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# TSNet: predicting transition state structures with tensor field networks and transfer learning

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Transition states are among the most important molecular structures in chemistry, critical to a variety of fields such as reaction kinetics, catalyst design, and the study of protein function. However, transition states are very unstable, typically only existing on the order of femtoseconds. The transient nature of these structures makes them incredibly difficult to study, thus chemists often turn to simulation. Unfortunately, computer simulation of transition states is also challenging, as they are first-order saddle points on highly dimensional mathematical surfaces. Locating these points is resource intensive and unreliable, resulting in methods which can take very long to converge. Machine learning, a relatively novel class of algorithm, has led to radical changes in several fields of computation, including computer vision and natural language processing due to its aptitude for highly accurate function approximation. While machine learning has been widely adopted throughout computational chemistry as a lightweight alternative to costly quantum mechanical calculations, little research has been pursued which utilizes machine learning for transition state structure optimization. In this paper TSNet is presented, a new end-to-end Siamese message-passing neural network based on tensor field networks shown to be capable of predicting transition state geometries. Also presented is a small dataset of  $S_N2$  reactions which includes transition state structures – the first of its kind built specifically for machine learning. Finally, transfer learning, a low data remedial technique, is explored to understand the viability of pretraining TSNet on widely available chemical data may provide better starting points during training, faster convergence, and lower loss values. Aspects of the new dataset and model shall be discussed in detail, along with motivations and general outlook on the future of machine learning-based transition state prediction.

## 1 Introduction

The transition state (TS) is central to several important chemical prediction tasks, including reaction mechanism studies, protein investigation, and catalyst design.<sup>2–5</sup> The common conception of a TS is a high energy configuration of reaction atoms along a particular reaction coordinate – a threshold along the trajectory from reactant to product which dictates reaction activation energy. Mathematically TS are represented by saddle points, also known as minimax points, on the potential energy surface (PES) of a given quantum system. Saddle points are complex critical points unique to surfaces with two degrees-of-freedom or more where all dimensions are either at a relative minimum or relative maximum. Of interest to most fields of chemistry are first-order saddle points where only a single dimension is at a maximum while all others are minimised. This maximised dimension corresponds to the reaction coordinate, *e.g.* the bond being broken or formed, and the minima which bookend this maximum represent the reactant and

product complexes of the reaction. In other words, the first-order saddle point sits as the maximal energy geometry along the minimum energy pathway (MEP) between reactant and product complexes. Locating a specific TS on any given PES is exceptionally difficult for a handful of reasons. First, since TS are off-equilibrium conformers corresponding to the breaking/forming of chemical bonds, electronic configuration must be considered – *i.e.* TS search is only viable with quantum mechanics (QM) based *ab initio* methodologies which are often incredibly costly and scale very poorly with the size of the quantum system.<sup>6</sup> Second, existing iterative Hessian-based search algorithms which optimize surfaces for minimax points potentially require many very expensive *ab initio* calls before convergence – and convergence is never guaranteed. Third, Hessian-based search algorithms are incredibly sensitive to starting point, meaning accurate estimations of true TS geometries must be provided to increase the probability of convergence upon desired structures.<sup>7</sup> Over the last several decades many TS search algorithms have been developed to address these listed obstacles. Algorithms of note include Schlegel and Peng's synchronous transit quasi-Newton (STQN) method,<sup>8,9</sup> the nudged elastic band (NEB) method,<sup>10,11</sup> and

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Zimmerman's growing string methods (GSM).<sup>12–14</sup> Other notable search implementations include eigen-vector following methods<sup>15–17</sup> and spline methods.<sup>18–21</sup> While implementation, reliability and use-cases differ among these search algorithms, they all largely follow the same schema: (1) generate a “guess” geometry of the TS from provided input structures (2) perform local Hessian-based surface optimization (typically some variation of the Broyden–Fletcher–Goldfarb–Shanno algorithm). Of the noted algorithms, GSMs are often reported as the most reliable,<sup>12</sup> however they still require a handful of *ab initio* force calls at each node. Depending on the number of nodes created along the string, a single search can accrue dozens of force calls – albeit far fewer than NEB and STQN. Peterson reported significantly lowered *ab initio* force calls with NEB when supplemented with a machine-learned PES, resulting in a much more rapid acquisition of valid structures.<sup>22</sup> The use of a machine-learned potential by Peterson is a footnote in the radical paradigm shift over the past decade towards novel machine learning (ML) algorithms for modelling chemical trends.<sup>1,23,24</sup> State-of-the-art neural networks (NNs) such as SchNet and PhysNet are capable of achieving energy predictions within error margins much smaller than 1 kcal mol<sup>−1</sup> trained upon data obtained using density functional theory (DFT) methods,<sup>25,26</sup> and recent work using transfer-learning has allowed for coupled-cluster levels of accuracy with small sets of training data.<sup>27</sup> We believe that the success observed mimicking *ab initio* with ML may be translated to TS search. However, while these milestones are incredibly promising, they all generally fall under the umbrella of “scalar regression”, where some representation of a quantum system serves as input and a single real number representing some aspect of that quantum system is outputted, *e.g.* internal energy. Less success has been observed developing predictive ML models which are capable of outputting structures – a requirement for modelling TS search.<sup>28</sup> Progress has been made creating generative ML models for the purposes of molecular design, which output data structures representative of equilibrium structures, such as SMILES strings.<sup>28–31</sup> However, these models are not transferrable to the task of TS prediction for a number of reasons. First, TSs are fundamentally very special non-equilibrium conformers of molecular systems, something not well represented by SMILES strings and simple binary connectivity graphs.<sup>32</sup> Second, generative models are often invariant to the translation and rotation, *i.e.* the orientation, of the input molecules<sup>33–35</sup> – meaning the output of the model does not change if the input is rotated and/or translated within its coordinate system. TS prediction ultimately requires equivariance to orientation, meaning the orientation of the output molecule must change equivalently to the orientation of the input molecule(s). The few existing equivariant ML models have only been developed within the last handful of year,<sup>36–38</sup> and only much more recently have they been applied to computational chemistry, such as with the very recent Cormorant architecture.<sup>39–41</sup> In order to translate the success of molecular predictive ML to structure prediction, effort must be put into the development of novel equivariant models capable of outputting verifiable and human-readable quantum systems. The solution we are proposing is an

end-to-end approach – replacing both PES modelling and traditional Hessian-based search. The alternative approach, like that proposed by Peterson, is to replace only PES modelling and utilize traditional saddle point search methods. There are benefits to this semi-learned approach, the main being that accurate ML-based PES generating models already exist.<sup>25,26</sup> However, these machine-learned potentials do not include TS structures in their training sets, meaning their performance for TS prediction would be dubious at best. Also, users will still be required to interact with traditional search algorithms which are very sensitive to input quality and often very difficult to operate effectively. Lastly, the success of end-to-end approaches in computer vision and natural language processing have shown considerable success along with other, more niche domains, such that they are quickly becoming the *de facto* methodologies in their respective fields.<sup>42–45</sup> While chances of observing similar success with-respect-to TS prediction are questionable, we believe this approach is worth exploring.

### 1.1 Project contributions

The goal of this study is to develop a ML model to predict accurate transition state structures on a potential energy surface (PES) between reactant and product complexes as input. This is challenging for several reasons. First, such a model requires interpreting molecular systems in multiple coordinate systems. Since reactant and product complexes are necessarily expressed in different coordinate systems (which cannot be unique), our ML models must both be capable of interpreting each of these and generating a third structure in yet another coordinate system. We find that tensor field networks based on the equivariant tensor field networks created by Thomas *et al.*<sup>38</sup> are amenable to this challenge and provide details of our chosen architecture below. Second, the ambiguity of directionality on the potential energy surface is an important challenge. It is, of course, irrelevant whether one labels the reactant complex as “reactants” or “products” and a transition structure prediction must be appropriately invariant. We find that incorporating a Siamese architecture into our network is an effective strategy for incorporating PES directional invariance within our model. Finally, though there are a growing number of examples of ML molecular property predictors and even generative models for whole-molecule prediction, we find a dearth of examples that explicitly treat non-equilibrium molecular structures. Though transition structures are critical for understanding kinetics, intersystem crossings, enzyme function, *etc.* we find that most (if not all) ML predictors focus on equilibrium structure and properties. As such, there is very little data from which to train on. We use a novel approach to transfer learning as a solution here, where we incorporate the benefits from training on a large equilibrium dataset to training on a small non-equilibrium dataset.

### 1.2 Similar works

While significant success has been observed applying ML to traditional fields of predictive computational chemistry, such as energy and force prediction, little research has been completed



with respect to ML applied to TS prediction. While there are many facets of TS search that may be bolstered with ML, truly desirable is the development of a novel ML algorithm capable of outputting TS structure predictions directly, with a high level of accuracy, reliability, and generalizability. Such a feat has been attempted only a handful of times. The earliest found inclusion of ML to TS-based research was the 2012 paper by Pozun *et al.* The paper detailed the development of a support vector machine (SVM), a type of traditional ML classifier, which classified various non-equilibrium structures as either belonging to a reactant, or product region of a mechanistic pathway. The decision boundary function learned by the SVM, which is a plane through feature space, may serve as an approximation for the dividing surface of the reaction. From this dividing surface, one may determine the rate of reaction as the approximate equilibrium flux out of this surface. While the results of this study could prove useful in a variety of fields, it is unrelated to the proposed work due to its focus on kinetics, and the lack of TS structures in the training dataset.<sup>46</sup> Slightly more related is the 2016 paper by Peterson, which described the use of traditional saddle-point search on machine-learned PESs. What is most interesting about this second work is the proposed hybrid QM-ML approach, where a majority of the saddle-point search is carried out upon a PES generated by a NN. While Peterson reported a drastic reduction in the number of required *ab initio* calls for successful calculations, the reliability of the method, *e.g.* the number of successfully obtained TSs, was not reported. Peterson's study was deemed dissimilar to the proposed study due to the lack of explicit TS structures in the dataset,<sup>47</sup> and the fact that TS predictions are still carried out with traditional search algorithms, just upon machine-learned surfaces.<sup>48</sup> Most recently, Pattanaik *et al.* Built a graph NN capable outputting TS distance matrices, which are used to generate a set of most probable Cartesians following non-linear least squares optimization.<sup>49</sup> Our approach is most similar to that followed by Pattanaik *et al.*, however there are a few key differences. First, we directly output Cartesians as opposed to distance matrices. This necessitates model equivariance and a standardized Cartesian dataset, however it affords our model the ability to differentiate enantiomers – something not possible with only a distance matrix. Second, the set of bond features used as input to the model proposed by Pattanaik *et al.* is a set of length 3 vectors, where the first entry of each vector is the bond distance for that particular bond averaged between reactant and product. The second and third entries indicate whether or not the bond has been broken or formed when going from reactant to product, and whether or not the bond was aromatic, respectively. Our input differs radically for a handful of reasons. First, TSNet is a Siamese architecture which offloads the combination of reactant and product features to the model. TSNet is also theoretically capable of operating on multi-reactant reactions, though the construction of a multi-reactant reaction dataset to test this capability is beyond the scope of this work. Second, we expand our interatomic distances using 800 unique atom-centered symmetry functions originally proposed by Behler, which have been shown to increase the predictive capabilities of chemical ML models.<sup>25,26,50</sup> The last major difference between

TSNet and Pattanaik's graph NN come from overall architectural differences. TSNet is a rotationally and translationally equivariant convolutional graph NN, while Pattanaik's model is an invariant multi-layer perceptron which operates atom-wise. It is too early in the development of both TSNet and Pattanaik's model to definitively say which model is superior with respect to predictive capabilities and computational cost. We leave an extensive benchmarking of TSNet and Pattanaik's model to future studies when larger TS datasets are more readily available.

### 1.3 Anticipated problems

Prior to the development of TSNet three major challenges were anticipated: (1) how should inputs and outputs be represented? (2) How would one present multiple inputs to a model? (3) How would one deal with the lack of readily available TS data? The first question has largely been answered already and is general to most chemical ML projects, while the latter two are much more specific to TS search, meaning they must be answered within this work. In the following sections, the answers to these questions shall be explored.

**1.3.1 Molecular representations.** The main factors which hindered the development of chemical predictive ML are the fundamental difficulties with representing quantum systems, such as molecules and reactions, in machine-interpretable formats, previously discussed.<sup>51</sup> While these detailed, invariant representations work well with scalar regression, structure prediction from graph inputs would ideally include orientation information. This puts the additional constraint of equivariance upon model selection, making existing state-of-the-art models poor choices for drop in usage and severely limits the number of possible architectures available. One such architecture fit for these purposes is a novel message passing neural network (MPNN) proposed by Thomas *et al.* which utilizes the spherical harmonics  $Y_m^{(l)}$  to encode rotational equivariance known as the tensor field networks (TFNs).<sup>38</sup> TFNs take an additional set of edge features as input; interatomic unit vectors. These unit vectors are centered on the origin of the input coordinate system and serve as inputs to create a basis of spherical harmonics functions from which learned filters are constructed and convolved with  $x_i$ . In a nutshell, this affords TFNs with the ability to output tensor fields, mathematical spaces where each point corresponds to a geometric tensor, for each atom in a particular molecule – making them ideal for predicting TS structures. Adding or removing atoms is not inherently supported by TFNs, though this is not a requirement for ML-based TS prediction as input and target systems already include all relevant atoms. Changing the number of atoms present in a particular system is a greater focus for a generative architecture, where the goal is to create new molecules. TS are effectively very special molecular conformers, so the inputted array of atomic types can be carried forward for outputted structures. A more detailed explanation of how TFNs operate may be found in Section 2.2.

