc</i> (No. 137)	Li ⁺	1.2×10^{-2}	~ 0.22 -0.28	1D Li ⁺ channels; highest conductivity among SSEs	68

Additionally, BVSE treats mobile ions as point charges and neglects polarization effects and correlated ion motion, limiting its accuracy in systems with strong ion-ion interactions or soft lattices.^{38–42} Therefore, BVSE-derived barriers should be interpreted as lower-bound estimates, useful for guiding experimental validation and identifying symmetry-enabled conduction channels rather than providing absolute transport metrics.^{38–42}

A similar symmetry-driven enhancement is observed in NaAlCl₄, where the transition from *Pnma* to *P2₁2₁2₁* improves conductivity by nearly an order of magnitude and reduces activation energy from 0.45 eV to 0.35 eV.^{2,50} This improvement is attributed to increased Na⁺ sublattice occupancy and reduced bottleneck constraints, despite the fact that the chloride lattice is mechanically stiffer than its bromide analogue. These findings underscore the critical role of symmetry tuning and defect engineering in optimizing the performance of halide-based SSEs. In particular, systems characterized by inherently soft lattices exhibit heightened sensitivity to structural perturbations, making them especially responsive to targeted modifications in symmetry and defect landscapes. Na₃YBr₆ is structurally flexible, has a poor conductivity (~4.6×10⁻⁸ S/cm) due to limited long-range Na⁺ connectivity, and a low activation energy (~0.15 eV).¹⁴ This suggests that vibrational freedom alone is insufficient without coherent migration pathways. Li₃InCl₆, a benchmark Li-ion halide SSE, offers higher conductivity (~1.5×10⁻³ S/cm) and moderate mechanical robustness, making it attractive for slurry-based integration.⁶⁵ Its monoclinic symmetry supports defect-assisted transport and air stability, features that could inform analogous Na⁺ systems.

In contrast, sulfide based SSEs such as Na₃PS₄ and Li₁₀GeP₂S₁₂ (LGPS) exhibit superior conductivities (~10⁻⁴ to 10⁻² S/cm) and lower activation energies (~0.22 - 0.28 eV), but at the cost of higher mechanical stiffness and moisture sensitivity.^{67,68} In particular LGPS remains the highest-performing SSE to date, with 1D Li⁺ channels.⁶⁸ However, its chemical instability and processing challenges limit its practical deployment without protective coatings or hybrid architectures. The halide derivatives, especially those adopting *P2₁2₁2₁* symmetry, offer a compelling balance of mechanical softness, defect tunability, and moderate conductivity.

These features position them as interesting candidates for next-generation Na-ion SSEs, particularly when paired with targeted doping strategies and interface engineering. The comparative data underscore the need to optimize both structural symmetry and defect chemistry to bridge the performance gap between halide and sulfide electrolytes. By strategically manipulating these parameters, it becomes possible to enhance ionic transport pathways and mitigate mechanical instabilities, thereby advancing the design of next-generation halide SSEs with improved conductivity and robustness.

Table 5: Comparative transport and structural properties of halide-based sodium SSEs.

