In this contribution we demonstrate that metal–organic frameworks (MOFs) with suitable underlying topological structure are amenable for the preparation of MOF-based substitutional solid-solutions (SSS) that follow Vegard’s law. Preparation of multivariate (MTV) libraries of Porous Interpenetrated Zirconia Organic Frameworks (PIZOF) using homeomorphic links of different length (18.6 and 19.3 Å) result in the formation of SSS that exhibit linearity between composition and structure. We prepared seven MTV MOF libraries (40 unique compositions) and observed relations in composition input/output ratio, composition/lattice parameter (Vegard’s law), and between composition obtained from NMR and from PXRD. We further show that SSS formation has dependence on linker solubility, stability, and steric hindrance caused by the presence of functional groups. We demonstrate that solid-solution strategies can be utilized as part of the synthetic toolkit for the preparation of reticular crystals with desired composition, structure, and properties.

In this work, we demonstrate that Vegard’s law is applicable to multivariate MOFs that have simple topological structures. For this, we utilize mixtures of homeomorphic links of different length and study the crystallographic traits of the prepared MOFs. Homeomorphisms in MOFs (Fig. 1a), refer to transformations that preserve the topological structure of the crystal (periodic net). Our hypothesis is that by varying the ratio of links in the form of $A_xB_{1-x}$, where A and B are homeomorphic links, should produce crystals with the relations expected in Vegard’s law. Our approach consists of choosing a MOF system whose underlying topology is described by periodic nets that are default and edge-1-transitive.

We selected Porous Interpenetrated Zirconia Organic Frameworks (PIZOF, Fig. 1b), a family of stable MOFs that bear the topology of the feu-c net. This net is a catenated form of feu, the default net for cuboctahedral vertices. The feu net is edge-1-transitive, it has a cubic space group symmetry in its maximum symmetry embedding, which provide for easy characterization of the lattice parameters via powder X-ray diffraction (PXRD) methods. The links used are linear and homeomorphic derivatives of oligo-phenylene and oligo-phenylene-ethynylene dicarboxylates. We selected QPDC-Me and PEPEP-X links (Fig. 1c) that contain heteroatom and solubilizing groups. The MOF made with QPDC (PIZOF-1) is the MOF matrix/solvent, and the PEPEP links are the solute (in PIZOF-2). The links have carbonate-carboxylate distances of 18.6 and 19.3 Å respectively, corresponding to an average difference in length of 3.8%. We hypothesize that size differ-

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Vegard’s law in multivariate libraries of porous interpenetrated zirconia organic frameworks†

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The ability to design stable multivariate metal–organic frameworks (MTV MOFs)† with targeted structure and composition requires for the crystal engineer to predict both structure and metrics based solely on the ratios and geometries of the molecular building blocks. This kind of control can be established by finding predictable relations between crystal structures and their compositions, as it is observed in inorganic substitutional solid solutions (SSS) that follow Vegard’s law.2 Vegard’s law states that in sphere-packed solids, there are empirical relations between composition and crystal properties, e.g., lattice parameters or cell volume.3 The application of Vegard’s law in inorganic SSS has enabled the preparation of many complex applied materials such as blue LEDs, Cr-ruby lasers, Li-ion batteries,4 high entropy alloys,7 and quantum dots.8 In MOF synthesis, Vegard’s law has been previously observed, but mostly to the mixing of metal ions.9–19 From the point of view of the organic links, only a handful of examples have been presented.20–28

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Results and discussion

We prepared MTV MOF libraries using standard MOF solvothermal crystallizations at 120 °C in glass reactors, by mixing ZrCl₄ and MTV links at specified input ratios in DMF/acetic acid/toluene mixtures (see ESIT). Our method produced both bulk microcrystalline powders and single crystals with compositions Zr₇O₄(OH)₄[links]₆, where the input composition of the links was varied in each library. Each library is a binary mixture of short and long links (QPDC and PEPEP, respectively, Fig. 1c) that have compositions with formula Zr₇O₄(OH)₄[QPDC₁₋ₓPEPEPₓ]₆, where x is the mol fraction of PEPEP links and x∈[0,1]. The input/output (I/O) compositions of these libraries were determined by ¹H NMR of digested samples, where we compared the ratio of the integrals for each monomer signal. The purpose of constructing I/O plots is to demonstrate control over the composition, i.e., the relations between the target and the experimental ratios. Our aim is to achieve 1:1 I/O incorporation, which can be assessed by the slope of this plot. Once we determine the experimental output composition, we can construct Vegard’s plots that relate the experimental parameters and compositions of the MOF.