**1.3.2 Representing multiple input systems.** There are two major types of TS search algorithms: double-ended, and single-



ended. Double-ended searches take two structures as input and search for the TS between them, while single-ended searches use either a reactant or product complex as a starting point to search for novel TS. As predictive ML is ultimately a form of data interpolation, single-ended TS search would be best left for generative models – however, TSNet may prove to be an adequate starting point for building a machine-learned single-ended search algorithm. As such, the model must be capable of taking multiple inputs. Dependence upon the order in which these multiple inputs are provided will likely introduce permutation variance. In other words, presenting a reactant, product pair in an order which the model has not yet seen could potentially result in a different prediction. A naïve solution could involve data augmentation, where input pairs are reversed – synthetically doubling the size of the dataset but not technically doubling information. However, there exist simplistic architectural changes which address the problem more elegantly. By using the same network on each input, then combining outputs commutatively, *e.g.* through summation, both inputs can be utilized in a manner invariant to permutation. This architecture is effectively identical to a Siamese neural network, an architecture proposed by Koch *et al.* in 2016 originally for training on very small categorical datasets, also known as one-shot learning.<sup>52</sup> However, while Siamese networks are more often used for classification, the architecture is designed exclusively for regression. While a Siamese-like architecture addresses multiple inputs, the question of which method of computing loss was one of the more difficult aspects of the development of the model. Computing loss directly on Cartesians was anticipated to cause issues, as a double-ended search algorithm must contend with three possibly very different coordinate systems (one for the reactant, TS, and product complexes). However, Cartesians are one of the very few ways in which molecules/reactions may be visualized, a must for the rudimentary validation pipeline of this work. Working directly with Cartesians requires careful consideration when creating training data to ensure coordinate systems are consistent across product, reactant, and TS. Many modern QM-based software provides the option to obtain structures in standard orientation, where molecules are translated such that their center of mass is on the origin of the coordinate system and rotated such that the molecule's principal axis of rotation aligns with the *z*-axis. By ensuring all structures are optimized in standard orientation, the multi-system problem may be largely avoided.

**1.3.3 Low data exploitative techniques.** In addition to the scarcity of architectures fit for making structure predictions, readily available datasets including TS structures are rare. To our knowledge, the only other dataset designed for ML which includes TS structures is an isomerization dataset created by Grambow *et al.*, developed in parallel to this study.<sup>53</sup> The reason for such data shortage is obvious: generating TS structures is exceptionally difficult. In fact, the low feasibility of TS search automation makes it likely that TS prediction will always be mired by a lack of data. However, low data exploitative techniques such as one-shot learning and transfer learning (TL) have shown considerable promise in other fields such as

natural language processing and computer vision.<sup>52,54</sup> Ultimately, while the scarcity of data is likely the biggest concern with ML-based TS search, there are several promising mitigation strategies. TL specifically is a very promising and easily implemented remedy to low data learning where knowledge from one source prediction task boosts performance on a second target task which is begrudged by a lack of training data. Unfortunately, predicting whether or not TL will provide a boost to performance upon the target dataset task through pre-training on a given source dataset is effectively impossible.<sup>55</sup> The only truly viable option for detecting whether or not a model will transfer properly is through trial-and-error and experimentation. Selecting a source dataset and prediction task as close to the target as possible is an easy way of increasing the probability of success, which is why a new prediction task was created from the reputable QM9 dataset.<sup>56,57</sup> Originally, the QM9 was created for molecular property prediction – however, such a task is likely too different for a meaningful transfer of information. Therefore, the decision was made to construct a new prediction task from the high-quality structures in the QM9 similar in nature to that used by Thomas *et al.* when first showcasing TFN capabilities for chemistry. While it is true that no physical motivations exist behind such a prediction task, the goal is not for the model to learn physics directly during pre-training, but to rather expose the model to what molecules and chemical systems should look like before training to construct TS structures.

## 2 Methods

### 2.1 Construction of the S<sub>N</sub>2-TS dataset

The semi-automated algorithm for dataset generation was developed using Python 3.6, a programming language well regarded for its utility as an adapter between applications.<sup>58</sup> A script written in Python that is capable of transcribing data from common file formats and executing external programs is often more digestible, reliable, and quick to develop than competing languages. Also, Python has become the *de facto* language for interacting with highly optimized ML backends, so integration between data generation and ML-based prediction is palatable. Seed data was recorded into the common *xyz* molecular file format by hand from various sources<sup>59–73</sup> before being funnelled into the semi-automated TS data generation pipeline, titled Gaussian Manager (GM) (<https://github.com/UPEIChemistry/GaussianManager>).

**2.1.1 Gaussian manager.** GM was based exclusively on the Gaussian 09 software suite.<sup>74</sup> Manually compiled seed data are gas-phase S<sub>N</sub>2-TS geometries obtained from several theoretical studies ranging from the early 1980s to the late 2000s.<sup>59–73</sup> GM begins by performing a second-order saddle point search using provided seed data as input “guesses” for starting points on the PES. Calculation times are often low due to the quality of the seed data. However, convergence errors were common across the dataset for saddle point search. These errors were largely handled by GM's error resolution. The error resolution functionality of GM is limited on account of the short development time of the application. With more time, GM can potentially



evolve into a more autonomous process, requiring very little human intervention. Gaussian has several exceptions, which may raise for a variety of reasons. GM attempts to handle the majority of exceptions typically thrown by IRC and saddle point search algorithms by either modifying self-consistent field parameters, or loosening convergence metrics to ensure calculation completion. Many error codes cannot be handled effectively, typically requiring manual review.

After TS optimization, GM performs an intrinsic reaction coordinate (IRC) calculation to obtain corresponding reactant and product geometries. To remedy convergence errors, IRC calculations were customized with the following keywords: loose, stepsize = 2, which ensured IRCs ran with very lenient convergence criteria, and the search computed only two steps along the negative mode, thus “pushing” the optimization towards either product or reactant complex. These IRCs are then followed by separate geometry optimizations for each of the reactant and product complexes. While most obtained reactant and product complex geometries appear visually to mimic the expected complexes some differ from expected structures,<sup>75</sup> particularly reactions containing atoms capable of hydrogen bonding. The chosen basis set to optimize the structures with was the cc-PVDZ double-zeta basis.<sup>76</sup> Calculations were performed at the Møller–Plesset 2 (MP2) level of theory.<sup>77</sup> The method and basis set were ultimately chosen to ensure calculations completed in a reasonable amount of time, while not sacrificing a significant amount of computational accuracy.

**2.1.2 Dataset preparation for machine learning.** Once all of the Gaussian output files for all reactions were obtained, they were split into arrays and compressed. Python 3.7, the NumPy high dimensional mathematics package,<sup>78</sup> and the h5py serialization package were used for dataset construction. While xyz files contain both atomic type and positional data, the information is technically stored as plaintext. By reading the xyz files with NumPy the entire dataset may be stored efficiently as high ranked arrays. However, due to type inconsistencies between type and positional data (integers vs. floats), separate arrays must be constructed, resulting in an atomic-type array and Cartesian array for the dataset – typical for other computational chemistry ML datasets. Also similar to other chemical datasets is the addition of dummy atoms to pad out smaller molecules such that all molecules in the dataset contain a consistent molecular size. This is required since high-dimensional arrays must be rectangular. The S<sub>N</sub>2-TS dataset is composed of three separate Cartesian arrays, corresponding to the positional data of the reactant, TS, and product structures; a single atomic-type array, as no atoms are added or removed upon transition from reactant to product; and three energy arrays, listing energy values of reactants, TSs, and products. These arrays are then stored into the hdf5 file format using the h5py Python package, which allows for quick serialization of large datasets and a convenient nested structure – affording high portability and organization. Due to its low number of structures, the S<sub>N</sub>2-TS dataset is quite lightweight, requiring only 216 Kb of storage space.

**2.1.3 Statistical analysis.** Dataset statistics were computed using Python 3.7 and the NumPy high dimensional

mathematics package. Relevant code for statistical evaluation and dataset construction may be found at: <https://github.com/PEIChemistry/critical-length-predictor>.

## 2.2 Tensor field networks

TSNet borrows heavily from TFNs proposed by Thomas *et al.*, complex multifaceted NNs built specifically for equivariant prediction upon point clouds (represented by graphs).<sup>38</sup> While other equivariant networks would likely suffice for TS prediction, the architecture from Thomas *et al.* was the most readily available and easily implemented architecture at the time TSNet was developed. In the following sections, an explanation of each distinct TFN layer, how TSNet differs from the original, and how they are integrated together to create the final learned “block” shall be given. Though based on similar motivations to the original, the overall construction of TSNet is very different. The four greatest differences are: (1) while the original implementation only accepted a single molecule/reaction at a time, TSNet utilizes dummy atoms and masking layers to allow for batching multiple molecule/reactions (2) the model uses residual skip connections (3) TSNet supports shared radial functions rather than only creating new sets of trainable weights every block (4) TSNet is built using a Siamese architecture, allowing it to accept any number of input systems. In the case of TSNet, we accept two inputs (reactant and product), though TSNet could easily be used for unimolecular reactions, or even trimolecular reactions.

**2.2.1 Point convolution.** Point convolution layers contain over 90% of the trainable weights of a TFN and serve as the entry point to each learned block in the network. TSNet’s implementation of the point convolution layer requires four distinct inputs:

**Input 1 – positional information.** Like most state-of-the-art architectures, TSNet utilizes an interatomic distance matrix expanded using Behler’s ACSFs, specifically radial basis functions. The shorthand ‘rbf’ is typically used when referring to positional information. The rbf array is of shape (molecules, atoms, atoms, basis functions) – a rank four array.

**Input 2 – orientation information.** Somewhat unique to TFNs is the use of interatomic unit vectors as input. The set of unit vectors for a single atom are computed as such:

$$V_i = \frac{c_i - c_j}{\|c_i - c_j\|_2} \mathbf{v}_j \quad (1)$$

where  $i$  and  $j$  index atoms,  $c_i$  is the  $x, y, z$  3-tuple for Cartesian coordinates of atom  $i$  and  $\|x\|_2$  is the Euclidean distance of the vector  $x$ . The term ‘vectors’ is used when referring to input 2. The array is of shape (molecules, atoms, atoms, 3), corresponding to the  $x, y, z$  3-tuple for each interatomic vector.

**Input 3 – one-hot representation of atomic type.** One-hot vectors are a very common binary data structure used in ML to represent the class of an object from a group of choices. For example, in a system that operates on all atoms up to an including fluorine, the one-hot vector representing a carbon atom would be a length nine vector with a one in the sixth position and zeroes everywhere else. An entire molecule or



reaction would require a matrix of one-hot vectors, one for each atom. In a nutshell, for TFNs, one-hots are used frequently to represent the types of atoms present in a particular reaction or molecule. This input was not required in the original implementation of TFNs, as they only operated on a single molecule/reaction at a time. In order to operate on collections of molecules at a time, dummy atoms must be utilized to pad out smaller molecules to ensure inputs are rectangular. Dummy atoms have atomic number zero and are all centered at the origin of the molecule's coordinate system. As TFNs operate atom-wise and share weights across all atoms of a molecule, dummy atoms can potentially influence model training in undesirable ways. The new TFN implementation requires one-hot vectors for corresponding positional and feature arrays to track which atoms are dummy atoms and remove any contributions attributed to them during training. TSNet utilizes Proffitt and Pearson's implementation of dummy atom masking,<sup>79</sup> which require one-hot vectors to track dummy atoms and zero any contribution they make to training. Dummy atoms are masked after every layer.

Input 4 – vertex features represented as a set of tensor fields. Feature arrays are the single way in which information flows between learned blocks in a TFN. Ultimately, TFNs operate upon and output tensor fields, thus vertex features must be represented as sets of tensor fields, *i.e.* they must be represented as arrays of shape (molecules, atoms, features, representation index). The final axis of a tensor field array, representation index (RI), is related to which set of spherical harmonics functions are

required to represent that particular feature array. In other words, RI is the same as the parameter  $m$  for the spherical harmonics, *e.g.* for  $Y_1^0$  RI is one (a scalar field) and for  $Y_3^1$  RI is three (a Euclidean vector field). Like the original implementation, TSNet only supports up to vector fields, however this is easily extendable to higher order tensors with minimal changes to the architecture. It should be noted that TFNs can actually operate on multiple feature arrays of differing RIs at a time. The only true vertex features on hand are atomic types which are only well represented by integers (or one-hot vectors, which are a very common way of representing integers). To translate atomic type to a tensor field one must expand the set of one-hot vectors for a collection of molecules which is of shape (molecules, atoms, highest atomic number) to a scalar field representation of atomic type, which is of shape (molecules, atoms, highest atomic number, 1). This operation is trivial mathematically but is critical to the model. This scalar field is then operated upon by an atom-wise dense layer to produce an array known as an 'embedding', which is exclusively machine-interpretable, of shape (molecules, atoms, units, 1) where 'units' refers to the number of trainable weights in the atom-wise dense layer. TSNet differs from the original in that it also includes a vector field embedding representation of atomic type which is of shape (molecules, atoms, units, 3) to facilitate residual skip connections between clusters of blocks of the network. Fig. 1 showcases the flow of information through the point convolution layer. All of the point convolution layers' trainable parameters are contained within a NN known as the

