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# Linear unit BN<sub>2</sub>: a novel birefringence-enhanced fundamental module with sp orbital hybridization†

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Inorganic planar  $\pi$ -conjugated groups are advantageous to generate large birefringence in optical functional materials, and many excellent materials contain CO<sub>3</sub>, BO<sub>3</sub> or B<sub>3</sub>O<sub>6</sub>, such as CaCO<sub>3</sub>,  $\alpha/\beta$ -BaB<sub>2</sub>O<sub>4</sub> ( $\alpha/\beta$ -BBO), and KB<sub>e</sub>BO<sub>3</sub>F<sub>2</sub> (KBBF). In view of their microscopic structures, the common characteristics are the planar structures, which are regarded as birefringence-enhanced fundamental modules (FMs). Nowadays, exploring novel birefringence-enhanced FMs is becoming a burning issue. Herein, we investigated the birefringence-enhanced FMs in B–N systems and found that the BN<sub>2</sub> linear unit could produce great birefringence. Through the investigation based on the Inorganic Crystal Structure Database, some compounds with the BN<sub>2</sub> linear group were screened out with the formulas A<sub>3</sub>BN<sub>2</sub> (A = Li, Na), A<sub>3</sub>BN<sub>3</sub> (A = Mg, Ca), and Ba<sub>3</sub>(BN<sub>2</sub>)<sub>2</sub>. Particularly, Ca<sub>3</sub>(BN<sub>2</sub>)N exhibits a great birefringence of about 0.411 at 1064 nm, which is 3.5, 2.5 and 2.0 times those of the most commercially used birefringent crystals  $\alpha$ -BaB<sub>2</sub>O<sub>4</sub> ( $\Delta n = 0.116$  at 1064 nm), CaCO<sub>3</sub> ( $\Delta n = 0.164$  at 1064 nm) and YVO<sub>4</sub> ( $\Delta n = 0.208$  at 1064 nm), respectively. To find the origins of the optical properties of compounds with the BN<sub>2</sub> linear group, the first-principles, REDA and polarizability anisotropy analysis methods were used. Owing to the structural arrangement and the polarization anisotropy of the BN<sub>2</sub> linear group, it can influence the birefringence significantly. This work will provide a general way for exploring birefringence-enhanced FMs in B–N compounds.

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## 1. Introduction

In the field of optical functional crystals, birefringence plays an important role in material processing, laser phase-matching, optical measurements, photolithography, and polarizing microscopic observation.<sup>1–13</sup> Standing out from the discovered birefringent crystal materials, CaCO<sub>3</sub>,<sup>14</sup> YVO<sub>4</sub>,<sup>15</sup> and  $\alpha$ -BaB<sub>2</sub>O<sub>4</sub>,<sup>16,17</sup> are widely applied in optical devices such as polarizers, optical isolators, and compensators.<sup>18–20</sup>

Nowadays, many studies about birefringent crystals with large optical anisotropy have been carried out. For instance, Ca(BO<sub>2</sub>)<sub>2</sub> (ref. 21) with the largest reported birefringence (0.247 at 193 nm) in the deep-ultraviolet (DUV) region is regarded as the most promising DUV birefringent material, while (NH<sub>4</sub>)<sub>2</sub>-C<sub>2</sub>O<sub>4</sub>·H<sub>2</sub>O<sup>22</sup> has also been identified as a promising UV

birefringent material with a large birefringence (0.248 at 546 nm). Meanwhile, introducing F atoms into micro groups (*e.g.* from PO<sub>4</sub> to PO<sub>3</sub>F and from SO<sub>4</sub> to SO<sub>3</sub>F) can enhance the polarizability anisotropy significantly, and subsequently increase the birefringence to a certain extent.<sup>23</sup> The original difference in the birefringence of Ba<sub>2</sub>BP<sub>3</sub>O<sub>11</sub> and Ba<sub>2</sub>BP<sub>2</sub>O<sub>8</sub>F (from 0.013 to 0.034) mainly comes from the BO<sub>3</sub>F tetrahedra.<sup>24</sup> Introducing different numbers of fluorine atoms into B–O groups can modulate the polarizability anisotropy.<sup>25–29</sup> Consequently, the introduction of the F atom has become a feasible strategy to enlarge the polarizability anisotropy and enhance the birefringence.

