

Cite this: *Digital Discovery*, 2025, 4, 204Received 5th September 2024
Accepted 25th November 2024

DOI: 10.1039/d4dd00286e

rsc.li/digitaldiscovery

Machine learning for accelerated prediction of lattice thermal conductivity at arbitrary temperature

Zihe Li,^a Mengke Li,^a Yufeng Luo,^a Haibin Cao,^a Huijun Liu ^{*a} and Ying Fang^{*b}

Efficient evaluation of lattice thermal conductivity (κ_L) is critical for applications ranging from thermal management to energy conversion. In this work, we propose a neural network (NN) model that allows ready and accurate prediction of the κ_L of crystalline materials at arbitrary temperature. It is found that the data-driven model exhibits a high coefficient of determination between the real and predicted κ_L . Beyond the initial dataset, the strong predictive power of the NN model is further demonstrated by checking several systems randomly selected from previous first-principles studies. Most importantly, our model can realize high-throughput screening on countless systems either inside or beyond the existing databases, which is very beneficial for accelerated discovery or design of new materials with desired κ_L .

1. Introduction

The lattice thermal conductivity (κ_L) plays an essential role in various application scenarios. For example, thermoelectric materials require low κ_L to enhance energy conversion efficiency,^{1–4} while high κ_L is needed to dissipate excessive thermal energy in electronic devices.^{5–7} It is therefore extremely important to discover or design specific systems with desired κ_L . Theoretically speaking, the κ_L can be accurately calculated by solving the phonon Boltzmann transport equation (BTE)^{8,9} within the framework of density functional theory (DFT), which is however limited by high computational cost, especially for systems with large unit cells and/or low symmetry. On the other hand, although classic molecular dynamics (MD) simulations^{10,11} can deal with large-scale systems, their accuracy is highly dependent on the choice of interatomic potentials.

Very recently, machine learning (ML) methods have attracted considerable attention in predicting the κ_L of given systems since they can deal with a huge search space at extremely low computational cost.^{12–25} For instance, Wang *et al.*¹⁴ developed various nonlinear regression ML models based on the κ_L of 5486 materials, which were computed by using the Automatic GIBBS Library (AGL) method. They found that the eXtreme Gradient Boosting (XGBoost) model exhibits the best prediction performance, which is utilized to screen candidate thermoelectric materials with ultra-low κ_L . By combining graph neural networks and random forest algorithms, Zhu *et al.*¹⁷ predicted

the room temperature κ_L of numerous inorganic compounds directly from their atomic structures, and a set of rare-earth chalcogenides were identified as a new class of promising thermoelectric materials. After a thorough algorithm comparison, Yang *et al.*¹⁹ found that Bayesian optimization²⁰ using the Gaussian process allows for fast and accurate measurement of κ_L over a wide temperature range. In addition, Qin *et al.*²⁴ constructed fifteen ML models for accurate prediction of κ_L , where the dataset consists of experimentally measured κ_L of 350 different materials and the input features include 8 basic properties of the compounds obtained from first-principles calculations. It should be noted that most of these studies were focused on the κ_L at 300 K, which is not beneficial for the discovery of systems with desired κ_L in a wide temperature range and large search space. Besides, some of the involved datasets contain κ_L calculated using semi-empirical models, which may lead to insufficient accuracy of the derived ML model. Moreover, to predict κ_L in a high-throughput style, it is necessary to adopt input features that can be readily obtained, which is however less considered in previous studies.

In this work, using a dataset completely obtained from first-principles calculations, we propose a neural network (NN) model by which the κ_L can be readily obtained at arbitrary temperature. The strong predictive power of our model is demonstrated by good agreement between the predicted and real κ_L , both inside and beyond the initial dataset. By leveraging the established NN model, we give a high-throughput prediction of the κ_L of 32 252 compounds from the Inorganic Crystal Structure Database (ICSD)²⁶ in a wide temperature range from 100 to 1000 K, where many promising candidates are quickly identified for effective thermoelectric conversion or heat dissipation.

^aKey Laboratory of Artificial Micro- and Nano-Structures of Ministry of Education and School of Physics and Technology, Wuhan University, Wuhan 430072, China. E-mail: phlhj@whu.edu.cn

^bSchool of Computer Science, Wuhan University, Wuhan 430072, China. E-mail: fangying@whu.edu.cn



2. Methods

Our ML approach has been performed by adopting a NN algorithm, where the fully connected pyramid architecture contains an input unit, hidden layers, and the quantitative labels of κ_L as the output unit.²⁷ To obtain an accurate NN model, the initial dataset is randomly divided into training (80%), validation (10%), and testing (10%) sets. The input features consist of 290 compositional descriptors derived from 58 elemental properties of the constituent atoms, which can be readily generated by the XENONPY package.^{28–30} As generally used in many ML methods, all the feature values are scaled to a range of 0–1. During the training process, the input unit receives the feature data, which is then manipulated by three hidden layers, each containing 150 neurons. When the data are moved from one neuron to another, their value will be multiplied by a particular weight parameter, and all the neurons are activated with an Exponential Linear Unit (ELU).³¹ It should be noted that the NN algorithm allows back propagation, which means that the weight and bias parameters of each neuron are updated so that the loss function can be minimized. To achieve the best prediction accuracy, the hyperparameters are fine-tuned by adopting the Adam

optimizer with a learning rate of 0.005. Meanwhile, the batch size is set as 128 and a dropout of 0.2 is used for each hidden layer.

