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# Attaining record-high magnetic exchange, magnetic anisotropy and blocking barriers in dilanthanofullerenes†

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While the blocking barrier ( $U_{\text{eff}}$ ) and blocking temperature ( $T_B$ ) for "Dysprocium" SMMs have been increased beyond liquid N<sub>2</sub> temperature, device fabrication of these molecules remains a challenge as low-coordinate Ln<sup>3+</sup> complexes are very unstable. Encapsulating the lanthanide ion inside a cage such as a fullerene (called endohedral metallofullerene or EMF) opens up a new avenue leading to several Ln@EMF SMMs. The *ab initio* CASSCF calculations play a pivotal role in identifying target metal ions and suitable cages in this area. Encouraged by our earlier prediction on Ln<sub>2</sub>@C<sub>79</sub>N, which was verified by experiments, here we have undertaken a search to enhance the exchange coupling in this class of molecules beyond the highest reported value. Using DFT and *ab initio* calculations, we have studied a series of Gd<sub>2</sub>@C<sub>2n</sub> (30 ≤ 2n ≤ 80), where an antiferromagnetic  $J_{\text{Gd...Gd}}$  of  $-43 \text{ cm}^{-1}$  was found for a stable Gd<sub>2</sub>@C<sub>38</sub>-D<sub>3h</sub> cage. This extremely large and exceptionally rare 4f...4f interaction results from a direct overlap of 4f orbitals due to the confinement effect. In larger cages such as Gd<sub>2</sub>@C<sub>60</sub> and Gd<sub>2</sub>@C<sub>80</sub>, the formation of two centre-one-electron (2c-1e<sup>-</sup>) Gd-Gd bonds is perceived. This results in a radical formation in the fullerene cage leading to its instability. To avoid this, we have studied heterofullerenes where one of the carbon atoms is replaced by a nitrogen atom. Specifically, we have studied Ln<sub>2</sub>@C<sub>59</sub>N and Ln<sub>2</sub>@C<sub>79</sub>N, where strong delocalisation of the electron yields a mixed valence-like behaviour. This suggests a double-exchange ( $B$ ) is operational, and CASSCF calculations yield a  $B$  value of  $434.8 \text{ cm}^{-1}$  and resultant  $J_{\text{Gd-rad}}$  of  $869.5 \text{ cm}^{-1}$  for the Gd<sub>2</sub>@C<sub>59</sub>N complex. These parameters are found to be two times larger than the world-record  $J$  reported for Gd<sub>2</sub>@C<sub>79</sub>N. Further *ab initio* calculations reveal an unprecedented  $U_{\text{cal}}$  of  $1183$  and  $1501 \text{ cm}^{-1}$  for Dy<sub>2</sub>@C<sub>59</sub>N and Tb<sub>2</sub>@C<sub>59</sub>N, respectively. Thus, this study offers strong exchange coupling as criteria for new generation SMMs as the existing idea of enhancing the blocking barrier via crystal field modulation has reached its saturation point.

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## Introduction

Single molecule magnets (SMMs) are of prime interest in molecular magnetism due to their potential application in memory storage devices, qubits, *etc.*<sup>1,2</sup> The figure of merit of an SMM is determined by the blocking barrier for magnetisation reversal ( $U_{\text{eff}}$ ) and blocking temperature ( $T_B$ ), the temperature below which opening of magnetic hysteresis is observed. These  $U_{\text{eff}}$  and  $T_B$  values are generally very high for lanthanides, thanks to their strong spin-orbit coupling.<sup>3–10</sup> The enhancement of  $T_B$  as high as 80 K in "Dysprocium" complexes was an important breakthrough, replenishing the hope for potential applications in information storage devices.<sup>11–15</sup> Among others, important bottlenecks that are likely to hamper the futuristic application of these SMMs are (i) enhancing the blocking

temperature beyond 80 K (ii) obtaining molecules that are stable under ambient conditions so that fabrication can be attempted (iii) retaining their intriguing magnetic properties upon fabrication – many of the best transition metal SMMs failed these criteria.<sup>16–21</sup>

To address the first challenge, among other strategies that could help enhance the barrier height/blocking temperature is the quenching of quantum tunnelling of magnetisation (QTM), which is prevalent at low temperatures. If a robust magnetic exchange between two Ln<sup>3+</sup> ions is induced, it can act as a perturbation to reduce the degeneracy of Kramers doublets (KDs). This quenches the QTM and gives rise to large  $U_{\text{eff}}$  and  $T_B$  values.<sup>22–24</sup> However, obtaining a large exchange coupling between two Ln<sup>3+</sup> metal ions is a formidable task as 4f orbitals are deeply buried, leading to a weak/no interaction in dinuclear or polynuclear Ln<sup>3+</sup> complexes.<sup>8,25–30</sup>

In this regard, lanthanide encapsulated fullerenes (called endohedral metallofullerenes or EMFs) are gaining tremendous attention for various reasons: (a) they offer stability to guest molecules which are otherwise unstable;<sup>31</sup> (b) thanks to their

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strong  $\pi$  cloud, fabrication of such molecules on graphene/HOPG/CNTs and other surfaces is straightforward;<sup>31–39</sup> (c) during this process guest molecules stay intact, and hence they are unlikely to lose their characteristics upon fabrication;<sup>31</sup> (d) as fullerenes are made of pure carbon, and the source of nuclear spin of the guest molecules can be controlled, they offer a nuclear spin free system – a key criterion for some qubit applications.<sup>31,40</sup> These key advantages mentioned here directly address the aforementioned goals (ii) and (iii), making them superior to traditional coordination chemistry/organometallic SMMs/SIMs.

One way to attain strong exchange coupling in lanthanide SMMs is to employ radical–Ln exchange which is substantially larger due to the direct exchange between 4f–2p orbitals.<sup>41–43</sup> In the search for a stronger exchange in Ln–radical systems, using a combination of DFT and *ab initio* methods, we have predicted a record high magnetic exchange coupling for a  $\text{Gd}_2@\text{C}_{79}\text{N}$  radical fullerene complex and also suggested a very large blocking barrier for the Dy analogue.<sup>44,45</sup> Both these predictions were proved in a span of few years independently by two groups,<sup>46–49</sup> and  $\text{Gd}_2@\text{C}_{79}\text{N}$  is found to have a very large spin relaxation time opening up a new avenue in spin-based qubits.<sup>40,46</sup> While a Ln–radical exchange could solve this problem,<sup>41</sup> the majority of the conventional lanthanide–radical systems are highly reactive and could pose a challenge in accomplishing the aforementioned goals (ii) and (iii).<sup>41,43,50–55</sup>

In this connection, if a robust exchange is induced between two  $\text{Ln}^{3+}$  ions, this will be very rewarding. One strategy to enhance the exchange coupling is to induce a weak  $\text{Ln}\cdots\text{Ln}$  bond, which is possible if two ions are brought very close to each other directly. The metal–metal bonds in transition metal complexes are common but are scarce for lanthanides.<sup>56–58</sup> Inspired from the report that even noble gas elements such as He form He–He bonds under confinement, we devise such models for lanthanides that can offer very large 4f–4f exchange interactions.<sup>59–63</sup> In line with this idea, we have explored various  $\text{Gd}_2@\text{C}_{2n}$  ( $2n = 30–52, 60, 80$ ) complexes in search of a stronger exchange and found  $\text{Gd}^{3+}\cdots\text{Gd}^{3+}$  exchange as high as  $-43\text{ cm}^{-1}$ . In the second approach, we have extended our study to air-stable azafullerene radical analogues such as  $\text{Ln}_2@\text{C}_{59/79}\text{N}$  ( $\text{Ln} = \text{Gd, Tb, Dy}$ ). Using *ab initio* calculations, we have computed the double-exchange parameter  $B$  in these azafullerene cages. We have exploited the presence of double exchange to design SMMs based on Dy and Tb and unveil a new line of prediction with models exhibiting a  $U_{\text{eff}}$  value exceeding  $1500\text{ cm}^{-1}$ .

