

# Scattering resonances in the rotational excitation of HDO by Ne and *normal*-H<sub>2</sub>: theory and experiment†

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The rotational excitation of a singly deuterated water molecule (HDO) by a heavy atom (Ne) and a light diatomic molecule (H<sub>2</sub>) is investigated theoretically and experimentally in the near-threshold regime. Crossed-molecular-beam measurements with a variable crossing angle are compared to close-coupling calculations based on high-accuracy potential energy surfaces. The two lowest rotational transitions,  $0_{00} \rightarrow 1_{01}$  and  $0_{00} \rightarrow 1_{11}$ , are probed in detail and a good agreement between theory and experiment is observed for both transitions in the case of HDO + Ne, where scattering resonances are however blurred out experimentally. In the case of HDO + H<sub>2</sub>, the predicted theoretical overlapping resonances are faithfully reproduced by experiment for the  $0_{00} \rightarrow 1_{11}$  transition, while the calculated strong signal for the  $0_{00} \rightarrow 1_{01}$  transition is not detected. Future work is needed to reconcile this discrepancy.

## 1 Introduction

Water ranks as the third most abundant neutral molecule in the interstellar medium, behind H<sub>2</sub> and CO. Consequently, H<sub>2</sub>O and its isotopologues, D<sub>2</sub>O and HDO, have been observed in different environments and play a key role in the oxygen and deuterium chemistry of star- and planet-forming regions.<sup>1–3</sup> Due to the water sublimation temperature, water is mainly frozen as ice in cold clouds and becomes abundant in the gas phase at higher temperatures, which defines the snowline.<sup>4</sup> Similarly, water can be produced in the gaseous phase or on the surface of interstellar dust grains in the coldest regions.<sup>1</sup> Deuterated water (HDO), on the other hand, is formed mainly on dust grains in cold clouds prior to core

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constants for the ground vibrational state were taken as  $A = 23.41395 \text{ cm}^{-1}$ ,  $B = 9.10340 \text{ cm}^{-1}$ , and  $C = 6.40628 \text{ cm}^{-1}$ .<sup>13</sup> The  $\text{H}_2$  molecule also has two fermionic nuclei and the *para* and *ortho* rotational levels correspond to the even and odd values of its angular momentum  $j_{\text{H}_2}$ , respectively. *Normal*- $\text{H}_2$  is a mixture of *ortho*- and *para*- $\text{H}_2$ , considering there is a Boltzmann distribution of the rotational levels in the *ortho* or *para* forms at the surrounding temperature, but a spin temperature is maintained at room temperature, as the *ortho*-to-*para* conversion is extremely slow. The *ortho*-to-*para* ratio in *normal*- $\text{H}_2$  is thus 3 : 1.

Previous theoretical studies of the  $\text{HDO}-\text{H}_2$  interactions are extensive.<sup>11,14</sup> Faure *et al.*<sup>14</sup> published the first rotational rate constants in the range 5–300 K. The calculations were recently improved<sup>11</sup> and radiative transfer calculations based on the new rate constants have shown that the  $\text{HDO}$  line at 225.9 GHz ( $3_{12} \rightarrow 2_{21}$ ) observed by ALMA towards the V883 Ori proto-planetary disk<sup>6</sup> is a weak maser.<sup>11</sup> It should be noticed that all the above studies rely on the full nine-dimensional potential energy surface (PES) computed by Valiron *et al.*<sup>33</sup> for the  $\text{H}_2\text{O}-\text{H}_2$  system and adapted for all water isotopes.<sup>15</sup> Theoretical calculations using this PES were compared to the only previous experimental study<sup>16</sup> on the rotational inelastic collisions between  $\text{HDO}$  and *normal*- $\text{H}_2$  or *para*- $\text{H}_2$ . The differential cross sections (DCSs) were observed by Sarma *et al.*<sup>16</sup> at collision energies of  $580 \text{ cm}^{-1}$  and  $440 \text{ cm}^{-1}$ . A good agreement was found between experiment and theory for the  $0_{00}$  to  $1_{11}$ ,  $2_{12}$ ,  $2_{21}$  and  $2_{20}$  transitions. To the best of our knowledge, there are no previous theoretical calculations or experiments on the  $\text{HDO}-\text{Ne}$  system.

The paper is organised as follows. Section 1 outlines the experimental setup and the procedure used. In Section 2, we describe the theoretical calculations for the  $\text{HDO}-\text{Ne}$  system and summarise those for the  $\text{HDO}-\text{H}_2$  system previously published.<sup>11</sup> Details on the PES for the  $\text{HDO}-\text{Ne}$  system are given in Section 3. The theoretical integral cross-sections (ICSSs), as well as DCSs for different rotational transitions of  $\text{HDO}$  ( $0_{00}$  and  $1_{01}$ ) scattered by  $\text{Ne}$ , are presented in section 3. In this section, a comparison of experimental and calculated integral cross-sections (ICSSs) for the  $0_{00}$  to  $1_{01}$  or  $1_{11}$  transitions at the thresholds for  $\text{HDO}$  scattered by  $\text{Ne}$  and *normal*- $\text{H}_2$  is also shown.

## 2 Experiments

### 2.1 Experimental set-up

Inelastic collisions down to low collision energies can be studied under single-collision conditions with the crossed-molecular-beam experiment in Bordeaux.<sup>17</sup> The angle between the two molecular beams generated in two different chambers is variable. This allows the collision energy between the molecules to vary:<sup>18</sup> the lowest value is for the lowest angle when the reactant beams have almost the same velocity. The molecules are then probed by a time-of-flight mass spectrometer (TOF-MS) following resonance-enhanced multiphoton ionization (REMPI).

The  $\text{HOD}$  beam was formed by expanding water vapour seeded in neon as a carrier gas through an Even–Lavie valve at 320 K from a high-pressure reservoir (11 bar), which contained a mixture with equal amounts of  $\text{H}_2\text{O}$  and  $\text{D}_2\text{O}$  maintained at a temperature of 45 °C. The carrier gas is neon to ensure better cooling of the water molecules.<sup>9</sup> Supersonic expansion by a second Even–Lavie pulsed





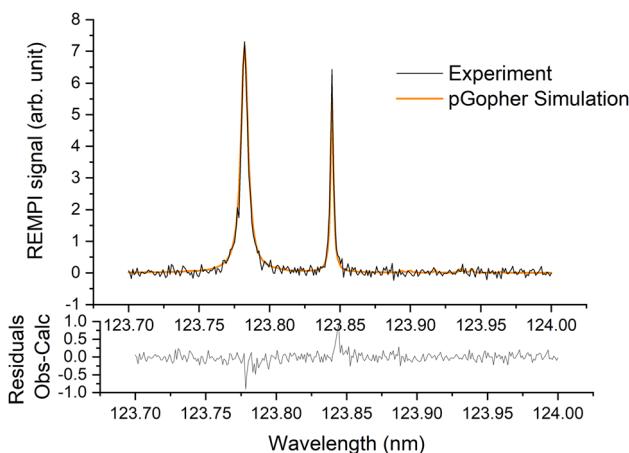


Fig. 2 Experimental (in black) and simulated (in orange) (2 + 1) REMPI spectra of the  $\text{C}^1\text{B}_1, v' = 0 \leftarrow \text{X}^1\text{A}_1, v = 0$  transition of HDO in the supersonic beam. See Yang *et al.*<sup>21</sup> for more details.