3. Results and discussion

To construct a reliable dataset, we have collected the κ_L of 103 crystalline systems at various temperatures (84–1200 K) from the literature,^{32,33} which in turn leads to 1795 entries for the NN training. It should be noted that all the κ_L values were obtained from accurate first-principles calculations by solving the linearized phonon BTE using an iterative process. Fig. 1(a) shows the distribution of the 1795 samples, where we find that the κ_L value spans several orders of magnitude. For example, the κ_L of CsK₂Sb (space group no. 225) is only 0.03 W m^{−1} K^{−1} at 1012 K. In contrast, the κ_L of SiC (space group no. 216) exceeds 5000 W m^{−1} K^{−1} at 148 K. Besides, we see from the inset that most κ_L are located in a narrow range of 0–10 W m^{−1} K^{−1}. Such an uneven distribution of input data may adversely affect the training process and thus the prediction accuracy of the NN model. Therefore, instead of directly using the original κ_L values, we adopt their natural logarithms so that an approximate normal

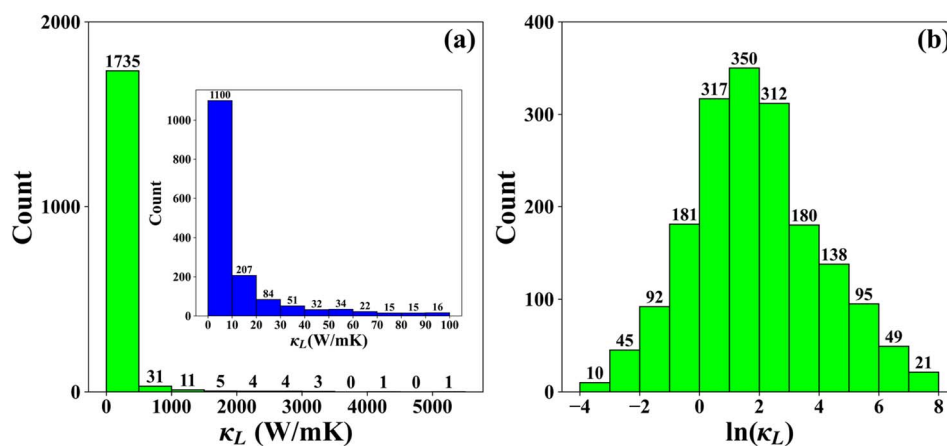


Fig. 1 The distribution of 1795 entries in the initial dataset according to (a) their lattice thermal conductivities and (b) the natural logarithmic values. The inset of (a) shows detailed distribution for those with small κ_L .

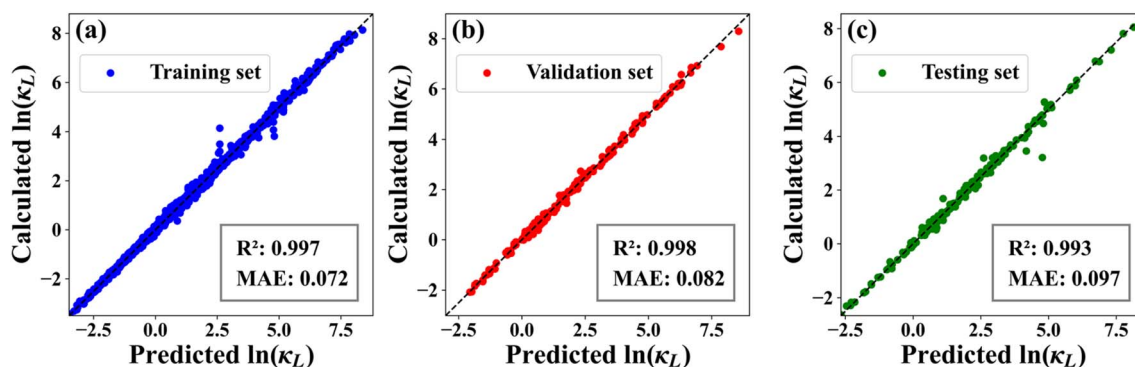


Fig. 2 The intuitive linear correlation between the real and NN-predicted lattice thermal conductivities (natural logarithmic values) for the (a) training, (b) validation, and (c) testing sets.



distribution can be obtained, as shown in the histogram in Fig. 1(b).

It is well known that the input features play a crucial role in determining the predictive power of the ML model. In the present work, we adopt 290 compositional descriptors generated from 58 elemental properties of the constituent atoms. In addition, the feature vector includes space groups of the systems and temperature by default. By utilizing the training and validation sets as benchmarks for hyperparameter tuning, we establish a well-optimized NN model to rapidly predict the κ_L of any given system at arbitrary temperature. Fig. 2(a) and (b) respectively show the intuitive linear correlation between the NN-predicted and real κ_L (on the natural logarithmic scale) for the training (1436 entries) and validation sets (179 entries), where we see that all the data points are located around the dashed line representing equality. Besides, the coefficient of determination (R^2) between the predicted and real $\ln(\kappa_L)$ is found to be 0.997 and 0.998 for the training and validation sets, respectively. Meanwhile, the corresponding mean absolute

errors (MAEs) are as small as 0.072 and 0.082 (note that the involved $\ln(\kappa_L)$ varies from -4 to 8). Even for the testing set (180 entries) that is not used during the training process, the NN model can still give strong prediction accuracy. As illustrated in Fig. 2(c), the R^2 between the predicted and real $\ln(\kappa_L)$ is as high as 0.993 with a small MAE of 0.097. All these findings suggest that the data-driven NN model is highly reliable and can be used to effectively predict the κ_L of crystalline materials.

