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Molecular light-upconversion: we have had a problem! When excited state absorption (ESA) overcomes energy transfer upconversion (ETU) in Cr(III)/Er(III) complexes†

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Nine-coordinate [ErN₉] or [ErN₃O₆] chromophores found in triple helical [Er(L)₃]³⁺ complexes (L corresponds to 2,2',6',2''-terpyridine (tpy), 2,6-(bisbenzimidazol-2-yl)pyridine (bzimpy), 2,6-diethylcarboxypyridine (dpa-ester) or 2,6-diethylcarboxamidopyridine (dpa-diamide) derivatives), [Er(dpa)₃]³⁻ (dpa is the 2,6-dipicolinate dianion) and [GaErGa(bpb-bzimpy)₃]⁹⁺ (bpb-bzimpy is 2,6-bis((pyridin-2-benzimidazol-5-yl)methyl-(benzimidazol-2-yl))pyridine) exhibit NIR (excitation at 801 nm) into visible (emission at 542 nm) linear light upconversion processes in acetonitrile at room temperature. The associated quantum yields $5.5(6) \times 10^{-11} \leq \phi_{\text{tot}}^{\text{up}}(\text{ESA}) \leq 1.7(2) \times 10^{-9}$ appear to be 1–3 orders of magnitude larger than those predicted by the accepted single-center excited-state absorption mechanism (ESA). Switching to the alternative energy transfer upconversion mechanism (ETU), which operates in multi-centers [CrErCr(bpb-bzimpy)₃]⁹⁺, leads to an improved quantum yield of $\phi_{\text{tot}}^{\text{up}}(\text{ETU}) = 5.8(6) \times 10^{-8}$, but also to an even larger discrepancy by 4–6 orders of magnitude when compared with theoretical models. All photo-physical studies point to Er(⁴I_{13/2}) as being the only available 'long-lived' ($1.8 \leq \tau \leq 6.3 \mu\text{s}$) and emissive excited state, which works as an intermediate relay for absorbing the second photon, but with an unexpected large cross-section for an intrashell 4f → 4f electronic transition. With this in mind, the ETU mechanism, thought to optimize upconversion via intermetallic Cr → Er communication in [CrErCr(bpb-bzimpy)₃]⁹⁺, is indeed not crucial and the boosted associated upconversion quantum yield is indebted to the dominant contribution of the single-center erbium ESA process. This curious phenomenon is responsible for the successful implementation of light upconversion in molecular coordination complexes under reasonable light power intensities, which paves the way for applications in medicine and biology. Its origin could be linked with the presence of metal–ligand bonding.

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Introduction

Light upconversion represents a rather counter-intuitive energetic process, which was theoretically predicted in 1931 by

Goeppert-Meyer¹ when considering the non-linear dependence of the refractive index on light intensity (Kerr effect).² Its experimental demonstration was delayed until the early sixties when sufficiently intense laser excitation beams became available for inducing second harmonic generation (SHG, a second-order non-linear optical (NLO) process)³ and two-photon absorption (TPA, a third-order NLO process).⁴ However, even for optimized polarized materials,⁵ these non-linear responses are so weak that NLO upconversion was found to be mainly useful for multiplying the frequency of intense laser beams. Consequently, NLO seems poorly adapted for the preparation of solar cell concentrators⁶ or for the design of upconverters able to transform deep penetrating low power near-infrared (NIR) beams into visible radiations of higher energy for biological or medical applications.⁷ The parallel discovery that light upconversion, relying strictly on suc-

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cessive linear optical response, is 5–8 orders of magnitude more efficient than NLO processes⁸ opened wide perspectives for technological applications based on (i) metal-based upconversion implemented in low-phonon ionic solids⁹ and (ii) triplet–triplet annihilation processes induced by the collision of two excited polyaromatic units.¹⁰ The common concept for linear light upconversion exploits a first efficient photonic excitation in order to reach long-lived intermediate excited states for energy storage prior to undergoing a second excitation (*via* photonic absorption or *via* collision), which gives finally access to an emissive excited state of higher energy. Focusing on metal-based upconversion, the second excitation process corresponds to the absorption of an additional photon with a non-negligible probability compared to the relaxation rate of the intermediate relay, a phenomenon referred to as excited state absorption (ESA).^{9b} The scale of regularly spaced multiplets found for trivalent open-shell lanthanides (Ln^{3+} with electronic configurations $[\text{Xe}]4f^n$, $n = 1\text{--}13$), and rationalized by the Russel–Saunders coupling scheme,¹¹ offers a privileged access for (linear) upconversion operating within a single molecular unit as long as the lifetime of the intermediate excited state (level |1> in Fig. 1b) is long enough for being compatible with a reasonable competition between the absorption of a second photon $k_A^{\text{exc}(1\rightarrow 2)}$ to reach the doubly excited state (level |2> in Fig. 1b) and the relaxation $k_A^{1\rightarrow 0}$ to the ground state (level |0> in Fig. 1b).^{12,13}

The low-energy phonons (a few tens of cm^{-1}) available in ionic oxides and fluorides are poorly efficient for inducing non-radiative relaxation between the spectroscopic levels of lanthanide cations (separated by several hundreds/thousands of cm^{-1}),^{9a–c} which makes these ionic solids ideal hosts for welcoming Ln^{3+} as dopants with the ultimate goal of inducing efficient (linear) upconversion processes in the solid state (maximum reported quantum yields about $\phi_{\text{tot}}^{\text{up}} = 9\text{--}12\%$).¹⁴