The first library contains only methyl solubilizing groups on the central rings of the links. We observed a linear I/O composition slope of 1.0891 ± 0.0581 (Fig. 2a) in all the composition range, indicating that these links produce a complete solid solution. We also fitted a logistic function (eqn (S1)) which typically accounts for interactions between the components of the mixture, observing good nonlinear fittings. We observe that both fittings are very close to each other suggesting QPDC/PEPEP link-link interactions that affect composition. The PXRD patterns of this library exhibits sharp diffraction lines (Fig. S18†), indicating high degree of crystallinity. The first and strongest diffraction line, the 111 peaks, exhibit a shift in peak position from 4.00 to 3.85 2θ-degrees (CuKα), indicating an expansion of the cell with increased PEPEP-Me content. This peak shift trend was also observed at higher angles, indicating a true change in lattice parameter, and not an instrumental artifact (like zero-shift). Indexing of both single crystals and powders provided cubic lattice parameters that are in good agreement with each other in triplicates, as well as with the expected values from the output composition following Vegard’s law (Fig. 2b). The Vegard’s plot indicates that there is a linear relation between the amount of PEPEP-Me and the lattice parameter. This is evidenced by the value of the correlation coefficient, R², the closer to 1 the more linear the correlation. In this library, powders and single crystals provide a coefficient of R² = 0.9871 and R² = 0.9695, respectively, indicating good linearity. All the prepared samples were porous, as exhibited in their N₂ gas isotherm (77 K), the only significant trend that we observed is that intermediate compositions, e.g. 50% and 60%, show higher uptakes, higher BET surface areas, higher pore volumes, and a higher frequency of larger pores (see section (S8)†). We speculate that SSS formation may induce nonlinear porosity trends that require a separate study.

References of <5% are favourable for SSS formation as it is for inorganic phases.³ Furthermore, we probed the influence of functionalization of the solute links. To this end, PEPEP links were functionalized with heteroatom groups on the central ring (Fig. 1c) to probe SSS formation with respect to their internal structure.

We prepared seven MTV libraries and observed two kinds of behaviour: links that form complete MTV solid solutions and follow Vegard’s law, and links that form partial MTV solid solutions that deviate from Vegard’s law. We speculate that link-link steric hinderance is among the determining factors for solid solution formation. These libraries allowed us to validate our method for composition determination from non-destructive diffraction methods. Providing new strategies for the preparation of complex crystals with predetermined composition, metrics, and properties.

Fig. 1 (a) Graph representation of a homeomorphic transformation in the fcu-c net as function of edges A and B in the form AₓB₁₋ₓ. (b) Crystal structures of the pure-link PIZOF-1 and PIZOF-2 MOFs used in this study. Colour code: gray spheres = carbon, white = hydrogen, red = oxygen, blue polyhedra = Zr, yellow spheres = pores. Colour tones represent each interpenetrating framework. (c) Homeomorphic dicarboxylate links of varied length and heteroatom functionalization used in this study. Table indicates functional groups.
We then investigated the effect of functional group substitution on the central ring of the PEPEP link with respect to crystal formation and SSS behaviour. We prepared seven PEPEP links that contained heteroatom groups: methoxy (O), ω-alkylferrocene (Fe), fluoro (F), chloro (Cl), bromo (Br), iodo (I), and trimethylsilyl (Si). These links are also homeomorphic, regardless of having different associated chemistries (e.g., steric and electronic), so they should exhibit SSS formation. The heteroatom solute links were prepared in a modular convergent fashion and were incorporated in libraries using similar synthesis strategies (see ESI†). The MTV libraries that successfully crystallize (Fig. 1c) exhibited two kinds of behaviours: Formation of complete solid solutions that follow Vegard’s law, and formation of partial solid solutions with deviation from Vegard’s law. Links PEPEP-OMe, PEPEP-F, and PEPEP-Fc form complete solid solutions with I/O slopes of 1.01 ± 0.07, 1.09 ± 0.13, and 1.07 ± 0.16 respectively (Fig. 3a), showing nearly linear trends vs. input composition. Both PEPEP-F and PEPEP-Fc deviate slightly from linearity at higher inputs, indicating that the links may be interacting during crystallization, as we previously observed when using acene links.14 We fitted the I/O plots of these two libraries with logistic functions (see Fig. S8 and Fig. S13†). In contrast, when a MTV was made with links of the same size that contained PEPEP-OMe and PEPEP-Fc as matrix and solvent, respectively, we observed a more linear 1 : 1 input–output correlation (m = 1).
0.9666, $R^2 = 0.9968$). This suggests that functionalization of the central ring plays a role in MTV library formation in conjunction with the differences in length. This effect was further accentuated when using sterically bulky substituents, like trimethylsilyl (TMS) in PEPEP-TMS. This QPDC-Me/PEPEP-TMS library exhibited crystal formation up to 40% input (see Fig. S21†), indicating a partial solid-solution with I/O slope of $m = 0.8224 \pm 0.0307$ and $R^2 = 0.9917$. Interestingly, the Vegard’s plot (Fig. S47†) shows linearity but only up to 10% output composition ($R^2 = 0.9788$), above this concentration the lattice parameter shrinks slightly before producing samples with large sample spread and lower indexing reliability. We believe these deviations in structure/composition are due to the appearance of defects caused by bulky TMS groups. The QPDC-Me/PEPEP-Br has a linear I/O slope of $m = 0.8548 \pm 0.0201$. Similarly, the Vegard’s plot (S65) also sees deviations from linearity at 10% input and above. EDS mapping of selected compositions (10% PEPEP-TMS, 80% PEPEP-Cl, and 90% PEPEP-F) observed even dispersion of heteroatoms in the mapped crystals. Further indicating mixing of the links as well as the lack of core–shell crystal formation. In-depth crystallographic, gas adsorption, and electron microscopy studies are ongoing to elucidate these observations.