Beyond the initial dataset, we have employed the NN model to predict the κ_L of 10 compounds that are randomly selected from the literature, as shown in Fig. 3(a) in a temperature range from 300 to 600 K. Although the values (40 entries in total) span over several orders of magnitude, the NN-predicted κ_L are in good agreement with those obtained from first-principles calculations. ^{34–41} For example, the room temperature κ_L of AlVFe_2 (space group no. 216) is calculated to be $48.0 \text{ W m}^{-1} \text{ K}^{-1}$,³⁹ which almost coincides with our NN-predicted result of $48.1 \text{ W m}^{-1} \text{ K}^{-1}$. Besides, the κ_L of GaN (space group no. 186) is predicted to be $102.5 \text{ W m}^{-1} \text{ K}^{-1}$ at 500 K, which is close to the

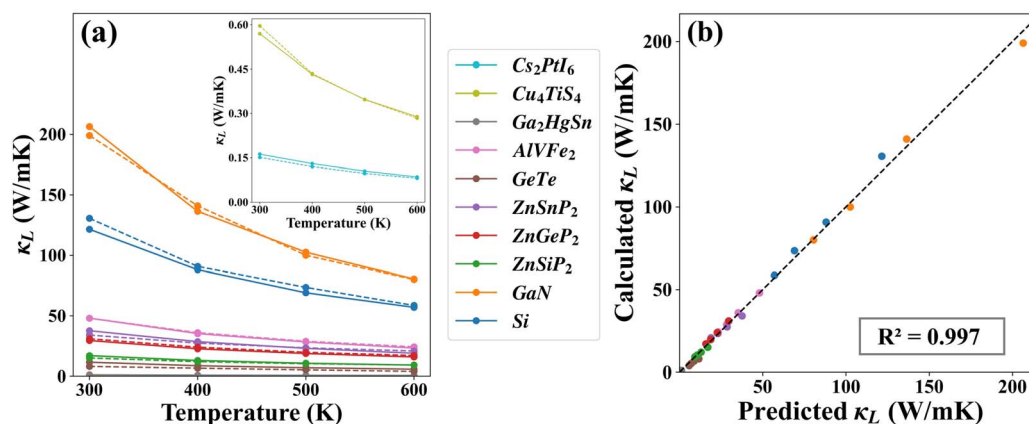


Fig. 3 (a) The NN-predicted κ_L (solid line) of ten randomly selected compounds beyond the initial dataset, plotted as a function of temperature. For comparison, the results from first-principles calculations (dashed line) are also shown. The inset shows an enlarged view of those with ultra-small κ_L . (b) The intuitive linear correlation between the NN-predicted κ_L and those from first-principles calculations.

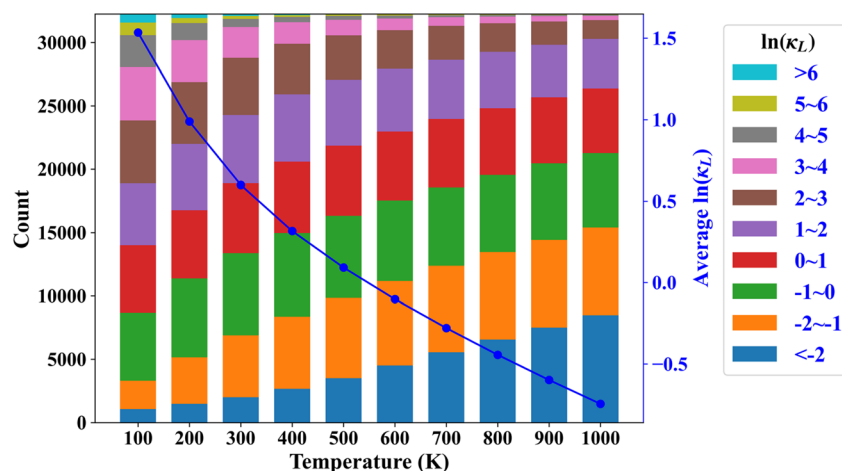


Fig. 4 The distribution of 32 252 systems in the ICSD according to their predicted lattice thermal conductivities (natural logarithmic values) at different temperatures. The blue circles refer to the average values.



calculated value of $100.0 \text{ W m}^{-1} \text{ K}^{-1}$.³⁷ At a higher temperature of 600 K, the NN-predicted κ_L of Cu_4TiS_4 (space group no. 219) is found to be $0.29 \text{ W m}^{-1} \text{ K}^{-1}$, which is almost identical to the first-principles result of $0.28 \text{ W m}^{-1} \text{ K}^{-1}$.⁴¹ To have a statistical analysis, Fig. 3(b) shows the intuitive linear correlation between the real and predicted values of κ_L for these 10 compounds at different temperatures. We see that all the data points are distributed around the dashed line with a slope of 1, and the R^2 between the real and predicted κ_L is as high as 0.997. All these observations indeed substantiate the strong predictive power of our NN model in evaluating the κ_L at various temperatures.