Fig. 1 (a) Molecular structure of $[\text{GaErGa}(\text{bpb-bzimpy})_3]^{9+}$ (ref. 12) and (b) associated kinetic scheme depicting the modelling of the one-ion excited state absorption (ESA) process occurring upon off-resonance irradiation into the activator-centered absorption band ($A = \text{Er}$) where $k_A^{\text{exc}(i\rightarrow j)}$ correspond to the excitation rate constants (eqn (1)) and $k_A^{i\rightarrow j}$ stand for the global decay rate constant of level i into level j .¹³ The pertinent kinetic matrix is given in Scheme S1a.†

The recurrent need for miniaturizing within the frame of biological applications resulted in an intense scientific activity, which aimed at transforming Ln-doped upconverting ionic solids into nanoparticles.^{9d–h} The unfavorable quenching due to the increase of the surface/volume ratio in the latter entities¹⁵ can be partially compensated (i) by coupling with plasmonic surfaces^{9f,k} and/or (ii) by statistically introducing some efficient light-sensitizers¹⁶ compatible with the operation of the more efficient energy transfer upconversion (ETU) mechanism (Fig. 3b).^{9b,17} In this context, the design of molecular lanthanide coordination complexes for upconversion was attempted at the turn of the century by Reinhard and Güdel with a detailed photophysical investigation of $\text{Na}_3[\text{Ln}(\text{dpa})_3]\cdot 13\text{H}_2\text{O}$ ($\text{Ln} = \text{Er}, \text{Tm}, \text{Yb}$; dpa = pyridine-2,6-dicarboxylate, see Fig. 5a).¹⁸ They concluded that the high-energy vibrations, characteristic for molecular objects, lead to intermediate metal-centered excited states with nano/microsecond lifetimes (instead of millisecond in ionic solids), which are not compatible with the detection of upconverted signals in these molecules.¹⁸ Synthetic chemists, probably unaware of this major physical deadlock, were nonetheless able to overcome this limitation, firstly with the preparation of multi-components supramolecular assemblies exhibiting light-upconversion assigned to the ETU mechanism (Fig. 3),^{12,19,20} secondly *via* the closely related cooperative upconversion (CU) mechanism²¹ and finally according to the basic excited state absorption pathway (ESA, Fig. 1).²² Reminiscent to the original analysis reported by Reinhard and Güdel,¹⁸ the modeling of the quantum yield for the ESA mechanism ($\phi_{\text{tot}}^{\text{up}}$ in Fig. 2a) using

$$\phi_{\text{tot}}^{\text{up}} = \frac{\text{emitted photons / time unit}}{\text{absorbed photons / time unit}} = \frac{k_{A,\text{rad}}^{2\rightarrow 0} \cdot N_A^{[2],\text{S-S}}}{k_A^{\text{exc}(0\rightarrow 1)} \cdot N_A^{[0],\text{S-S}} + k_A^{\text{exc}(1\rightarrow 2)} \cdot N_A^{[1],\text{S-S}}}$$

$$\xrightarrow[\text{Scheme S1a}]{\text{Kinetic matrix}} \phi_{\text{tot}}^{\text{up}} = \left(\frac{k_A^{\text{exc}(1\rightarrow 2)}}{k_A^{\text{exc}(1\rightarrow 2)} + k_A^{1\rightarrow 0}} \right) \left(\frac{k_{A,\text{rad}}^{2\rightarrow 0}}{k_{A,\text{rad}}^{2\rightarrow 0} + k_A^{2\rightarrow 1}} \right) = \eta_{\text{ESA}} \phi_A$$

Fig. 2 (a) Definition and modeling of the global upconversion quantum yield ($\phi_{\text{tot}}^{\text{up}}$) obtained under steady-state (S-S) excitation for the ESA mechanism depicted in Fig. 1b. $k_{A,\text{rad}}^{2\rightarrow 0}$ corresponds to the radiative decay constant, η_{ESA} represents the efficiency of the ESA mechanism and ϕ_A stands for the activator-based intrinsic quantum yield. (b) Simulation of the upconversion quantum yield ($\phi_{\text{tot}}^{\text{up}}$) upon increasing incident pump intensity for the standard erbium activator found in $[\text{GaErGa}(\text{bpb-bzimpy})_3]^{9+}$ at room temperature (Fig. 1a). Excitation fixed at $\lambda_p = 801 \text{ nm}$ ($\text{Er}(^4\text{I}_{9/2} \leftarrow ^4\text{I}_{15/2})$, absorption cross-section $\sigma_A^{0\rightarrow 1} = 6.2 \times 10^{-22} \text{ cm}^2$ ($\epsilon_{801} = 0.163 \text{ M}^{-1} \text{ cm}^{-1}$), $k_A^{1\rightarrow 0} = \left(\tau_{\text{Er}}^{4\text{I}_{13/2}} \right)^{-1} = (4.50 \mu\text{s})^{-1}$, $k_A^{2\rightarrow 0} = \left(\tau_{\text{Er}}^{4\text{S}_{3/2}} \right)^{-1} = (1.6 \text{ ms})^{-1}$, $k_A^{2\rightarrow 0} + k_A^{2\rightarrow 1} = \left(\tau_{\text{Er}}^{4\text{S}_{3/2}} \right)^{-1} = (40 \text{ ns})^{-1}$.¹² $\sigma_A^{1\rightarrow 2} = \sigma_A^{0\rightarrow 1} = 6.2 \times 10^{-22} \text{ cm}^2$ ($\epsilon_{801} = 0.163 \text{ M}^{-1} \text{ cm}^{-1}$), $\sigma_A^{1\rightarrow 2} = \sigma_A^{0\rightarrow 1}$ is arbitrarily (but reasonably) fixed for the simulation.