REMPI signal intensities over 5000 to 10 000 data. To take into account the density-to-flux transformation under our work conditions,<sup>20</sup> the values were divided by the relative velocity of the water and  $\text{H}_2$  or Ne beams and the mean interaction time between the two beam pulses, considering a forward angular distribution of scattered water molecules as demonstrated later by the theory for the  $\text{HOD} + \text{Ne}$  system. This conversion is described in detail in ref. 17 and 20. In

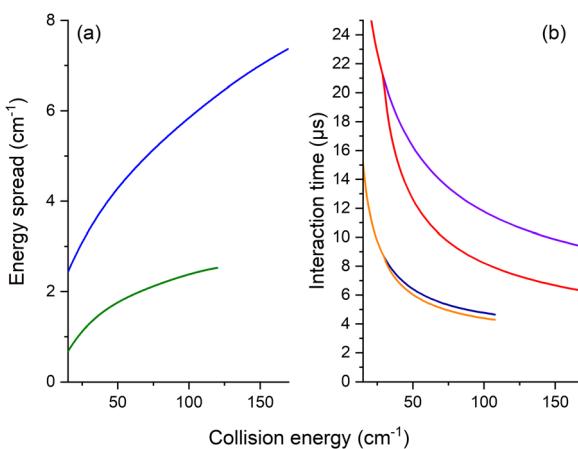


Fig. 3 Experimental collision-energy spread (a) and interaction time (b) as a function of the collision energy. (a) Half-width at 1/e of the collision-energy distribution assuming velocities and collision angular spreads as Gaussian functions for the two systems  $\text{HOD} + \text{Ne}$  (in blue) and *normal*- $\text{H}_2$  (in green). (b) Interaction time (right panel) considering an isotropic distribution of the scattered HDO (in violet, by Ne and in navy blue, by  $\text{H}_2$ ) and a forward distribution (in red, by Ne and in orange, by  $\text{H}_2$ ); the threshold is at  $29.8 \text{ cm}^{-1}$  for the  $1_{11} \leftarrow 0_{00}$  rotational transition.

Fig. 3, the mean interaction time corrections are presented *versus* the collision energy. We note that the angular distribution (forward or isotropic) was found to have an almost negligible impact in the case of HOD + *normal*-H<sub>2</sub>, contrary to the HDO + Ne system.

Moreover, for comparison with the experiment, the theoretical values were convoluted with the experimental collision-energy spread presented in Fig. 3. The poor collision-energy spread for the HDO + Ne system is due to the reduced mass, which is 9.746 a.m.u. in this case compared to 1.822 a.m.u. for the HDO + H<sub>2</sub> system. The convolution with the experimental collision-energy spread was computed from the interpolated calculated cross-sections. A short sensibility test was carried out, by changing each experimental parameter used to calculate the density-to-flux transformation and the experimental collision-energy spread. The energy resolution depends mainly on the beam velocity spreads and the pulse duration of the Ne or H<sub>2</sub> beam. The product of the relative velocity and the interaction time was essentially sensitive to the beam angular spreads, the beam velocities and the pulse duration of the Ne or H<sub>2</sub> beam. The influences of the other experimental characteristics remain within the statistic experimental error bars. Also reported on the experimental ICSs are the energy uncertainties due to the uncertainties in the beam velocities and the beam angle. These collision-energy uncertainties will just shift the excitation functions on the collision-energy scale.

### 3 Theoretical calculations

#### 3.1 HOD + Ne potential energy surface

The coordinates employed for describing the potential energy surface (PES) of the HOD-Ne complex are shown in Fig. 4.  $R$  connects the centre of mass of the HOD molecule with Ne,  $\theta$  is the angle between the  $z$ -axis and  $R$ , and  $\varphi$  is the azimuthal angle. The interatomic distances and bending angle of the HOD molecule are set to the vibrationally averaged values in the ground state ( $r_{\text{OH}} = 0.9727 \text{ \AA}$ ,  $r_{\text{OD}} = 0.9684 \text{ \AA}$  and  $\gamma = 104.34^\circ$ ).<sup>15</sup> The *ab initio* calculations were performed at the explicitly correlated coupled-cluster level of theory, including single, double and perturbative triple excitations (CCSD(T)-F12),<sup>23</sup> together with the cc-pVTZ-F12 and cc-pVQZ-F12 basis sets.<sup>24</sup> The energies at the completed basis set (CBS) limit were determined using the two-point extrapolation formula:<sup>25</sup>

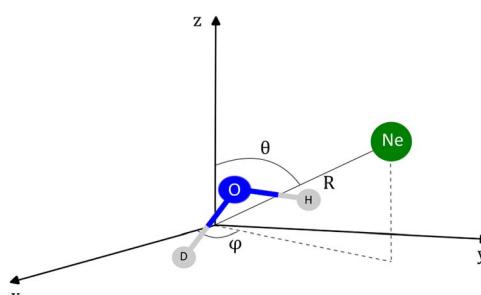


Fig. 4 Coordinates employed in the description of the HOD + Ne system. The origin of the coordinate system corresponds to the center of mass of the HDO molecule.



$$\left( E_{\text{CBS}} = \frac{E_X X^3 - E_Y Y^3}{X^3 - Y^3} \right),$$

where  $X = 4$  and  $Y = 3$  are the cardinality of the basis set. All the *ab initio* energies were computed with the MOLPRO package.<sup>26</sup>

A grid of 6175 energies at the CBS limit was computed for the system. The grid includes 25  $R$  values from 1.5 Å to 10.0 Å, with  $\theta$  and  $\varphi$  varying from 0° to 180° in steps of 10° and 15°, respectively. It should be noted that for certain angular configurations, *ab initio* calculations could not produce energies for a few low  $R$  values due to the proximity of Ne and the atoms within the triatomic molecule. Consequently, local extrapolation was utilized.