Generally speaking, crystal structures and the corresponding microscopic groups are inextricably linked with the optical properties of materials, for instance, the arrangement of anionic frameworks or diverse combinations with metal cation polyhedrons will influence the birefringence in significant ways.<sup>30–32</sup> Consequently, a feasible strategy to enhance the birefringence is introducing some functional modules with large optical anisotropy such as coplanar groups with  $\pi$ -conjugation systems (CO<sub>3</sub>, BO<sub>3</sub>, B<sub>3</sub>O<sub>6</sub>, C<sub>3</sub>N<sub>3</sub>O<sub>3</sub>, *etc.*).<sup>22,33–37</sup> At the same time, the isolated parallel distribution, quasi-one dimensional (1D) chain, and quasi-2D plane of anionic groups are proper arrangement modes that contribute to the microscopic effects in FMs. The current research on  $\pi$ -conjugated microscopic groups mainly focuses on the B–O unit (BO<sub>2</sub>, BO<sub>3</sub>, B<sub>2</sub>O<sub>5</sub> and

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B<sub>3</sub>O<sub>6</sub>)<sup>38</sup> and C–O unit (CO<sub>3</sub>, C<sub>2</sub>O<sub>4</sub>, C<sub>2</sub>O<sub>6</sub>, C<sub>4</sub>O<sub>4</sub>, and C<sub>6</sub>O<sub>6</sub>)<sup>22</sup> which make great contributions to the birefringence due to their large anisotropy of polarizabilities. Despite the enormous promise in their chemical and physical functionalities, the sheer amounts of boron nitrides are much smaller than those of the oxides, for reasons well known to chemists,<sup>39,40</sup> and the units containing N have rarely been utilized in the optical functional crystals field.

$$\Delta\alpha = \sqrt{\frac{1}{2} \left[ (\alpha_{xx} - \alpha_{yy})^2 + (\alpha_{xx} - \alpha_{zz})^2 + (\alpha_{yy} - \alpha_{zz})^2 + 6(\alpha_{xy}^2 + \alpha_{xz}^2 + \alpha_{yz}^2) \right]}$$

Nevertheless, it does not mean that the units containing N cannot make a contribution to the optical properties.

To the best of our knowledge, there are only a few systematic studies that unveil the contribution of B–N groups towards enhancing birefringence. Based on the analysis of B–O groups, we propose an idea that BN<sub>2</sub> is a linear group containing sp orbital hybridization that contributes greatly to birefringence. Through the investigation of the BN<sub>2</sub> unit based on the Inorganic Crystal Structure Database (Version 4.7.1 build 20211220-1548, ICSD), seven ternary nitride borates with the formulas A<sub>3</sub>BN<sub>2</sub> (A = Li, Na), A<sub>3</sub>BN<sub>3</sub> (A = Mg, Ca), and Ba<sub>3</sub>(BN<sub>2</sub>)<sub>2</sub> were screened out.<sup>41–47</sup> In order to explore the origin of the optical properties of the selected compounds, multiple analytical methods have been applied. The birefringence was calculated from the dielectric functions by the first-principles method, along with polarizability anisotropy and REDA analysis to further prove the contribution of the BN<sub>2</sub> unit to birefringence. Essentially, a new birefringent-enhanced FM was screened.

## 2. Computational details

The electronic structures and optical properties of the target compounds were calculated by first-principles calculations based on the density functional theory (DFT) method<sup>48</sup> implemented in the CASTEP package.<sup>49</sup> The exchange-correlation functional<sup>50</sup> was Perdew–Burke–Ernzerhof (PBE) within the generalized gradient approximation (GGA). The norm-conserving pseudopotentials (NCP)<sup>51</sup> were selected for structural optimization and calculating the electronic and optical properties. The electronic configurations were treated as follows: Li 2s<sup>1</sup>, B 2s<sup>2</sup> 2p<sup>1</sup>, N 2s<sup>2</sup> 2p<sup>3</sup>, Na 2s<sup>2</sup> 2p<sup>6</sup> 3s<sup>1</sup>, Mg 2s<sup>2</sup> 2p<sup>6</sup> 3s<sup>2</sup>, Ca 3s<sup>2</sup> 3p<sup>6</sup> 4s<sup>2</sup>, and Ba 5s<sup>2</sup> 5p<sup>6</sup> 6s<sup>2</sup>, the cutoff energy was set at 880 eV for Ca<sub>3</sub>(BN<sub>2</sub>)N, 990 eV for Mg<sub>3</sub>BN<sub>3</sub>, and 770 eV for Li<sub>3</sub>BN<sub>2</sub> (P2<sub>1</sub>/c), Li<sub>3</sub>BN<sub>2</sub> (P4<sub>2</sub>/mmm), Li<sub>3</sub>BN<sub>2</sub> (I4<sub>1</sub>/amd), Na<sub>3</sub>BN<sub>2</sub>, and Ba<sub>3</sub>(BN<sub>2</sub>)<sub>2</sub>. The scissor operators of 1.18 eV for Li<sub>3</sub>BN<sub>2</sub> (P2<sub>1</sub>/c), 1.21 eV for Li<sub>3</sub>BN<sub>2</sub> (P4<sub>2</sub>/mmm), 1.22 eV for Li<sub>3</sub>BN<sub>2</sub> (I4<sub>1</sub>/amd), 1.24 eV for Na<sub>3</sub>BN<sub>2</sub>, 1.18 eV for Mg<sub>3</sub>BN<sub>3</sub>, 0.61 eV for Ca<sub>3</sub>(BN<sub>2</sub>)N and 1.00 eV for Ba<sub>3</sub>(BN<sub>2</sub>)<sub>2</sub> were applied to move the simulated conduction bands to the right place. Furthermore, to reach the convergence of this calculation, the Brillouin zone comprised 3 × 3 × 3 Monkhorst–Pack *k*-point sampling with a separation of 0.04 Å<sup>-1</sup>. The other calculation parameters and convergent criteria were employed as the default values of the CASTEP code.