Fig. 3 (a) Molecular structure of [CrErCr(bpb-bzimpy)₃]⁹⁺ (ref. 19) and (b) associated kinetic scheme depicting the modelling of the sensitizer/activator energy transfer upconversion (ETU) process occurring upon off-resonance irradiation into the sensitizer-centered absorption band in a SAS system (S = Cr, A = Er) where $k_S^{\text{exc}(0 \rightarrow 1)}$ corresponds to the sensitized-based excitation rate constant (eqn (1)), $k_S^{i \rightarrow j}$ and $k_A^{i \rightarrow j}$ stand for the sensitizer-based, respectively activator-based global decay rate constants of level i into level j . $W_{S \rightarrow A}^i$ correspond to the first-order sensitizer-to-activator energy transfer (ET) rate constants.¹³ The pertinent kinetic matrix is given in Scheme S1b.[†]

Fig. 4 (a) Definition and modeling of the global upconversion quantum yield ($\phi_{\text{tot}}^{\text{up}}$) obtained under steady-state (S-S) excitation for the ETU mechanism depicted in Fig. 3b. η_{ETU} represents the efficiency of the ETU mechanism and ϕ_A stands for the activator-based intrinsic quantum yield. (b) Simulation of the upconversion quantum yield ($\phi_{\text{tot}}^{\text{up}}$) upon increasing incident pump intensity simulated for the erbium activator found in [CrErCr(bpb-bzimpy)₃]⁹⁺ (Fig. 3a). Excitation fixed at $\lambda_p = 718 \text{ nm}$ $\text{Cr}({}^2\text{T}_1 \leftarrow {}^4\text{A}_2)$, absorption cross-section $\sigma_S^{0 \rightarrow 1} = 3.84 \times 10^{-22} \text{ cm}^2$ ($\epsilon_{718} = 0.101 \text{ M}^{-1} \text{ cm}^{-1}$), $k_S^{1 \rightarrow 0} = (\tau_{\text{CrYCr}}^{4\text{I}_{3/2}})^{-1} = (296 \mu\text{s})^{-1}$ at 293 K and $(2.81 \text{ ms})^{-1}$ at 150 K, $k_A^{1 \rightarrow 0} = (\tau_{\text{GaErGa}}^{4\text{I}_{3/2}})^{-1} = (4.50 \mu\text{s})^{-1}$ at 293 K and $(4.30 \mu\text{s})^{-1}$ at 150 K, $k_A^{2 \rightarrow 0} = (\tau_{\text{GaErGa}}^{4\text{S}_{3/2}})^{-1} = (1.6 \text{ ms})^{-1}$, $k_A^{2 \rightarrow 0} + k_A^{2 \rightarrow 1} = (\tau_{\text{GaErGa}}^{4\text{S}_{3/2}})^{-1} = (40 \text{ ns})^{-1}$ at 3 K, $W_{S \rightarrow A}^1 = 232 \text{ s}^{-1}$ at 293 K and 169 s^{-1} at 150 K.¹² $W_{S \rightarrow A}^2 = 1000 \text{ s}^{-1}$ is arbitrarily (but reasonably) fixed for the simulation.

standard experimental values for the different relaxation rate constants in a molecular Er^{3+} complex, as those found in [GaErGa(bpb-bzimpy)₃]⁹⁺ (Fig. 1),¹² indeed predicts faint quantum yields $10^{-13} \leq \phi_{\text{tot}}^{\text{up}} \leq 10^{-11}$ (Fig. 2b) under reasonable excitation power intensities $1 \leq P \leq 30 \text{ W cm}^{-2}$ (Fig. 2b, the excitation rate constants $k_A^{\text{exc}(i \rightarrow j)}$ is obtained with eqn (1), where λ_p is the pump wavelength, P is the incident pump intensity (in W cm^{-2}), $\sigma_A^{i \rightarrow j}$ is the absorption cross section of the activator-centered $i \rightarrow j$ transition (in cm^2) related to the

decadic molar absorption coefficient $\epsilon^{i \rightarrow j}$ (in $\text{M}^{-1} \text{ cm}^{-1}$) according to $\sigma^{i \rightarrow j} = 3.8 \times 10^{-21} \epsilon^{i \rightarrow j} \frac{2.3}{h}$, h is the Planck constant and c is the speed of light in vacuum).²⁴

$$k_A^{\text{exc}(i \rightarrow j)} = \frac{\lambda_p}{hc} P \sigma_A^{i \rightarrow j} \quad (1)$$

With these predictions in mind, only massive excitation intensities could give the lie to Reinhard and Güdel and the detection of a faint, but measurable green $\text{Er}({}^4\text{S}_{3/2} \rightarrow {}^4\text{I}_{15/2})$