The grid of energies was fitted to an analytical function, like the one used for H<sub>2</sub>S + He<sup>27</sup> and H<sub>2</sub>O/D<sub>2</sub>O + Ne,<sup>28</sup> with the angular part expressed as the product of normalized associated Legendre functions  $\bar{P}_l^m(\cos\theta)$  and cosine functions:

$$V(R, \theta, \varphi) = \sum_{l=0}^{10} \sum_{m=0}^{\min(l,4)} v_{lm}(R) \bar{P}_l^m(\cos\theta) \cos(m\varphi)$$

The  $v_{lm}(R)$  coefficients were computed from a least-squares procedure and fitted using the reproducing kernel Hilbert space (RKHS) method,<sup>29</sup> using:

$$v_{lm}(R) = \sum_{k=1}^{N_R} \alpha_k^{lm} q^{2.5}(R, R_k)$$

where  $N_R$  is the number of  $R$  values of the grid, and  $q^{2.5}$  is the one-dimensional kernel:<sup>29</sup>

$$q^{2.5}(R, R_k) = \frac{2}{21R_{>}^6} - \frac{R_{<}}{14R_{>}^7}$$

with  $R_{<}$  and  $R_{>}$  defining the lower and larger values, respectively, between  $R$  and  $R_k$ . The  $\alpha_k^{lm}$  coefficients were computed by solving the linear equation system  $\mathbf{q}(R_k, R_k) \alpha^{lm} = \mathbf{v}_{lm}(R_k)$ , where  $k$  and  $k'$  label different radial configurations of the grid. The van der Waals  $\sim R^{-6}$  long-range behaviour is ensured using the  $q^{2.5}(R, R_k)$  kernel.

### 3.2 HOD + Ne dynamical calculations

The Newmat code<sup>30</sup> was used to carry out the dynamical calculations of the HOD + Ne system. As in our previous work on H<sub>2</sub>O/D<sub>2</sub>O + Ne,<sup>28</sup> the rotational eigenfunctions and eigenenergies were determined by diagonalizing the rigid rotor Hamiltonian of an asymmetric top molecule:

$$\mathbf{H}_{\text{rot}} = A_x \mathbf{j}_x^2 + A_y \mathbf{j}_y^2 + A_z \mathbf{j}_z^2$$

In the usual symmetrised basis set:

$$|jkm, \varepsilon\rangle = [2(1 + \delta_{k0})]^{-\frac{1}{2}} (|jkm\rangle + (-1)^\varepsilon |jk - m\rangle)$$



where  $|jkm\rangle$  is a symmetric top wave function and  $A_x = A$ ,  $A_y = C$  and  $A_z = B$ .<sup>15</sup> These wave functions were used to perform close-coupling calculations, with the calculated eigenenergies being substituted for experimental ones.<sup>31</sup>

The basis set includes 7 values of HDO rotational angular momentum ( $0 \leq j \leq 6$ ), which leads to energies up to  $872.77 \text{ cm}^{-1}$ . We carried out scattering calculations within the collision-energy range of  $[15.5, 315.5] \text{ cm}^{-1}$ , utilizing  $0.1 \text{ cm}^{-1}$  steps for energies below  $115.5 \text{ cm}^{-1}$  and  $1 \text{ cm}^{-1}$  steps above. The log derivative method<sup>32</sup> was used to propagate the radial wave functions for  $3.0 \leq R \leq 20a_0$  ( $a_0$  is the Bohr radius). The relative convergence of the cross sections as a function of the total angular momentum  $J$  was imposed to be better than  $10^{-3}$ , leading to a maximum value of  $J = 66$  for the maximum energy value investigated. The reduced mass of the HDO-Ne system is  $9.747\,406 \text{ a.m.u.}$

### 3.3 HDO + H<sub>2</sub> dynamical calculations

Scattering quantum close-coupling calculations of the rotational inelastic cross sections were previously published by Faure *et al.*<sup>11</sup> to provide rate constants over a large kinetic temperature range (5–300 K). For this study, the same V08 PES of Valiron *et al.*<sup>33</sup> was used, but more refined scattering calculations were performed. The calculations were conducted at the quantum close-coupling level with the MOLSCAT program.<sup>34</sup> The rigid-rotor PES for HDO-H<sub>2</sub> was derived from the full-dimensional (9D) PES of H<sub>2</sub>O-H<sub>2</sub> determined at the CCSD(T) level of theory and further refined with higher-accuracy explicitly correlated CCSD(T)-R12 calculations.<sup>33</sup> To provide precise descriptions of the rotational thresholds and resonances, calculations were conducted up to total energies of  $220 \text{ cm}^{-1}$  with a grid increment of  $0.25 \text{ cm}^{-1}$ . Cross sections for *normal*-H<sub>2</sub> were obtained through calculations for the ground *para*-( $j_{\text{H}_2} = 0$ ) and *ortho*-( $j_{\text{H}_2} = 1$ ) states of H<sub>2</sub> and an *ortho*-to-*para* ratio of 3 was employed. The highest HDO rotational angular momentum in the basis set was  $j_{\text{HDO}} = 5$ , while two rotational levels of H<sub>2</sub> were included for the *para*-( $j_{\text{H}_2} = 0, 2$ ) and *ortho*-( $j_{\text{H}_2} = 1, 3$ ) symmetries of H<sub>2</sub>. In the calculations of Faure *et al.*,<sup>11</sup> total energies of up to  $2000 \text{ cm}^{-1}$  (with a less-fine energy grid) were considered, but the level  $j_{\text{H}_2} = 3$  of *ortho*-H<sub>2</sub> was neglected. The inclusion of this level was found to be important for a very detailed description of resonances in this work. The maximum value of the total angular momentum  $J$  used in the calculations was  $J = 37$  at the highest collisional energy. The value of the H<sub>2</sub> rotational constant was  $B_0 = 59.322 \text{ cm}^{-1}$ . Finally, the reduced mass of the system HDO-H<sub>2</sub> was calculated to be  $1.82252 \text{ a.m.u.}$

## 4 Results and discussion

### 4.1 HDO + Ne PES

The accuracy of the fitting procedures was assessed by measuring the root-mean-square deviation (RMSD). The RMSD for negative energies was determined to be  $8.5 \cdot 10^{-4} \text{ cm}^{-1}$ . Subsequently, the RMSD values obtained for energy intervals of  $[0, 1000] \text{ cm}^{-1}$  and  $[1000, 10\,000] \text{ cm}^{-1}$  were  $7.7 \cdot 10^{-2} \text{ cm}^{-1}$  and  $25.6 \text{ cm}^{-1}$  correspondingly.