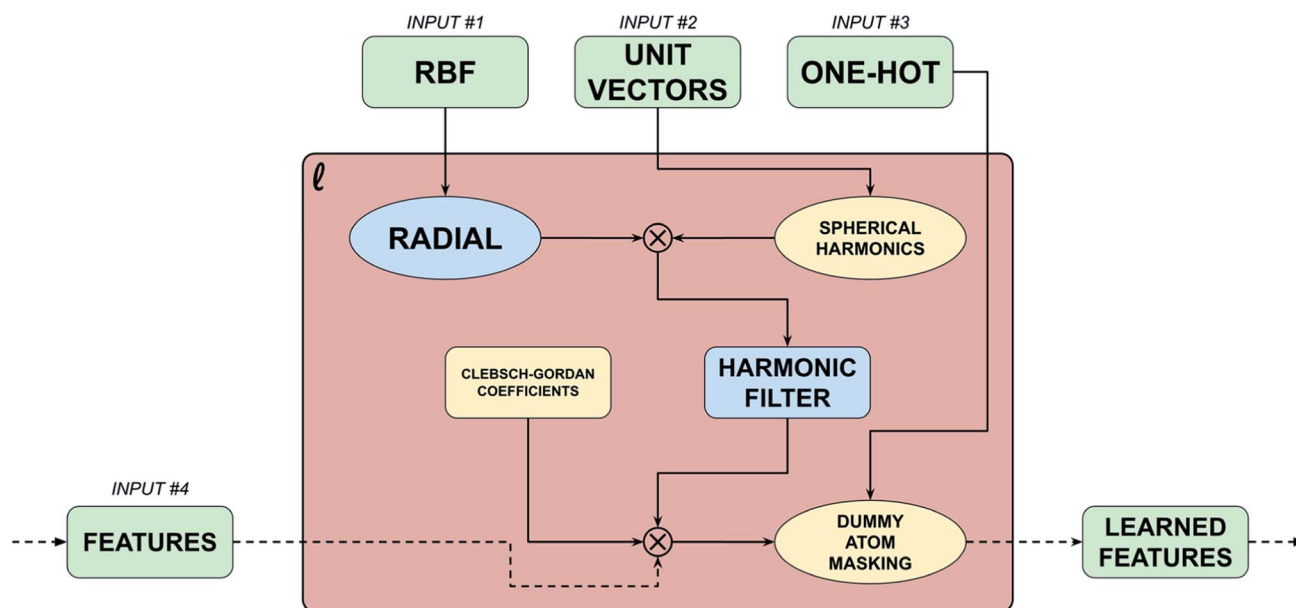


Fig. 1 Point convolution layer. Illustration of the point convolution layer showcasing the flow of information. Ellipses are functions, while rectangles are arrays. Input 1 (RBF) containing positional information, is inputted to an interatomic-wise dense neural network known as the radial. Each point convolution layer has a unique radial; however, the implementation supports radial sharing. Input 2 (Vectors) containing orientation information are used to create a basis of spherical harmonics based on a provided value of  $l$  which is combined with the learned output from the radial using a tensor product to produce a harmonic filter. Harmonic filters are spherically symmetric learned representations including both positional and orientation information on the input molecules/reactions, and are rank 5 tensors of shape (molecules, atoms, atoms, radial units, filter index). Input 4 (Features) is convolved with the harmonic filter and the Clebsch–Gordan coefficients using another tensor product to produce a set of learned features which are of shape (molecules, atoms, learned features, output index). Input 3 (One-hot) is used to mask out dummy atom activations from the learned features.



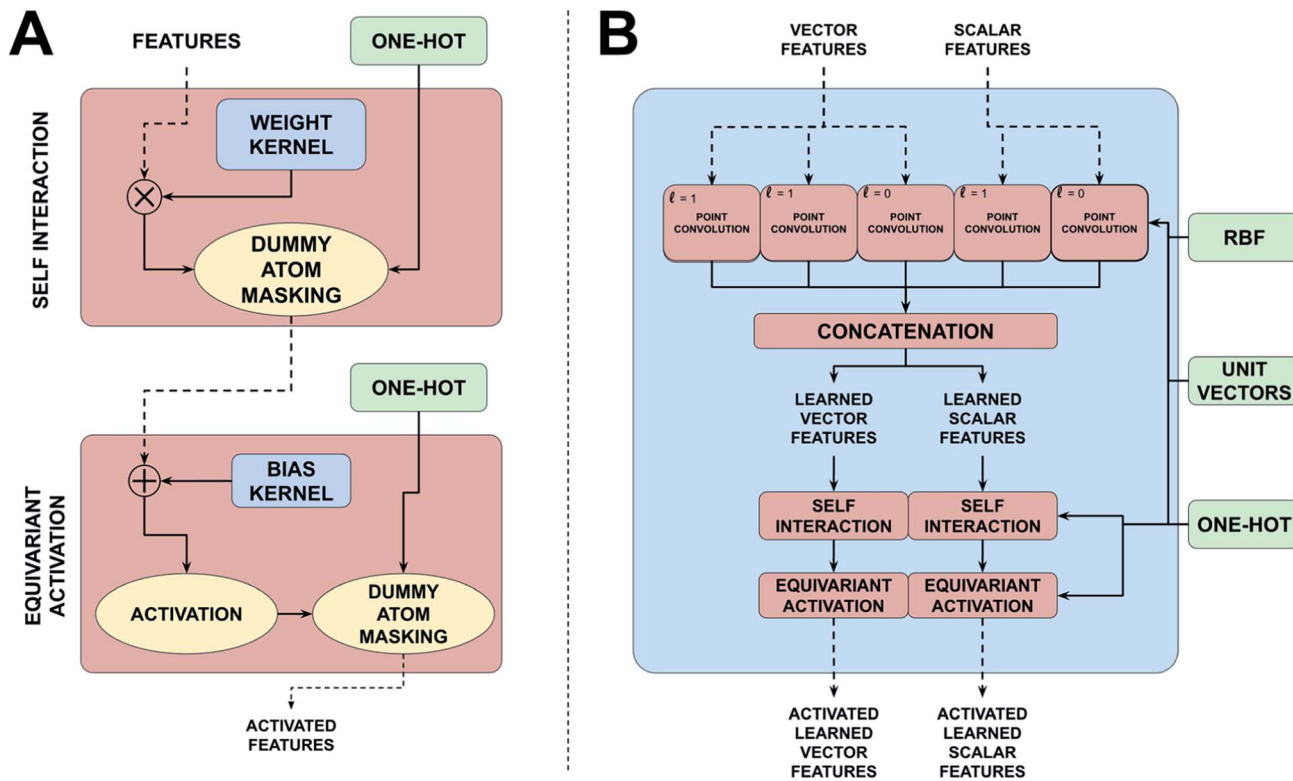


Fig. 2 Molecular convolution block (A) illustration of self interaction and equivariant activation layers. Self interaction layers are similar to traditional  $1 \times 1$  convolutions, while equivariant activation layers ensure higher order tensor field feature arrays are activated across all representation indices. The shifted softplus activation from SchNet is used:  $\text{ssp}(x) = \ln(0.5e^x + 0.5)$ . (B) Illustration of the complete molecular convolution block contain all layers for scalar and vector field inputs. These blocks are analogous to traditional convolutional layers in their mathematical function, with the added benefit of equivariant afforded through the spherical harmonics.

'radial' which operates interatomic-wise rather than atom-wise – with weights shared across all atoms in a molecule. This is distinct from networks like that proposed by Profitt and Pearson, which utilises a positional array in which the second atoms axis has been summed over to create a set of atomic environment vectors which describe the molecule. The radial extracts learned features from the interatomic distance features, *i.e.* the basis functions, and the second atoms axis is summed over during the tensor product between the harmonic filter, the vertex features, and the Clebsch–Gordan coefficients. Point convolution layers contain over 90% of the weights in a TFN.

**2.2.1.1 Concatenation.** The harmonic filter which is combined with vertex features is also a tensor field, and there are mathematical restrictions on how tensor fields of varying irreducible representations must be combined. The combinations TSNets supports are:

- (1) Scalar  $\times$  scalar  $\rightarrow$  scalar.
- (2) Scalar  $\times$  vector  $\rightarrow$  scalar.
- (3) Scalar  $\times$  vector  $\rightarrow$  vector.
- (4) Vector  $\times$  vector  $\rightarrow$  scalar.
- (5) Vector  $\times$  vector  $\rightarrow$  vector.

Combinations (1), (3), and (5) are simply element-wise multiplication, while (2) and (4) are dot products. Since TFNs can accept multiple feature arrays of differing representation order (*i.e.* differing values of  $l$ ) and TSNets supports harmonic filters up to  $l = 1$ , all five combinations are possible depending

on input orders and desired filter orders. For example, providing both scalar field and vector field features and selecting both scalar and vector harmonic filters to be used results in five output feature arrays: three scalar fields, two vector fields. Keeping feature arrays of identical representation order separate is nonsensical, so they are concatenated along the features axis resulting in as many feature arrays as there are differing representation orders.

**2.2.2 Self interaction and equivariant activation.** Fig. 2A shows how information flows between self interaction and activation layers. Self interaction layers are conceptually similar to  $1 \times 1$  convolutions, where a weight kernel facilitates communication across the vertices of the graph by mixing the features axis of the learned feature arrays. Equivariant activation layers are quite simplistic as well, simply applying a bias to inactivated feature arrays from self interaction layers before input to an activation function. The activation function, the shifted softplus, originally from SchNet was maintained for the new implementation.<sup>25</sup> The shifted softplus has the form:

$$\text{ssp}(x) = \ln(0.5e^x - 0.5) \quad (2)$$

**2.2.3 Molecular convolution block.** All of the TFN layers integrate together to create the complete molecular convolution block (MCB), presented in Fig. 2B. MCBs may be fed one into



the other using a typical feed forward approach, or constructed with residual skip connections. Even the construction of recurrent architectures is possible, though this has yet to be attempted. Like typical convolutional neural networks, a number of dense layers are appended to a collection of MCBs to reduce the number of points in each atomic tensor field to the required output for the prediction task. For Cartesian prediction, points in each atom's vector field are reduced to one, corresponding to the single  $x, y, z$  coordinate 3-tuple for each atom. It should be noted that while the main use case of TSNet is chemistry, the architecture itself is agnostic to prediction task – TFN in general may be utilized upon any form of data well represented by point clouds.

### 2.3 TSNet development

**2.3.1 Implementation.** While the original implementation of TFNs was built for TensorFlow 1.x, the original repository (<https://github.com/tensorfieldnetworks/tensorfieldnetworks>) was forked and rebuilt from scratch using Python 3.7 and Keras for TensorFlow 2.3.<sup>80,81</sup> Keras is a user-friendly, streamlined frontend for TensorFlow 2 which allows for highly modular network construction through the layer interface. The new TFN repository can be found at <https://github.com/UPEIChemistry/tensor-field-networks> and contains two sub-packages: `tfn.layers`, and `tfn.tools`. The first package, `layers`, includes all TFN Keras layers as well as the aggregate MCB layer, along with some

additional utilities for construction of TFNs. The second package, `tools`, contains a number of premade models and scripts for loading datasets, such as the QM9 and the  $S_N2$ -TS dataset. Data preparation was completed using Python 3.7, and the NumPy and h5py libraries.

**2.3.2 Architecture.** A complete view of the network architecture is presented in Fig. 3. A very fine grid of radial basis functions (RBFs) was chosen compared to SchNet and Profit's architecture to ensure distinction between very similar conformers. Grid resolution was 0.02 Å, with basis function width  $\omega$  of 0.2 Å. Like Profit, a distance range of  $-1.0$  Å to 15.0 Å was probed. Basis functions centered below zero activate for "self-interactions" (not to be confused with the TFN self-interaction layer), where an atom is connected to itself with a bond length of 0 Å. Allowing the grid to probe below 0 Å ensures self-interactions activate a similar number of basis-functions as compared to larger distances. MCBs in the new TFN architecture are connected with residual skip connections, unlike the original implementation, all of which use 64 self-interaction units internally. The trunks in Fig. 3A are identical networks with shared weights. Their outputs are elementwise summed, and the resultant is fed into a handful of self-interaction layers which reduce the number of points in each atom's tensor field to one to produce the final predicted Cartesian array. The architecture overall has approximately 2.4 million weights, most of which is concentrated within the various radial functions in each block. It should be noted that