Fig. 5 Synthesis and molecular structures of the triple-helical complexes possessing (a) $[\text{ErN}_3\text{O}_6]$ and (b) $[\text{ErN}_9]$ chromophores considered in this work. The molecular structures are taken from the crystal structures of $(\text{NH}_4)_3[\text{Er}(\text{dpa})_3](\text{CF}_3\text{SO}_3)_2$ (1), $[\text{Er}(\text{dpa-ester})_3](\text{ClO}_4)_3$ (2), $[\text{Er}(\text{dpa-diamide})_3](\text{ClO}_4)_3$ (3), $[\text{Er}(\text{Et-bzimpy})_3](\text{ClO}_4)_3 \cdot 2\text{CH}_3\text{CN}$ (4),²⁸ $[\text{Er}(\text{tpy})_3](\text{ClO}_4)_3$ (5),²⁸ $[\text{Er}(\text{Et-tpy})_3](\text{ClO}_4)_3 \cdot 1.5\text{CH}_3\text{CN}$ ²⁸ and $[\text{GaErGa}(\text{bpb-bzimpy})_3]_2(\text{CF}_3\text{SO}_3)_{18} \cdot 30\text{C}_3\text{H}_5\text{N}$.¹² Green sphere = Er(III), greyish-red sphere = Ga(III).

upconverted signals (545 nm) from a 0.02 M solution of $[\text{Er}(\text{dpa})_3]^{3-}$ in D_2O indeed required 10^9 W cm^{-2} ($= 1 \text{ GW cm}^{-2}$) near-infrared (800–980 nm) laser excitation.²⁵ Similarly, Sorensen and Faulkner had to focus a high-power OPO tunable NIR femtosecond laser onto simple Tm^{3+} solvates in DMSO for inducing some weak visible luminescence, which could be unambiguously assigned to second and third-order NLO responses whereas linear upconversion based on linear optics only negligibly contributed to the visible luminescence.²⁶ Surprisingly, the few preliminary quantum yields determined experimentally for ESA occurring in mono-nuclear molecular erbium complexes with $[\text{ErN}_9]$ chromophores in solution lie in the $10^{-9} \leq \phi_{\text{tot}}^{\text{up}} \leq 10^{-8}$ range (measured for a fixed incident excitation power around 21 W cm^{-2})²² and appear to be 2–3 orders of magnitude larger than those predicted in Fig. 2b with the help of the accepted ESA mechanism.

The situation becomes even more critical when one considers that Charbonnière reported $\phi_{\text{tot}}^{\text{up}} = 1.4 \times 10^{-8}$ (at $P = 10.3$

W cm^{-2}) for a $[\text{Tb}(\text{YbL})_2]$ assembly dissolved in deuterated water,^{21a,b} and recently $\phi_{\text{tot}}^{\text{up}} = 10^{-7}$ (at $P = 2.9 \text{ W cm}^{-2}$) for a nonanuclear Yb_8Tb cluster,^{21c} in which only a poorly efficient cooperative energy (CU) transfer mechanism^{9b} may explain the feeding of the high-energy emissive $\text{Tb}({}^5\text{D}_4)$ level. A simulation of the steady-state quantum yields expected for the ETU mechanism pertinent to upconversion implemented in $[\text{CrErCr}(\text{bpb-bzimpy})_3]^{9+}$ (Cr = sensitizer, Er = activator, Fig. 4a) indeed results in negligible upconversion quantum yields at room temperature ($10^{-15} \leq \phi_{\text{tot}}^{\text{up}} \leq 10^{-14}$, red trace in Fig. 4b), which are improved at 150 K ($10^{-14} \leq \phi_{\text{tot}}^{\text{up}} \leq 10^{-12}$, blue trace in Fig. 4b) because the lifetime of the intermediate excited state of the chromium sensitizer increases by one order of magnitude. Again, the predicted quantum yields are much smaller (4–6 orders of magnitude) than the few pertinent experimental data reported for the less efficient CU mechanism.

Paraphrasing astronaut Jim Lovell, who confirmed the discovery of the explosion that severely damaged the Apollo 13

spacecraft by saying “Ah, Houston, we have had a problem”, we report here our efforts for recording reliable and accurate experimental quantum yields for the ESA mechanism operating in a series of triple-helical $[\text{Er}(\text{L})_3]$ complexes possessing nine-coordinate $[\text{ErN}_6\text{O}_3]$ (Fig. 5a) and $[\text{ErN}_9]$ chromophores (Fig. 5b) with tunable crystal fields and variable protections of the erbium activator. A thorough exploration of the origin of the discrepancy between modelling and experiments is described together with some cures compatible with a pertinent rationalization of single-site ESA, but also multi-centered ETU and CU upconversion mechanisms operating in multimetallic molecules and metallosupramolecular assemblies.

Results and discussion

Synthesis, molecular structures, crystal field parameters and ‘phonon bath’ in triple-helical erbium complexes