As mentioned above, the input features for our NN model can be readily obtained from 58 elemental properties of the constituent atoms, which is very beneficial to predict κ_L of any crystalline system at negligible computational cost. For instance, the κ_L of 32 252 systems in the ICSD can be quickly obtained in a wide temperature range from 100 to 1000 K. Fig. 4 plots the distribution of these systems according to their predicted $\ln(\kappa_L)$, where we see that the quantity of systems possessing lower κ_L becomes increased while that with higher κ_L is decreased at elevated temperature. As a consequence, the average $\ln(\kappa_L)$ decreases with increasing temperature. Such an observation is consistent with the general understanding that the κ_L is usually inversely proportional to the temperature for most systems. More importantly, our high-throughput prediction provides a good opportunity to discover new materials with desired κ_L that are suitable for different application scenarios. For example, it is well-known that good thermoelectric materials require low κ_L to enhance the energy conversion efficiency. If we focus on room temperature, we find that 22 050 compounds have ultra-small κ_L in the range of $0.1\text{--}5 \text{ W m}^{-1} \text{ K}^{-1}$. Among them, 4957 systems exhibit moderate band gaps ($0.1\text{--}2.0 \text{ eV}$), which implies that they could be possible high-performance thermoelectric candidates. In particular, it is found that 582 compounds are composed of non-toxic and earth-abundant elements,⁴² which is strongly desirable and highly competitive for thermoelectric applications. Table 1 summarizes some of these candidate systems, where the room temperature κ_L is further restricted to be lower than $0.15 \text{ W m}^{-1} \text{ K}^{-1}$. On the other hand, we see from Table 2 that there are 50 systems with very high κ_L (exceeding $300 \text{ W m}^{-1} \text{ K}^{-1}$) at 300 K, which are suggested to be very promising candidates for heat dissipation. A typical example is the diamond with a NN-predicted κ_L as high as $2091.76 \text{ W m}^{-1} \text{ K}^{-1}$, which is consistent with that measured experimentally⁴³ and further confirms the reliability of our ML approach. It should be noted that we focus on the systems with finite band gaps and the effect of electron-phonon interactions is not considered. Besides, it is surprising to find that some systems exhibit κ_L even larger than that of the diamond, such as C_4Os_2 ($2148.42 \text{ W m}^{-1} \text{ K}^{-1}$, space group no. 194) and COs ($3797.02 \text{ W m}^{-1} \text{ K}^{-1}$, space group no. 216), which deserve further theoretical and experimental investigations. It should be emphasized that although we are dealing with room temperature, a similar picture can also be found at other temperatures, as already implied in Fig. 4.

It is important to note that the above-mentioned 32 252 systems all feature integer stoichiometry. Beyond the ICSD or

Table 1 The 40 compounds screened from the ICSD with moderate band gaps (in units of eV) and ultra-small κ_L (in units of $\text{W m}^{-1} \text{ K}^{-1}$) at 300 K, which are desirable for thermoelectric conversion. The corresponding space group is also shown

Compound	Space group	κ_L	Gap
$\text{Cs}_{16}\text{O}_{16}\text{Zn}_8$	14	0.103	1.94
$\text{Fe}_4\text{K}_{12}\text{O}_8$	92	0.105	1.29
$\text{I}_{10}\text{K}_2\text{Sn}_4$	140	0.106	1.50
$\text{Ca}_4\text{Sn}_4\text{Sr}_4$	62	0.110	0.34
$\text{Cs}_8\text{I}_{24}\text{Sn}_4$	225	0.113	0.12
$\text{Fe}_4\text{Rb}_{12}\text{S}_{12}$	64	0.113	1.34
$\text{Cs}_8\text{Cu}_2\text{K}_4\text{O}_{16}\text{Si}_4$	136	0.113	1.65
$\text{C}_{12}\text{Cs}_4\text{Fe}_2\text{K}_2\text{N}_{12}$	14	0.114	0.14
$\text{Cr}_8\text{Fe}_4\text{O}_{32}\text{Rb}_4$	62	0.116	1.89
$\text{Cs}_2\text{Mo}_6\text{O}_{18}$	12	0.120	0.77
$\text{Fe}_4\text{Rb}_8\text{S}_{10}$	2	0.120	0.93
$\text{Cs}_4\text{S}_6\text{Ti}_2$	36	0.121	1.70
$\text{Fe}_4\text{Rb}_{12}\text{S}_{12}$	14	0.121	1.31
$\text{Cs}_6\text{S}_{27}\text{Ti}_6$	146	0.124	1.45
$\text{Ba}_6\text{Cr}_4\text{O}_{18}\text{W}_2$	194	0.125	1.75
$\text{Ba}_6\text{Cr}_2\text{O}_{10}$	140	0.125	1.47
$\text{Cs}_4\text{Li}_2\text{Mn}_2\text{O}_8$	36	0.126	1.84
$\text{Fe}_{10}\text{K}_{14}\text{S}_{20}$	15	0.126	1.02
$\text{Ba}_4\text{Fe}_4\text{Li}_2\text{N}_6$	15	0.127	0.10
$\text{Fe}_6\text{Na}_{14}\text{O}_{16}$	2	0.128	1.99
$\text{Ba}_6\text{Cr}_4\text{Mo}_2\text{O}_{18}$	194	0.129	1.12
Cr_5CsS_8	12	0.132	0.71
$\text{Fe}_4\text{Rb}_2\text{S}_6$	63	0.134	0.25
$\text{Ba}_8\text{Cr}_4\text{O}_{16}$	62	0.135	1.75
$\text{Fe}_2\text{K}_6\text{O}_5$	8	0.135	1.27
$\text{Ba}_8\text{Cr}_4\text{Nb}_4\text{O}_{24}$	194	0.136	1.78
$\text{Fe}_4\text{K}_4\text{Na}_8\text{O}_{12}$	62	0.138	1.98
$\text{Fe}_4\text{K}_8\text{O}_{10}$	14	0.138	2.00
$\text{Fe}_4\text{K}_{12}\text{O}_{16}$	62	0.139	0.29
$\text{Fe}_2\text{K}_4\text{Na}_2\text{O}_6$	67	0.139	1.99
I_6Sn_3	12	0.142	1.70
$\text{Fe}_2\text{K}_6\text{O}_6$	12	0.142	1.80
$\text{Fe}_4\text{K}_8\text{O}_{16}$	62	0.143	1.54
$\text{Fe}_2\text{Na}_{12}\text{S}_8$	36	0.144	1.87
$\text{Fe}_2\text{Na}_8\text{O}_6$	9	0.145	1.10
$\text{Fe}_{10}\text{Na}_6\text{O}_{18}$	15	0.146	1.54
$\text{Fe}_2\text{K}_3\text{NaO}_8$	164	0.147	1.40
$\text{Fe}_4\text{K}_{12}\text{S}_{12}$	14	0.147	1.25
$\text{Cs}_4\text{Cu}_4\text{S}_{16}$	19	0.148	1.87
$\text{Cu}_4\text{K}_4\text{O}_{36}\text{Ta}_{12}$	53	0.149	1.44