## Results and discussion

Achieving large exchange coupling in lanthanides is challenging as the 4f orbitals of lanthanides are deeply buried and interact weakly with ligand orbitals. The highest magnetic exchange between two  $\text{Ln}^{3+}$  ions is estimated in a  $\{\text{Gd}_2\text{Cr}_2\}$  complex where  $J_{\text{Gd–Gd}}$  is  $+1.4\text{ cm}^{-1}$  ( $\hat{H} = -J\hat{S}_1\hat{S}_2$ ).<sup>28</sup> As the  $\text{Ln}\cdots\text{Ln}$  distance plays a crucial role in controlling the 4f–4f exchange interaction, a large  $J$  is expected if two  $\text{Ln}^{3+}$  ions are confined in a fullerene cage. With this goal, we begin our study with  $\text{Gd}_2$  endohedral fullerenes by varying the cage size from  $\text{C}_{30}$  to  $\text{C}_{80}$ .

We have analysed the structure, binding energy, and magnetic properties within the DFT framework for two low energy conformers of the fullerene cages among various close-lying isomers.

### Structure and bonding in $\text{Gd}_2@\text{C}_{2n}$ ( $2n = 30–48, 52, 40, 80$ )

The  $\text{C}_{30}$  fullerene is the smallest cage where encapsulation leads to a stable geometry, as steric strain dominates over the metal–cage stabilisation in  $\text{C}_{28}$  and lower cages (Fig. 1, Table S1 and Appendix S1–S25†). For  $\text{Gd}_2@\text{C}_{30}$ , a  $C_{2v}$  isomer is found to be stable by  $52.3\text{ kJ mol}^{-1}$  compared to the  $D_{5h}$  isomer due to stronger Gd–C interactions in the former as affirmed by the AIM analysis (see Table 1 for larger cages and Tables S1–S4 in the ESI†). In larger cages, the stability can be rationalised using (i) the number of APRs (Table S2†) and (ii) the nature of Gd–C interaction as obtained from the AIM analysis (see Fig. 1, S1–S26 and Tables S3–S27 in the ESI†).

Considering the  $\text{Gd}^{3+}$  ionic radius,<sup>64</sup> a  $\text{Gd}\cdots\text{Gd}$  distance less than  $2.5\text{ \AA}$  (van der Walls radii) is likely to suggest a weak interaction or even a metal–metal bond. Such interactions are expected to reflect on  $J_{\text{Gd–Gd}}$  values with smaller values indicate weaker  $\text{Gd}\cdots\text{Gd}$  interactions and not a metal–metal bond. Therefore, to compare the metal–metal interaction in  $\text{Gd}_2@\text{C}_{2n}$  with  $30 \leq 2n \leq 52$ , the magnetic exchange  $J_{\text{Gd–Gd}}$  between two  $\text{Gd}^{3+}$  ions has been estimated using DFT calculations (B3LYP/TZV,  $\hat{H} = -J\hat{S}_{\text{Gd}1}\hat{S}_{\text{Gd}2}$ , see computational details and Table 1). The  $J_{\text{Gd–Gd}}$  is found to be antiferromagnetic in all  $\text{Gd}_2@\text{C}_{2n}$  ( $2n \leq 52$ ) EMFs with the exception of  $\text{Gd}_2@\text{C}_{44}\text{-C}_s$ ,  $\text{Gd}_2@\text{C}_{48}\text{-C}_{2v}$ , and  $\text{Gd}_2@\text{C}_{52}\text{-D}_{2d}$  EMFs having a ferromagnetic coupling (Table 1). The value in Table 1 suggests the decrease in antiferromagnetic interaction with the decrease in cage size. Within the same cage, the  $J_{\text{Gd–Gd}}$  value increases for a higher symmetry isomer. The largest antiferromagnetic  $J_{\text{Gd–Gd}}$  was estimated for  $\text{Gd}_2@\text{C}_{30}\text{-D}_{5h}$  ( $-62.7\text{ cm}^{-1}$ ). This is several orders of magnitude larger than the experimentally known largest 4f–4f interaction. For the  $\text{Gd}_2@\text{C}_{30}\text{-C}_{2v}$  isomer, the  $J_{\text{Gd–Gd}}$  decreases to  $-49.6\text{ cm}^{-1}$  despite a shorter  $\text{Gd}\cdots\text{Gd}$  distance compared to the  $D_{5h}$  isomer.

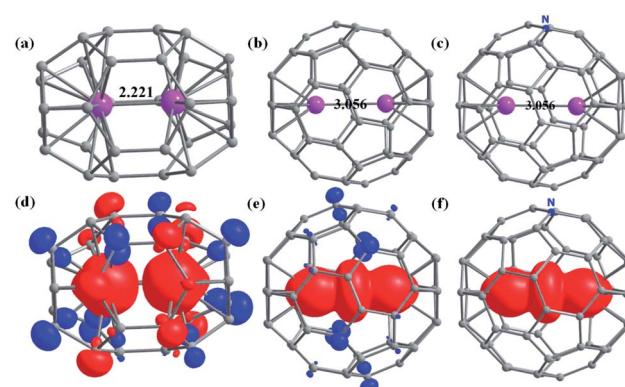


Fig. 1 The optimized structures of (and Gd–C bond length range) (a)  $\text{Gd}_2@\text{C}_{30}\text{-D}_{5h}$  ( $2.140–2.350\text{ \AA}$ ), (b)  $\text{Gd}_2@\text{C}_{60}\text{-I}_h$  ( $2.400–2.407\text{ \AA}$ ), and (c)  $\text{Gd}_2@\text{C}_{59}\text{N-C}_s$  ( $2.400–2.407\text{ \AA}$ ). The corresponding spin density plots for the high spin state are given in figures (d–f) with an isosurface value of  $0.006\text{ e}^{-}\text{bohr}^{-3}$ . Colour code: Gd-pink, C-grey, N-blue.