The global minimum of the system is located in a deuterium-bonded configuration that is linear ( $-65.15 \text{ cm}^{-1}$ , at  $R = 3.17 \text{ \AA}$ ,  $\theta = 73^\circ$ ,  $\varphi = 0^\circ$ ). From a comparison with the minima found for H<sub>2</sub>O and D<sub>2</sub>O systems in our previous

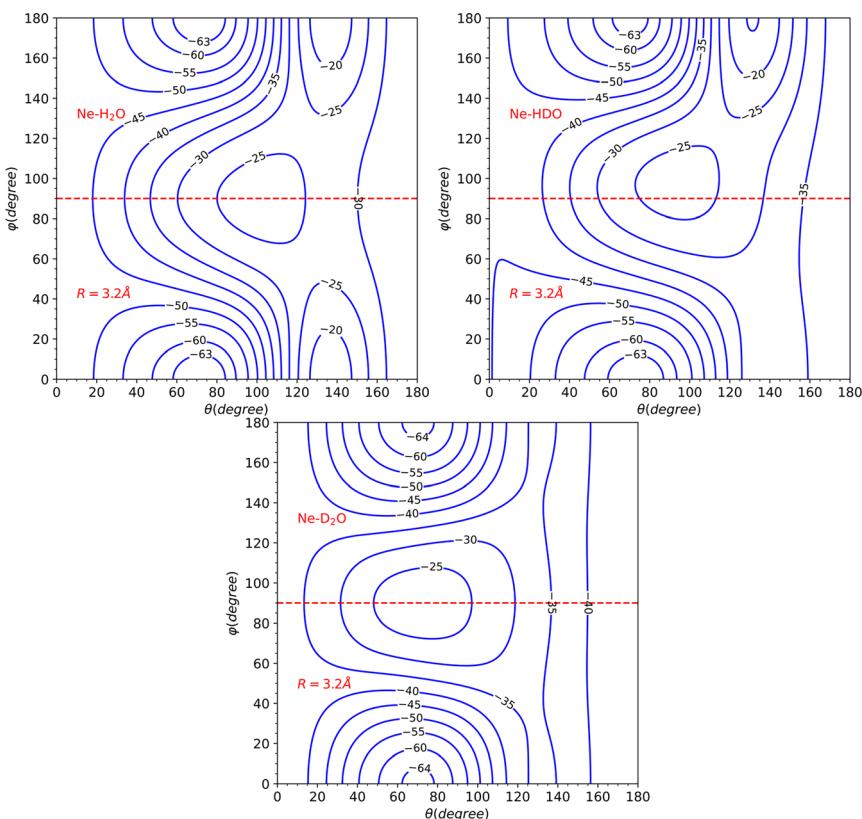


**Table 2** Equilibrium energies,  $De$ , and geometries (variables as defined in the text and Fig. 4) of the Ne–HOD, Ne–H<sub>2</sub>O and Ne–D<sub>2</sub>O systems. The values for Ne–HOD were obtained in this work, while the values for the two other systems were taken from our previous work<sup>28</sup>

System	$De$ (cm <sup>-1</sup> )	$Re$ (Å)	$\theta_e$ (°)	$\varphi_e$ (°)
HOD + Ne	65.15	3.18	73.50°	0.00°
H <sub>2</sub> O + Ne	65.29	3.20	73.00°	0.00°
D <sub>2</sub> O + Ne	65.29 <sup>a</sup>	3.24	71.50°	0.00°

<sup>a</sup> The D<sub>2</sub>O + Ne PES was obtained from the H<sub>2</sub>O + Ne PES taking into account the shift of the centre of mass (see ref. 28 for more details).

work<sup>28</sup> (see Table 2), we notice that these three stationary points are very close in geometrical configuration and well depth. Nevertheless, it should be noted that despite the similarities observed among the three minima in the PES, the most significant changes in the dynamics are determined by other features such as the shape of the barriers and the anisotropy.

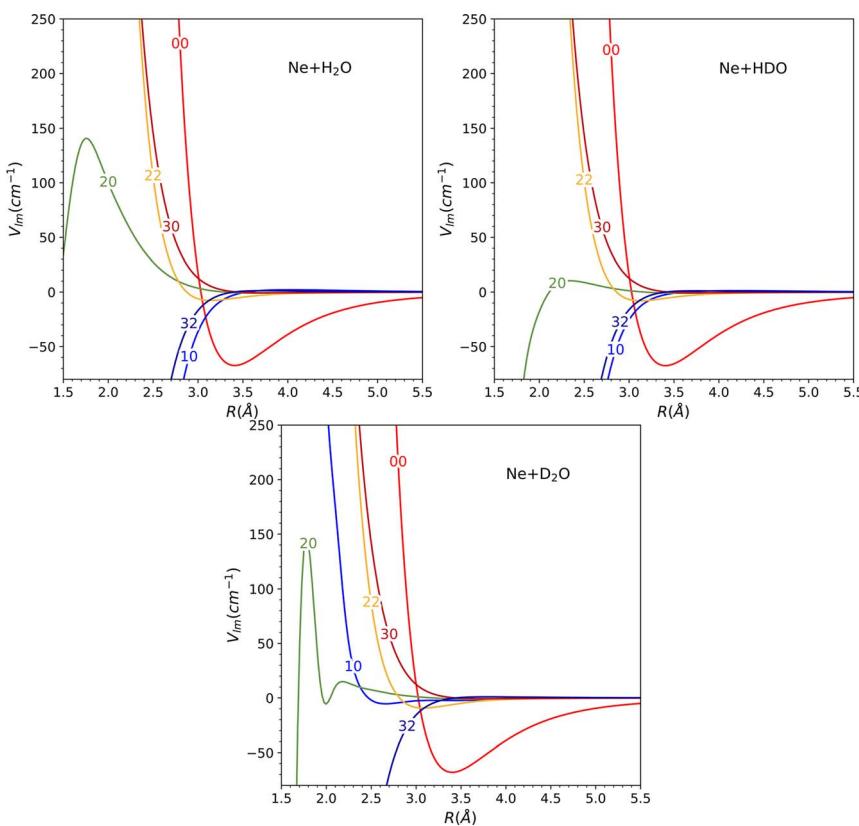


**Fig. 5** Contour plots for the H<sub>2</sub>O–Ne, HOD–Ne and D<sub>2</sub>O–Ne PESs for  $R = 3.2$  Å. A red dashed line is placed at  $\varphi = 90^\circ$  to mark the symmetry of the H<sub>2</sub>O/D<sub>2</sub>O–Ne systems and the break in the symmetry of the HOD–Ne system. The labels of the contour lines correspond to the energy in cm<sup>-1</sup>.



The topological variations amongst the systems can be more effectively visualized through Fig. 5. This displays a contour plot of the three systems, with  $R = 3.2 \text{ \AA}$ . To aid discussion of PES changes resulting from the H/D interchange, a red dashed line at  $\varphi = 90^\circ$  has been included. From the plot of HDO, which corresponds to the top-right plot of Fig. 5, it can be observed that the contour lines behave similarly to those of  $\text{H}_2\text{O}$ , for the interval of  $90^\circ \leq \varphi \leq 180^\circ$ , which corresponds to the Ne atom approaching the H atom. The latter half of the interval for  $0^\circ \leq \varphi \leq 90^\circ$ , however, displays more variations compared to the  $\text{D}_2\text{O}$  system, implying that the system's dynamics will bear more similarities to that of  $\text{H}_2\text{O}$ .

This hypothesis receives further support by comparing the  $v_{lm}(R)$  coefficients (refer to Fig. 6). For the  $\text{H}_2\text{O}$  and HDO system, the  $l = 2, m = 0$  partial wave exhibits the only marked difference, while for  $\text{D}_2\text{O}$ , the  $l = 1, m = 0$  partial wave differs significantly, being repulsive instead of attractive as seen for the water and singly-deuterated water systems. We should note that only even  $m$  partial waves are presented, since they are the only ones observed in the  $\text{H}_2\text{O}$  and  $\text{D}_2\text{O}$  systems.