Structure	E _g (eV)	NaCl Schottky energy (eV/def)	D ₀ (cm ² /s)	σ ₀ (S cm ⁻¹)	E _a ^a (eV)	Ref.
NaAlBr ₄	4.29	0.96	4.48×10 ⁻⁶	0.11	0.10	This work
Na ₂ Sr ₃ Cl ₈	4.85	1.03	1.07×10 ⁻⁷	3.29×10 ⁻³	0.15	69
Na ₂ MgCl ₄	4.74	0.97	1.65×10 ⁻⁹	1.70×10 ⁻⁴	0.17	34,44
Na ₂ Mg ₃ Cl ₈	4.90	1.49	3.00×10 ⁻⁸	1.26×10 ⁻³	0.18	27,69
Na ₂ ZnCl ₄	3.50	0.96	2.33×10 ⁻⁹	2.15×10 ⁻⁴	0.18	74
Na ₂ Ca ₃ Cl ₈	5.03	1.11	1.11×10 ⁻⁷	3.78×10 ⁻³	0.20	69
Na ₂ SrCl ₄	4.50	0.84	1.16×10 ⁻⁹	8.07×10 ⁻⁵	0.21	44
Na ₂ CaCl ₄	4.45	0.79	9.81×10 ⁻¹⁰	6.94×10 ⁻⁵	0.21	44
Na ₂ Zn ₃ Cl ₈	2.98	1.21	7.74×10 ⁻⁹	3.07×10 ⁻⁴	0.27	69
Na ₆ BaCl ₈	4.30	0.82	2.46×10 ⁻¹¹	3.22×10 ⁻⁶	0.38	43
Na ₆ SrCl ₈	4.53	0.97	2.05×10 ⁻¹¹	2.69×10 ⁻⁶	0.39	43
Na ₆ CaCl ₈	4.63	0.99	1.64×10 ⁻¹¹	2.15×10 ⁻⁶	0.39	43
Na ₆ ZnCl ₈	4.06	1.28	9.12×10 ⁻¹²	1.19×10 ⁻⁶	0.41	43
Na ₆ MgCl ₈	5.10	1.26	8.33×10 ⁻¹²	2.14×10 ⁻⁶	0.56	43,70

Among the evaluated materials, NaAlBr₄ remains the standout sodium-ion conductor, exhibiting the highest ionic conductivity (0.11 S/cm), which is over an order of magnitude greater than the best-performing chloride analogues such as Na₂Ca₃Cl₈ (3.78×10⁻³ S/cm) and Na₂Sr₃Cl₈ (3.29×10⁻³ S/cm). This superior conductivity is underpinned by its remarkably low activation energy (E_a = 0.10 eV) and the highest diffusion coefficient (D₀ = 4.48×10⁻⁶ cm²/s), indicating a highly mobile sodium sublattice and facile Na⁺ migration.

Structurally, the incorporation of Br⁻ expands the lattice due to its larger ionic radius, which softens the framework and reduces Coulombic interactions with Na⁺. These effects collectively lower the energy barriers for ion transport and enhance defect accommodation.

In contrast, chloride-based compounds generally exhibit higher activation energies (0.15–0.56 eV) and lower conductivities, with several materials such as Na₂MgCl₄, Na₂ZnCl₄, and Na₂CaCl₄ showing conductivity values below 10⁻⁴ S/cm.^{34,44} Even structurally similar compounds like Na₂Mg₃Cl₈ and Na₂Zn₃Cl₈, despite moderate improvements, remain significantly less conductive than NaAlBr₄. Notably, the Na₆MCl₈ (M = Ba, Sr, Ca, Zn, Mg) family exhibits the lowest conductivities and highest activation energies, with σ₀ values in the 10⁻⁶ S/cm range and E_a approaching 0.56 eV for Na₆MgCl₈, further emphasizing the transport limitations of these frameworks.^{43,70}

Similar behaviour has been reported in other bromide-containing systems such as Li₃OBr and Na₃OBr, which outperform their chloride counterparts (Li₃OCl and Na₃OCl) in terms of ionic conductivity and activation energy.^{71,72} In these systems, Br⁻ substitution similarly enhances lattice polarizability and expands migration pathways, leading to reduced energy barriers and improved alkali-ion mobility. This recurring trend across both lithium and sodium-based oxyhalides underscores the beneficial role of bromide chemistry in optimizing SSE performance. Moreover, anion substitution strategies such as Li₃OCl_{1-x}Br_x have proven

effective in tuning conductivity by leveraging the synergistic effects of mixed halide environments.⁷¹⁻⁷³ Partial replacement of Cl⁻ with Br⁻ introduces local lattice distortions and increases polarizability, which collectively facilitate Na⁺ or Li⁺ migration. These mixed-anion systems often exhibit lower activation energies and enhanced ionic conductivities compared to their pure halide analogues, offering a versatile design pathway for tailoring transport properties without compromising structural integrity. Such substitutional approaches highlight the broader utility of bromide incorporation—not only as a direct replacement but also as a strategic modifier within complex halide frameworks.