**Table 1** The estimated  $J$  and the binding energy ( $\text{kJ mol}^{-1}$ ) of chosen conformers in  $\text{Gd}_2@\text{C}_{2n}$ . Next to the symmetry label, the  $\text{Gd}\cdots\text{Gd}$  distance is given in parentheses (Å). The value of spin density of each metal centre in the HS configuration of all  $\text{Gd}_2@\text{C}_{2n}$  has been given below the exchange values in parentheses. All the  $J$  and  $B$  values are shown in  $\text{cm}^{-1}$

$\text{Gd}_2@\text{C}_{2n}$	$J_{\text{Gd}\cdots\text{Gd}}$ ( $\text{cm}^{-1}$ )		Binding energy ( $\text{kJ mol}^{-1}$ )		$\Delta E$ ( $\text{kJ mol}^{-1}$ )	
$2n = 30$	$C_{2v}$ (2.185) $J_{\text{Gd}\cdots\text{Gd}} = -49.8$ (6.96; 6.95)	$D_{5h}$ (2.224) $J_{\text{Gd}\cdots\text{Gd}} = -62.7$ (6.94; 6.94)	$C_{2v}$ 1160.9	$D_{5h}$ 1149.8	$C_{2v}$ 0.0	$D_{5h}$ 52.3
$2n = 32$	$C_2$ (2.207) $J_{\text{Gd}\cdots\text{Gd}} = -12.0$ (6.93; 6.95)	$D_3$ (2.272) $J_{\text{Gd}\cdots\text{Gd}} = -15.5$ (6.93; 6.93)	$C_2$ 907.3	$D_3$ 620.2	$C_2$ 166.5	$D_3$ 0.0
$2n = 34$	$C_s$ (2.283) $J_{\text{Gd}\cdots\text{Gd}} = -11.6$ (6.94; 6.94)	$C_2$ (2.266) $J_{\text{Gd}\cdots\text{Gd}} = -13.7$ (6.94; 6.94)	$C_s$ 279.1	$C_2$ 392.1	$C_s$ 73.0	$C_2$ 0.0
$2n = 36$	$C_s$ (2.400) $J_{\text{Gd}\cdots\text{Gd}} = -8.3$ (6.92; 6.97)	$D_{2d}$ (2.269) $J_{\text{Gd}\cdots\text{Gd}} = -26.3$ (6.96; 6.95)	$C_s$ 61.0	$D_{2d}$ 330.0	$C_s$ 0.0	$D_{2d}$ 37.2
$2n = 38$	$C_1$ (2.443) $J_{\text{Gd}\cdots\text{Gd}} = -7.0$ (6.97; 6.93)	$D_{3h}$ (2.734) $J_{\text{Gd}\cdots\text{Gd}} = -43.4$ (6.99; 6.99)	$C_1$ -108.1	$D_{3h}$ -63.0	$C_1$ 0.0	$D_{3h}$ 478.1
$2n = 40$	$C_{2v}$ (2.376) $J_{\text{Gd}\cdots\text{Gd}} = -4.9$ (7.00; 7.00)	$D_2$ (2.400) $J_{\text{Gd}\cdots\text{Gd}} = -10.3$ (6.99; 6.99)	$C_{2v}$ 0.7	$D_2$ -173.2	$C_{2v}$ 320.6	$D_2$ 0.0
$2n = 42$	$C_1$ (2.495) $J_{\text{Gd}\cdots\text{Gd}} = -3.0$ (6.99; 6.99)	$D_3$ (2.430) $J_{\text{Gd}\cdots\text{Gd}} = -7.6$ (7.00; 7.00)	$C_1$ -337.3	$D_3$ -193.1	$C_1$ 0.0	$D_3$ 60.8
$2n = 44$	$C_s$ (2.608) $J_{\text{Gd}\cdots\text{Gd}} = 0.2$ (6.98; 7.02)	$D_2$ (2.549) $J_{\text{Gd}\cdots\text{Gd}} = -8.4$ (7.00; 7.00)	$C_s$ -386.9	$D_2$ -308.0	$C_s$ 157.1	$D_2$ 0.0
$2n = 46$	$C_1$ (2.728) $J_{\text{Gd}\cdots\text{Gd}} = -1.4$ (6.99; 7.00)	$C_s$ (2.796) $J_{\text{Gd}\cdots\text{Gd}} = -0.3$ (6.98; 6.98)	$C_1$ -494.8	$C_s$ -537.0	$C_1$ 42.1	$C_s$ 0.0
$2n = 48$	$C_1$ (2.836) $J_{\text{Gd}\cdots\text{Gd}} = -0.4$ (7.00; 7.00)	$C_{2v}$ (3.002) $J_{\text{Gd}\cdots\text{Gd}} = 0.7$ (6.99; 6.99)	$C_1$ -560.2	$C_{2v}$ -534.3	$C_1$ 0.0	$C_{2v}$ 410.6
$2n = 52$	$C_s$ (3.282) $J_{\text{Gd}\cdots\text{Gd}} = -1.3$ (7.02; 6.99)	$D_{2d}$ (2.324) $J_{\text{Gd}\cdots\text{Gd}} = 2.7$ (7.04; 7.04)	$C_s$ -595.2	$D_{2d}$ -991.4	$C_s$ 224.3	$D_{2d}$ 0.0
$2n = 60^b$	$\text{Gd}_2@\text{C}_{60}\text{I}_h$ (3.056) $J_1 = 869.8, J_2 = 0.08, J_3 = 40.2, B = 434.8$ (7.53; 7.53)	$\text{Gd}_2@\text{C}_{59}\text{N-C}_s$ (3.056) $J_1 = 869.8, J_2 = 0.08, B = 434.8$ (7.54; 7.54)	$\text{Gd}_2@\text{C}_{60}\text{I}_h$ -369.4 <sup>a</sup>	$\text{Gd}_2@\text{C}_{59}\text{N-C}_s$ -389.1 <sup>a</sup>		
$2n = 80^b$	$D_{5h}$ (3.818) $J_1 = 404.6, J_2 = 0.03, B = 202.1, J_3 = 351.3, B = 175.6$ (7.54; 7.54)	$C_{2v}$ (4.074) $J_1 = 351.3, J_2 = 0.03, J_3 = -41.3$ (7.54; 7.54)	$D_{5h}$ -987.6 <sup>a</sup>	$C_{2v}$ -840.5 <sup>a</sup>	$D_{5h}$ 0.0	$C_{2v}$ 96.7
$2n = 80^b$	$\text{Gd}_2@\text{C}_{79}\text{N-C}_s\text{-1}$ (3.816) $J_1 = 404.6, J_2 = 0.03, B = 202.1$ (7.55; 7.55)	$\text{Gd}_2@\text{C}_{79}\text{N-C}_s\text{-2}$ (4.107) $J_1 = 351.3, J_2 = 0.03, B = 175.6$ (7.52; 7.53)	$\text{Gd}_2@\text{C}_{79}\text{N-C}_s\text{-1}$ -958.0 <sup>a</sup>	$\text{Gd}_2@\text{C}_{79}\text{N-C}_s\text{-2}$ -733.7 <sup>a</sup>	$\text{Gd}_2@\text{C}_{79}\text{N-C}_s\text{-1}$ 0.0	$\text{Gd}_2@\text{C}_{79}\text{N-C}_s\text{-2}$ 157.7

<sup>a</sup> The binding energy has been calculated with respect to electronic energy. In all other isomers, the binding energy has been calculated with respect to electronic and thermal free energies. <sup>b</sup> Here 'n' represents the total number of atoms, including the one nitrogen atom.