**Fig. 6**  $v_{lm}(R)$  expansion coefficients for the  $\text{H}_2\text{O}-\text{Ne}$ ,  $\text{HDO}-\text{Ne}$ , and  $\text{D}_2\text{O}-\text{Ne}$  PESs. Labels in the curves are associated with  $lm$  partial waves. Only even  $m$  values are allowed for  $\text{H}_2\text{O}$  and  $\text{D}_2\text{O}$  due to the  $C_{2v}$  symmetry of these two molecules, while for HDO, even though odd  $m$  values are now allowed, just the even  $m$  values are represented to simplify the comparison.



This is due to the fact that the molecule exhibits  $C_{2v}$  symmetry in these two cases. However, the inclusion of the odd  $m$  coefficients in the HDO PES expansion, which were not depicted in the plot to facilitate comparison, is likely to result in significant variations in the dynamics.

#### 4.2 HDO + Ne inelastic collisions

The theoretical calculations focus on exciting the fundamental rotational level  $0_{00}$  and the related transitions for exciting and relaxing from the first excited rotational state,  $1_{01}$ . This is due to the experimental probing of only the first two excited rotational levels,  $1_{01}$  and  $1_{11}$ .

**4.2.1 The rotational transition  $0_{00}$  to  $1_{11}$ .** Calculated state-to-state cross sections for exciting the fundamental rotational level  $0_{00}$  to the excited rotational states are illustrated in Fig. 7. The transition  $1_{11} \leftarrow 0_{00}$  exhibits the largest cross-section, followed by the excitation  $2_{02} \leftarrow 0_{00}$ . It is noteworthy that all the water isotopes display a similar ordering for these largest cross-sections, providing evidence of similarities between the PESs. However, the symmetry between *para* and *ortho* in  $\text{H}_2\text{O}$  or  $\text{D}_2\text{O}$  systems has been broken, allowing previously forbidden transitions to occur. A few of these transitions, like  $1_{01} \leftarrow 0_{00}$  and  $2_{12} \leftarrow 0_{00}$ , have significant cross sections. The energy threshold appears to have a small impact on the ordering of cross sections based on magnitude.

To compare the theoretical data with the experimental results shown in Fig. 8, we applied the experimental collision-energy spread to the theoretical cross-sections. Moreover, the experimental ICSs were scaled by one factor so that the area in the  $[30\text{--}165]\text{ cm}^{-1}$  range is the same as for the theoretical ICS calculated. It should be noted that in the HDO beam, more than 95% of the population was in the ground rotational state. Therefore, when the  $1_{11}$  level was probed, the main contribution originated from the  $1_{11} \leftarrow 0_{00}$  transition. The experiment successfully reproduced the transition threshold at  $29.75\text{ cm}^{-1}$ , but the calculated resonance features were blurred out in the experimental ICSs due to the energy

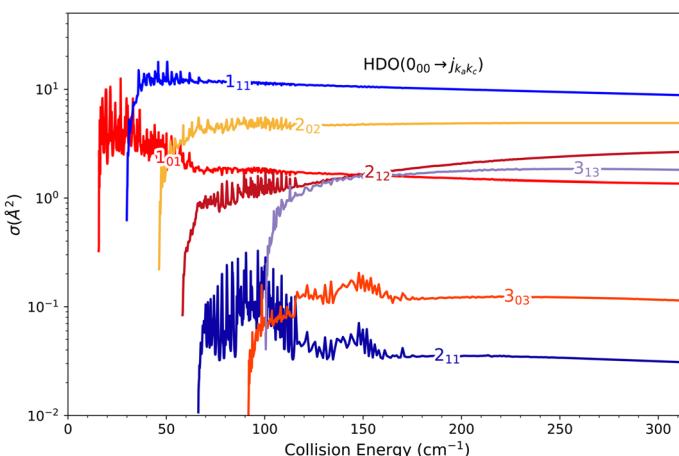
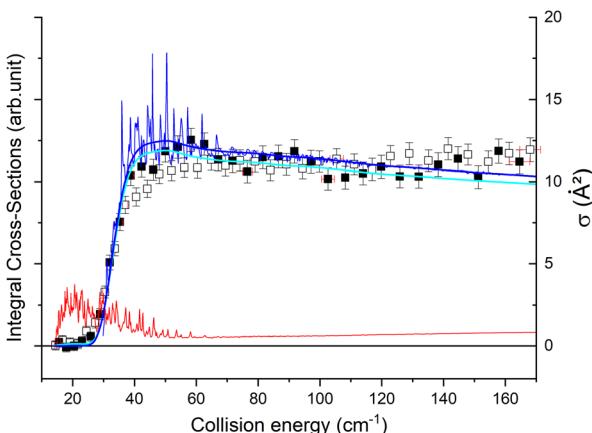


Fig. 7 Cross sections for the excitation of the fundamental rotational level of the HDO molecule by collision with Ne. The curves on the log-linear plot correspond to transitions from the  $0_{00}$  level to the first eight rotational excited states.





**Fig. 8** Experimental and theoretical cross-sections for the  $\text{HDO} + \text{Ne} \rightarrow \text{HDO (1}_{11}\text{)} + \text{Ne}$  inelastic collisions. Squares are experimental ICSSs for the  $1_{11}$  rotational level, with error bars corresponding to statistical uncertainties at 95% of the confidence interval (white or black squares are the means of around 7000 or 9200 data, respectively). Shown as solid lines, the total theoretical contributions are convoluted to the experimental collision-energy spread, with partial contributions of the  $1_{11} \leftarrow 0_{00}$  transition (as a thin blue line) and the  $1_{11} \leftarrow 1_{01}$  transition (as a thin red line), assuming an initial rotational population from the  $1_{01}$  state of 0% (as a bold blue line) or 5% (as a bold cyan line).

spread. To verify the reproducibility, two sets of experimental data were recorded at different periods of the year: the disparity in values was, however, mainly due to fluctuations in the laser power (as well as humidity in the laboratory). This might also explain why the experimental values exceed the theoretical cross-sections above  $120 \text{ cm}^{-1}$  (as the experimental acquisition was always carried out from the highest beam angle to the lowest angle).