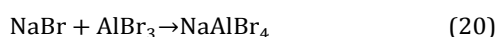
Future directions will likely focus on expanding this substitutional chemistry to multi-anion and multi-cation systems, exploring synergistic effects with other anions (e.g., I⁻, F⁻), and integrating bromide-based electrolytes into full-cell architectures. Additionally, coupling computational screening with targeted synthesis will be essential to identify new bromide-rich frameworks that balance conductivity, stability, and processability for next-generation solid-state batteries.

3.5 Synthesis strategies of NaAlBr₄

Synthetic strategies for NaAlCl₄ (space group *P2₁2₁2₁*) and NaAlBr₄ (space group *Pnma*) have been reported.^{16,50} NaAlCl₄ was obtained via ball milling of equimolar NaCl-AlCl₃ precursors, yielding a Na⁺ conductivity of 3.9×10⁻⁶ S/cm at 30°C and an activation energy of 0.46 eV.⁵⁰ Notably, annealing at 100 and 200°C led to a deterioration in transport properties.⁵⁰ A similar ball milling method was used to obtain NaAlBr₄ in *Pnma* space group, resulting in a Na⁺ conductivity of 1.2×10⁻⁵ S/cm at 30°C with experimental activation energy of 0.43 eV.¹⁶ In addition, molecular simulation yields lower activation energies of 0.26 and 0.16 eV for Na⁺ vacancy and Na⁺ interstitial mechanism, respectively.¹⁶ Miyazaki et al. highlight the improvement of transport properties of NaAlBr₄ against NaAlCl₄, but both structures have different space group (*Pnma* and *P2₁2₁2₁*, respectively).¹⁶

While experimental methods have successfully yielded NaAlBr₄ in the *Pnma* space group with enhanced ionic conductivity, a complementary theoretical evaluation assuming the *P2₁2₁2₁* space group can provide insight into the energetics of its formation and guide future synthesis strategies.

The formation of NaAlBr₄ can be evaluated theoretically by using the energy of possible reactions for the synthesis of the NaAlBr₄ considering the energetics difference between reactants and product constituents. For instance, as follows:



resulting in pristine NaAlBr₄. The reaction energy (ΔE_R) is evaluated by equation (21):

$$\Delta E_R = \sum(E_{\text{Prod}} - E_{\text{React}}) \quad (21)$$

where $E_{\text{Prod}} = E_L^{\text{NaAlBr}_4}$ and $E_{\text{React}} = E_L^{\text{AlBr}_3} + E_L^{\text{NaBr}}$ represent the lattice energy of product and reactants, respectively, and the sums runs over the product/reactant constituents.

The lattice energies of reactant and product constituents are computed by using the force field and DFT methods adopting the same setup described in Section 2. The results show a ΔE_R of -0.28 and -1.89 eV for force field and DFT approaches, respectively. Taking the thermodynamic data of $E_L^{\text{NaBr}} = -7.32$ kJ/mol (-0.076 eV/f.u), $E_L^{\text{AlBr}_3} = -524.70$ kJ/mol (-5.44 eV/f.u),⁴⁴ and estimating the $E_L^{\text{NaAlBr}_4} = -574.76$ kJ/mol (-5.96 eV/f.u), the reaction (21) yield $\Delta E_R = -0.34$ eV/f.u which lies in the mid-range between the value obtained from FF and DFT computations. This analysis implies that the reaction (20) is kinetically favourable with low energetics cost, and thus may proceed under mild conditions without requiring significant thermal activation. These results align well with those discussed earlier concerning convex hull energy.

In summary, NaAlBr₄ with orthorhombic *P2₁2₁2₁* symmetry can be synthesized via mechanochemical routes such as high-energy ball milling, using stoichiometric mixtures of NaBr and AlBr₃ under inert atmosphere to prevent hydrolysis and ensure phase purity.⁵⁰ To introduce M²⁺ doping (e.g., Mg²⁺, Ca²⁺), partial substitution of NaBr with the corresponding MBr₂ precursor enables incorporation at Na⁺-sites, generating Na⁺ vacancies [$x\text{MBr}_2 + (1-x)\text{NaBr} + \text{AlBr}_3 \rightarrow \text{Na}_{1-x}\text{M}_x\text{AlBr}_4$] that may enhance ionic transport. Similarly, divalent doping at Al³⁺-site compensating the charge with Na⁺ interstitial [$x\text{MBr}_2 + (1-x)\text{AlBr}_3 + \text{NaBr} \rightarrow \text{Na}_{1+x}\text{M}_x\text{Al}_{1-x}\text{Br}_4$] can further enhance the transport properties. The resulting defect landscape and local structural distortions are expected to influence both conductivity and stability, warranting further investigation. Careful control of dopant concentration and milling parameters is essential to avoid amorphous or secondary phase formation. Post-synthesis annealing under controlled atmosphere can improve crystallinity while preserving the targeted space group symmetry. This approach provides a scalable pathway for tailoring halide-based conductors via controlled defect engineering.