According to (i) the considerable cumulative thermodynamic stability constants measured for the formation of triple-helical $[\text{Er}(\text{dpa})_3]^{3-}$ in water ($\log(\beta_{1,3}^{\text{Er,L}}) = 22.13$)²⁷ and for $[\text{Er}(\text{L})_3]^{3+}$ in acetonitrile ($\text{L} = \text{dpa-ester}$ with $\log(\beta_{1,3}^{\text{Er,L}}) = 17.3$,²⁹ $\text{L} = \text{dpa-amide}$ with $\log(\beta_{1,3}^{\text{Er,L}}) \approx \log(\beta_{1,3}^{\text{Y,L}}) = 22.7$,³⁰ $\text{L} = \text{tpy}$ with $\log(\beta_{1,3}^{\text{Er,L}}) = 22.5$,²⁸ $\text{L} = \text{Et-tpy}$ with $\log(\beta_{1,3}^{\text{Er,L}}) = 21.8$,²⁸ $\text{L} = \text{Et-bzimpy}$ with $\log(\beta_{1,3}^{\text{Er,L}}) = 26$,²⁸ Fig. 5) and (ii) the extreme kinetic inertness of $[\text{GaErGa}(\text{bpb-bzimpy})_3]^{9+}$,³¹ we conclude that all these complexes (Fig. S1a†), except $[\text{Er}(\text{dpa-diester})_3]^{3+}$ (Fig. S1b†), are quantitatively formed in solution (>99%) for $|\text{Er}|_{\text{tot}}/|\text{L}|_{\text{tot}} = 1:3$ and total ligand concentrations of 3–10 mM, the specific conditions that are used for recording the photophysical studies. For the less stable triple-helical $[\text{Er}(\text{dpa-diester})_3]^{3+}$ complex, its speciation corresponds to more than 80% of the total ligand content in the same conditions (Fig. S1b†). These complexes can be also isolated in the solid state and the crystal structures of those containing $[\text{ErN}_9]$ chromophores have been previously solved by X-ray diffraction (Fig. 5b).^{12,28} For $[\text{ErN}_3\text{O}_6]$ units, X-ray quality crystals of $(\text{NHET}_3)_5[\text{Er}(\text{dpa})_3](\text{CF}_3\text{SO}_3)_2$ (**1**), $[\text{Er}(\text{dpa-ester})_3](\text{ClO}_4)_3$ (**2**), $[\text{Er}(\text{dpa-amide})_3](\text{ClO}_4)_3$ (**3**) could be obtained by slow diffusion of diethylether into concentrated butyronitrile solutions (Fig. 5a and Fig. S2–S4, Tables S1–S7†).

As expected, the molecular structure of the triple-helical $[\text{Er}(\text{dpa})_3]^{3-}$ anion exactly mirrors those reported for $\text{Na}_3[\text{Ln}(\text{dpa})_3] \cdot 13\text{H}_2\text{O}$ ^{18,32} and for $(\text{imidazol-H})_3[\text{Ln}(\text{dpa})_3] \cdot 3\text{H}_2\text{O}$,³³ but the crystals of $(\text{NHET}_3)_5[\text{Er}(\text{dpa})_3](\text{CF}_3\text{SO}_3)_2$ are soluble in acetonitrile with no sign of significant water content. The $[\text{Er}(\text{dpa-amide})_3]^{3+}$ building block found in **3** is almost superimposable with that previously reported for $[\text{La}(\text{dpa-amide})_3](\text{ClO}_4)_3 \cdot 2.5\text{C}_2\text{H}_5\text{CN}$ ³⁰ while $[\text{Er}(\text{dpa-ester})_3]^{3+}$, to the best of our knowledge, is the first reported crystal structure along the 2,6-diesterpyridine series. All nine-coordinate $\text{Er}(\text{III})$ centers adopt slightly distorted tricapped trigonal prismatic geometries (SHAPE’s factors $1.59 \leq S \leq 3.30$, Table S8†)³⁴ with the pyridine nitrogen atom of each wrapped ligand occupying a

capping position in the final polyhedra (Fig. S5†). The Er–N and Er–O bond lengths are poorly dispersed (Table S8†) and correspond to those expected for triple-helical $[\text{ErN}_9]$ and $[\text{ErN}_3\text{O}_6]$ complexes with tridentate ligands.^{12,19,20b,28} Given that the existence of ‘long’ Er-centered excited state lifetimes in molecular complexes, which are critical for implementing linear upconversion, requires (i) a minimum splitting of the J manifolds produced by the crystal field effect and (ii) a global lack of energy matching between the high-energy oscillators of the ligands and the average energy gap between the successive $^{2S+1}L_J$ spectroscopic levels,^{35,36} the crystal field parameters (Table S9†) and associated energy splitting of the J manifolds in complexes **1–5** have been described by SO-CASSCF and SO-CASPT2 calculations (Fig. S6†).³⁷ Interestingly, the computed global crystal field strengths S^{38} are larger for $[\text{ErN}_3\text{O}_6]$ units ($S(\text{Er}(\text{dpa})_3) = 217(16) \text{ cm}^{-1} > S(\text{Er}(\text{dpa-amide})_3) = 186 \text{ cm}^{-1} > S(\text{Er}(\text{dpa-ester})_3) = 171 \text{ cm}^{-1}$) than for $[\text{ErN}_9]$ chromophores ($S(\text{Er}(\text{Et-bzimpy})_3) = 157 \text{ cm}^{-1} \approx S(\text{Er}(\text{tpy})_3) = 155 \text{ cm}^{-1}$, Table S9†). Consequently the total splitting of the Er ($^{2S+1}L_J$) manifolds is broader when tridentate NO_2 ligands are bound to $\text{Er}(\text{III})$ (Tables S10–S14†), thus offering more probabilities for non-radiative relaxation induced by high-energy vibrations (and shorter intermediate excited lifetimes) in $[\text{ErN}_3\text{O}_6]$ units than in $[\text{ErN}_9]$ analogues. Although less pertinent for optical³⁹ than for magnetic properties,⁴⁰ the main difference between $[\text{ErN}_3\text{O}_6]$ and $[\text{ErN}_9]$ chromophores lies in the sign of B_0^2 which is negative for $[\text{ErN}_9]$ (oblate arrangement of the donor atoms with the principal magnetic axis parallel to the pseudo-threefold Z axis) and positive for $[\text{ErN}_3\text{O}_6]$ (prolate arrangement of the donor atoms with the principal magnetic axis perpendicular to the Z axis).³³ Finally, according to Reinhard and Güdel,¹⁸ the weighted average of the vibrations participating in the nonradiative relaxation process (= ‘phonon bath’) in molecular $[\text{Ln}(\text{dpa})_3]^{3-}$ can be set to $h\nu_{\text{eff}} \approx 2000 \text{ cm}^{-1}$, an approximation which can be extended for complexes **1–5** according to the vibrational IR spectra (Fig. S7†).