**4.2.2 The differential cross sections related to the rotational transition from  $0_{00}$  to  $1_{11}$ .** The calculations of the differential cross sections (DCSs) for the inelastic collision between Ne and HDO have been performed with the usual formula:

$$\frac{d\sigma}{d\hat{R}}(j'\tau' \leftarrow j\tau | \hat{R}) = \frac{k_{j\tau}}{k_{j\tau}(2j+1)} \sum_{m_j=-j}^j \sum_{m_{j'}=-j'}^{j'} \left| f(j'\tau' \leftarrow j\tau | \hat{R}) \right|^2$$

where  $\tau = k_a - k_{c'} \hat{R} = (\theta, \varphi)$  and  $k_{j\tau} = \sqrt{2\mu(E - E_{j\tau})}$ , while  $(j', \tau')$  and  $(j, \tau)$  define the final and initial level of the HDO molecule.  $f(j'\tau' \leftarrow j\tau | \hat{R})$  is the scattering amplitude, which, in the case of the collision between a rigid asymmetric top and an atom, is defined as:

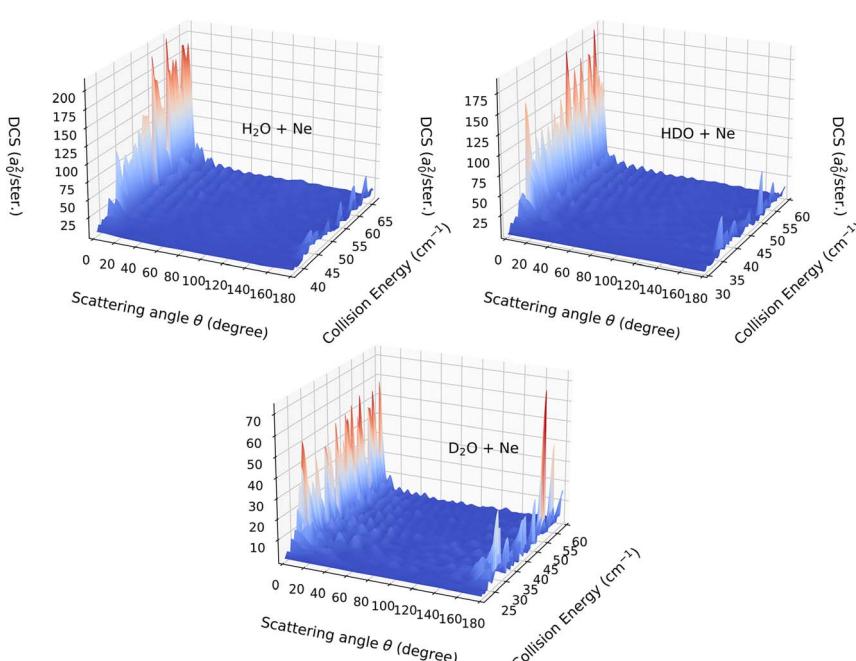
$$f(j'\tau' \leftarrow j\tau | \hat{R}) = \sqrt{\frac{\pi}{k_{j\tau} k_{j'\tau'}}} \sum_{J, M_J, M_{J'}} i^{l-l'+1} (-1)^{j+j'-l-l'} (2J+1) \sqrt{2l+1} \times \begin{pmatrix} j & l & J \\ m_j & 0 & -M \end{pmatrix} \\ \times \begin{pmatrix} j' & l' & J \\ m_{j'} & m_{l'} & -M \end{pmatrix} T_{j\tau l j'\tau' l'} {}^J Y_{l' m_{l'}}(\hat{R})$$

where  $T_{j\tau l j'\tau' l'}$  are the transition matrix elements at a given collision energy and  $Y_{l' m_{l'}}(\hat{R})$  is a spherical harmonic function.

In the present paragraph dedicated to the DCS associated with the  $1_{11} \leftarrow 0_{00}$  transition, we extend the comparison between the HDO, H<sub>2</sub>O and D<sub>2</sub>O systems. The collision-energy ranges used to calculate the DCS were [37.5, 67.0] cm<sup>-1</sup>, [30.0, 60.0] cm<sup>-1</sup> and [21.0, 60.0] cm<sup>-1</sup> for H<sub>2</sub>O, HDO and D<sub>2</sub>O, respectively. The choice of the energy interval allows a near-threshold study for each system.

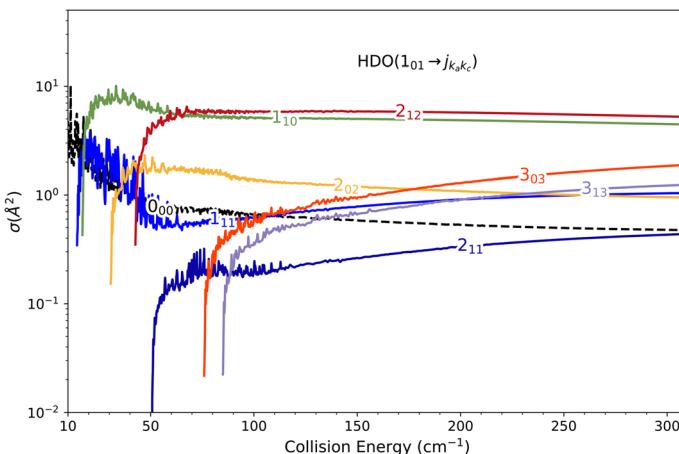
Fig. 9 displays the computed DCSs. For the entire energy interval under consideration, the DCSs are observed to be strongly forward peaked for collisions with deuterated water (HDO) and H<sub>2</sub>O, while significant back-scattering contributions are noted for collisions with D<sub>2</sub>O. The uniqueness of the latter system stems from the  $v_{10}(R)$  potential term, which is the only relevant  $v_{lm}(R)$  term for this transition. As seen in Fig. 6, it is entirely attractive and primarily contributes to the forward-scattering direction for both H<sub>2</sub>O and HDO. For D<sub>2</sub>O, however, this interaction is predominantly repulsive, with a minor potential well of around 5 cm<sup>-1</sup> depth, the repulsion being a significant contributor to the backward-scattering phenomenon.

**4.2.3 The rotational transition  $0_{00}$  to  $1_{01}$ .** Theoretical state-to-state cross sections for the excitation and relaxation of the first excited rotational level  $1_{01}$  are displayed in Fig. 10. The transitions  $2_{12} \leftarrow 1_{01}$  and  $1_{10} \leftarrow 1_{01}$  currently have the largest related cross sections, being approximately equal in magnitude. It is interesting to note that, despite the loss of *para/ortho* symmetry, the most significant cross sections for exciting the  $0_{00}$  and  $1_{01}$  levels correspond to those with similar symmetry in H<sub>2</sub>O or D<sub>2</sub>O molecules.<sup>28</sup> This proximity in symmetry may be linked to the similarities of the H<sub>2</sub>O-Ne and HDO-Ne PESs, implying that partial waves with even values of  $m$  dominate the system's dynamics.



**Fig. 9** Differential cross section for the  $(1_{11} \leftarrow 0_{00})$  transitions induced by the collision of H<sub>2</sub>O, HDO or D<sub>2</sub>O with Ne.