We would encourage experimental researchers to synthesize via ball milling pristine and divalent doping NaAlBr₄, and to evaluate its performance as a solid electrolyte in Na-ion batteries.

4. Concluding Remarks

The present study involves a comprehensive computational investigation of the orthorhombic NaAlBr₄ compound with a space group *P2₁2₁2₁*, integrating structural, thermodynamic, mechanical, defect and transport analyses to assess its perspective as a halide-based solid-state electrolyte (SSE) for sodium-ion batteries.

The symmetry lowering from the conventional *Pnma* phase to the *P2₁2₁2₁* configuration is found to be both structurally and energetically favourable. This transformation enables the formation of one-dimensional Na⁺ conduction ribbons along the *b*-axis, which are absent in the higher-symmetry phase. These channels provide a crystallographic basis for enhanced ionic mobility and

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directional transport, underscoring the importance of symmetry control in designing functional conduction pathways.

Thermodynamic stability is evaluated through decomposition reaction energies derived from DFT computations. Calculated extraction energies confirm that NaAlBr₄ is stable against elemental decomposition, particularly with respect to Na⁺ and Br⁻ removal. These values indicate strong bonding and resistance to phase breakdown under ambient conditions. Additionally, the global instability index suggests minimal bond strain and excellent valence matching, reinforcing the structural coherence and thermodynamic resilience of the *P*₂₁*2*₁*2*₁ phase. In addition, convex-hull computations and the evaluation of reaction energies confirm the stability and synthesizability of NaAlBr₄ in *P*₂₁*2*₁*2*₁ space group. Together, these metrics confirm that the enhanced transport features observed in the distorted structure are intrinsic to a thermodynamically stable ground state. Mechanical stability is confirmed through elastic tensor analysis using both classical force field and density functional theory methods. The material considered exhibits moderate elastic anisotropy and high compressibility, indicating mechanical compliance suitable for interface integration in battery architectures.

Defect chemistry analysis reveals that both the NaBr Schottky-type and Na⁺ Frenkel defects are the most thermodynamically favourable intrinsic defects, contributing to native carrier generation. Divalent doping with Zn²⁺ and Mg²⁺ at the Na⁺-site further reduces defect formation energies and promotes the formation of mobile Na⁺ vacancies. These findings suggest that defect engineering, both intrinsic and external, can strategically be employed to enhance the ionic conductivity, offering a tuneable route for optimizing transport behaviour in halide frameworks.

Transport properties are evaluated using bond valence site energy method, revealing low activation energies for Na⁺ migration along the *b*-axis channels. The predicted ionic conductivity reaches up to a value of 0.11 S/cm at room temperature, positioning NaAlBr₄ among the most efficient halide SSEs. The directional nature of the conduction pathways, enabled by symmetry breaking, reinforces the importance of crystallographic control in designing high-performance ionic conductors.

In summary, the orthorhombic NaAlBr₄ emerges as a thermodynamically stable, mechanically compliant, and defect-tuneable SSE with symmetry-enabled conduction pathways and high ionic conductivity. These insights provide us with a robust framework for a rational design of halide-based electrolytes through symmetry modification and defect engineering, contributing to the development of next generation sodium-ion batteries with enhanced performance and reliability.

Conflicts of interest

There are no conflicts to declare.

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Data Availability Statement

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- 1) Several computational programs have been used. References of these programs are given in the list of references.
- 2) Additional datasets supporting this article are given as part of the Supplementary Information (SI) file which includes the inputs for computations of some specific properties.
- 3) Specific results can be obtained from the corresponding author.