To explore the optically anisotropic hierarchy and the functionality of the B–N units on birefringence, we investigated their polarizability anisotropy characteristics using the DFT method. The B–N units were obtained from primary crystal information without structure optimization. The polarization anisotropy was acquired from static polarization according to the following formula:

$$\chi_{\alpha\beta\gamma}^{(2)} = \chi_{\alpha\beta\gamma}^{(2)}(\text{VE}) + \chi_{\alpha\beta\gamma}^{(2)}(\text{VH})$$

where  $\chi_{\alpha\beta\gamma}^{(2)}(\text{VE})$  and  $\chi_{\alpha\beta\gamma}^{(2)}(\text{VH})$  are computed as follows:

$$\chi_{\alpha\beta\gamma}^{(2)}(\text{VH}) = \frac{e^3}{2\hbar^2 m^3} \sum_{v'c} \int \frac{d^3k}{4\pi^3} P(\alpha\beta\gamma) \text{Im} \left[ p_{vv'}^\alpha p_{v'c}^\beta p_{cv}^\gamma \right] \left( \frac{1}{\omega_{cv}^3 \omega_{v'c}^2} + \frac{2}{\omega_{vc}^4 \omega_{c'v}^4} \right)$$

$$\chi_{\alpha\beta\gamma}^{(2)}(\text{VE}) = \frac{e^3}{2\hbar^2 m^3} \sum_{vc'} \int \frac{d^3k}{4\pi^3} P(\alpha\beta\gamma) \text{Im} \left[ p_{vc}^\alpha p_{c'c}^\beta p_{c'v}^\gamma \right] \left( \frac{1}{\omega_{cv}^3 \omega_{vc}^2} + \frac{2}{\omega_{vc}^4 \omega_{c'v}^4} \right)$$

Here,  $\alpha$ ,  $\beta$  and  $\gamma$  are the Cartesian components,  $\chi_{\alpha\beta\gamma}^{(2)}(\text{VH})$  and  $\chi_{\alpha\beta\gamma}^{(2)}(\text{VE})$  denote the contribution from the VE process and the VH process, respectively,  $v$  and  $v'$  represent the valence bands,  $c$  and  $c'$  represent the conduction bands, and  $P(\alpha\beta\gamma)$ ,  $\hbar\omega_{ij}$  and  $p_{ij}^\alpha$  represent the full permutation, the difference of band energy and momentum matrix elements, respectively. Furthermore, the Born effective charges were also obtained using density functional perturbation theory (DFPT) with a linear response method performed using the CASTEP code.

## 3. Results and discussion

### 3.1. Crystal and electronic structure

To accelerate the process of advanced material discovery, computer-aided material design and prediction have recently emerged as particularly useful techniques and have exhibited various successful cases.<sup>55,56</sup> In this work, in order to find the target compounds, the nitride borates with alkali and alkaline earth metal cations were screened in terms of the type of FMs with the BN<sub>2</sub> units by investigating the ICSD. In total, 7 structures were screened out, as listed in Table 1.