This is due to stronger 4f–4f overlaps (Tables S28–S29†). Although the  $\text{Gd}\cdots\text{Gd}$  distances are very similar for  $\text{C}_{30}$  and  $\text{C}_{32}$ , the  $J_{\text{Gd}\cdots\text{Gd}}$  value is significantly smaller in  $\text{Gd}_2@\text{C}_{32}$  (see Table 1) due to symmetry constraints and the associated 4f–4f overlaps (see Tables S30–S49 in ESI†). Further increase in the cage size only nominally decreases the  $J_{\text{Gd}\cdots\text{Gd}}$  values with several exceptions, though lower symmetry models follow the trend (see Fig. S27†). A net ferromagnetic interaction is observed in  $\text{Gd}_2@\text{C}_{44}\text{-C}_s$ ,  $\text{Gd}_2@\text{C}_{48}\text{-C}_{2v}$  and  $\text{Gd}_2@\text{C}_{52}\text{-D}_{2d}$  cages due to a meagre contribution to the antiferromagnetic part of  $J$  (see Tables S42, S46 and S48†). Orbital orthogonality of 4f-orbitals and dipolar contributions due to shorter  $\text{Gd}\cdots\text{Gd}$  distance leads to a net ferromagnetic coupling in these examples. A very large 4f–4f overlap suggests a possibility of direct 4f–4f interactions between two lanthanide ions, which are hard to observe in classical coordination chemistry. The binding energy becomes positive for  $\text{Gd}_2@\text{C}_{2n}$  with  $2n \leq 36$  and negative for  $\text{Gd}_2@\text{C}_{2n}$  with  $2n > 36$  (see Table 1) except for the  $\text{Gd}_2@\text{C}_{40}\text{-C}_{2v}$  isomer, where it is thermoneutral (0.7  $\text{kJ mol}^{-1}$ ). Thus, it suggests that the large antiferromagnetic interaction is feasible for the isomers of  $\text{Gd}_2@\text{C}_{2n}$  with  $2n > 36$ .

The magnitude of the spin density of the two  $\text{Gd}^{3+}$  ions increases with an increase in ring size, supported by the contour plots of the electron density map obtained from AIM analysis (Fig. S28–S54 and Tables S50, S51†). Particularly a sudden jump in the magnitude of spin density is noted for  $\text{Gd}_2@\text{C}_{60}$ , with nearly one electron found between the two  $\text{Gd}$  ions (see Fig. 1, S50–S52†). Our NBO analysis reveals that this electron is delocalised in the formally empty orbitals, which are hybridised among 6s, 6p, and 5d orbitals ( $6s6p^{0.11}5d^{0.33}$ , see Fig. S55†). Thus, it suggests a strong valence delocalisation where one unpaired electron is delocalised to vacant 5d/6s/6p orbitals of each  $\text{Gd}^{3+}$  ion leading to a type-III class of mixed valence systems ( $\text{Gd}^{2.5+}\cdots\text{Gd}^{2.5+}$ , see later).<sup>65</sup>

### Mechanism of the formation of $\text{Gd}_2@\text{C}_{2n}$

To further investigate the unusual behaviour wherein the cage size decides the magnitude of the spin density present between the  $\text{Gd}^{3+}$  ion, we have analysed the formation of  $\text{Gd}_2@\text{C}_{2n}$  from the HOMO–LUMO gap perspective. In the formation of dimesitallofullerene  $\text{Gd}_2@\text{C}_{2n}$ , we can presume that two  $\text{Gd}$  atoms donate three electrons each from their frontier orbitals (5d and



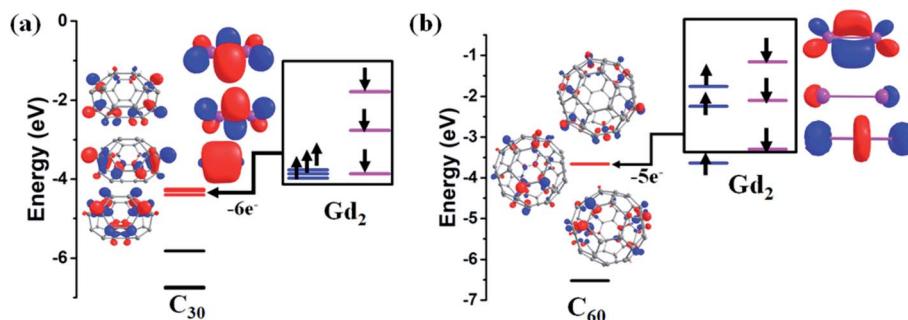


Fig. 2 The MO diagrams corresponding to the formation of (a)  $\text{Gd}_2@\text{C}_{30}-\text{C}_{2v}$  and (b)  $\text{Gd}_2@\text{C}_{60}-\text{I}_{\text{h}}$  isomers. The three black and red horizontal lines correspond to the energy of the occupied and empty orbitals of the  $\text{C}_{2n}$  fullerene ring, respectively. The blue and pink horizontal lines correspond to the energy of  $\alpha$  and  $\beta$  orbitals of the  $\text{Gd}_2$  fragment. We have shown the three lowest unoccupied  $\alpha$  orbitals of the  $\text{C}_{2n}$  fullerene cage with an isosurface value of  $0.055 \text{ e}^- \text{ bohr}^{-3}$ . The three highest occupied  $\alpha$  orbitals for  $\text{Gd}_2$  in the  $\text{C}_{30}$  and  $\text{C}_{60}$  fullerene cage are also shown (isosurface  $0.06 \text{ e}^- \text{ bohr}^{-3}$ ). Colour code: Gd-pink, C-grey.

6s orbitals) to the three lowest unoccupied molecular orbitals (LUMOs) of the  $\text{C}_{2n}$  cage resulting in  $\text{Gd}_2^{6+}@\text{C}_{2n}^{6-}$ .<sup>66</sup> Thus, the formation of  $\text{Gd}_2@\text{C}_{2n}$  depends on the energy gap between the frontier orbitals of  $\text{Gd}_2$  and the LUMOs of the  $\text{C}_{2n}$  cage. If the LUMOs of the  $\text{C}_{2n}$  cage are found to be lower in energy than the frontier orbitals of the  $\text{Gd}_2$  fragment, a large stabilisation occurs after the electron transfer. Quite interestingly, this is the case for the  $\text{C}_{2n}$  cage with  $2n \leq 52$ , which favours the transfer of six electrons from  $\text{Gd}_2$  (with the Gd–Gd distance  $<3.0 \text{ \AA}$ , ignoring 4f orbitals, the valence electron configuration is  $\sigma_g^2\sigma_u^1\pi_u^1\pi_u^2$ )<sup>67</sup> to the  $\text{C}_{2n}$  cage (Fig. 2a for  $\text{Gd}_2@\text{C}_{30}$  and Fig. S56 and S57† for  $\text{Gd}_2@\text{C}_{52}-\text{D}_{2d}$  and  $\text{Gd}_2@\text{C}_{48}-\text{C}_{2v}$ ). As the ring size increases, the LUMOs of the  $\text{C}_{2n}$  cage destabilised. In  $\text{Gd}_2@\text{C}_{60}$  with the Gd–Gd distance of  $3.056 \text{ \AA}$ , the bonding in the  $\text{Gd}_2$  fragment before encapsulation is found to be  $\sigma_g^2\pi_u^3\sigma_u^1$  (ignoring the 4f orbitals, Fig. 2b). After encapsulation, the five electrons are fully transferred to the cage except one  $\sigma_g^1$  electron (here the  $\beta$  electron in Fig. 2b for  $\text{Gd}_2@\text{C}_{60}-\text{I}_{\text{h}}$ ) resulting in a  $2\text{e}-1\text{e}^-$  bond between two Gd atoms. This is due to the comparable energy of the beta (6s/5d)  $\sigma_g^1$  orbital with the LUMO of the  $\text{C}_{2n}$  cage.