other materials databases, it is possible to construct countless samples with fractional stoichiometry by alloying or doping, which provides additional degrees of freedom to tune the κ_L . Within the framework of DFT, it is rather time-consuming or even prohibitive to calculate the κ_L of alloyed or doped systems because very large supercells are usually involved. This is especially the case for high-entropy materials, which hold promise for various applications by selecting specific elements and altering stoichiometry. Fortunately, such a challenging task can be readily fulfilled by using our NN model since the required 290 compositional descriptors are directly derived from the 58 elemental properties of the constituent atoms. Taking a binary system $\text{A}_{w_A}\text{B}_{w_B}$ as an example, where the stoichiometry w_A and w_B could be an integer or a fractional, if the elemental properties of A and B atoms are respectively denoted



Table 2 The 50 systems screened from the ICSD with finite band gaps and very high κ_L (in units of $\text{W m}^{-1} \text{K}^{-1}$) at 300 K, which are desirable for efficient heat dissipation. The corresponding space group is also shown

Compound	Space group	κ_L
B ₄ N ₄	62	327.14
CSn	216	338.96
He	191	351.72
B ₂ P ₂	186	359.27
C ₂ N ₄	36	373.29
B ₄ N ₄	9	378.98
B ₄ N ₄	8	379.82
CHN	44	388.99
B ₃ N ₃	160	413.05
SiSn	216	414.30
C ₂ Si ₂	186	421.59
He	225	424.36
He	229	433.51
He ₂	194	438.02
B ₁₂	166	449.77
B ₆ Si	221	473.05
CN ₂	119	492.50
AsB ₂ P	115	514.13
B ₁₂ C ₃	166	519.33
BP	216	537.37
C ₁₆	194	547.26
CHN	107	550.65
C ₁₆	62	603.44
C ₁₆	67	605.57
B ₂ N ₂	194	607.09
CSi	216	608.69
B ₂ N ₂	187	618.17
B ₂ N ₂	186	619.76
BSb	216	620.35
C ₁₄	166	687.27
C ₈	12	690.93
C ₄ Os ₄	198	749.25
C ₁₂	194	795.39
BN	216	816.54
C ₈	65	896.92
B ₂ C ₄ N ₂	17	902.67
CRu	216	904.49
AsB	216	916.91
C ₁₀	166	969.22
CGe	216	981.78
C ₈	194	1176.85
C ₈	206	1185.38
C ₈	229	1192.31
C ₄	139	1208.59
C ₂	166	1495.80
C ₄	194	1635.08
BC ₂ N	25	1912.43
C ₂	227	2091.76
C ₄ Os ₂	194	2148.42
COs	216	3797.02

by $f_{A,i}$ and $f_{B,i}$ ($i = 1, 2, \dots, 58$), the 290 compositional descriptors can be calculated using:

$$f_{\max,i} = \max(f_{A,i}, f_{B,i}) \text{ (max-pooling)} \quad (1)$$

$$f_{\min,i} = \min(f_{A,i}, f_{B,i}) \text{ (min-pooling)} \quad (2)$$

$$f_{\text{sum},i} = w_A f_{A,i} + w_B f_{B,i} \text{ (weighted sum)} \quad (3)$$

$$f_{\text{ave},i} = w_A^* f_{A,i} + w_B^* f_{B,i} \text{ (weighted average)} \quad (4)$$

$$f_{\text{var},i} = w_A^* (f_{A,i} - f_{\text{ave},i})^2 + w_B^* (f_{B,i} - f_{\text{ave},i})^2 \text{ (weighted variance)} \quad (5)$$

where w_A^* and w_B^* refer to the normalized composition summing up to one. In other words, the NN model is applicable for any compound with either integer or fractional stoichiometry, which allows us to discover or design new materials with target κ_L in an even larger search space.