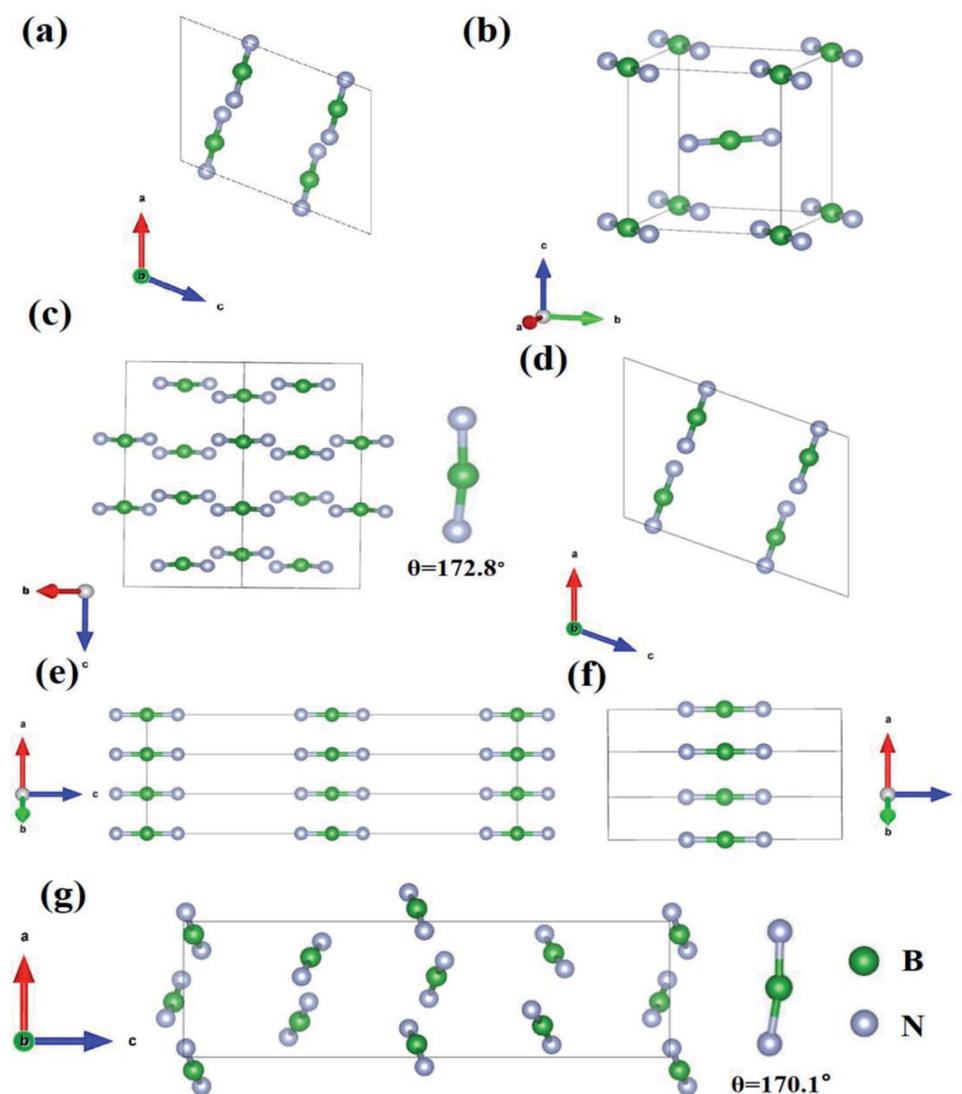
**Table 1** The space group, bandgap, and birefringence of the selected  $\text{BN}_2$  compounds

Compounds	Space group	Group	$E_g$ -GGA (eV)	$E_g$ -HSE06 (eV)	Birefringence (@1064 nm)
$\text{Li}_3\text{BN}_2$	$P2_1/c$	$\text{BN}_2$	3.4	4.58	0.198
$\text{Li}_3\text{BN}_2$	$P4_2/mmm$	$\text{BN}_2$	3.36	4.57	0.103
$\text{Li}_3\text{BN}_2$	$I4_1/amd$	$\text{BN}_2$	3.16	4.38	0.116
$\text{Na}_3\text{BN}_2$	$P2_1/c$	$\text{BN}_2$	1.52	2.76	0.157
$\text{Mg}_3\text{BN}_3$	$P6_3/mcm$	$\text{BN}_2$	1.5	2.55	0.244
$\text{Ca}_3(\text{BN}_2)\text{N}$	$P4/mmm$	$\text{BN}_2$	0.55	1.16	0.411
$\text{Ba}_3(\text{BN}_2)_2$	$P2_12_12_1$	$\text{BN}_2$	2.46	3.46	0.185

As we can see, Fig. 1 shows the structure and arrangement of the seven compounds. Obviously, the  $\text{BN}_2$  units are all short straight linear groups. The two of  $\text{Li}_3\text{BN}_2$  and  $\text{Na}_3\text{BN}_2$ , crystallize in the  $P2_1/c$  and  $P4_2/mmm$  space groups, and are composed

of isolated and parallel  $\text{BN}_2$  units in opposite directions and A-site Li and Na cations (Fig. 1a, c and d).  $\text{Mg}_3\text{BN}_3$  and  $\text{Ca}_3(\text{BN}_2)\text{N}$  belong to the space groups of  $P6_3/mcm$  and  $P4/mmm$ , respectively, and the  $\text{BN}_2$  units are parallel to each other along the  $c$  axis, while the Mg and Ca cations show symmetric distribution.  $\text{Li}_3\text{BN}_2$  and  $\text{Ba}_3(\text{BN}_2)_2$  belong to the  $I4_1/amd$  and  $P2_12_12_1$  space groups, respectively, and the  $\text{BN}_2$  units are not only aligned in a parallel manner but also have angles of  $172.8^\circ$  and  $170.1^\circ$ , as shown in Fig. 1c and g.