### Estimation of magnetic exchange in $\text{Gd}_2@\text{C}_{59}\text{N}-\text{C}_s$ , $\text{Gd}_2@\text{C}_{79}\text{N}-\text{C}_s\text{-}1$ , and $\text{Gd}_2@\text{C}_{79}\text{N}-\text{C}_s\text{-}2$

The most sensitive parameter that yields insight into the spin density distribution discussed in the last section is the corresponding exchange coupling  $J_{\text{Gd-Gd}}$ . Here we intend to compute this parameter and analyse this with respect to the cage size. The mechanism of formation of  $\text{Gd}_2@\text{C}_{2n}$  suggests the presence of one unpaired electron between two  $\text{Gd}^{3+}$  ions and another conjugate electron in the fullerene cage for  $\text{Gd}_2@\text{C}_{60}-\text{I}_{\text{h}}$ ,  $\text{Gd}_2@\text{C}_{80}-\text{D}_{5h}$ , and  $\text{Gd}_2@\text{C}_{80}-\text{C}_{2v}$  isomers (see Table S50†). For these molecules, a complex set of magnetic coupling emerges: (i) the coupling between  $\text{Gd}^{3+}$  and the radical that reside inside the cage ( $J_1$ ), (ii) the second one describes the coupling between two  $\text{Gd}^{3+}$  ions ( $J_2$ ), (iii) the third one describes the coupling between two radicals ( $J_3$ ) and (iv) in addition to these isotropic exchange coupling values, a strong electron delocalisation of the radical between two  $\text{Gd}^{3+}$  ions suggests a double-exchange (parameter  $B$ ) being operative between two Gd ions (in a fully delocalised case,  $\text{Gd}_2^{2.5+}$ ). All these exchanges have been illustrated in Scheme S2.† This is similar to a type-III mixed-valence

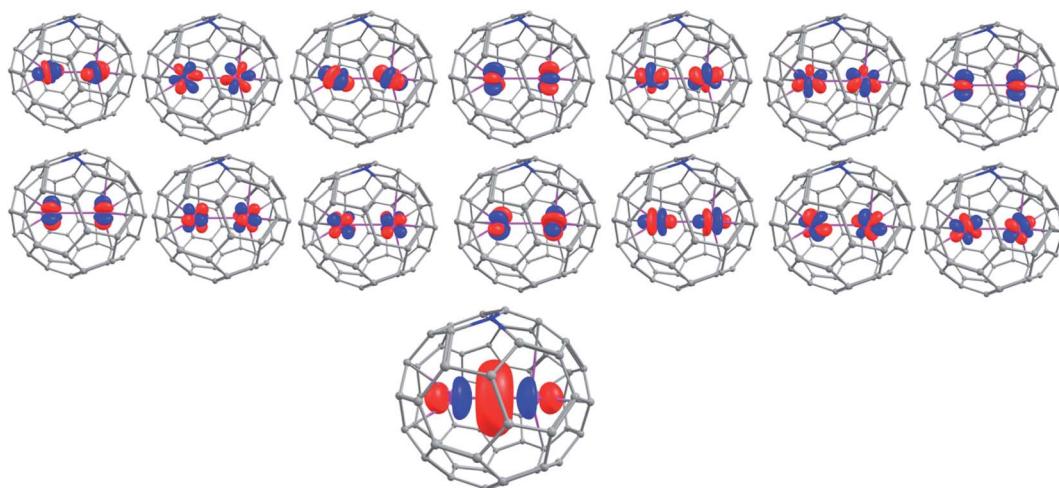


Fig. 3 The fifteen active orbitals of  $\text{Gd}_2@\text{C}_{59}\text{N}$  with CAS (15,15) active space for the  $S = 15/2$  state. Colour code: Gd-pink, C-grey, N-blue.



system<sup>68,69</sup> where the spin Hamiltonian parameters are estimated using the following Hamiltonian<sup>70</sup>

$$H = -J(S_A \cdot S_B \cdot O_A + S_A \cdot S_B \cdot O_B) + BT_{AB}, \quad (1)$$

where  $J$  and  $B$  denote the exchange interaction and delocalisation parameter, respectively,  $S_A$  and  $S_B$  are the total spin multiplicity of centres  $A$  and  $B$ , respectively,  $O_A$  and  $O_B$  are the localisation operator, and  $T_{AB}$  is the electron transfer operator (see computational details for more information).

The presence of one unpaired electron in the fullerene cage of  $\text{Gd}_2@\text{C}_{60}-\text{I}_h$ ,  $\text{Gd}_2@\text{C}_{80}-\text{D}_{5h}$ , and  $\text{Gd}_2@\text{C}_{80}-\text{C}_{2v}$  leads to polymerisation or aggregation, and often, these complexes are not isolable.<sup>71,72</sup> There are two strategies available to demonstrate their existence (i) by transforming them into a chemically stable form with one-electron reduction/substitution at the ring position. This has been adapted to stabilise the  $\text{Dy}_2@\text{C}_{80}-\text{I}_h$  molecule by chemically transforming it to  $\text{Dy}_2@\text{C}_{80}(\text{CH}_3\text{Ph})$ .<sup>72-75</sup> (ii) By substituting one of the carbon with the nitrogen atom yielding azafullerenes such as  $\text{Ln}_2@\text{C}_{79}\text{N}$  and other analogues.<sup>49,70,76-79</sup> Here, we have adopted the second approach where one carbon atom is substituted by nitrogen in  $\text{Gd}_2@\text{C}_{60/80}$  isomers yielding  $\text{Gd}_2@\text{C}_{59/79}\text{N}$  molecules (see Appendix S26-S28† for optimised coordinates) possessing  $C_s$  symmetry (here  $\text{Gd}_2@\text{C}_{79}\text{N}-\text{C}_s\text{-}1$  is derived from  $\text{Gd}_2@\text{C}_{80}-\text{D}_{5h}$  and  $\text{Gd}_2@\text{C}_{79}\text{N}-\text{C}_s\text{-}2$  is derived from  $\text{Gd}_2@\text{C}_{80}-\text{C}_{2v}$ , see Table 1). Upon substitution, as expected, the spin density of the cage in  $\text{Gd}_2@\text{C}_{59/79}\text{N}$  was seized (see Fig. 1e and f for the 665 isomer, see ref. 44). While  $\text{Gd}_2@\text{C}_{79}\text{N}$  is a well-characterised and thoroughly studied molecule,  $\text{Gd}_2@\text{C}_{59}\text{N}$  is not known. However, the X-ray structure of  $\text{C}_{59}\text{N}$  and encapsulation of some metal ions are experimentally studied, and their existence has been proved beyond ambiguity.<sup>35,80-86</sup> Particularly,  $\text{K}_6\text{C}_{59}\text{N}$  has been isolated and characterised thoroughly. This suggests that the  $\text{C}_{59}\text{N}^{6-}$  cage is a stable molecular fragment and can encapsulate  $\text{Ln}^{3+}$  cations similar to those hypothesised here.<sup>87-89</sup>