Some heteroatom-containing links did not follow our hypothesis, exhibiting irregular MOF or MTV formation. PEPEP links with chlorines ortho to each other (PEPEP-Cl), exhibited limited solubility and thus unpredictable crystallization and SSS formation. Whereas the MOFs with 100% PEPEP-Cl crystallized successfully, SSS formation of the MTV was nonuniform, obtaining mixed phase samples of unpredictable composition and with recrystallized PEPEP-Cl link. This emphasizes the need for soluble links when preparing MTV MOFs. Also, we observed that PEPEP-I decomposes during MOF synthesis at all compositions and temperatures. This was evidenced by the mother liquor turning a deep purple during solvothermal synthesis similar to PEPEP-Br. Attempts to grow MTV crystals at lower temperatures and longer times were not successful. Attemptsto prepare MTVs using terphenyl dicarboxylate (TPDC) and QPDC-Me as links with size difference of 21% (14.697 Å for TPDC) resulted in phase separation of PIZOF-1 and UiO-68 mixtures. Further studies will investigate size difference tolerance and SSS formation.

From Vegard’s plots, we used the indexed lattice parameter ($a_{\text{indexed}}$) to solve for the composition according to the ideal trend between pure phases:

$$x_{\text{XRD}} = \frac{a_{\text{indexed}} - a_{\text{PIZOF-1}}}{a_{\text{PIZOF-2}} - a_{\text{PIZOF-1}}}$$

where $x_{\text{XRD}}$ is the experimental composition from indexing, $a_{\text{indexed}}$ is the indexed lattice parameter, $a_{\text{PIZOF-1}}$ and $a_{\text{PIZOF-2}}$ are the lattice parameters of the pure phases. We then plotted the experimental composition obtained from NMR vs. the experimental composition from indexed PXRDs of all samples. We found a global linear trend with a slope of $m = 1.0215 \pm 0.0201$ and $R^2 = 0.9653$ for the QPDC-Me/PEPEP-Me system (Fig. 4a). This linear correlation falls very close to the ideal analytical parameters ($m = 1, b = 0, R^2 = 1$), indicating that it is possible to determine MTV compositions from diffraction, as long as functional groups in the links are of sufficient steric bulk to allow SSS formation (e.g. up to 10 mol% in PEPEP-TMS and 7.5% in PEPEP-Br). This observation indicates that accurate composition of a sample can be obtained from non-destructive PXRD methods with accuracy. We believe that the demonstration of these linear relations in tailored crystals to be the foundation for the preparation of more complex MOF-based phases for applications that require high compositional and crystallographic precision.

**Conclusions**

We demonstrated that by choosing a MOF crystal with the precise topological features, it is possible to apply Vegard’s law when using mixtures of homeomorphic links at varied ratios. For this, we successfully prepared seven libraries of MTV MOFs based on the PIZOF family of MOFs, observing linear trends between input and output link composition, between output composition and lattice parameters, and between composition from NMR spectroscopy and from PXRD indexing. We further explored the effect of functionalization of the central ring of the longer link, observing the SSS formation to be dependent on link-link interactions, particularly from steric hindrance. This work provides a synthesis approach for the
design of frameworks with complex compositions utilizing topology and crystallography as a guide, extending the applicability of Vegard’s law from sphere-packed solids to molecular frameworks. Our strategy presents a toolbox that enables the design of MOF crystals with tunable topology, complex composition, and controlled lattice parameters that will further enable the design of even more complex MOFs towards direct applications. Future goals include to determine tolerance for size difference in links, explore the influence of defects, and the effects of the topology.

**Author contributions**

FJU, JIF, JTB, TJ conceived the project. JIF, JTB, KRL, DCF, AR and RM performed synthesis and characterization. SDM, JBB performed single crystal studies. FJU, JIF, JTB wrote manuscript.

**Conflicts of interest**

The authors declare no competing financial interests.

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