Since there is no data about the experimental bandgap, we employed the HSE06 hybrid functional to accurately predict the bandgaps of these compounds. Most of the title compounds possess a direct bandgap, while  $\text{Li}_3\text{BN}_2$  ( $P2_1/c$ ),  $\text{Li}_3\text{BN}_2$  ( $P4_2/mmm$ ) and  $\text{Ba}_3(\text{BN}_2)_2$  possess indirect ones. The calculated bandgaps using the GGA and HSE06 methods are shown in Table 1. The results from the HSE06 hybrid functional are 4.58 eV for  $\text{Li}_3\text{BN}_2$  ( $P2_1/c$ ), 4.57 eV for  $\text{Li}_3\text{BN}_2$  ( $P4_2/mmm$ ), 4.38 eV for  $\text{Li}_3\text{BN}_2$  ( $I4_1/amd$ ),



**Fig. 1** The arrangements of  $\text{BN}_2$  in the compounds (a)  $\text{Li}_3\text{BN}_2$ -14, (b)  $\text{Li}_3\text{BN}_2$ -136, (c)  $\text{Li}_3\text{BN}_2$ -141, (d)  $\text{Na}_3\text{BN}_2$ , (e)  $\text{Mg}_3\text{BN}_3$ , (f)  $\text{Ca}_3(\text{BN}_2)\text{N}$  and (g)  $\text{Ba}_3(\text{BN}_2)_2$ .



2.76 eV for  $\text{Na}_3\text{BN}_2$ , 2.55 eV for  $\text{Mg}_3\text{BN}_3$ , 1.16 eV for  $\text{Ca}_3(\text{BN}_2)\text{N}$ , and 3.46 eV for  $\text{Ba}_3(\text{BN}_2)_2$ , which are larger than those from GGA because of the discontinuity of the derivative on the exchange-correlation energy within the GGA functional. The GGA bandgaps of those compounds are listed in Fig. S1.†

The partial density of states (PDOS) as well as the orbitals near the Fermi level show the electronic states that are related to the optical properties. The total and partial density of states with the respective atoms were analyzed to clearly understand the electronic structures of the title compounds, and calculated to analyze the distribution of atomic orbitals in the energy range from  $-15$  to  $15$  eV, as shown in Fig. S2.† Similar electronic structures demonstrated for the target compounds. It clear to see that the bands of the title compounds near the bandgap are mainly determined by the 2p orbital of the N at the valence bands (VBs) maximum, and the bottom of the conduction bands (CBs) primarily dominated by the B 2p orbital, which means that the B–N group will play an important role in the optical properties of the title compounds.

To reveal the birefringence-related functions of the  $\text{BN}_2$  anionic group, we performed an intensive comparison with  $\text{BN}_3$ ,  $\text{BO}_2$  and  $\text{BO}_3$ . In order to explore these anionic groups, we investigated the electronic structure and optical properties of the  $\text{BN}_2$ ,  $\text{BN}_3$ ,  $\text{BO}_2$  and  $\text{BO}_3$  anionic groups using DFT implemented by the Gaussian 09 package<sup>57</sup> at the 6-31G basis set. As is well known to all, the optical property is closely related to the characteristic occupied and unoccupied states near the Fermi level. Fig. 2 shows the corresponding polarizability anisotropy and highest occupied molecular orbital–lowest unoccupied molecular orbital (HOMO–LUMO) gaps of the four anionic groups. The polarizability anisotropy  $\Delta\alpha$  values are 23.7, 13.7, 14.4, 7.0 for  $\text{BN}_2$ ,  $\text{BN}_3$ ,  $\text{BO}_2$  and  $\text{BO}_3$ , respectively. It is obvious that the  $\text{BN}_2$  unit possesses the biggest polarizability anisotropy among them. For  $\text{BN}_2$ ,  $\text{BN}_3$ ,  $\text{BO}_2$  and  $\text{BO}_3$ , the HOMO–LUMO gaps are 5.1, 3.1, 9.2 and 8.3 eV, respectively. In particular, the  $\text{BO}_3$  unit has been recognized as a birefringence-preferential group, such as in  $\text{Ca}_3(\text{BO}_3)_2$  (ref. 58) with a large birefringence of 0.104 (at 1064 nm). It was also confirmed that  $\Delta\alpha$  increases with the  $\pi$ -electron population of the conjugated system, which indicates the possibility of large birefringence for the corresponding compounds with B–N units. These calculated results suggest that the  $\text{BN}_2$  unit

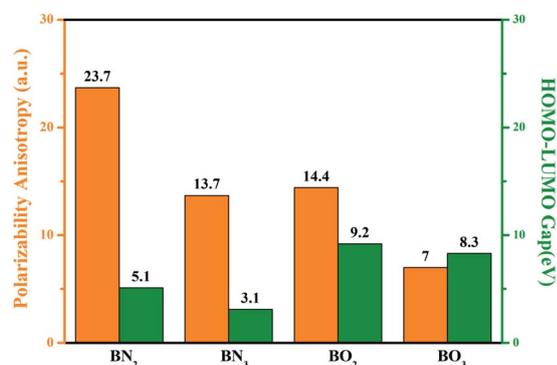


Fig. 2 Comparison of the polarizability anisotropy and HOMO–LUMO gap for the  $\text{BN}_2$ ,  $\text{BN}_3$ ,  $\text{BO}_2$ , and  $\text{BO}_3$  anionic groups.

possesses a larger polarizability anisotropy and maintains a moderate HOMO–LUMO gap as well. Thus, we investigated the compounds with  $\text{BN}_2$  units and analyzed their optical properties.