This type-III mix valence moiety of  $\text{Gd}_2@\text{C}_{59}\text{N}-\text{C}_s$ ,  $\text{Gd}_2@\text{C}_{79}\text{N}-\text{C}_s\text{-}1$ , and  $\text{Gd}_2@\text{C}_{79}\text{N}-\text{C}_s\text{-}2$  isomer represents a multireference wave function as the unpaired electron is not localised on a particular centre. Therefore, a multireference method such as the state-average CAS(15,15)SCF set up was employed to estimate the double exchange parameter ( $B$ ) (Fig. 3, see computational details).<sup>90</sup> As per the CASSCF calculations, the additional radical electron resides in a hybrid orbital containing coefficients from 6s, 5p<sub>z</sub>, 6p<sub>z</sub>, and 5d<sub>z<sup>2</sup></sub> orbitals of  $\text{Gd}_1$  and  $\text{Gd}_2$  centres (see Table S55† for composition). The set of spin Hamiltonian parameters obtained from the CASSCF calculations are as follows,  $\text{Gd}_2@\text{C}_{59}\text{N}-\text{C}_s$  ( $\text{Gd}_2@\text{C}_{79}\text{N}-\text{C}_s\text{-}1$ ) [ $\text{Gd}_2@\text{C}_{79}\text{N}-\text{C}_s\text{-}2$ ]:  $J_1 = +869.8$  (+404.6) [+351.3]  $\text{cm}^{-1}$ ,  $J_2 = 0.08$  (0.03) [0.03]  $\text{cm}^{-1}$  and  $B = +434.8$  (+202.1) [+175.6]  $\text{cm}^{-1}$  (Table 1). For all three complexes, the  $J_1$  interaction is found to be extremely large, and this is due to the involvement of the diffuse virtual 6s and 6p<sub>z</sub> and 5d<sub>z<sup>2</sup></sub> orbitals of Gd ions, while the  $J_2$  coupling between two  $\text{Gd}^{3+}$  ions is found to be very small as the 4f orbitals are only weakly interacting here. It is worth mentioning that we have previously reported a very large  $J_1$  value of +400  $\text{cm}^{-1}$  ( $\hat{H} = -J\hat{S}_1\hat{S}_2$ ) in  $\text{Gd}_2@\text{C}_{79}\text{N}$  using the

UB3LYP/TZV setup.<sup>44</sup> The estimated  $J_1$  value by our *ab initio* approach for  $\text{Gd}_2@\text{C}_{79}\text{N}$  lies in the range of 350–405  $\text{cm}^{-1}$ , and this is in line with the DFT calculations and experimental reports ( $-350 \pm 20 \text{ cm}^{-1}$  using  $\hat{H} = J_1(\hat{S}_{\text{Gd}_1}\hat{S}_{\text{rad}} + \hat{S}_{\text{Gd}_2}\hat{S}_{\text{rad}}) + J_2(\hat{S}_{\text{Gd}_1}\hat{S}_{\text{Gd}_2})$  in ref. 40 and a  $J_1$  value of  $170 \pm 10 \text{ cm}^{-1}$  using the Hamiltonian  $\hat{H} = -2J_1(\hat{S}_{\text{Gd}_1}\hat{S}_{\text{rad}} + \hat{S}_{\text{Gd}_2}\hat{S}_{\text{rad}}) - 2J_2(\hat{S}_{\text{Gd}_1}\hat{S}_{\text{Gd}_2})$  in ref. 76; see ESI† for the discussion of  $J_3$ ). These large exchange values have potential application in qubits as they enhance the quantum coherence required for qubit applications.<sup>40</sup>

### A case study of magnetic anisotropy in $\text{Dy}_2@\text{C}_{59}\text{N}$ and $\text{Tb}_2@\text{C}_{59}\text{N}$

As heterofullerenes yield larger  $J_s$  and homofullerene yields relatively smaller antiferromagnetic  $J_s$ , the former is the best suited to design SMMs. The antiferromagnetic  $J_s$  in homofullerene yields diamagnetic ground states, and smaller ferromagnetic  $J_s$  observed in larger cage sizes such as  $\text{C}_{52}$  did not yield any appealing SMM characteristics. To harness SMMs in this class, heterodinuclear lanthanides with unequal  $m_J$  states were modelled. Models such as  $\text{PrEr}@\text{C}_{38}-\text{D}_{3h}$  yield a reasonable  $U_{\text{cal}}$  value with robust QTM quenching (ca. 109  $\text{cm}^{-1}$ , see ESI†) but are not substantial to serve as a synthetic target.

Therefore, we aim to estimate the magnetic anisotropy in the  $\text{Dy}_2@\text{C}_{59}\text{N}$  and  $\text{Tb}_2@\text{C}_{59}\text{N}$ . It is noteworthy to mention that the record-breaking magnetic anisotropy is previously achieved in  $\text{Dy}_2@\text{C}_{79}\text{N}$  and  $\text{Tb}_2@\text{C}_{79}\text{N}$  molecules.<sup>44,77</sup> The metal centre in  $\text{Dy}_2@\text{C}_{59}\text{N}$  is found to interact in an  $\eta^6$  fashion with the  $\text{C}_{59}\text{N}$  cage, which creates a strong uniaxial anisotropy (see Fig. 1c) as a long  $\text{Dy} \cdots \text{Dy}$  bond (3.056 Å) induces a weak ligand field in the opposite site of cage binding. Thus, the coordination can be compared with  $\text{Dy}^{3+}-\text{O}$ , which perfectly suits the oblate ground state.<sup>91,92</sup> The easy axis of magnetisation is found to be nearly collinear with the  $\text{Dy}-\text{Dy}$  axis with a very small angle (2.0 (1.1°) for  $\text{Dy}_1(\text{Dy}_2)$ , Fig. S72†). The calculated  $g_z$  values of KD1 (~19.97) imply an Ising ground state for both the Dy centres (Tables S70, S71 and Fig. S71†), with the relaxation predicted to

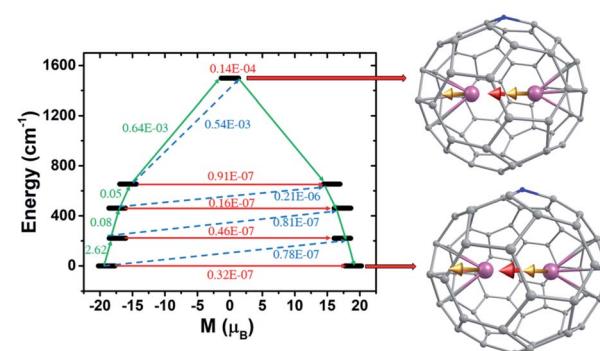


Fig. 4 The POLY\_ANISO computed relaxation mechanism of  $\text{Tb}_2@\text{C}_{59}\text{N}-\text{C}_s$ . The anisotropy axis of the metal (represented by yellow) and radical (represented by red) centre are shown on the right. The thick black line represents the magnetic moment of KDs. The red arrows imply the QTM for ground KD and TA-QTM for higher excited KDs. The blue dotted arrows indicate a possible Orbach process. The green arrows represent the mechanism of magnetic relaxation. Colour code: Tb-blue violet, C-grey, N-blue.