Molecular light-upconversion operating in single-center triple-helical erbium complexes

The NIR-Visible absorption spectra of triple-helical $[\text{GaErGa}(\text{bpb-bzimpy})_3]^{9+}$ (Fig. 6a and b), $[\text{Er}(\text{L})_3]^{3+}$ ($\text{L} = \text{Et-bzimpy}$, Et-tpy, tpy, dpa-amide, dpa-ester) and $[\text{Er}(\text{dpa})_3]^{3-}$ in acetonitrile (Fig. S8 and S9†) are all similar and display weak metal-centered $\text{Er}(^{2S+1}L_J \leftarrow ^4I_{15/2})$ transitions ($0.1 \leq \epsilon_{\text{max}} \leq 5 \text{ M}^{-1} \text{ cm}^{-1}$) characteristic of the well-known energy diagram depicted in Fig. 6c.^{11,12} The radiative rate constant $k_{\text{rad}}^{J'-J}$, and related radiative lifetime $\tau_r^{J'-J} = 1/k_{\text{rad}}^{J'-J}$, associated with the emission between each excited $\text{Er}(^{2S+1}L_J)$ level and the ground $\text{Er}(^4I_{15/2})$ level can be calculated from the absorption spectrum $\epsilon(\tilde{\nu})$ (in $\text{M}^{-1} \text{ cm}^{-1}$) using eqn (2), where $\int \epsilon(\tilde{\nu}) d\tilde{\nu}$ is the integrated spectrum of the incriminated absorption transition recorded in solution, J and J' refer to the ground ($J = 15/2$) and excited states, respectively, n is the refractive index of the medium, N_A is Avogadro’s number (in mol^{-1}), c is the speed of light in

Table 1 Radiative lifetimes (τ_{rad}) for the $\text{Er}({}^4\text{S}_{3/2} \rightarrow {}^4\text{I}_{15/2})$ and $\text{Er}({}^4\text{I}_{13/2} \rightarrow {}^4\text{I}_{15/2})$ transitions computed with eqn (2) and (3), experimental excited lifetimes (τ_{tot}) for the $\text{Er}({}^4\text{S}_{3/2})$ and $\text{Er}({}^4\text{I}_{13/2})$ levels and associated intrinsic quantum yields ($\phi_{\text{Er}} = \tau_{\text{tot}}/\tau_{\text{rad}}$) in $[\text{GaErGa}(\text{bpb-bzimpy})_3]^{9+}$, $[\text{Er}(\text{L})_3]^{3+}$ ($\text{L} = \text{Et-bzimpy}$, Et-tpy , tpy , dpa-amide , dpa-ester) and $[\text{Er}(\text{dpa})_3]^{3-}$ at 298 K

Complexes	$\lambda_{\text{exc}}/\text{nm}$ Level	$\text{Er}({}^4\text{S}_{3/2})$ $\tau_{\text{rad}}/\text{ms}$	405 $\text{Er}({}^4\text{S}_{3/2})$ $\tau_{\text{tot}}/\text{ns}$	405 $\text{Er}({}^4\text{S}_{3/2})$ ϕ_{Er}	$\text{Er}({}^4\text{I}_{13/2})$ $\tau_{\text{rad}}/\text{ms}$	355 $\text{Er}({}^4\text{I}_{13/2})$ $\tau_{\text{tot}}/\mu\text{s}$	355 $\text{Er}({}^4\text{I}_{13/2})$ ϕ_{Er}	805 $\text{Er}({}^4\text{I}_{13/2})$ $\tau_{\text{tot}}/\mu\text{s}$	975 $\text{Er}({}^4\text{I}_{13/2})$ $\tau_{\text{tot}}/\mu\text{s}$
$[\text{Er}(\text{Et-bzimpy})_3]^{3+}$	Solid	—	^b	—	—	5.57(6)	—	4.786(7)	—
	Solution ^a	1.31(9)	—	$3.0(3) \times 10^{-5}$ ^d	7.12(5)	—	$7.8(1) \times 10^{-4}$	6.299(5)	5.8(2)
$[\text{GaErGa}(\text{bpb-bzimpy})_3]^{9+}$	Solid	—	40(2) ^c	—	—	4.04(4)	—	3.955(7)	—
	Solution	1.6(1)	—	$2.5(2) \times 10^{-5}$ ^d	9.4(5)	—	$4.3(2) \times 10^{-4}$	5.109(5)	4.8(1)
$[\text{Er}(\text{tpy})_3]^{3+}$	Solid	—	^b	—	—	1.88(2)	—	2.092(3)	—
	Solution	0.75(5)	—	$5.3(4) \times 10^{-5}$ ^d	8.1(6)	—	$2.3(2) \times 10^{-4}$	2.005(1)	1.9(1)
$[\text{Er}(\text{Et-tpy})_3]^{3+}$	Solid	—	^b	—	—	1.94(2)	—	2.309(3)	—
	Solution	0.38(3)	—	$1.0(1) \times 10^{-4}$ ^d	7.01(5)	—	$2.77(4) \times 10^{-4}$	2.250(1)	2.16(3)
$[\text{Er}(\text{dpa})_3]^{3-}$	Solid	—	^b	—	—	2.217(1)	—	1.772(2)	—
	Solution	0.98(7)	—	$4.1(4) \times 10^{-5}$ ^d	6.9(5)	—	$3.2(2) \times 10^{-4}$	2.450(1)	2.39(6)
$[\text{Er}(\text{dpa-ester})_3]^{3+}$	Solid	—	^b	—	—	3.270(3)	—	2.78(4)	—
	Solution	1.01(5)	—	$4.0(3) \times 10^{-5}$ ^d	9.2(6)	—	$3.6(2) \times 10^{-4}$	3.919(2)	3.2(1)
$[\text{Er}(\text{dpa-amide})_3]^{3+}$	Solid	—	^b	—	—	3.067(1)	—	3.118(2)	—
	Solution	0.81(6)	—	$4.9(4) \times 10^{-5}$ ^d	7.4(5)	—	$4.1(3) \times 10^{-4}$	3.441(3)	3.03(9)