Although our NN model can be used to accurately predict the κ_L of any crystalline system at arbitrary temperature, it is somewhat similar to a “black box” which is not beneficial to understand the inherent physical mechanism. To address this issue, we become aware that materials with lower κ_L usually have weaker chemical bonds, lower phonon frequencies, and complex unit cells.^{44–46} In principle, such characteristics can be described by two simple structural parameters, namely, the average atomic volume (V_{ave}) and the average atomic mass (m_{ave}). By respectively using V_{ave} and m_{ave} as the horizontal and vertical coordinates, we plot in Fig. 5 the distribution of the above-mentioned 32 252 compounds, where the corresponding room temperature κ_L (natural logarithmic value) is indicated by a color scale. It is interesting to note that the distribution can be approximately viewed as a triangle, where systems with low κ_L tend to be distributed in the upper right corner and those with high κ_L are more likely to be found in the lower left corner. The physical origin is that a larger V_{ave} usually indicates longer distances between atoms in a given system and thus weaker bond strength, while heavier m_{ave} of the constituent atoms in general corresponds to lower phonon frequency. It should be mentioned that such kinds of systems also tend to have complex unit cells. As a consequence, the systems with simultaneously large V_{ave} and m_{ave} would exhibit small κ_L and appear in the upper right corner of the triangle. All these findings demonstrate that our NN model has effectively captured and

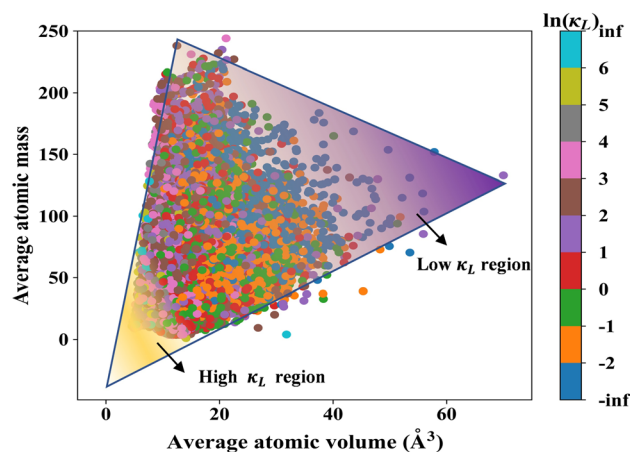


Fig. 5 The distribution of 32 252 compounds in the two-dimensional space defined by the average atomic volume and average atomic mass, where the color of the data points represents the NN-predicted lattice thermal conductivities (natural logarithmic values) at 300 K.



learned the inherent connection between the κ_L and the fundamental structural properties of crystalline materials. Accordingly, the predicted results are highly reliable and very beneficial for accelerated discovery of promising systems with desired κ_L in a large exploration space.

4. Summary

In summary, using the NN algorithm, we propose a machine learning model for rapid and accurate prediction of the κ_L of any crystalline system at arbitrary temperature. It is found that the NN model shows strong predictive power in the training, validating, and testing sets, as demonstrated by the high coefficient of determination and low mean absolute error. By leveraging such a data-driven model with physical intuition, we give a high-throughput prediction of the κ_L of 32 252 compounds in a wide temperature range from 100 K to 1000 K, where many systems with ultra-small or extremely high κ_L are quickly identified. Our work not only enables accelerated discovery of candidate materials with desired κ_L , but also highlights their diverse applications such as thermoelectric conversion and heat dissipation.

Data availability

This study was carried out using publicly available data from ref. 32 and 33.

Conflicts of interest

There are no conflicts of interest to declare.

Acknowledgements

We acknowledge financial support from the National Natural Science Foundation of China (Grant No. 62074114 and 12474019). The numerical calculations in this work have been done on the platform in the Supercomputing Center of Wuhan University.