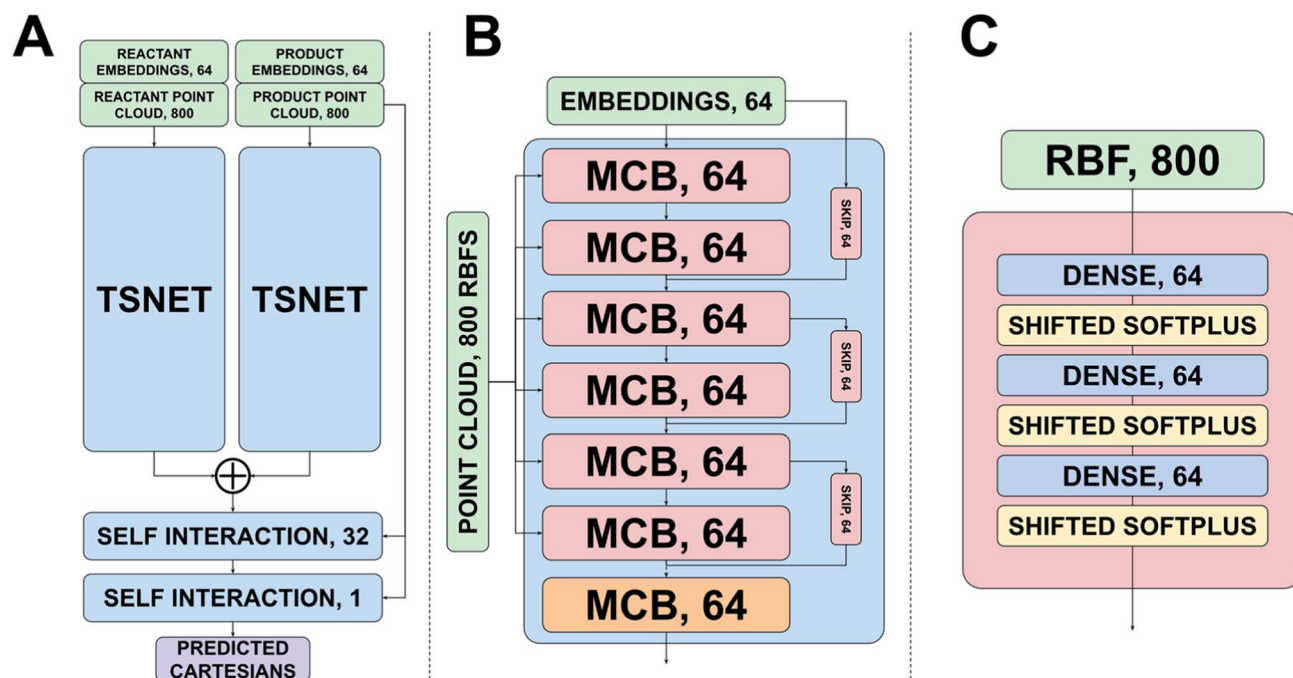


Fig. 3 TSNet architecture. (A) Illustration of the entire TSNet architecture. Weights are shared across both trunks and outputs are summed together before being inputted to a handful of capping dense layers. Output of the network is the predicted Cartesians for the input reactant, product pair. (B) Internals of the TFN trunks from (A). Molecular Convolution Blocks (MCBs) are arranged in a residual style connection with three clusters of blocks at two blocks each. 64 self interaction units are used per block. The final orange block supports combinations of inputs/filters such that only vector fields are outputted, resulting in fewer internal radials. (C) Internals of the radial used in each block in (B). 800 radial basis functions are used on a grid with a resolution of 0.02 angstroms (Å), probing from  $-1.0$  to 15.0 Å. A cosine cutoff function was used after 15 Å. Each block in (B) contains multiple radials (C), resulting in an architecture with approx. 2.4 million trainable parameters.



the implementation supports radial sharing, which can reduce the number of trainable parameters ten-fold.

We should also point out that TSNet does not make use of the energy data for our structures. Though this is tempting, we have found in our tests that incorporating electronic energy data leads to a decrease in performance, since that comes with the necessary addition of a new loss function. When loss is split among structural measures and energetic measures, the performance on structure worsens. As such, since there are many excellent models for the rapid prediction of quantum energetics, we have focussed our efforts on structural predictions.

**2.3.3 Transfer learning.** TL allows one to combine sequences of training, providing the ability to pre-train models on larger source datasets before fitting a typically smaller, target dataset. The core logic of the TL pipeline is responsible for weight transfer, which searches model layers at any given node in the process and replaces flagged layer weights with kernels from the previous node. Additional flags exist to “freeze” certain layers, ensuring pre-trained weights are not overwritten during training of the following node. While general to an arbitrary number of source datasets (hence the title, “pipeline”), pipeline training was held to only two nodes: the source being the QM9, and the target being the  $S_{\text{N}2}$ -TS dataset. Training within each node is completely configurable independent of placement within the pipeline. Pre-training is essentially a replacement to random weight initialisation, providing a better guess at parameter values for a particular target task to both boost performance and reduce convergence times.

### 2.3.4 Training and validation

**2.3.4.1  $S_{\text{N}2}$ -TS dataset.** A benefit of adding support for training on batches of molecules rather than one molecule per batch is an increase in the amount of data pushed through the network during a single iteration of the optimizer. In fact, due to the miniscule amount of data in the  $S_{\text{N}2}$ -TS dataset, training may be performed on the entire trainset rather than in batches, resulting in effectively noiseless steps along the cost landscape and smooth convergence. Selecting a validation split from a small dataset may result in a split comprised entirely of outliers, providing a skewed view on model performance.  $K$ -fold cross validation with  $k = 5$  was used to address this.  $K$ -fold cross validation reports model performance as the average over  $k$  different train/validation splits, reducing the influence of unfortunate splits. Training on the  $S_{\text{N}2}$ -TS dataset was completed in 1000 epochs using the Adam optimizer with an initial learning rate of  $1 \times 10^{-3}$ .<sup>82</sup> The learning rate was halved whenever the training reached a plateau to further assure complete convergence upon local minima on the cost landscape. The mean absolute error (MAE) loss function with L2 regularization was utilized for both the QM9 and  $S_{\text{N}2}$ -TS dataset:

$$\mathcal{L} = \frac{1}{m} \sum_i^m \sum_j^a \sum_k^3 |c_{ijk} - \hat{c}_{ijk}| + \lambda \sum_l^m w_l^2 \quad (3)$$

where  $m$  is the batch size,  $a$  is the total number of atoms for molecule  $i$ , and  $k$  loops over the  $x, y, z$  3-tuple. The scalar  $c_{ijk}$  is

the predicted Cartesian coordinate for  $k$ -axis of atom  $j$  from the  $i$ -th system of the dataset, and  $\hat{c}_{ijk}$  is the corresponding true Cartesian coordinate. The second term of the loss is the L2-norm regularization term, which hinders any single weight from reaching high values to combat overfitting.  $\lambda$  is a tuneable scalar to control the level of regularization used during training typically with a value in the range  $[0,0.1]$ . It is common practice to utilize a small amount of regularization regardless of any perceived overfitting, thus a default  $\lambda$  value of 0.01 is used. Models were trained using the Compute Canada HPC cluster Beluga on a single Nvidia 16 Gb Tesla v100 GPU.

**2.3.4.2 QM9 dataset.** The QM9 dataset was obtained from: <http://quantum-machine.org/datasets/>. Targets for the QM9 were discarded in favour of a new prediction task more closely related to TS structure prediction. For each molecule in the dataset, a single random atom from the first three atoms of the Cartesian array was selected and perturbed by 0.75 Å along the positive and negative  $x, y$ , and  $z$  axes, while all other atoms were perturbed by 0.15 Å. This resulted in a set of three systems: the reverse structure, the forward structure, and the equilibrium structure, which are labelled as pseudo-reactant, product, and TS, respectively. For the pseudo-reaction double-ended prediction task, reactant, and product Cartesians along with a single atomic number one-hot array (since they are identical for all three systems) serve as input while the TS Cartesian array serves as the target. Training was completed on the QM9 in only 50 epochs – however, due to the size of the QM9 dataset, each epoch contains many more optimizer iterations. Batch size for the QM9 was 48, and training was completed with the Adam optimizer at an initial learning rate of  $1 \times 10^{-3}$  using a 90 : 10 train/validation ratio. Loss and regularization were performed following the strategies defined in Section 2.3.4.1.

**2.3.4.3  $\omega$ B97X-D3 isomerization dataset.** The higher theory  $\omega$ B97X-D3 isomerization dataset developed by Grambow *et al.* was used following a similar approach to Pattanaik *et al.* to benchmark TSNet. We split the  $\omega$ B97X-D3 dataset into an 80 : 10 : 10 train/validation/test split, further following Pattanaik *et al.* We trained for 100 epochs with a batch size of 32 reactions. Optimization, loss, and regularization were performed following the strategies defined in Section 2.3.4.1.

## 3 Results and discussion

### 3.1 $S_{\text{N}2}$ -TS analytics

Significant thought was put into selecting the  $S_{\text{N}2}$  reaction for generation of a TS dataset. While other saddle points exist upon the PES, such as rotational TSs which only involve the rotation of a particular group of a single species, there was initial concern that such a prediction task may be too trivial. Also, “product” and “reactant” states for rotations may be exceedingly similar in structure, which could cause concern for models which depend on picking out differences between inputs. Ultimately, a greater impact was foreseen from a model capable of performing predictions upon reactions in which bonds are broken and formed. Gas-phase  $S_{\text{N}2}$  reactions are some of the most simplistic reaction mechanisms: a nucleophile approaches the center to form a bond, while the bond between



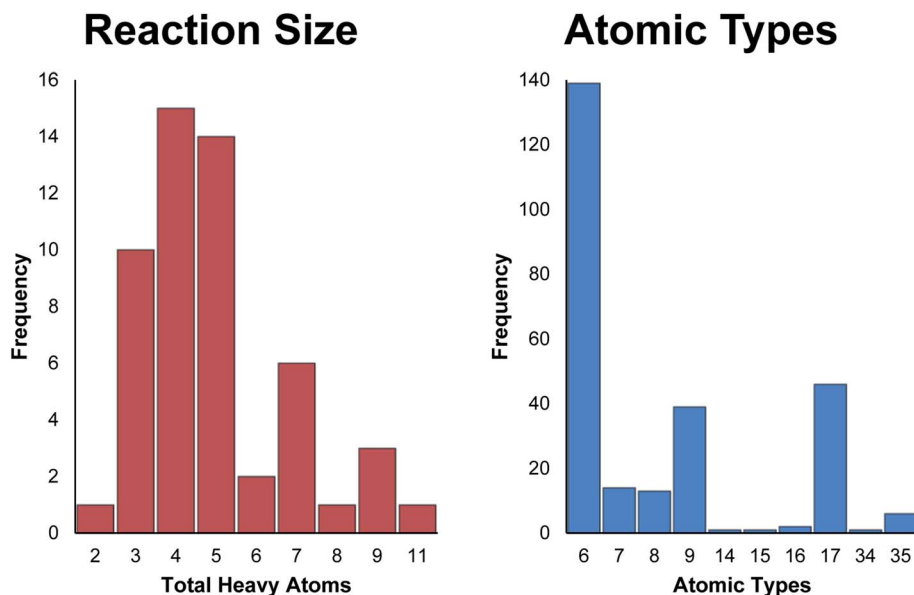


Fig. 4  $S_N2$ -TS histograms. Reaction size and atomic type histograms for the  $S_N2$ -TS dataset. Only heavy atoms (*i.e.* everything other than hydrogen) are counted in both charts. The vast majority of structures within the dataset contain three to five heavy atoms of mostly carbon, fluorine, and chlorine. All reaction centers are carbocations.

the leaving group and the center breaks – all in a single concerted motion. In a nutshell, the decision to use  $S_N2$  reactions was made due to their complexity over rotations and other more trivial molecular motions, yet simplicity when compared to other chemical reactions. The  $S_N2$ -TS dataset was generated using the second order Møller-Plesset perturbation theory method with the cc-pVDZ double-zeta basis set.<sup>76,83</sup> This level of theory and basis were chosen strategically to ensure relative accuracy while keeping calculation times low.

Only 53  $S_N2$  reactions passed manual validation from a pool of 114 seed structures, the majority of which are small systems of 6–8 atoms. Each reaction includes three structures: reactant complex, TS, and product complex. The dataset as a whole includes 518 atoms and 17 226 interatomic distances totalled across all three structures of each reaction. Most reactions are symmetric, *e.g.* both nucleophile and leaving group are identical. Why so many calculations failed to converge is unknown, but it could relate to the simplicity of many reactions in the dataset. Hessian-based optimization methods are typically quite sensitive to the curvature of the surface being searched. It is possible that the PES around the small, symmetric reactions from the  $S_N2$ -TS is shallow enough that many steps must be taken before convergence is observed. In other words, the PES may be too flat such that the Hessian does not provide enough of a definitive direction for critical point location. This would lead to the optimizer taking effectively random steps across the surface, where each point is not radically different from the next energetically. Theoretically, an exact critical point representing reactant and product complexes should exist on the “true” surface, however a weak approximation may represent very shallow curvature as flat, hindering convergence. Further testing using methods of higher accuracy than MP2 must be conducted before anything conclusive about IRC convergence

failure can be stated. Fig. 4 presents size and atomic type distributions for the dataset. Fig. 5 includes TS structures for four reactions from the  $S_N2$ -TS dataset, selected to showcase example variety. All structures are presented and saved in standard orientation. IRC calculations for locating product/reactant complexes from optimized TS structures regularly failed during data acquisition. The alternative approach described above in Section 2.1.1 provided some success, however it does not guarantee convergence upon the exact complex, unlike true IRC calculations. This “noisy” optimization procedure is the sole reason for why such a low yield of final reactions were obtained from seed structures. While sufficient at providing a preliminary view of performance for TSNet, the  $S_N2$ -TS dataset could benefit from a greater number of more diverse structures.