^a In acetonitrile. ^b Too weak to be measured. ^{20b} ^c Recorded at 3–10 K. ^{20b} ^d Computed by using $\tau_{\text{tot}}^{4\text{S}_{3/2}} = 40(2)$ ns.

vacuum (in cm s^{-1}) and $\tilde{\nu}_{\text{m}}$ is the barycenter of the transition (in cm^{-1}) given in eqn (3).⁴¹

$$k_{\text{rad}}^{J'-J} = \frac{1}{\tau_{\text{rad}}^{J'-J}} = 2303 \times \frac{8\pi c n^2 \tilde{\nu}_{\text{m}}^2 (2J+1)}{N_{\text{A}} (2J'+1)} \int \epsilon(\tilde{\nu}) d\tilde{\nu} \quad (2)$$

$$\tilde{\nu}_{\text{m}} = \frac{\int \tilde{\nu} \cdot \epsilon(\tilde{\nu}) d\tilde{\nu}}{\int \epsilon(\tilde{\nu}) d\tilde{\nu}} \quad (3)$$

The experimental $\tau_{\text{rad}}^{J'-J=15/2}$ extracted for the $\text{Er}({}^{2\text{S}+1}\text{L}_J)$ excited levels located within the 6000–20 000 cm^{-1} domain cover the 1–20 ms range in agreement with the symmetry-forbidden character of the intrashell (f-f) electric dipole transitions (Fig. 6a and c and S8† and Tables 1 and S15†). As expected from the dependence of the Einstein coefficient for spontaneous emission with $\tilde{\nu}_{\text{m}}$,^{23a,41} the global radiative lifetimes decrease with increasing energy gaps. The allowed ligand-centered $\pi^* \leftarrow n, \pi$ absorption bands ($2 \times 10^4 \leq \epsilon \leq 20 \times 10^4 \text{ M}^{-1} \text{ cm}^{-1}$) cover the UV part of the absorption spectra (24 000–40 000 cm^{-1})^{12,28} and mask the Er-centered transitions expected to occur in this domain. In this context, the low-energy tail of the latter ligand-based absorption can be easily detected in the visible part of the absorption spectra recorded for $[\text{GaErGa}(\text{bpb-bzimpy})_3]^{9+}$ (Fig. 6a) or for $[\text{Er}(\text{Et-bzimpy})_3]^{9+}$ (Fig. S8†).

Ligand-centered UV-excitation (355 to 400 nm) of $[\text{GaErGa}(\text{bpb-bzimpy})_3]^{9+}$,¹² $[\text{Er}(\text{L})_3]^{3+}$ ($\text{L} = \text{Et-bzimpy}$, Et-tpy , tpy , dpa-amide , dpa-ester)²⁸ or $[\text{Er}(\text{dpa})_3]^{3-}$ (ref. 18) sensitizes the Er(III) metal *via* the antenna effect, which provides some rare dual Er-based emissions in these molecular complexes (Fig. S10–S13†).^{28,35} The (very) weak visible band ($\lambda_{\text{em}} = 540\text{--}560 \text{ nm}$, Fig. S10 and S12†) can be assigned to the $\text{Er}({}^4\text{S}_{3/2} \rightarrow {}^4\text{I}_{15/2})$ transition, while the more intense near infrared band ($\lambda_{\text{em}} = 1500\text{--}1540 \text{ nm}$, Fig. S11 and S13†) corresponds to the common $\text{Er}({}^4\text{I}_{13/2} \rightarrow {}^4\text{I}_{15/2})$ luminescent transition.²⁸