### 3.2. Optical properties

Using the GGA approach and scissors operator *via* the calculations with the CASTEP code, the birefringence  $\Delta n$  ( $\Delta n = n_{\text{max}} - n_{\text{min}}$ ), which depends on the difference of the maximum and minimum values of the refractive index along the different optical principal axes, was obtained, and the title compounds possessed a large birefringence of about 0.1–0.4 (at 1064 nm). The birefringence values of  $\text{Li}_3\text{BN}_2$  in different space groups are 0.198 ( $P2_1/c$ ), 0.103 ( $P4_2/mmm$ ), and 0.116 ( $I4_1/amd$ ). For  $\text{Na}_3\text{BN}_2$ ,  $\text{Mg}_3\text{BN}_3$  and  $\text{Ba}_3(\text{BN}_2)_2$ , their birefringence values are 0.157, 0.244, 0.185, respectively. The bandgap was calculated by GGA, and the birefringence of the compounds with  $\text{BN}_2$  units is shown in Fig. 3. Simultaneously, the results indicate that the birefringence of the title compounds is much larger than that of the birefringent materials.  $\text{Ca}_3(\text{BN}_2)\text{N}$  especially exhibits a gigantic birefringence of about 0.411 at 1064 nm, which is 3.5, 2.5, and 2.0 times those of the most commercially used birefringent crystals  $\alpha\text{-BaB}_2\text{O}_4$  ( $\Delta n = 0.116$  at 1064 nm),  $\text{CaCO}_3$  ( $\Delta n = 0.164$  at 1064 nm), and  $\text{YVO}_4$  ( $\Delta n = 0.208$  at 1064 nm), respectively. Meanwhile, in comparison with the chain  $\text{CNO}$ ,<sup>59</sup> which possesses a birefringence of 0.3–0.4, and the compounds with planar  $\text{C}_3\text{N}_3\text{O}_3$ , such as  $\beta\text{-Sr}_3(\text{C}_3\text{N}_3\text{O}_3)_2$  ( $\Delta n = 0.36$  at 1064 nm) and  $\text{Ca}_3(\text{C}_3\text{N}_3\text{O}_3)_2$  ( $\Delta n = 0.35$  at 1064 nm),<sup>60–62</sup> the birefringence of  $\text{Ca}_3(\text{BN}_2)\text{N}$  is also larger than theirs, which indicates that the proposed compounds containing  $\text{BN}_2$  units such as  $\text{Ca}_3(\text{BN}_2)\text{N}$  are promising candidates for optical applications.

Since  $\text{Ba}_3(\text{BN}_2)_2$  belongs to a non-centrosymmetric group, the NLO coefficients were obtained with *ab initio* electronic structure calculations. Owing to the Kleinman symmetry, the  $P2_12_12_1$  space group for the investigated compounds yields three non-zero NLO coefficients. Similar to the calculation of linear optical properties, the same scissors were used to generate the NLO coefficients. At the zero-frequency limit, the

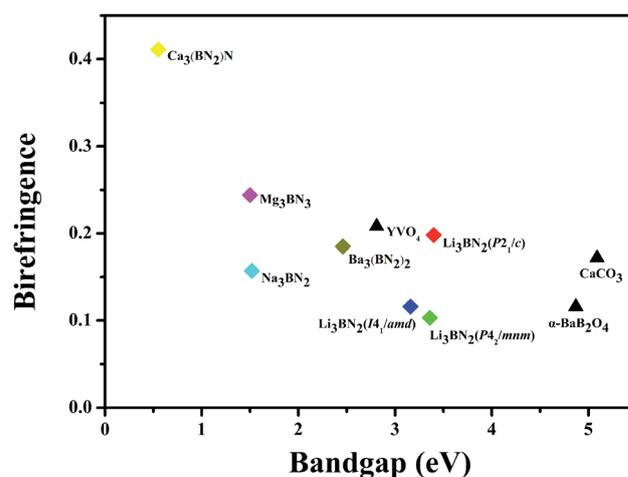


Fig. 3 Bandgap and birefringence (@1064 nm) of the compounds with  $\text{BN}_2$  units and birefringent crystal materials.



calculated largest NLO coefficient  $d_{14}$  is  $0.174 \text{ pm V}^{-1}$  ( $\sim 0.44 \times \text{KDP}$ ) for  $\text{Ba}_3(\text{BN}_2)_2$ , and B–N interactions contribute the most to the SHG coefficients (Fig. S4†).