References

- 1 D. M. Rowe, *CRC Handbook of Thermoelectric*, CRC Press, 1995.
- 2 K. F. Hsu, S. Loo, F. Guo, W. Chen, J. S. Dyck, C. Uher, T. Hogan, E. K. Polychroniadis and M. G. Kanatzidis, Cubic $\text{AgPb}_m\text{SbTe}_{2+m}$: Bulk Thermoelectric Materials with High Figure of Merit, *Science*, 2004, **303**, 818–821.
- 3 K. Biswas, J. He, I. D. Blum, C. I. Wu, T. P. Hogan, D. N. Seidman, V. P. Dravid and M. G. Kanatzidis, High-performance bulk thermoelectrics with all-scale hierarchical architectures, *Nature*, 2012, **489**, 414–418.
- 4 L. D. Zhao, S. H. Lo, Y. Zhang, H. Sun, G. Tan, C. Uher, C. Wolverton, V. P. Dravid and M. G. Kanatzidis, Ultralow thermal conductivity and high thermoelectric figure of merit in SnSe crystals, *Nature*, 2014, **508**, 373–377.
- 5 A. L. Moore and L. Shi, Emerging challenges and materials for thermal management of electronics, *Mater. Today*, 2014, **17**, 163–174.
- 6 X. Qian, J. Zhou and G. Chen, Phonon-engineered extreme thermal conductivity materials, *Nat. Mater.*, 2021, **20**, 1188–1202.
- 7 Z. He, Y. Yan and Z. Zhang, Thermal management and temperature uniformity enhancement of electronic devices by micro heat sinks: A review, *Energy*, 2021, **216**, 119223.
- 8 D. A. Broido, M. Malorny, G. Birner, N. Mingo and D. A. Stewart, Intrinsic lattice thermal conductivity of semiconductors from first principles, *Appl. Phys. Lett.*, 2007, **91**, 231922.
- 9 W. Li, J. Carrete, N. A. Katcho and N. Mingo, ShengBTE: A solver of the Boltzmann transport equation for phonons, *Comput. Phys. Commun.*, 2014, **185**, 1747–1758.
- 10 P. K. Schelling, S. R. Phillpot and P. Keblinski, Comparison of atomic-level simulation methods for computing thermal conductivity, *Phys. Rev. B:Condens. Matter Mater. Phys.*, 2002, **65**, 144306.
- 11 Z. Fan, L. F. C. Pereira, H. Q. Wang, J. C. Zheng, D. Donadio and A. Harju, Force and heat current formulas for many-body potentials in molecular dynamics simulations with applications to thermal conductivity calculations, *Phys. Rev. B:Condens. Matter Mater. Phys.*, 2015, **92**, 094301.
- 12 A. Seko, H. Hayashi, K. Nakayama, A. Takahashi and I. Tanaka, Representation of compounds for machine-learning prediction of physical properties, *Phys. Rev. B*, 2017, **95**, 144110.
- 13 L. Chen, H. Tran, R. Batra, C. Kim and R. Ramprasad, Machine learning models for the lattice thermal conductivity prediction of inorganic materials, *Comput. Mater. Sci.*, 2019, **170**, 109155.
- 14 X. Wang, S. Zeng, Z. Wang and J. Ni, Identification of Crystalline Materials with Ultra-Low Thermal Conductivity Based on Machine Learning Study, *J. Phys. Chem. C*, 2020, **124**, 8488–8495.
- 15 C. Loftis, K. Yuan, Y. Zhao, M. Hu and J. Hu, Lattice Thermal Conductivity Prediction Using Symbolic Regression and Machine Learning, *J. Phys. Chem. A*, 2021, **125**, 435–450.
- 16 H. Miyazaki, T. Tamura, M. Mikami, K. Watanabe, N. Ide, O. M. Ozkendir and Y. Nishino, Machine learning based prediction of lattice thermal conductivity for half-Heusler compounds using atomic information, *Sci. Rep.*, 2021, **11**, 13410.
- 17 T. Zhu, R. He, S. Gong, T. Xie, P. Gorai, K. Nielschb and J. C. Grossman, Charting lattice thermal conductivity for inorganic crystals and discovering rare earth chalcogenides for thermoelectrics, *Energy Environ. Sci.*, 2021, **14**, 3559.
- 18 P. R. Chowdhury and X. Ruan, Unexpected thermal conductivity enhancement in aperiodic superlattices discovered using active machine learning, *npj Comput. Mater.*, 2022, **8**, 12.
- 19 L. Yang, A. Gil, P. S. H. Leong, J. O. Khor, B. Akhmetov, W. L. Tan, S. Rajoo, L. F. Cabeza and A. Romagnoli, Bayesian optimization for effective thermal conductivity