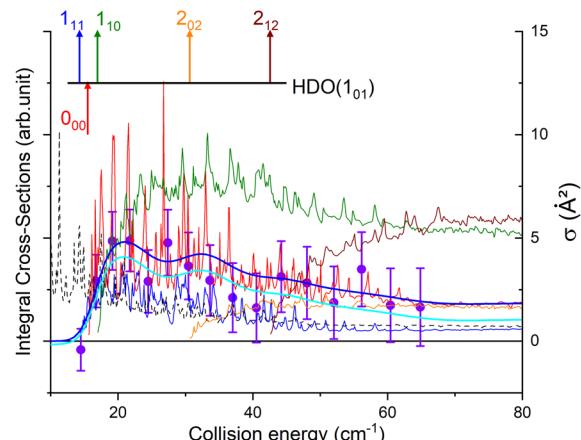


**Fig. 10** Cross sections for the relaxation (black dashed line) and the excitation (solid lines) of the  $1_{01}$  rotational level of the HDO molecule by collision with Ne. Excitation cross sections correspond to transitions to the seven closest in energy excited states.

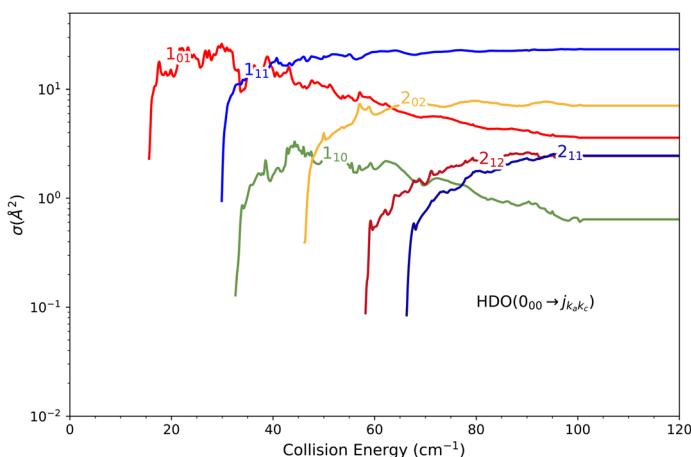
Direct comparison with the experiment is not possible as the HDO beam may contain water in its first excited rotational state  $1_{01}$ . To obtain the effective ICSs, it is thus necessary to combine the state-to-state calculated cross-sections. The HDO ( $1_{01}$ ) formation is due to the  $1_{01} \leftarrow 0_{00}$  transition, weighted by 1 or 0.95, depending on the ground HDO rotational level population. When there is 5% rotationally excited HDO in the water beam, HDO ( $1_{01}$ ) also disappears due to the transitions from this  $1_{01}$  level to the fundamental level or all the excited levels up to  $2_{12}$ . The sum of these cross-sections is therefore subtracted with a weight of 0.05 from the HDO ( $1_{01}$ ) production cross-section. The effective ICSs are then convolved with the experimental energy dispersion. The two computed excitation functions shown in Fig. 11 thus correspond, respectively, to the two extreme scenarios: the presence of 0% or 5% HDO in state  $1_{01}$  in the water beam. Although the statistical uncertainties at 95% confidence remain high, due to the low HDO ( $1_{01}$ ) intensity detected, the two undulations due to resonances predicted by theory can be seen. To verify the ratio of effective cross-sections between the  $1_{01}$  to the  $1_{11}$  states, a study at  $44\text{ cm}^{-1}$  was carried out. Some of the experimental data used are presented in the ESI file.† REMPI intensities were acquired from both levels  $1_{01}$  and  $1_{11}$  while maintaining perfectly identical experimental conditions, with the exception of MCP amplification and the wavelength of the ionizing laser (its power having varied very little between the 2 wavelengths). The ratio of  $1_{01}$  to  $1_{11}$  population variations due to HDO + Ne collisions is roughly 0.2, with a relative uncertainty of 50%. This was deduced from pGopher simulations with the spectroscopic data from Yang *et al.*<sup>21</sup> The ratio of the effective theoretical cross-sections convolved with the experimental energy dispersion at this collision energy is found to be in the range of 0.196 to 0.248 (depending on the population in state  $1_{01}$  in the beam).

#### 4.3 HDO + H<sub>2</sub> inelastic collisions

The state-to-state cross sections for the experimental energy range are displayed in Fig. 12. The HDO + *normal*-H<sub>2</sub> cross sections are similar in magnitude to those



**Fig. 11** Experimental and theoretical cross-sections for the  $\text{HDO} + \text{Ne} \rightarrow \text{HDO} (1_{01}) + \text{Ne}$  inelastic collisions. Circles are experimental ICSs for the  $1_{01}$  rotational level, with error bars corresponding to statistical uncertainties at 95% of the confidence interval (a circle represents the mean of around 9200 data). Shown as solid lines, the effective theoretical contributions are convoluted to the experimental collision-energy spread assuming an initial rotational population from the  $1_{01}$  state of 0% (as a bold blue line) or 5% (as a bold cyan line), with partial contributions of the  $1_{01} \leftarrow 0_{00}$  transition (as a red line), the  $1_{01} \rightarrow 0_{00}$  transition (as a black dashed line), the  $1_{11} \leftarrow 1_{01}$  transition (as a blue line), the  $1_{10} \leftarrow 1_{01}$  transition (as a green line), the  $2_{02} \leftarrow 1_{01}$  transition (as an orange line) and the  $2_{12} \leftarrow 1_{01}$  transition (as a wine-colored line). The respective thresholds of the transitions are indicated by arrows.

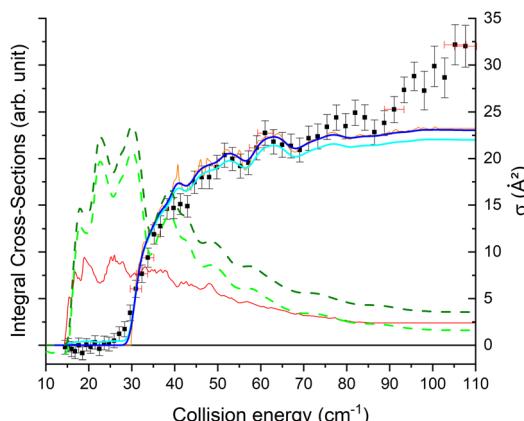


**Fig. 12** Cross sections for the excitation of the fundamental  $0_{00}$  rotational level of the HDO molecule by collision with *normal*-H<sub>2</sub>. The curves on the log-linear plot correspond to transitions to the first six rotational excited states.

of the HDO + Ne system, except for the  $1_{01} \leftarrow 0_{00}$  transition, which is approximately 5 times stronger at the peak when averaged over resonances. Moreover, for the HDO + *normal*-H<sub>2</sub> system, sharp resonances are not observed, in contrast to the HDO + Ne system. These differences can be rationalized by the fact that the