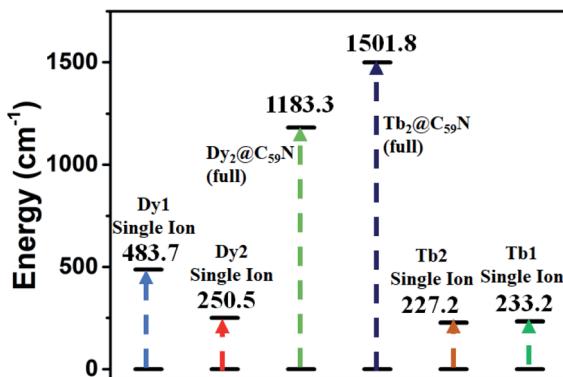


Fig. 5 Diagrammatic representation of the estimated single ion and exchange-coupled  $U_{\text{cal}}$  for  $\text{Dy}_2@\text{C}_{59}\text{N}-\text{C}_s$  and  $\text{Tb}_2@\text{C}_{59}\text{N}-\text{C}_s$ .

proceed *via* the first excited state for  $\text{Dy}_2$  ( $U_{\text{cal}} = 250.5 \text{ cm}^{-1}$ ) and third excited state for the  $\text{Dy}_1$  centre ( $U_{\text{cal}} = 483.7 \text{ cm}^{-1}$ , Fig. S71 and Tables S70, S71†). The very large axial crystal field parameter<sup>93</sup>  $B_{kq}$  ( $k = 0, q = 0$ ) compared to the non-axial crystal field parameter  $B_{kq}$  ( $k = 0, q \neq 0$ ) suggests significant axiality for both the  $\text{Dy}$  centres in  $\text{Dy}_2@\text{C}_{59}\text{N}$  (see Table S72†). Furthermore, the axial CF parameters are found to be slightly larger in  $\text{Dy}_2@\text{C}_{59}\text{N}$  compared to  $\text{Dy}_2@\text{C}_{79}\text{N}$ , suggesting a larger axiality of the former compared to the latter.<sup>44</sup>

The *ab initio* calculations on  $\text{Tb}_2@\text{C}_{59}\text{N}$  reveal a negligible tunnel splitting in the ground pKDs ( $0.025(0.013) \text{ cm}^{-1}$  for  $\text{Tb}_1(\text{Tb}_2)$ , see Tables S73, S74 and Fig. S73†). Further, the ground state  $g_z$  value ( $g_z = 17.921(17.919)$  for  $\text{Tb}_1(\text{Tb}_2)$  centre<sup>94,95</sup>) suggests the Ising nature of the ground state. The ground anisotropy axis of the  $\text{Tb}_1(\text{Tb}_2)$  centre is oriented along the pseudo  $C_6$  axis of the hexagonal ring and nearly collinear with the  $\text{Tb}-\text{Tb}$  axis (the tilting angle becomes  $0.45$  and  $1.50^\circ$  for  $\text{Tb}_1$  and  $\text{Tb}_2$  centres, respectively, Fig. 4). However, the significant tunnel splitting ( $0.115$  and  $0.102 \text{ cm}^{-1}$  for  $\text{Tb}_1(\text{Tb}_2)$ , Fig. S73†) in the first excited pKDs reinforces the magnetisation relaxation *via* this state. This leads to the  $U_{\text{cal}}$  value of  $227.6$  and  $233.2 \text{ cm}^{-1}$  for  $\text{Tb}_1$  and  $\text{Tb}_2$  centres, respectively (see Tables S73, S74 and S70†).

To explore the mechanism of magnetisation relaxation in the exchange-coupled  $\text{Dy}_2@\text{C}_{79}\text{N}$  and  $\text{Tb}_2@\text{C}_{79}\text{N}$  systems, we have simulated the exchange-coupled energy spectrum using the POLY\_ANISO module (see Table 1 and computational details). For the computed ground state, a large magnetic moment of *ca.*  $21$  and  $19 \mu_B$  for  $\text{Dy}_2@\text{C}_{59}\text{N}$  and  $\text{Tb}_2@\text{C}_{59}\text{N}$  respectively, was obtained (see Fig. 4 and S72†) with negligible tunnel splitting or QTM effects. The first, second, and third excited states are found to possess negligible tunnel splitting/TA-QTM, which is reflected in negligible  $g_x/g_y$  and very large  $g_z$  values (see Tables S75, S76, Fig. 4 and S72†). The magnetic moment in the fourth excited state is very small, and it results in sizeable QTM for  $\text{Dy}_2@\text{C}_{59}\text{N}$  and  $\text{Tb}_2@\text{C}_{59}\text{N}$ . Therefore, the magnetisation relaxation for this exchange-coupled system is expected *via* the fourth excited state yielding a record-high  $U_{\text{cal}}$  value of  $1183.3$  and  $1501.8 \text{ cm}^{-1}$  for  $\text{Dy}_2@\text{C}_{59}\text{N}$  and  $\text{Tb}_2@\text{C}_{59}\text{N}$ , respectively (see Fig. 4, 5, and S72†). These gigantic  $U_{\text{cal}}$  values are two times

larger than  $\text{Dy}_2@\text{C}_{79}\text{N}/\text{Tb}_2@\text{C}_{79}\text{N}$  estimates, thanks to a very large ferromagnetic exchange.<sup>44,77</sup> The other relaxation process due to intermolecular interactions is expected to be minimal due to confinement, which is likely to yield large  $T_B$  values. As our predictions on  $\text{Gd}_2@\text{C}_{79}\text{N}$  and  $\text{Dy}_2@\text{C}_{79}\text{N}$  are proved by experiments lately, with  $\text{Dy}_2@\text{C}_{79}\text{N}$  yielding an attractive blocking temperature (24 K), these smaller cages, if made, could enhance  $T_B$  values even further.<sup>40,44,47,76</sup>

## Conclusions

To this end, we have employed an array of theoretical tools in search of finding lanthanide encapsulated fullerenes with very large blocking barriers and blocking temperatures. Various ideas, such as enhancing the coupling between two lanthanide ions by bringing them close to each other in the confined space, have been tested, and the main conclusions drawn from this works are summarised below.

(i) Sourcing the large  $J_{\text{Gd-Gd}}$  exchange *via* confinement: in search of increasing the magnetic exchange ( $J_{\text{Gd-Gd}}$ ) between two lanthanide ions *via* confinement, we have varied the cage size from  $\text{C}_{30}$  to  $\text{C}_{80}$  where the  $\text{Gd}\cdots\text{Gd}$  distance ranging from  $2.185 \text{ \AA}$  to  $4.107 \text{ \AA}$  is observed. Here smaller cages ( $\text{C}_{2n}$ ,  $2n \leq 52$ ) yield a weaker  $\text{Gd}\cdots\text{Gd}$  interactions with a stable  $\text{Gd}_2@\text{C}_{38}-\text{D}_{3h}$  complex having a record-high exchange for any 4f-4f interaction ( $J_{\text{Gd}\cdots\text{Gd}} = -43.4 \text{ cm}^{-1}$ ). A strong 4f-4f orbital overlap between two  $\text{Gd}^{3+}$  ions suggests the  $\text{Gd}-\text{Gd}$  bond formation under confinement. As the exchange is antiferromagnetic, these are not ideal for SMMs, however among hetero dilanthanide EMFs, some promising SMMs are identified.