As a post-processing tool, SHG-density (SHG<sup>63</sup> stands for second harmonic generation) was performed to further explore the contribution of atoms or groups to the SHG response through the CASTEP package. SHG-density is divided into occupied and unoccupied states of virtual-electron (VE) and virtual-hole (VH), respectively. Then, the distribution of density can be visualized by highlighting the map of SHG-density in the occupied and unoccupied states. For the  $\text{Ba}_3(\text{BN}_2)_2$  VE and VH processes, VE occupies 57% in the SHG process for  $d_{14}$ . Therefore, both of the SHG processes of VE and VH with the occupied and unoccupied states were examined, as shown in Fig. S4.† In  $\text{Ba}_3(\text{BN}_2)_2$ , the density of SHG in the VE occupied and unoccupied states originate from B–N interaction and a small contribution from Ba–N interaction, respectively, while Ba–N interaction plays an important role in the VH process. The result indicates that the atoms in the asymmetric sublattice have an important contribution to the SHG response.<sup>64</sup>

### 3.3. Optical properties analysis

To the best of our knowledge, the methods of analysis and birefringence estimation can also be employed to evaluate the ability of the functional units, such as the response electron

distribution anisotropy (REDA) model,<sup>65,66</sup> real-space atom-cutting (RSAC) technique,<sup>67</sup> Born effective charge analysis method,<sup>68</sup> Bader charge integration<sup>69</sup> and the sum of polarizability anisotropy (SPA).<sup>70</sup>

The optical anisotropy of a crystal depends on the direction of the covalent bond in the anionic groups. Therefore, birefringence is sensitive to the anisotropy of the response electron distribution, corresponding to the REDA index  $= \sum_g [N_c Z_a \Delta \rho^b / (n_1 E_g)] g$  of the anionic groups contained in the same crystal. Here,  $N_c$  is the coordination number of the nearest neighbor cations to the central anion,  $Z_a$  is the formal chemical valence of the anion,  $E_g$  is the optical bandgap,  $\Delta \rho^b = \rho_{\text{max}}^b - \rho_{\text{min}}^b$ ,  $\rho_{\text{max}}^b$  and  $\rho_{\text{min}}^b$  are the maximum and minimum of the covalent electron density of the covalent bond on the optical principal axes of a crystal, and  $n_1$  is the minimum refractive index. Pan *et al.* proved the rationality of the REDA method, and that the birefringence is proportional to the REDA index. To check the above analysis and quantification, we employed the REDA method to calculate the bonding electron density difference ( $\Delta \rho^b$ ) of the covalent bonds along the optical principal axes in  $\text{Li}_3\text{BN}_2$  ( $P2_1/c$ ,  $P4_2/mnm$ ,  $I4_1/amd$ ),  $\text{Na}_3\text{BN}_2$ ,  $\text{Mg}_3\text{BN}_3$ ,  $\text{Ca}_3(\text{BN}_2)\text{N}$  and  $\text{Ba}_3(\text{BN}_2)_2$ . The  $\Delta \rho^b$  values of  $\text{BN}_2$  are  $2.12\text{--}4.49 (\times 10^{-2})$ , and we calculated the  $\Delta \rho^b$  values of the remaining anionic groups (Li–N, Na–N, Mg–N, Ca–N and Ba–N), as shown in Table 2. As a result, the  $\text{BN}_2$  units contribute the most to  $\Delta \rho^b$ , which

Table 2 Bonding electron density difference ( $\Delta \rho^b$ ) for anionic groups in compounds calculated by the REDA method

Compounds	Group	$\Delta \rho^b (\times 10^{-2})$ of $\text{BN}_2$	$\Delta \rho^b (\times 10^{-2})$ of A (Li, Na, Mg, Ca, Ba) $\text{N}_n$	Birefringence (@1064 nm)
$\text{Li}_3\text{BN}_2(P2_1/c)$	$\text{BN}_2$	3.07	0.09	0.198
$\text{Li}_3\text{BN}_2(P4_2/mnm)$	$\text{BN}_2$	2.51	−0.18	0.103
$\text{Li}_3\text{BN}_2(I4_1/amd)$	$\text{BN}_2$	4.49	1.46	0.116
$\text{Na}_3\text{BN}_2$	$\text{BN}_2$	2.15	−0.13	0.157
$\text{Mg}_3\text{BN}_3$	$\text{BN}_2$	3.28	−0.77	0.244
$\text{Ca}_3(\text{BN}_2)\text{N}$	$\text{BN}_2$	2.65	−0.16	0.411
$\text{Ba}_3(\text{BN}_2)_2$	$\text{BN}_2$	2.12	0.47	0.185