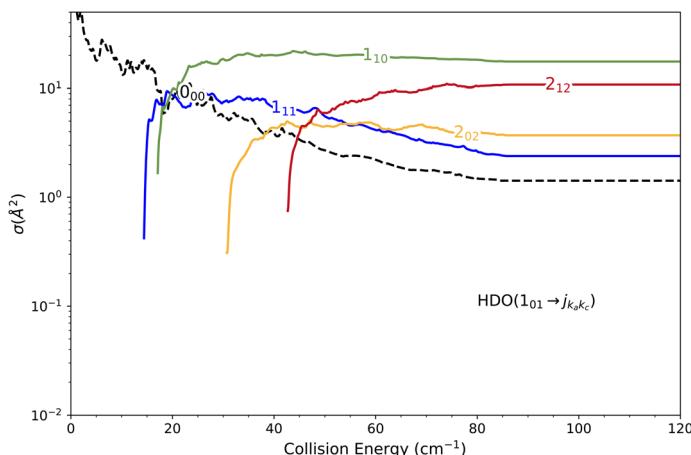
well for HDO–Ne is  $65\text{ cm}^{-1}$  deep, whereas for HDO–H<sub>2</sub> it is  $237\text{ cm}^{-1}$ . In the *normal*-H<sub>2</sub> system, it is also relevant to consider the angular momentum (and quadrupole) of H<sub>2</sub> ( $j_{\text{H}_2} = 1$ ). Consequently, more scattering resonances may be found when H<sub>2</sub> is the collider, and their overlap broadens the excitation function in the HDO + *normal*-H<sub>2</sub> system. In the case of HDS + H<sub>2</sub>,<sup>35</sup> the  $1_{01} \leftarrow 0_{00}$  transition has an effective maximum cross-section whose magnitude is close to that of the  $1_{11} \leftarrow 0_{00}$  transition, as observed here for HDO + H<sub>2</sub>.

Experimentally, a weak signal from the  $1_{01}$  rotational level is clearly visible when probing the water beam, indicating the presence of HDO in its first excited rotational state, mainly on the border of the water gas pulse (see the ESI file).† Nonetheless, no increase is detectable when colliding with *normal*-H<sub>2</sub>. This leads us to the conclusion that the cross-section for the  $1_{01} \leftarrow 0_{00}$  transition should be lower than about  $7\text{ \AA}^2$  (our limit for the HDO + Ne system), which is a factor of 2–3 lower than the theoretical value. In Fig. 13, a simulation of the excitation function has been carried out, taking into account the experimental dispersion of the energy and the population and depopulation of the  $1_{01}$  rotational state (theoretical state-to-state cross-sections are reported in Fig. 14), as carried out for the HDO + Ne system. The simulation also considered two scenarios: initial  $1_{01}$  state populations of 0% and 5% in the beam. The maximum in the cross-section for the  $1_{01} \leftarrow 0_{00}$  transition is still of the same order of magnitude as the  $1_{11} \leftarrow 0_{00}$  cross-sections, in disagreement with our lack of experimental signal. In contrast, the agreement between theory and experiment is excellent when probing the  $1_{11}$  state (Fig. 13). The transition threshold is replicated accurately and the ICS oscillations



**Fig. 13** Experimental and theoretical cross-sections for the HDO + *normal*-H<sub>2</sub> → HDO ( $1_{01}$  or  $1_{11}$ ) + H<sub>2</sub> inelastic collisions. Squares are experimental ICSs for the  $1_{11}$  rotational level, with error bars corresponding to statistical uncertainties at 95% of the confidence interval (a square represents the mean of around 5000 data). Shown as solid lines, the total theoretical contributions are convoluted to the experimental collision-energy spread assuming an initial rotational population from the  $1_{01}$  state of 5% (as a bold cyan line) or 0% (as a bold blue line), with partial contributions of the  $1_{11} \leftarrow 0_{00}$  transition (as an orange line) and the  $1_{11} \leftarrow 1_{01}$  transition (as a red line). Shown as dashed lines, the simulated total theoretical contributions are convoluted to the experimental collision-energy spread for the  $1_{01}$  rotational level, assuming an initial rotational population from the  $1_{01}$  state of 5% (in green) or 0% (in olive), as done for the HDO + Ne system.





**Fig. 14** Cross sections for the excitation (as solid lines) and relaxation (as a dashed line) of the first excited rotational level of the HDO molecule by collision with *normal*-H<sub>2</sub>. The curves correspond to transitions from the 1<sub>01</sub> level to the six closest in energy rotational states.

caused by overlapping resonances are in perfect concordance up to about 80 cm<sup>-1</sup>. For comparison, the experimental data have been normalised with a single rescaling value corresponding to the cross-section at 51.6 cm<sup>-1</sup>. It also indicates that the water beam contains less than 5% HDO in the first excited 1<sub>01</sub> state: there is no signal below 25 cm<sup>-1</sup> collision energy. The excellent agreement between theory and experiment for the 1<sub>11</sub> ← 0<sub>00</sub> ICSs makes the disagreement for the 1<sub>01</sub> ← 0<sub>00</sub> even more puzzling. Moreover, for collision energies above 80 cm<sup>-1</sup>, it can be noticed that the experimental 1<sub>11</sub> ← 0<sub>00</sub> cross-section is significantly larger than the theoretical prediction. In this collision-energy regime, a large resonance is, however, not expected. More experimental acquisitions are needed to reduce the error bars. In conclusion, in both cases, additional work is clearly necessary to reconcile these discrepancies.

## 5 Conclusions

Scattering resonances in the rotational excitation of HDO by Ne and *normal*-H<sub>2</sub> were studied through a joint theoretical and experimental approach. For the 1<sub>11</sub> ← 0<sub>00</sub> transition of HDO + Ne collisions, a good agreement was found for the overall shape of the cross-sections and the near-threshold rise, but the large experimental collision-energy spread due to the reduced mass of the system did not allow us to see the calculated resonances. In the case of the 1<sub>01</sub> ← 0<sub>00</sub> transition, even if the experimental signal was in the noise, the small undulations due to resonances were nicely reproduced by the calculations. Moreover, this system afforded the supplementary opportunity to theoretically compare the collisions of Ne with the three water isotopes, H<sub>2</sub>O, D<sub>2</sub>O, and HDO. We discovered that HDO and H<sub>2</sub>O produce very similar outcomes, whereas D<sub>2</sub>O differs from the other two. From this perspective, we demonstrated that straightforward reasoning, hinging on the symmetry of the potential terms  $v_{lm}(R)$ , facilitated the comprehension of

the scattering outcomes. In the case of HDO + *normal*-H<sub>2</sub>, an excellent agreement was found for the  $1_{11} \leftarrow 0_{00}$  cross-section between the theoretical and experimental results. However, for the  $1_{01} \leftarrow 0_{00}$  cross-section, theory predicted an effective maximum cross-section whose magnitude is close to that of the  $1_{11} \leftarrow 0_{00}$  transition, although no signal was experimentally detected. Thus, additional work is clearly necessary to reconcile this discrepancy.

## Author contributions

All the authors have participated in the investigation, validation and writing. RMGV, AB, ODA, AF and TS carried out the formal analysis of the theoretical and experimental data. AB, ODA, AF and TS acquired the funding.

## Conflicts of interest

There are no conflicts to declare.

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