- measurement of thermal energy storage: An experimental and numerical approach, *J. Energy Storage*, 2022, **52**, 104795.
- 20 M. F. Lazin, C. R. Shelton, S. N. Sandhofer and B. M. Wong, High-dimensional multi-fidelity Bayesian optimization for quantum control, *Mach. Learn.: Sci. Technol.*, 2023, **4**, 045014.
 - 21 Z. Guo, P. R. Chowdhury, Z. Han, Y. Sun, D. Feng, G. Lin and X. Ruan, Fast and accurate machine learning prediction of phonon scattering rates and lattice thermal conductivity, *npj Comput. Mater.*, 2023, **9**, 95.
 - 22 Y. Srivastava and A. Jain, End-to-end material thermal conductivity prediction through machine learning, *J. Appl. Phys.*, 2023, **134**, 225101.
 - 23 Z. Wang, J. Ma, R. Hu and X. Luo, Predicting lattice thermal conductivity of semiconductors from atomic-information-enhanced CGCNN combined with transfer learning, *Appl. Phys. Lett.*, 2023, **122**, 152106.
 - 24 G. Qin, Y. Wei, L. Yu, J. Xu, A. D. Rodriguez, H. Wang, Z. Qin and M. Hu, Predicting lattice thermal conductivity from fundamental material properties using machine learning techniques, *J. Mater. Chem. A*, 2023, **11**, 5801.
 - 25 Y. Luo, M. Li, H. Yuan, H. Liu and Y. Fang, Predicting lattice thermal conductivity via machine learning: a mini review, *npj Comput. Mater.*, 2023, **9**, 4.
 - 26 ICSD, <https://icsd.products.fiz-karlsruhe.de>, accessed June 2024.
 - 27 M. Anthony and P. L. Bartlett, *Neural Network Learning: Theoretical Foundations*, Cambridge University Press, 1999.
 - 28 S. Ju, R. Yoshida, C. Liu, S. Wu, K. Hongo, T. Tadano and J. Shiomi, Exploring diamondlike lattice thermal conductivity crystals via feature-based transfer learning, *Phys. Rev. Mater.*, 2021, **5**, 053801.
 - 29 C. Liu, E. Fujita, Y. Katsura, Y. Inada, A. Ishikawa, R. Tamura, K. Kimura and R. Yoshida, Machine Learning to Predict Quasicrystals from Chemical Compositions, *Adv. Mater.*, 2021, **33**, 2102507.
 - 30 XenonPy, <https://github.com/yoshida-lab/XenonPy>, accessed June 2024.
 - 31 D. A. Clevert, T. Unterthiner, and S. Hochreiter, Fast and Accurate Deep Network Learning by Exponential Linear Units (ELUs), *arXiv*, 2016, preprint, arXiv:1511.07289, DOI: [10.48550/arXiv.1511.07289](https://doi.org/10.48550/arXiv.1511.07289).
 - 32 R. Juneja, G. Yumnam, S. Satsangi and A. K. Singh, Coupling the High-Throughput Property Map to Machine Learning for Predicting Lattice Thermal Conductivity, *Chem. Mater.*, 2019, **31**, 5145–5151.
 - 33 R. Jaafreh, Y. S. Kang and K. Hamad, Lattice Thermal Conductivity: An Accelerated Discovery Guided by Machine Learning, *ACS Appl. Mater. Interfaces*, 2021, **13**, 57204–57213.
 - 34 J. Garg, N. Bonini and N. Marzari, High Thermal Conductivity in Short-Period Superlattices, *Nano Lett.*, 2021, **11**, 5135–5141.
 - 35 J. He, M. Amsler, Y. Xia, S. S. Naghavi, V. I. Hegde, S. Hao, S. Goedecker, V. Ozoliņš and C. Wolverton, Ultralow Thermal Conductivity in Full Heusler Semiconductors, *Phys. Rev. Lett.*, 2016, **117**, 046602.
 - 36 L. Wei, X. Lv, Y. Yang, J. Xu, H. Yu, H. Zhang, X. Wang, B. Liu, C. Zhang and J. Zhou, Theoretical Investigation on the Microscopic Mechanism of Lattice Thermal Conductivity of ZnXP_2 ($X = \text{Si, Ge, and Sn}$), *Inorg. Chem.*, 2019, **58**, 4320–4327.
 - 37 Q. Zheng, C. Li, A. Rai, J. H. Leach, D. A. Broido and D. G. Cahill, Thermal conductivity of GaN, ^{71}GaN , and SiC from 150 K to 850 K, *Phys. Rev. Mater.*, 2019, **3**, 014601.
 - 38 M. Sajjad, Q. Mahmood, N. Singh and J. A. Larsson, Ultralow Lattice Thermal Conductivity in Double Perovskite Cs_2PtI_6 : A Promising Thermoelectric Material, *ACS Appl. Energy Mater.*, 2020, **3**, 11293–11299.
 - 39 K. Berland, O. M. Løvvik and R. Tranås, Discarded gems: Thermoelectric performance of materials with band gap emerging at the hybrid-functional level, *Appl. Phys. Lett.*, 2021, **119**, 081902.
 - 40 T. Xing, C. Zhu, Q. Song, H. Huang, J. Xiao, D. Ren, M. Shi, P. Qiu, X. Shi, F. Xu and L. Chen, Ultralow Lattice Thermal Conductivity and Superhigh Thermoelectric Figure-of-Merit in (Mg, Bi) Co-Doped GeTe, *Adv. Mater.*, 2021, **33**, 2008773.
 - 41 T. Zhang, T. Yu, S. Ning, Z. Zhang, N. Qi, M. Jiang and Z. Chen, Extremely Low Lattice Thermal Conductivity Leading to Superior Thermoelectric Performance in Cu_4TiSe_4 , *ACS Appl. Mater. Interfaces*, 2023, **15**, 32453–32462.
 - 42 O. Caballero-Calero, J. R. Ares and M. Martín-González, Environmentally Friendly Thermoelectric Materials: High Performance from Inorganic Components with Low Toxicity and Abundance in the Earth, *Adv. Sustainable Syst.*, 2021, **5**, 2100095.
 - 43 S. Li, Q. Zheng, Y. Lv, X. Liu, X. Wang, P. Y. Huang, D. G. Cahill and B. Lv, High thermal conductivity in cubic boron arsenide crystals, *Science*, 2018, **361**, 579–581.
 - 44 E. S. Toberer, A. Zevkink and G. J. Snyder, Phonon engineering through crystal chemistry, *J. Mater. Chem.*, 2021, **21**, 15843.
 - 45 W. G. Zeier, A. Zevkink, Z. M. Gibbs, G. Hautier, M. G. Kanatzidis and G. J. Snyder, Thinking Like a Chemist: Intuition in Thermoelectric Materials, *Angew. Chem., Int. Ed.*, 2016, **55**, 6826–6841.
 - 46 T. Zhu, Y. Liu, C. Fu, J. P. Heremans, J. G. Snyder and X. Zhao, Compromise and Synergy in High-Efficiency Thermoelectric Materials, *Adv. Mater.*, 2017, **29**, 1605884.