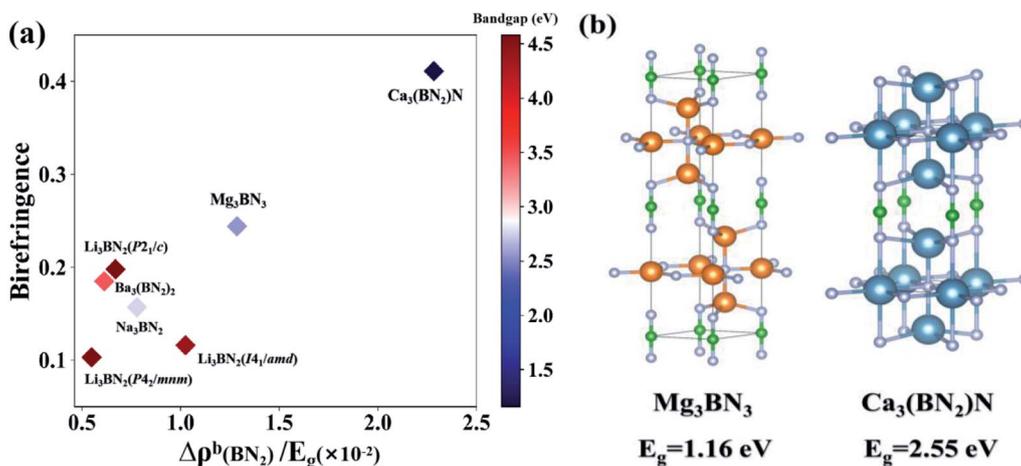


Fig. 4 (a) A comparison of  $\Delta \rho^b(\text{BN}_2)/E_g$  (HSE06) with the birefringence of the selected  $\text{BN}_2$  compounds. (b) The structural comparison of  $\text{Mg}_3\text{BN}_3$  and  $\text{Ca}_3(\text{BN}_2)\text{N}$ .



indicates that the  $\text{BN}_2$  units are the main sources of birefringence in the title compounds. As is well known, the  $\Delta\rho^b$  value obtained by the REDA method contains the arrangement of anionic groups, and the arrangement of  $\text{BN}_2$  units in the title compounds are aligned in a parallel manner; the parallel or antiparallel arrangement is beneficial to the superposition of micro polarizability anisotropy. These results are generally in agreement with the increase of the macro birefringence.

Particularly, comparing  $\text{Mg}_3\text{BN}_3$  and  $\text{Ca}_3(\text{BN}_2)\text{N}$ , we find that the birefringence of  $\text{Ca}_3(\text{BN}_2)\text{N}$  is about 1.7 times that of  $\text{Mg}_3\text{BN}_3$ , while the values of  $\Delta\rho^b$  of  $\text{BN}_2$  in  $\text{Mg}_3\text{BN}_3$  and  $\text{Ca}_3(\text{BN}_2)\text{N}$  are 3.28 and 2.65, respectively. Therefore, the  $\text{BN}_2$  units contribute about 130% in  $\text{Mg}_3\text{BN}_3$  and 110% in  $\text{Ca}_3(\text{BN}_2)\text{N}$ , which are roughly in agreement. According to the formula of REDA approximation, except for the values of the bandgap, the other indicators in the formula of the two compounds are very close to each other. Consequently, we performed a comparison of  $\Delta\rho^b(\text{BN}_2)/E_g$  with the birefringence, as shown in Fig. 4. As we can see, the proportion of  $\Delta\rho^b(\text{BN}_2)/E_g$  is 2.284 in  $\text{Ca}_3(\text{BN}_2)\text{N}$ , almost 1.7 times that in  $\text{Mg}_3\text{BN}_3$  (1.286), which indicates that the tendency is consistent with the variation of birefringence. Simultaneously, there exists a significant difference between the bandgap of the two compounds, which is the main reason that  $\text{Ca}_3(\text{BN}_2)\text{N}$  exhibits an extraordinarily larger birefringence than  $\text{Mg}_3\text{BN}_3$  while they have a similar contribution of  $\text{BN}_2$  units.

## 4. Conclusion

In summary, we investigated the birefringence-enhanced FMs in a B–N system, and a linear group  $\text{BN}_2$  unit was proposed. Through the investigation based on the Inorganic Crystal Structure Database, seven ternary boron nitrides with the formulas  $\text{A}_3\text{BN}_2$  (A = Li, Na),  $\text{A}_3\text{BN}_3$  (A = Mg, Ca), and  $\text{Ba}_3(\text{BN}_2)_2$  were screened. Their birefringence values are in the range of 0.1–0.4 at 1064 nm, especially the birefringence of  $\text{Ca}_3(\text{BN}_2)\text{N}$ , which is 0.411 at 1064 nm, exceeding that of the commercial birefringent materials  $\alpha\text{-BaB}_2\text{O}_4$ ,  $\text{CaCO}_3$  and  $\text{YVO}_4$ . Based on the analysis of the REDA and polarizability anisotropy, it was concluded that the large birefringence mainly originates from the  $\text{BN}_2$  unit. Hence, the functionality of the  $\text{BN}_2$  unit could be utilized in birefringence-enhanced FMs, which exhibits the possibility of applying B–N modules to the birefringent field. This study provides a feasible way to design optical materials with enlarged birefringence.

## Conflicts of interest

The authors declare that they have no conflict of interest.

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