### 3.2 TSNet performance

The architectural motivation behind TSNet is multifaceted, pulling inspiration from ResNet,<sup>84</sup> SchNet,<sup>25</sup> the weight-sharing network from Proffitt and Pearson,<sup>79</sup> and the original TFN architecture proposed by Thomas *et al.*<sup>38</sup> The decision to use only 64 self-interaction units for embeddings, MCB layers and radials was motivated by the high memory constraints of the new TFN implementation. Despite access to high-end hardware through Compute Canada, TSNet regularly encountered memory errors when using more than 64 units across the entire network architecture – thus limiting the size of each individual MCB. Training a larger version of TSNet will likely require access to GPU hardware with greater than 16 Gb of onboard vRAM. Presence of residual skip connections also motivated architectural decisions, as skip connections allow the model to determine during training which layers are necessary for



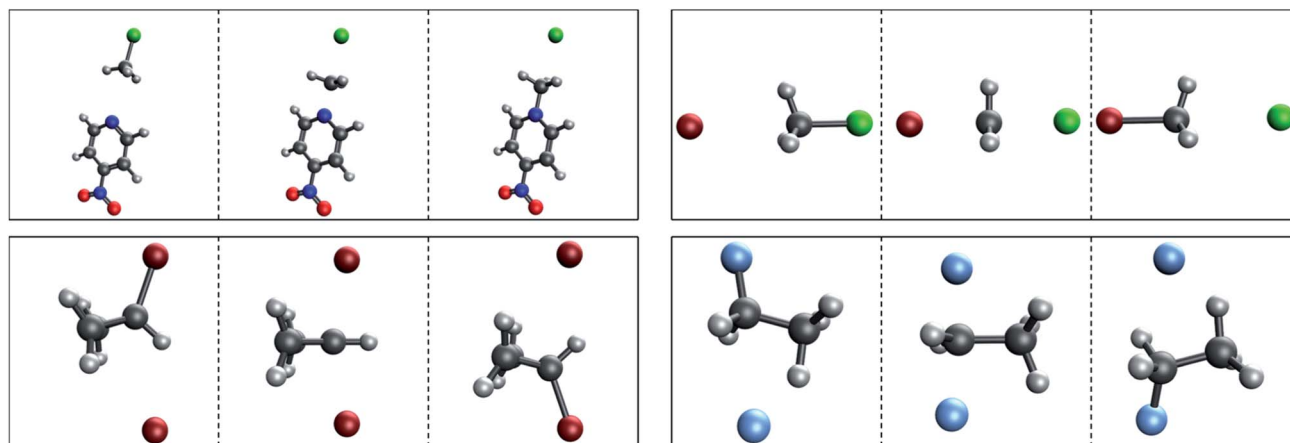


Fig. 5 Select reactions from the  $S_N2$ -TS dataset. 4 selected reactions from the  $S_N2$ -TS dataset. The first reaction in the top left corner of the figure is the largest system of the dataset, with 11 heavy (non-hydrogen) atoms. Note the slight  $sp^3$  nature to the carbon center in the TSs of the first two reactions presented. This is consistent with Hammond's postulate. For the final two reactions, which are symmetric, the carbon center is completely flat, meaning the TSs exist exactly halfway through the trajectory between reactant and product.

training, push layer weights which are not necessary towards zero, dropping their involvement in prediction. Ultimately, the largest model within hardware constraints was chosen which trained smoothly and allowed the model to decide which layers were not necessary during training. One of the biggest additions to TSNet over the original TFNs is the support for radial sharing. In addition to TSNet TSNet-shared was tested, which utilizes a single shared radial across all MCBs within the network, resulting in a 10-fold reduction of trainable parameters.

Some complications arise from computing loss from Cartesian arrays, as the difference between true and predicted includes coordinate system differences in addition to molecular differences. A separate distance error metric was developed to determine molecular difference within coordinate system difference inclusion, which computes average absolute difference between the distance matrix of true and predicted structures:

$$\text{Distance error} = \frac{1}{mn^2} \sum_i^m \sum_j^n \sum_k^n |D_{ijk}^{\text{predicted}} - D_{ijk}^{\text{true}}| \quad (4)$$

where  $i$  indexes  $m$  number of reactions,  $j, k$  index  $n$  number of atoms within each reaction, and  $D^{\text{true}}$  and  $D^{\text{predicted}}$  are  $m \times n \times$

$n$  distance matrices for the true and predicted TS structure, respectively. Training using this metric as the loss function resulted in incredibly poor training and performance (see Table 1). The underlying reason behind this failure is unknown, however the current assumption is related to how distance matrices are non-unique representations (*i.e.* there are an infinite number of Cartesian arrays which produce identical distance matrices). A model trained to minimize distance error is effectively allowed to cheat, producing nonsense Cartesians which are close to the correct distance matrix, but are in fact not representative of the true structure of the TS. Findings suggest that successful molecular conformer prediction requires the use loss functions which include a sort of systematic coordinate error, where a theoretical perfect predictor which produces indistinguishable TS structures may achieve loss values above zero simply because the predictions are not within the same coordinate system as the true structure. While removal of this systematic error is ideal, use of distance error as a metric is a fair replacement, allowing for the identification of models which are still performant, despite their inability to translate/rotate predictions into an arbitrary "true" coordinate system. Ultimately loss and distance values should scale proportionally,

Table 1  $k = 5$  cross validation results from  $S_N2$ -TS. Cross validation results for TSNet and TSNet-shared upon the  $S_N2$ -TS dataset, pre-trained and trained from scratch. Loss values are coordinate angstroms (Å), while distance error values are Euclidean distance Å. Standard deviation values are in parentheses. Overall best distance error values are achieved by TSNet trained from scratch, while the best loss values are achieved by TSNet when pre-trained with the midpoint prediction task from the QM9. TSNet-distance which trains using distance error performs the worst

Model	Train		Validation	
	Mean loss	Mean distance error	Mean loss	Mean distance error
TSNet	0.1455 (0.01)	<b>0.02672 (0.01)</b>	0.4576 (0.03)	<b>0.1831 (0.03)</b>
Pre-trained TSNet (midpoint)	<b>0.06956 (0.004)</b>	0.07868 (0.006)	<b>0.3631 (0.04)</b>	0.2095 (0.03)
TSNet-shared	0.2977 (0.03)	0.07301 (0.02)	0.6598 (0.1)	0.1950 (0.04)
Pre-trained TSNet-shared (midpoint)	0.07941 (0.007)	0.04978 (0.01)	0.4181 (0.05)	0.1925 (0.06)
Pre-trained TSNet (energy)	0.1327 (0.03)	0.04387 (0.01)	0.5124 (0.07)	0.2838 (0.1)
TSNet-distance	—	0.6198 (0.2)	—	1.828 (0.3)



but in some instances a handful of interesting interactions may happen. The three major relationships between MAE and distance error are as such:

- Low MAE, low distance error → model fits structures & coordinate system.
- Low MAE, high distance error → model fits coordinate system, not structures.
- High MAE, low distance error → model fits structures, not coordinate system.

Relationships one and three are ideal, while two is likely indicative of overfitting. In addition to computing distance error, manual validation must be performed in order to obtain the full view of a model's performance, as predicted chemical systems are too complex to score reliably with a single number. A fully automated pipeline including various molecular comparison metrics and sub-graph matching would ideally be utilized in place of manual validation; however, such a complex validation pipeline is beyond the scope of this paper and the  $S_{\text{N}}2$ -TS dataset is small enough for visualization to be viable.

**3.2.1  $S_{\text{N}}2$ -TS.** Cross validation results for TSNet and TSNet-shared may be found in Table 1. Cross validation shows that TSNet slightly outperforms TSNet-shared on average when trained from scratch upon the  $S_{\text{N}}2$ -TS dataset, though performance is mostly comparable. Train performance is excellent across both models. On average for TSNet, interatomic distances are off by only 0.02672 Å, meaning all predicted atomic positions are only off by a small fraction of an angstrom from where they are in the true TS structures. The interatomic distance error is 0.07941 Å for TSNet-shared. Both of these distance error values result in predicted structures which are virtually indistinguishable from true TS structures to the human eye. Fig. 6 further demonstrates how indistinguishable training set predictions are from true TS structures.

Representative train/validation curves for TSNet may be found in Fig. 7. Output inspection revealed virtually zero noticeable differences between true and predicted structures for both models as well. Validation performance suggests that TSNet is overfitting the  $S_{\text{N}}2$ -TS dataset, and manual inspection uncovers that validation predictions are worse than train predictions. This overfitting is likely not a flaw of TSNet for

a number of reasons. First, the small size and low diversity of the dataset suggests TSNet and TSNet-shared perform worse on the validation set simply because equivalent systems are not abundant within the train set. Second, overfitting behaviour is not observed during the midpoint prediction task with the QM9. TSNet is also trained by default with L2-regularization using a  $\lambda$  value of 0.01, and no noticeable changes to train and validation performance were observed when increasing regularization strength. Boosting TSNet validation performance is likely only possible with a larger, more diverse dataset of reactions. However, the performance of TSNet is promising, and worth further exploration.

**3.2.2  $\omega$ B97X-D3 isomerization dataset.** TSNet performance on the  $\omega$ B97X-D3 isomerization dataset was better than the graph NN developed by Pattanaik *et al.*, with an average test distance error of 0.2236 Å compared to 0.28 Å reported by Pattanaik *et al.* for test set reactions which successfully optimized, and a reported 0.43 Å loss for test set reactions which did not optimize. Results may be found in Table 2. This comparison is largely valid, as the distance error metric (see eqn (4)) is identical to the loss function used by Pattanaik *et al.* We leave both further benchmarking and testing the viability of TSNet as a guess generator for traditional TS search algorithms for future studies, as it is beyond the scope of this report.

**3.2.3 Midpoint prediction task.** In order to take advantage of the latent knowledge of the QM9 for TS prediction purposes, some modifications must be made to the original QM9 prediction task. The QM9 was modified in a similar fashion to Thomas *et al.*,<sup>38</sup> making a new vector field prediction task more closely related to TS predictions. In a nutshell, the midpoint prediction task selects at random an atom from each molecule in the dataset, perturbing it forward and backward by 0.75 Å, taking snapshots of the entire molecule at each extrema. All other atoms are perturbed in a similar fashion by only 0.15 Å. From these modifications, each molecule produces a set of 3 related structures, which serve as pseudo reactant, product, and TS for pre-training of the model. While this new prediction task is not based upon any true physical motivations, the ultimate goal was to create dataset from which the model can learn what molecules typically look like exclusively through exposure. For

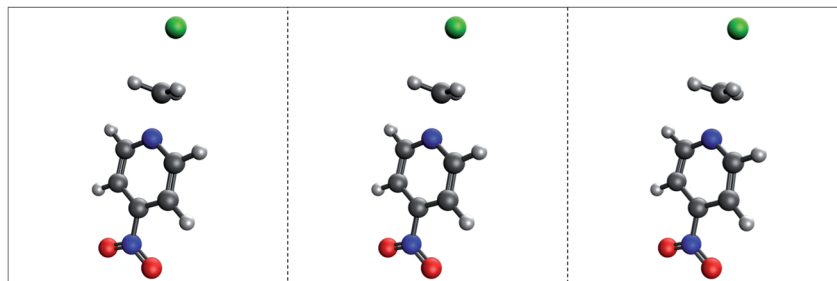
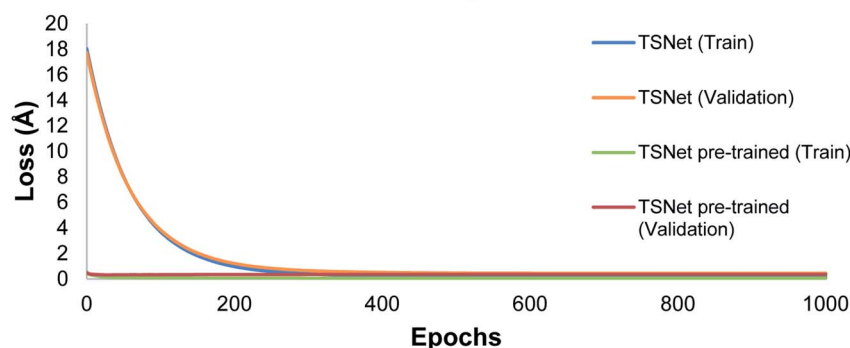


Fig. 6 Train results for TSNet and TSNet-shared. Train results for TSNet and TSNet-shared for  $\text{C}_5\text{H}_4\text{N}_2\text{O}_2 + \text{CH}_3\text{Cl}$ , the largest system in the dataset. The leftmost structure is the true TS for the reaction, while the center structure is the prediction made by TSNet, and the right structure is the prediction made by TSNet-shared. The quality of these predictions are representative of all other reactions from the  $S_{\text{N}}2$ -TS dataset when trained upon. Similar results are observed for train prediction using pre-trained variants of TSNet and TSNet-shared. Note how TSNet and TSNet-shared are able to reproduce the slight  $\text{sp}^3$  nature of the reaction center, an important aspect of this particular TS.



## TSNet Averaged Loss



## TSNet Averaged Distance Error

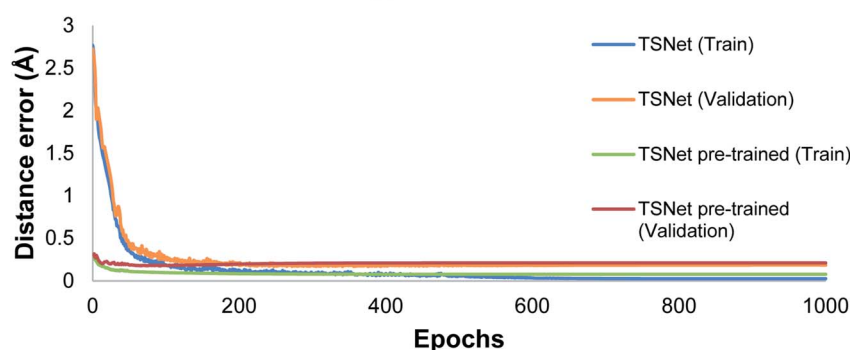


Fig. 7 TSNet train/validation curves. Averaged loss and distance error curves for TSNet on the  $S_{N2}$ -TS dataset both trained from scratch and pre-trained using the midpoint task. Very similar curves were observed for TSNet-shared, and were thus omitted. Pre-training for both metrics results in better starting points for predictions, however, pre-trained models do not outperform equivalent models trained from scratch during validation. Ultimately, worse validation performance is likely due to the small size of the  $S_{N2}$ -TS dataset.