(ii) *Ab initio* estimation of double exchange in endohedral azafullerenes: the larger cages ( $\text{Gd}_2@\text{C}_{60}$  and  $\text{Gd}_2@\text{C}_{80}$ ) lead to the formation of a two-centre-one-electron  $\text{Gd}-\text{Gd}$  bond due to the comparable energy of the highest occupied orbitals of  $\text{Gd}_2$  and lowest unoccupied orbitals of the fullerene cage. Here we have studied  $\text{Gd}_2@\text{C}_{59/79}\text{N}$  complexes where the delocalisation of the electron between two  $\text{Gd}$  centres is treated *via* a double-exchange parameter. A protocol to compute the double-exchange using *ab initio* CASSCF calculations is proposed, and this methodology yields spin Hamiltonian parameters that are in excellent agreement with experiments for  $\text{Gd}_2@\text{C}_{79}\text{N}$ . The application of this method in  $\text{Gd}_2@\text{C}_{59}\text{N}$  unveils a massive  $J_{\text{Gd-Gd}}$  exchange ( $J_{\text{Gd-Gd}} = +869 \text{ cm}^{-1}$ ) which is two times larger than the record-high  $J$  reported for  $\text{Gd}_2@\text{C}_{79}\text{N}$ .

(iii) Record-high blocking barrier for  $\text{Dy}_2@\text{C}_{59}\text{N}$  and  $\text{Tb}_2@\text{C}_{59}\text{N}$ : the huge ferromagnetic  $J_{\text{Gd-Gd}}$  exchange found in the  $\text{C}_{59}\text{N}$  cage quenches the QTM significantly and yields a very high  $U_{\text{cal}}$  value of  $1502 \text{ cm}^{-1}$  for  $\text{Tb}_2@\text{C}_{59}\text{N}$  – the largest reported for any lanthanide EMF. This opens up the possibility of generating large magnetic anisotropy without relying on a stronger ligand field.

## Computational details

All the DFT calculations have been performed using the Gaussian09 suite with the B3LYP functional.<sup>96,97</sup> There are several isomers with different symmetries possible for a chosen



fullerene cage. To estimate the effect of symmetry of the fullerene cage in the associated magnetic properties, we have chosen two low energy conformers of the  $C_{2n}$  ( $n = 15\text{--}24, 26, 30, 40$ ) fullerene from <http://www.nanotube.msu.edu/> and encapsulated two Gd atoms.<sup>98</sup> The optimisation of the resulting  $\text{Gd}_2@\text{C}_{2n}$  has been performed using the UB3LYP/CSDZ(Gd), SVP (rest) methodology.<sup>99,100</sup> Further single point energy calculations were performed with an Ahlrichs' triple- $\xi$  valence (TZV) basis set to obtain an excellent numerical estimate of energy/magnetic coupling.<sup>101</sup> A quadratic convergence SCF method was used throughout all the calculations.<sup>102</sup> One high spin (HS, the spin on both  $\text{Gd}^{3+}$  centres is "up") and one broken symmetry (BS, the spin on one  $\text{Gd}^{3+}$  centre is "up" and another  $\text{Gd}^{3+}$  centre is "down") configuration was used to estimate the magnetic exchange. The magnetic exchange has been calculated using the

Hamiltonian  $\hat{H} = -J\hat{S}_1\hat{S}_2$ , where  $J = \frac{E_{\text{BS}} - E_{\text{HS}}}{2S_1S_2 + S_2}$ .<sup>103,104</sup> Additionally, we have performed AIM (atoms in molecules) analysis with the AIM2000 programme package to determine the coordination number of the  $\text{Gd}^{3+}$  ion inside the fullerene cage along with the nature of Gd–C and Gd–Gd bonds.<sup>105</sup>

To estimate the double exchange in  $\text{Gd}_2@\text{C}_{79}\text{N}$  and  $\text{Gd}_2@\text{C}_{59}\text{N}$  molecules, *ab initio* CASSCF calculations have been performed using the MOLCAS 8.4 programme package.<sup>106</sup> We have employed  $[\text{Ln}\cdot\text{ANO-RCC}\cdots 8s7p5d3f2g1h]$ ,  $[\text{C}\cdot\text{ANO-RCC}\cdots 3s2p1d]$  and  $[\text{N}\cdot\text{ANO-RCC}\cdots 3s2p1d]$  contraction schemes in the basis set for Gd, C and N, respectively.<sup>106,107</sup> The DKH Hamiltonian was used to take into account the scalar relativistic effect.<sup>108</sup> The Cholesky decomposition technique was used to reduce the size of the disk space.<sup>109</sup> The CASSCF calculations have been performed with the CAS (15,15) active space.<sup>90</sup> The active space includes seven 4f orbitals from each Gd atom and one orbital for the unpaired electron. Within the active space, we have computed the energy of the  $S = 13/2, 11/2, 9/2, 7/2, 5/2$  and  $3/2$  states with two roots while the energy of  $S = 15/2$  and  $1/2$  states has been estimated using only one root. Further details on the computational methods are elaborated in the ESI.†

The magnetic anisotropy in the  $\text{Gd}_2@\text{C}_{38}\text{-D}_{3h}$  and  $\text{Gd}_2@\text{C}_{52}\text{-D}_{2d}$  isomer has been estimated by replacing the isotropic Gd metal centres with Dy, Er and Pr. For  $\text{Gd}_2@\text{C}_{59}\text{N}$  model, anisotropic calculations were performed using Dy and Tb ions. The CASSCF calculations have been performed with minimal CAS( $n,7$ ) active space ( $n$  = number of 4f electrons) for Pr, Dy, Tb and Er using the MOLCAS 8.4 programme package.<sup>106</sup> We have computed the energies of the 21 triplets and 28 singlets of  $\text{Pr}^{3+}$ , 7 septets, 140 quintets and 195 triplets for  $\text{Tb}^{3+}$ , 21 sextets for  $\text{Dy}^{3+}$ , and 35 quartets and 112 doublets for  $\text{Er}^{3+}$  within the size of the active space. Thereafter, the computed spin-free states (7 septets, 105 quintets and 112 triplets for  $\text{Tb}^{3+}$ ) have been mixed in RASSI-SO to obtain the spin-orbit coupled energies. Finally, the  $g$  tensors, QTM/TA-QTM, *etc.* of the metal centre have been computed by SINGLE\_ANISO, which interfaces with the RASSI-SO. After calculating the magnetic anisotropy of the individual metal centres, they have been coupled by POLY\_ANISO using the Lines model to compute the energy of the exchange-coupled system.<sup>110</sup> The magnetic exchange computed with the DFT and *ab initio* approach has been scaled with 5/7, 6/7, and 3/7 for Dy,

Tb and Er centres, respectively, to estimate exchange coupled energy levels.

## Author contributions

GR conceived the idea and SD performed all the calculations and interpret and analyse the data. Both wrote the manuscript together.

## Conflicts of interest

There are no conflicts to declare.

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