Table 2  $\omega$ B97X-D3 results. Results for TSNet and TS-shared when trained upon the  $\omega$ B97X-D3 isomerization dataset. Loss values are coordinate angstroms (Å) unless otherwise stated, while distance error values are Euclidean distance Å. Both TSNet and TSNet-shared perform well on the  $\omega$ B97X-D3 dataset compared to the graph NN developed by Pattanaik *et al.*

Model	Train		Validation		Test	
	Mean loss	Mean distance error	Mean loss	Mean distance error	Mean loss	Mean distance error
TSNet	0.1961	0.2130	0.2099	0.2236	0.2090	0.2236
TSNet-shared	0.1972	0.2131	0.2103	0.2238	0.2098	0.2238

example, a model which has been exposed to enough conjugated systems should understand that typically they must be planar simply because the model has observed enough planar conjugated systems – the model has no understanding of the orbital overlap which motivates conjugation. In essence, the midpoint prediction task asks a model to view two perturbed systems and produce the true system which links them, *i.e.* the equilibrium structure from the QM9, encoding what molecules should look like within model parameters. Performance of TSNet on the midpoint task is effectively perfect (results in Table 3). No difference between true and predicted structures

were discerned during manual validation for both models in either the train, or the validation splits.

**3.2.4 Transfer learning.** The naïve approach to TL using the original scalar prediction task posed by the QM9 performed the worst out of all tested models, as anticipated, achieving average validation loss and distance error values of 0.5124 Å and 0.2838 Å, respectively. Pre-training TSNet and TSNet-shared with the midpoint task provided both models with better initial weights than Glorot initialization, producing predicted structures with distinguishable moieties prior to training on the  $S_{N2}$ -TS dataset (see Fig. 8). Pre-training also results in faster convergence to better loss values on both train and validation sets during cross



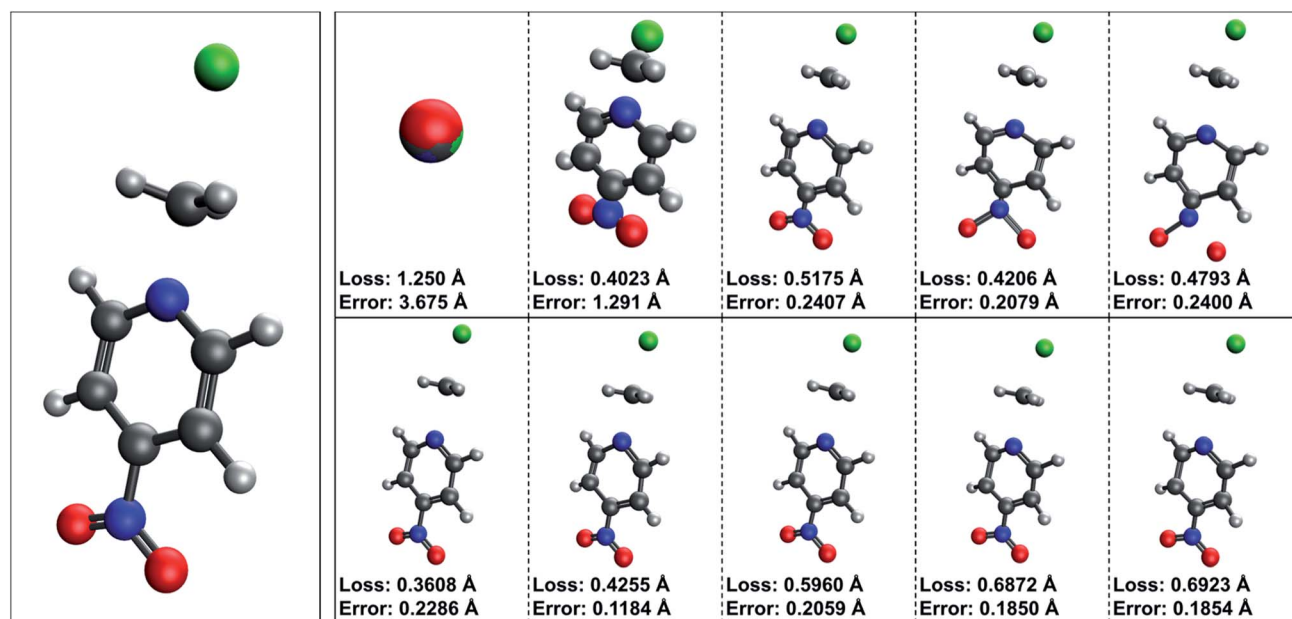
**Table 3** QM9 results. Results for TSNet and TS-shared when trained upon the QM9. Loss values are coordinate angstroms (Å) unless otherwise stated, while distance error values are Euclidean distance Å. Both TSNet and TSNet-shared perform exceptionally well on the midpoint prediction task, producing validation prediction indistinguishable from “true” target structures. TSNet performance predicting internal energy values is over an order of magnitude worse than state-of-the-art networks. Effort was not put into constructing a model capable of producing high accuracy energy predictions, as the ultimate goal was for transition state geometry prediction

Model	Train		Validation	
	Mean loss	Mean distance error	Mean loss	Mean distance error
TSNet (midpoint)	0.1286	0.03893	0.1355	0.05694
TSNet-shared (midpoint)	0.1962	0.03589	0.2209	0.06257
TSNet (energy)	0.1575 (eV)	—	0.3976 (eV)	—

validation. However, pre-training does not result in lower distance errors, which is a better metric for prediction quality. Manual inspection of various validation results (Fig. 8) shows that predictions made from pre-trained models are of comparable quality to models trained from scratch.

TSS for reactions are mostly made up of interatomic distances which are at equilibrium, with a handful of critical distances surrounding the reaction center, most notably the bonds being broken/formed. In a sense, by pretraining with the midpoint task, TSNet encodes knowledge about “equilibrium” chemistry within its weights – leading to superior predictions prior to training compared to an equivalent model trained from scratch. In other words, pretraining TSNet and TSNet-shared with the midpoint task allows each model to effectively skip

the first approx. 400 epochs of training observed when training from scratch on the  $S_{N2}$ -TS dataset. However, there are too few examples within the  $S_{N2}$ -TS dataset for TSNet to hook into the underlying pattern which defines the  $S_{N2}$  reaction trajectory and produce highly accurate predictions. This leads to optimizing what is effectively noise, causing overfitting. Following the relationship between loss and distance error defined above in Section 3.3, pre-training with the midpoint task before training on the  $S_{N2}$ -TS dataset in its current state creates models which better fit the coordinate system, rather than actual TS structures. Re-iterating the point made in Section 3.3.1, likely the only way of boosting performance is through the acquisition of more data. However, the idea of using TL to pretrain machine-learned TS structure predictors like TSNet is worthy of



**Fig. 8** TSNet predictions during training. Snapshots of transition state predictions for  $C_6H_7N_2O_2Cl$  (the largest system in the  $S_{N2}$ -TS dataset) taken prior to training, at epochs 10, 100, 500, and after training completed, presented in chronological order.  $C_6H_7N_2O_2Cl$  is part of the first cross validation fold, and these predictions were made using models trained on all folds except the first fold. The true structure for  $C_6H_7N_2O_2Cl$  is presented as the leftmost structure. The first row of TS structures was produced by TSNet trained from scratch, while the second row was produced by TSNet pre-trained using the QM9 midpoint task. TSNet trained from scratch covers a massive distance during training, but the model fails to produce high accuracy predictions post training. The initial prediction from pre-trained TSNet is essentially the midpoint between reactant and product complex – however, note how TSNet when pre-trained fails to reconstruct the slight  $sp^3$  nature to the carbon reaction center, key to this particular transition state. Due to the small size of the  $S_{N2}$ -TS dataset, features crucial to this particular transition state are missing from TSNet predictions.



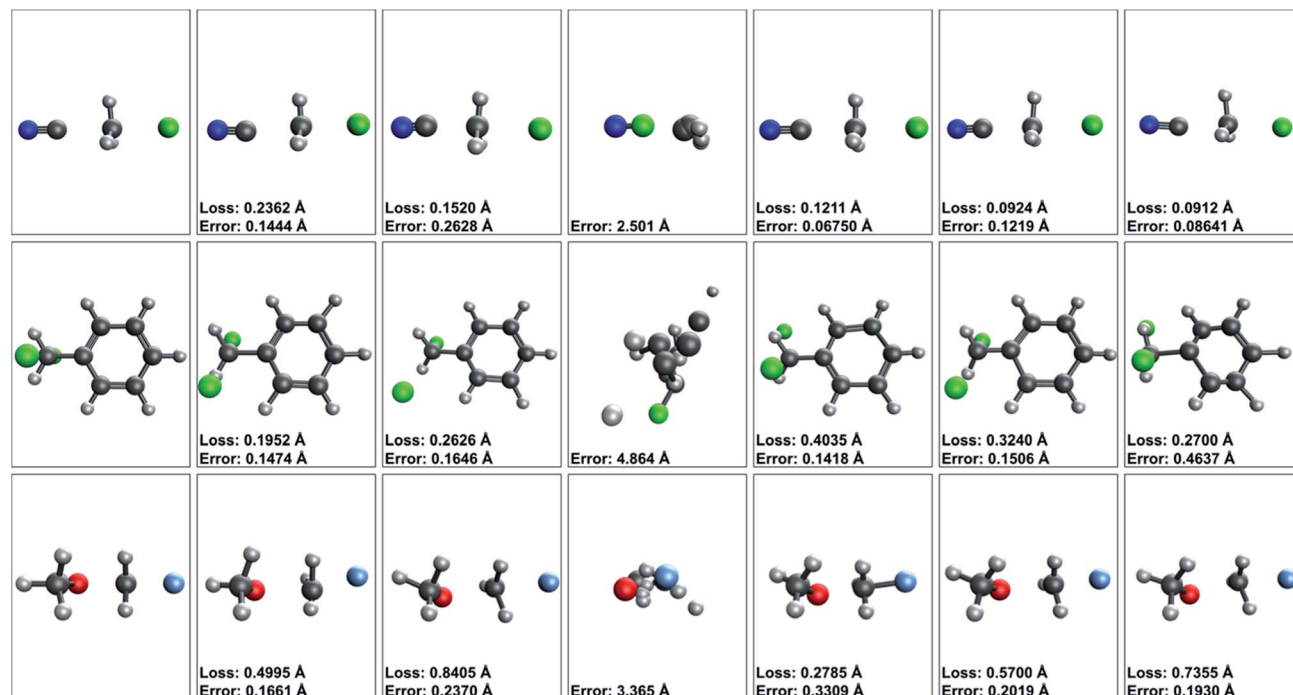


Fig. 9 Select validation results for all tested TSNet variants. Each of the three selected validation structures were selected from the first, second, and third validation fold, respectively. Comparisons within the same fold are fair, as all TSNet variants are trained with identical train sets. Comparisons between structures are less fair, as the models have different train sets for different folds. To keep it simple, comparisons across rows are fair, comparisons down columns are not. Across each row predictions are presented from each variant of TSNet in comparison to the true TS geometry. From left to right for each row structures are: the true TS, TSNet, TSNet-shared, TSNet-distance, TSNet (pre-trained w/ midpoint), TSNet-shared (pre-trained w/midpoint), and TSNet (pre-trained w/energies). Loss units are in coordinate angstroms (Å) while (distance) error values are in Euclidean distance Å. Note how across all presented validation structures, TSNet-distance fails to produce any distinguishable features. Better performance is observed from all other variants, with TSNet both trained from scratch and pre-trained with the midpoint task producing arguably the best results. Poor performance across all models on the final structure presented,  $C_2H_6OF$ , is likely due to a slight rotation in the methoxy group during the trajectory from reactant complex to product.

further investigation. Pretraining can, in a sense, remove the need to learn “equilibrium” chemistry, providing it prior to training on TS data. There is simply a lack of enough TS structures and clean reactant and product complexes to achieve highly accurate predictions during validation (Fig. 9).

## 4 Conclusion

The innovations of this paper are threefold. First, a novel dataset of  $S_N2$  reactions including TS structures has been created, the first of its kind. Second, a new MPNN based on TFNs titled TSNet has been developed which perform admirably on this dataset, given its size. Lastly, a TL method has been tested which allows for pretraining of TSNet using the QM9, a dataset over 4 orders of magnitude larger than the  $S_N2$ -TS dataset, providing a reduction to loss values and a boost to prediction quality prior to training on the  $S_N2$ -TS dataset. Already TSNet has outgrown the  $S_N2$ -TS dataset – and the need for more data is paramount. Also, TS structures for much more complex reactions should be acquired to further test out TSNet’s capabilities. While data acquisition is difficult for this particular task, if enough theoreticians make inhouse TS data readily available, data scientists may be able to quickly compile datasets of respectable size for training purposes. Also, in the past

decade advancements have been made with respect to better automated saddle point search methods,<sup>85–87</sup> meaning better pipelines for data acquisition may be constructed to reduce the need for human intervention. In addition to data generation and acquisition, new TL procedures should be developed to shrink the minimum amount of target TS data necessary to achieve high performance. Pre-training with the QM9 midpoint prediction task has shown that pretraining tasks do not necessarily have to be predicated by the laws of physics so long as the model is ultimately exposed to meaningful chemical structures. Ideally physically motivated prediction tasks would be used for pretraining, such as predicting frames from molecular dynamics trajectories – however, pseudo-physical tasks like the QM9 midpoint task are much easier to generate. While TSNet has been built for TS structure prediction, the framework upon which the model is built is general to many applications in chemistry. Ultimately, TSNet and TFNs are capable of any form of conformer prediction, mean they may be used for machine-learned molecular dynamics algorithms, calculation of stable conformers for known and potentially novel compounds, *etc.* The lightweight nature of ML *vs.* QM could also make QM/MM techniques obsolete, as ML can achieve QM level accuracies in a fraction of the compute times.



Despite the quick paradigm shift to ML for computational chemistry, the field is still extremely young and full of potential. Hopefully, the very promising performance of TSNet will spark interest in the development of both purely machine-learned TS predictors, and composite methods which blend ML and traditional search, resulting in a new cast of inexpensive methods capable of predicting TS structures with a high degree of fidelity, and reliability.

## Data availability

The data and code developed in this work is available at <https://github.com/UPEIChemistry/tensor-field-networks>.

## Author contributions

RJ and JP designed the study. RJ was responsible for data curation, model design, and the original implementation of TSNet. WZ was responsible for review and editing of the manuscript in the revision stage. JP was responsible for overall project administration and supervision as well as funding acquisition.

## Conflicts of interest

There are no conflicts to declare.

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## References

- 1 Y. Lecun, Y. Bengio and G. Hinton, Deep Learning, *Nature*, 2015, 436–444, DOI: 10.1038/nature14539.
- 2 A. Thawabteh, F. Lelario, L. Scranò, S. A. Bufo, S. Nowak, M. Behrens, A. Di Pizio, M. Y. Niv and R. Karaman, Bitterless Guaifenesin Prodrugs-Design, Synthesis, Characterization, in Vitro Kinetics, and Bitterness Studies, *Chem. Biol. Drug Des.*, 2019, 93(3), 262–271, DOI: 10.1111/cbdd.13409.
- 3 E. Karlsson, E. Andersson, J. Dogan, S. Gianni, P. Jemth and C. Camilloni, A Structurally Heterogeneous Transition State Underlies Coupled Binding and Folding of Disordered Proteins, *J. Biol. Chem.*, 2019, 294(4), 1230–1239, DOI: 10.1074/jbc.RA118.005854.
- 4 A. J. Neel, M. J. Hilton, M. S. Sigman and F. D. Toste, Exploiting Non-Covalent  $\pi$  Interactions for Catalyst Design, *Nature*, 2017, 637–646, DOI: 10.1038/nature21701.
- 5 Y. Zhu and D. G. Drueckhammer, Transition State Modeling and Catalyst Design for Hydrogen Bond-Stabilized Enolate Formation, *J. Org. Chem.*, 2005, 70(19), 7755–7760, DOI: 10.1021/jo0513818.
- 6 E. T. Stewart, *Quantum Chemistry*, ed. N. Folchetti, K. P. Hamman and C. DuPont, Pearson Prentice Hall, Upper Saddle River, NJ, 6th edn, 1970, vol. 226, DOI: 10.1038/226383b0.
- 7 J. Nocedal and S. Wright, *Numerical Optimization; Springer Series in Operations Research and Financial Engineering*, Springer, New York, 2006.
- 8 C. Peng and H. B. Schlegel, Combining Synchronous Transit and Quasi-Newton Methods to Find Transition States, *Isr. J. Chem.*, 1993, 33(4), 449–454, DOI: 10.1002/ijch.199300051.
- 9 P. Culot, G. Dive, V. H. Nguyen and J. M. Ghuyssen, A Quasi-Newton Algorithm for First-Order Saddle-Point Location, *Theor. Chim. Acta*, 1992, 82, 189–205, DOI: 10.1007/BF01113251.
- 10 S. A. Trygubenko and D. J. Wales, A Doubly Nudged Elastic Band Method for Finding Transition States, *J. Chem. Phys.*, 2004, 120(5), 2082–2094, DOI: 10.1063/1.1636455.
- 11 G. Henkelman, B. P. Uberuaga and H. Jónsson, Climbing Image Nudged Elastic Band Method for Finding Saddle Points and Minimum Energy Paths, *J. Chem. Phys.*, 2000, 113(22), 9901–9904, DOI: 10.1063/1.1329672.
- 12 P. Zimmerman, Reliable Transition State Searches Integrated with the Growing String Method, *J. Chem. Theory Comput.*, 2013, 9(7), 3043–3050, DOI: 10.1021/ct400319w.
- 13 P. M. Zimmerman, Single-Ended Transition State Finding with the Growing String Method, *J. Comput. Chem.*, 2015, 36(9), 601–611, DOI: 10.1002/jcc.23833.
- 14 M. Jafari and P. M. Zimmerman, Reliable and Efficient Reaction Path and Transition State Finding for Surface Reactions with the Growing String Method, *J. Comput. Chem.*, 2017, 38(10), 645–658, DOI: 10.1002/jcc.24720.
- 15 C. J. Cerjan and W. H. Miller, On Finding Transition States, *J. Chem. Phys.*, 1981, 75(6), 2800–2801, DOI: 10.1063/1.442352.
- 16 J. Simons, P. Jørgensen, H. Taylor and J. Ozment, Walking on Potential Energy Surfaces, *J. Phys. Chem.*, 1983, 87(15), 2745–2753, DOI: 10.1021/j100238a013.
- 17 A. Banerjee, N. Adams, J. Simons and R. Shepard, Search for Stationary Points on Surfaces, *J. Phys. Chem.*, 1985, 89(1), 52–57, DOI: 10.1021/j100247a015.
- 18 S. A. Ghasemi and S. Goedecker, An Enhanced Splined Saddle Method, *J. Chem. Phys.*, 2011, 135(1), 014108, DOI: 10.1063/1.3605539.
- 19 R. Granot and R. Baer, A Spline for Your Saddle, *J. Chem. Phys.*, 2008, 128(18), 184111, DOI: 10.1063/1.2916716.
- 20 A. C. Vaucher and M. Reiher, Minimum Energy Paths and Transition States by Curve Optimization, *J. Chem. Theory Comput.*, 2018, 14(6), 3091–3099, DOI: 10.1021/acs.jctc.8b00169.
- 21 M. A. Heuer, A. C. Vaucher, M. P. Haag and M. Reiher, Integrated Reaction Path Processing from Sampled Structure Sequences, *J. Chem. Theory Comput.*, 2018, 14(4), 2052–2062, DOI: 10.1021/acs.jctc.8b00019.



- 22 A. A. Peterson, Acceleration of Saddle-Point Searches with Machine Learning, *J. Chem. Phys.*, 2016, **145**(7), 074106, DOI: 10.1063/1.4960708.
- 23 J. Behler, Perspective: Machine Learning Potentials for Atomistic Simulations, *J. Chem. Phys.*, 2016, **145**(17), 170901, DOI: 10.1063/1.4966192.
- 24 A. C. Mater and M. L. Coote, Deep Learning in Chemistry, *J. Chem. Inf. Model.*, 2019, **59**(6), 2545–2559, DOI: 10.1021/acs.jcim.9b00266.
- 25 K. T. Schütt, P. J. Kindermans, H. E. Saucedo, S. Chmiela, A. Tkatchenko and K. R. Müller, SchNet: A Continuous-Filter Convolutional Neural Network for Modeling Quantum Interactions, *Adv. Neural Inf. Process. Syst.*, 2017, 992–1002.
- 26 O. T. Unke and M. Meuwly, PhysNet: A Neural Network for Predicting Energies, Forces, Dipole Moments, and Partial Charges, *J. Chem. Theory Comput.*, 2019, **15**(6), 3678–3693, DOI: 10.1021/acs.jctc.9b00181.
- 27 J. S. Smith, B. T. Nebgen, R. Zubatyuk, N. Lubbers, C. Devereux, K. Barros, S. Tretiak, O. Isayev and A. E. Roitberg, Approaching Coupled Cluster Accuracy with a General-Purpose Neural Network Potential through Transfer Learning, *Nat. Commun.*, 2019, **10**(1), 2903, DOI: 10.1038/s41467-019-10827-4.
- 28 D. C. Elton, Z. Boukouvalas, M. D. Fuge and P. W. Chung, Deep Learning for Molecular Design – A Review of the State of the Art, *Mol. Syst. Des. Eng.*, 2019, 828–849, DOI: 10.1039/c9me00039a.
- 29 B. Sanchez-Lengeling, C. Outeiral, G. L. Guimaraes and A. Aspuru-Guzik, Optimizing Distributions over Molecular Space. An Objective-Reinforced Generative Adversarial Network for Inverse-Design Chemistry (ORGANIC), ChemRxiv 2017.
- 30 N. W. A. Gebauer, M. Gastegger and K. T. Schütt, Symmetry-Adapted Generation of 3d Point Sets for the Targeted Discovery of, *Molecules*, 2019, arXiv:1906.00957.
- 31 G. N. C. Simm, R. Pinsler and J. M. Hernández-Lobato, Reinforcement Learning for Molecular Design Guided by Quantum Mechanics, 2020, arXiv:2002.07717.
- 32 M. Simonovsky and N. Komodakis, *GraphVAE: Towards Generation of Small Graphs Using Variational Autoencoders*, 2018, DOI: 10.5121/csit.2012.2417.
- 33 G. L. Guimaraes, B. Sanchez-Lengeling, C. Outeiral, P. L. C. Farias and A. Aspuru-Guzik, Objective-Reinforced Generative Adversarial Networks (ORGAN) for Sequence Generation, *Models*, 2017, arXiv:1705.10843v2.
- 34 M. Benhenda, *ChemGAN Challenge for Drug Discovery: Can AI Reproduce Natural Chemical Diversity?*, 2017.
- 35 E. Putin, A. Asadulaev, Q. Vanhaelen, Y. Ivanenkov, A. V. Aladinskaya, A. Aliper and A. Zhavoronkov, Adversarial Threshold Neural Computer for Molecular *de Novo* Design, *Mol. Pharm.*, 2018, **15**(10), 4386–4397, DOI: 10.1021/acs.molpharmaceut.7b01137.
- 36 T. S. Cohen and M. Welling, Steerable CNNs, *5th Int. Conf. Learn. Represent. ICLR 2017 – Conf. Track Proc.*, 2016.
- 37 M. Weiler, M. Geiger, M. Welling, W. Boomsma and T. Cohen, 3D Steerable CNNs: Learning Rotationally Equivariant Features in Volumetric Data, *Adv. Neural Inf. Process. Syst.*, 2018, 10381–10392.
- 38 N. Thomas, T. Smidt, S. Kearnes, L. Yang, L. Li, K. Kohlhoff and P. Riley, *Tensor Field Networks: Rotation- and Translation-Equivariant Neural Networks for 3D Point Clouds*, 2018.
- 39 B. Anderson, T.-S. Hy and R. Kondor, *Cormorant: Covariant Molecular Neural Networks*, 2019.
- 40 G. N. C. Simm, R. Pinsler, G. Csányi and J. M. Hernández-Lobato, Symmetry-Aware Actor-Critic for 3D Molecular Design, 2020, arXiv:2011.12747.
- 41 R. Kondor, N-Body Networks: A Covariant Hierarchical Neural Network Architecture for Learning Atomic Potentials, 2018, arXiv:1803.01588.
- 42 G. Trigeorgis, F. Ringeval, R. Brueckner, E. Marchi, M. A. Nicolaou, B. Schuller and S. Zafeiriou, Adieu Features? End-to-End Speech Emotion Recognition Using a Deep Convolutional Recurrent Network, in *ICASSP, IEEE International Conference on Acoustics, Speech and Signal Processing – Proceedings*, Institute of Electrical and Electronics Engineers Inc., 2016, pp. 5200–5204, DOI: 10.1109/ICASSP.2016.7472669.
- 43 D. Amodei, S. Ananthanarayanan, R. Anubhai, J. Bai, E. Battenberg, C. Case, J. Casper, B. Catanzaro, Q. Cheng, G. Chen, J. Chen, J. Chen, Z. Chen, M. Chrzanowski, A. Coates, G. Diamos, K. Ding, N. Du, E. Elsen, J. Engel, W. Fang, L. Fan, C. Fougner, L. Gao, C. Gong, A. Hannun, T. Han, L. Vaino Johannes, B. Jiang, C. Ju, B. Jun, P. LeGresley, L. Lin, J. Liu, Y. Liu, W. Li, X. Li, D. Ma, S. Narang, A. Ng, S. Ozair, Y. Peng, R. Prenger, S. Qian, Z. Quan, J. Raiman, V. Rao, S. Satheesh, D. Seetapun, S. Sengupta, K. Srinet, A. Sriram, H. Tang, L. Tang, C. Wang, J. Wang, K. Wang, Y. Wang, Z. Wang, Z. Wang, S. Wu, L. Wei, B. Xiao, W. Xie, Y. Xie, D. Yogatama, B. Yuan, J. Zhan and Z. Zhu, *Deep Speech 2: End-to-End Speech Recognition in English and Mandarin*, PMLR, 2016.
- 44 S. Ekins, A. C. Puhl, K. M. Zorn, T. R. Lane, D. P. Russo, J. J. Klein, A. J. Hickey and A. M. Clark, Exploiting Machine Learning for End-to-End Drug Discovery and Development, *Nat. Mater.*, 2019, **18**, 435–441, DOI: 10.1038/s41563-019-0338-z.
- 45 X. Shi, Z. Chen, H. Wang, D.-Y. Yeung, W.-K. Wong and W.-C. Woo, Convolutional LSTM Network: a machine learning approach for precipitation nowcasting, In *Proceedings of the 28th International Conference on Neural Information Processing Systems – Vol. 1 (NIPS'15)*, MIT Press, Cambridge, MA, 2015, pp. 802–810.
- 46 Z. D. Pozun, K. Hansen, D. Sheppard, M. Rupp, K. R. Müller and G. Henkelman, Optimizing Transition States via Kernel-Based Machine Learning, *J. Chem. Phys.*, 2012, **136**(17), 174101, DOI: 10.1063/1.4707167.
- 47 A. A. Peterson, Acceleration of Saddle-Point Searches with Machine Learning, *J. Chem. Phys.*, 2016, **145**(7), 074106, DOI: 10.1063/1.4960708.
- 48 A. Khorshidi and A. A. Peterson, Amp: A Modular Approach to Machine Learning in Atomistic Simulations, *Comput. Phys. Commun.*, 2016, **207**, 310–324, DOI: 10.1016/j.cpc.2016.05.010.



- 49 L. Pattanaik, J. B. Ingraham, C. A. Grambow and W. H. Green, Generating Transition States of Isomerization Reactions with Deep Learning, *Phys. Chem. Chem. Phys.*, 2020, **22**(41), 23618–23626, DOI: 10.1039/d0cp04670a.
- 50 J. Behler, Atom-Centered Symmetry Functions for Constructing High-Dimensional Neural Network Potentials, *J. Chem. Phys.*, 2011, **134**(7), 074106, DOI: 10.1063/1.3553717.
- 51 A. P. Bartók, R. Kondor and G. Csányi, On Representing Chemical Environments, *Phys. Rev. B: Condens. Matter Mater. Phys.*, 2012, **87**(18), 184115, DOI: 10.1103/PhysRevB.87.184115.
- 52 G. Koch, R. Zemel and R. Salakhutdinov, *Siamese Neural Networks for One-Shot Image Recognition*, 2015.
- 53 C. A. Grambow, L. Pattanaik and W. H. Green, Reactants, Products, and Transition States of Elementary Chemical Reactions Based on Quantum Chemistry, *Sci. Data*, 2020, **7**(1), 1–8, DOI: 10.1038/s41597-020-0460-4.
- 54 C. Tan, F. Sun, T. Kong, W. Zhang, C. Yang and C. Liu, A Survey on Deep Transfer Learning, in *Lecture Notes in Computer Science (including subseries Lecture Notes in Artificial Intelligence and Lecture Notes in Bioinformatics)*, LNCS, Springer Verlag, 2018, vol. 11141, pp. 270–279, DOI: 10.1007/978-3-030-01424-7\_27.
- 55 J. Yosinski, J. Clune, Y. Bengio and H. Lipson, How Transferable Are Features in Deep Neural Networks?, *Adv. Neural Inf. Process. Syst.*, 2014, 3320–3328.
- 56 L. Ruddigkeit, R. Van Deursen, L. C. Blum and J. L. Reymond, Enumeration of 166 Billion Organic Small Molecules in the Chemical Universe Database GDB-17, *J. Chem. Inf. Model.*, 2012, **52**(11), 2864–2875, DOI: 10.1021/ci300415d.
- 57 R. Ramakrishnan, P. O. Dral, M. Rupp and O. A. Von Lilienfeld, Quantum Chemistry Structures and Properties of 134 Kilo Molecules, *Sci. Data*, 2014, **1**(1), 1–7, DOI: 10.1038/sdata.2014.22.
- 58 G. van Rossum, *Python Tutorial*, CWI Rep. CS-R9526 1995, No. CS-R9526, 1995, pp. 1–65.
- 59 S. Wolfe, D. J. Mitchell and H. Bernhard Schlegel, Theoretical Studies of  $S_N2$  Transition States. 1. Geometries, *J. Am. Chem. Soc.*, 1981, 7692–7694, DOI: 10.1021/ja00415a068.
- 60 I. Lee, C. Kim, D. Chung and B.-S. Lee, *J. Org. Chem.*, 1994, **59**(16), 4490–4494.
- 61 Y. S. Park, C. K. Kim, B. -S. Lee, I. Lee, W. M. Lim and W. K. Kim, Theoretical Studies on the Reactions of Substituted Phenolate Anions with Formate Esters, *J. Phys. Org. Chem.*, 1995, **8**(5), 325–334, DOI: 10.1002/poc.610080502.
- 62 A. Streitwieser, G. S. C. Choy and F. Abu-Hasanayn, Theoretical Study of Ion Pair  $S(N)2$  Reactions: Ethyl vs. Methyl Reactivities and Extension to Higher Alkyls, *J. Am. Chem. Soc.*, 1997, **119**(21), 5013–5019, DOI: 10.1021/ja961673d.
- 63 Y. A. Borisov, E. E. Arcia, S. L. Mielke, B. C. Garrett and T. H. Dunning, A Systematic Study of the Reactions of OH- with Chlorinated Methanes. 1. Benchmark Studies of the Gas-Phase Reactions, *J. Phys. Chem. A*, 2001, **105**(32), 7724–7736, DOI: 10.1021/jp011447c.
- 64 S. Parthiban, G. De Oliveira and J. M. L. Martin, Benchmark Ab Initio Energy Profiles for the Gas-Phase  $S_N2$  Reactions  $Y^- + CH_3X \rightarrow CH_3Y + X^-$  ( $X, Y = F, Cl, Br$ ). Validation of Hybrid DFT Methods, *J. Phys. Chem. A*, 2001, **105**(5), 895–904, DOI: 10.1021/jp0031000.
- 65 I. Lee, C. K. Kim, C. K. Sohn, H. G. Li and H. W. Lee, A High-Level Theoretical Study on the Gas-Phase Identity Methyl Transfer Reactions, *J. Phys. Chem. A*, 2002, **106**(6), 1081–1087, DOI: 10.1021/jp013690h.
- 66 J. Ren and J. I. Brauman, Energy Deposition in  $S_N2$  Reaction Products and Kinetic Energy Effects on Reactivity, *J. Phys. Chem. A*, 2002, **106**(15), 3804–3813, DOI: 10.1021/jp0141070.
- 67 K. K. Chang, G. L. Hong, B. S. Lee, K. K. Chan, W. L. Hai and I. Lee, Gas-Phase Identity Nucleophilic Substitution Reactions of Cyclopropenyl Halides, *J. Org. Chem.*, 2002, **67**(6), 1953–1960, DOI: 10.1021/jo0164047.
- 68 T. D. Fridgen and T. B. McMahon, Enthalpy Barriers for Asymmetric  $S_N2$  Alkyl Cation Transfer Reactions between Neutral and Protonated Alcohols, *J. Phys. Chem. A*, 2003, **107**(5), 668–675, DOI: 10.1021/jp026818j.
- 69 A. Dybala-Defratyka, M. Rostkowski, O. Matsson, K. C. Westaway and P. Paneth, A New Interpretation of Chlorine Leaving Group Kinetic Isotope Effects; a Theoretical Approach, *J. Org. Chem.*, 2004, **69**(15), 4900–4905, DOI: 10.1021/jo049327z.
- 70 H. J. Zhu, Y. Ren, J. Ren and S. Y. Chu, Theoretical Investigation of Ion Pair  $S_N2$  Reactions of Alkali Isothiocyanates with Alkyl Halides. Part 1. Reaction of Lithium Isothiocyanate and Methyl Fluoride with Inversion Mechanism, *Int. J. Quantum Chem.*, 2005, **101**(1), 104–112, DOI: 10.1002/qua.20213.
- 71 A. P. Bento, M. Solà and F. M. Bickelhaupt, Ab Initio and DFT Benchmark Study for Nucleophilic Substitution at Carbon ( $S_N2@C$ ) and Silicon ( $S_N2@Si$ ), *J. Comput. Chem.*, 2005, **26**(14), 1497–1504, DOI: 10.1002/jcc.20261.
- 72 S. R. K. Ainarapu, A. P. Wiita, L. Dougan, E. Uggerud and J. M. Fernandez, Single-Molecule Force Spectroscopy Measurements of Bond Elongation during a Bimolecular Reaction, *J. Am. Chem. Soc.*, 2008, **130**, 6479–6487, DOI: 10.1021/ja800180u.
- 73 A. P. Bento, M. Sola and F. M. Bickelhaupt, E2 and S2 Reactions of  $X + CHCHX$  ( $X = F, Cl$ ); an Ab Initio and DFT Benchmark Study, *J. Chem. Theory Comput.*, 2008, **4**(6), 929–940, DOI: 10.1021/ct700318e.
- 74 M. Frisch, G. Trucks, H. Schlegel, G. Scuseria, M. Robb, J. Cheeseman, G. Scalmani, V. Barone, B. Mennucci, G. Petersson, H. Nakatsuji, M. Caricato, X. Li, H. Hratchian, A. Izmaylov, J. Bloino, G. Zheng, J. Sonnenberg, M. Hada, M. Ehara, K. Toyota, R. Fukuda, J. Hasegawa, M. Ishida, T. Nakajima, Y. Honda, O. Kitao, H. Nakai, T. Vreven, J. Montgomery, J. Peralta, F. Ogliaro, M. Bearpark, J. Heyd, E. Brothers, K. Kudin, V. Staroverov, R. Kobayashi, J. Normand, K. Raghavachari, P. Rendell, J. Burant, S. Iyengar, J. Tomasi, M. Cossi, N. Rega, J. Millam, M. Klene, J. Knox, J. Cross, V. Bakken,



- C. Adamo, J. Jaramillo, R. Gomperts, R. Stratmann, O. Yazyev, A. Austin, R. Cammi, C. Pomelli, J. Ochterski, R. Martin, K. Morokuma, V. Zakrzewski, G. Voth, P. Salvador, J. Dannenberg, S. Dapprich, A. Daniels, O. Farkas, J. Foresman, J. Ortiz, J. Cioslowski and D. Fox, *Gaussian ~09 Revision D.01*, January 1, 2014.
- 75 R. J. Ouellette and J. D. Rawn, Nucleophilic Substitution and Elimination Reactions, in *Principles of Organic Chemistry*, Elsevier, 2015, pp. 189–208, DOI: 10.1016/b978-0-12-802444-7.00007-0.
- 76 T. H. Dunning, Gaussian Basis Sets for Use in Correlated Molecular Calculations. I. The Atoms Boron through Neon and Hydrogen, *J. Chem. Phys.*, 1989, **90**(2), 1007–1023, DOI: 10.1063/1.456153.
- 77 M. Head-Gordon, J. A. Pople and M. J. Frisch, MP2 Energy Evaluation by Direct Methods, *Chem. Phys. Lett.*, 1988, **153**(6), 503–506, DOI: 10.1016/0009-2614(88)85250-3.
- 78 C. R. Harris, K. J. Millman, S. J. van der Walt, R. Gommers, P. Virtanen, D. Cournapeau, E. Wieser, J. Taylor, S. Berg, N. J. Smith, R. Kern, M. Picus, S. Hoyer, M. H. van Kerkwijk, M. Brett, A. Haldane, J. F. del Río, M. Wiebe, P. Peterson, P. Gérard-Marchant, K. Sheppard, T. Reddy, W. Weckesser, H. Abbasi, C. Gohlke and T. E. Oliphant, Array Programming with NumPy, *Nature*, 2020, 357–362, DOI: 10.1038/s41586-020-2649-2.
- 79 T. A. Proffitt and J. K. Pearson, A Shared-Weight Neural Network Architecture for Predicting Molecular Properties, *Phys. Chem. Chem. Phys.*, 2019, **21**(47), 26175–26183, DOI: 10.1039/c9cp03103k.
- 80 M. Abadi, A. Agarwal, P. Barham, E. Brevdo, Z. Chen, C. Citro, G. S. Corrado, A. Davis, J. Dean, M. Devin, S. Ghemawat, I. Goodfellow, A. Harp, G. Irving, M. Isard, Y. Jia, R. Jozefowicz, L. Kaiser, M. Kudlur, J. Levenberg, D. Mané, R. Monga, S. Moore, D. Murray, C. Olah, M. Schuster, J. Shlens, B. Steiner, I. Sutskever, K. Talwar, P. Tucker, V. Vanhoucke, V. Vasudevan, F. Viégas, O. Vinyals, P. Warden, M. Wattenberg, M. Wicke, Y. Yu and X. Zheng, *Google Research, TensorFlow: Large-Scale Machine Learning on Heterogeneous Distributed Systems*, 2015.
- 81 F. Chollet, others, *Keras*, GitHub, 2015.
- 82 D. P. Kingma and J. L. Ba, Adam: A Method for Stochastic Optimization, in *3rd International Conference on Learning Representations, ICLR 2015 – Conference Track Proceedings; International Conference on Learning Representations*, ICLR, 2015.
- 83 C. Møller and M. S. Plesset, Note on an Approximation Treatment for Many-Electron Systems, *Phys. Rev.*, 1934, **46**(7), 618–622, DOI: 10.1103/PhysRev.46.618.
- 84 K. He, X. Zhang, S. Ren and J. Sun, Deep Residual Learning for Image Recognition, in *Proceedings of the IEEE Computer Society Conference on Computer Vision and Pattern Recognition*, IEEE Computer Society, 2016, pp. 770–778, DOI: 10.1109/CVPR.2016.90.
- 85 E. Martínez-Núñez, An Automated Method to Find Transition States Using Chemical Dynamics Simulations, *J. Comput. Chem.*, 2015, **36**(4), 222–234, DOI: 10.1002/jcc.23790.
- 86 L. D. Jacobson, A. D. Bochevarov, M. A. Watson, T. F. Hughes, D. Rinaldo, S. Ehrlich, T. B. Steinbrecher, S. Vaitheeswaran, D. M. Philipp, M. D. Halls and R. A. Friesner, Automated Transition State Search and Its Application to Diverse Types of Organic Reactions, *J. Chem. Theory Comput.*, 2017, **13**(11), 5780–5797, DOI: 10.1021/acs.jctc.7b00764.
- 87 E. L. Kolsbjerg, M. N. Groves and B. Hammer, An Automated Nudged Elastic Band Method, *J. Chem. Phys.*, 2016, **145**(9), 094107, DOI: 10.1063/1.4961868.