Both transitions display linear $\log(I)\text{--}\log(P)$ plots between the emitted intensity (I) and the incident UV excitation power (P) with slopes close to one, which is diagnostic for the operation of linear light-downshifting in these complexes (Fig. S10–S13†).^{9c} Because of the only faint visible (green) Er-centered emission, the determination of experimental lifetimes for the $\text{Er}({}^4\text{S}_{3/2})$ excited level represents a real technical challenge, which could be addressed by a time-gated CCD-camera only for the ‘most intense’ emitter along the series at low temperature (3–10 K), namely $[\text{GaErGa}(\text{bpb-bzimpy})_3](\text{CF}_3\text{SO}_3)_9$ with $\tau_{\text{tot}}^{4\text{S}_{3/2}} = 40(2) \text{ ns}$; a value confirmed for its dinuclear analogue $[\text{GaEr}(\text{pb-bzimpy})_3](\text{CF}_3\text{SO}_3)_6$ with $\tau_{\text{tot}}^{4\text{S}_{3/2}} = 38(2) \text{ ns}$.^{20b} The associated intrinsic quantum yields $\phi_{\text{Er}}^{4\text{S}_{3/2}} = k_{\text{rad}}^{4\text{S}_{3/2}}/\tau_{\text{tot}}^{4\text{S}_{3/2}} = \tau_{\text{tot}}^{4\text{S}_{3/2}}/\tau_{\text{rad}}^{4\text{S}_{3/2}} = (40 \times 10^{-9}/1.6 \times 10^{-3}) = .5 \times 10^{-5}$ calculated for $[\text{GaErGa}(\text{bpb-bzimpy})_3](\text{CF}_3\text{SO}_3)_9$ can be thus taken as a valuable estimate for the maximum efficiency of $\phi_{\text{Er}}^{4\text{S}_{3/2}}$ in these complexes (Table 1, column 5). Although weak, the intensity of the near infrared $\text{Er}({}^4\text{I}_{13/2} \rightarrow {}^4\text{I}_{15/2})$ transition is compatible with standard time-gated detection techniques and systematically gives mono-exponential decay traces with $1.9 \leq \tau_{\text{tot}}^{4\text{I}_{13/2}} \leq 6 \mu\text{s}$ characteristic lifetimes (Table 1 column 7 and Fig. S14†) and $2 \times 10^{-4} \leq \phi_{\text{Er}}^{4\text{I}_{13/2}} = k_{\text{rad}}^{4\text{I}_{13/2}}/\tau_{\text{tot}}^{4\text{I}_{13/2}} = \tau_{\text{tot}}^{4\text{I}_{13/2}}/\tau_{\text{rad}}^{4\text{I}_{13/2}} \leq 8 \times 10^{-4}$ intrinsic quantum yields (Table 1, column 8). In line with the hypothesis that erbium complexes with smaller crystal field strength are less prone to undergo efficient non-radiative vibrational relaxation processes,^{35,36} the lifetimes measured for the $\text{Er}({}^4\text{I}_{13/2})$ level are maximum for $[\text{Er}(\text{Et-bzimpy})_3]^{3+}$ and $[\text{GaErGa}(\text{bpb-bzimpy})_3]^{9+}$ (Table 1, column 7 and Fig. S15†).

Upon continuous near-infrared diode laser excitation at 801 nm (12 480 cm^{-1}) into the $\text{Er}({}^4\text{I}_{9/2} \rightarrow {}^4\text{I}_{15/2})$ transition at reasonable power intensities, the $[\text{GaErGa}(\text{bpb-bzimpy})_3]^{9+}$, $[\text{Er}(\text{L})_3]^{3+}$ ($\text{L} = \text{Et-bzimpy}$, Et-tpy , dpa-amide , dpa-ester) and

Fig. 6 (a) NIR-VIS absorption spectrum of $[\text{GaErGa}(\text{bpb-bzimpy})_3]^{9+}$ (0.01 M in acetonitrile at 293 K) showing the $\text{Er}(^2\text{S}^{+1}\text{L}_J \leftarrow ^4\text{I}_{15/2})$ transitions and the associated radiative lifetimes (in ms) between parenthesis, (b) highlight of the $\text{Er}(^4\text{I}_{9/2} \leftarrow ^4\text{I}_{15/2})$ centered at 801 nm and (c) energy diagram of low-energy erbium-centered levels.

Fig. 7 (a) Upconverted visible $\text{Er}(^2\text{H}_{11/2} \rightarrow ^4\text{I}_{15/2})$ and $\text{Er}(^4\text{S}_{3/2} \rightarrow ^4\text{I}_{15/2})$ emissions observed for $[\text{GaErGa}(\text{bpb-bzimpy})_3]^{9+}$, $[\text{Er}(\text{L})_3]^{3+}$ ($\text{L} = \text{Et-bzimpy}$, Et-tpy , dpa-amide , dpa-ester) and $[\text{Er}(\text{dpa})_3]^{3-}$ recorded upon laser excitation of the $\text{Er}(^4\text{I}_{9/2} \leftarrow ^4\text{I}_{15/2})$ transition at $\lambda_{\text{exc}} = 801 \text{ nm}$ ($\tilde{\nu}_{\text{exc}} = 12\,284 \text{ cm}^{-1}$) and using incident pump intensity $P = 25 \text{ W cm}^{-2}$ in acetonitrile solution at 298 K ($c \sim 10 \text{ mM}$ with similar optical density at 801 nm, Fig. S19†). The blank (red curve) was recorded from pure acetonitrile solvent using the same incident pump intensity $P = 25 \text{ W cm}^{-2}$. (b) Associated energy diagram with the proposed kinetic mechanism.

$[\text{Er}(\text{dpa})_3]^{3-}$ complexes exhibit upconverted visible $\text{Er}(^2\text{H}_{11/2} \rightarrow ^4\text{I}_{15/2})$ and $\text{Er}(^4\text{S}_{3/2} \rightarrow ^4\text{I}_{15/2})$ emissions in the solid state (Fig. S16–S18†) and in solution (Fig. 7a). The associated $\log(I)$ – $\log(P)$ plots are linear with slopes close to 2.0, which is diag-

nostic for the operation of light-upconversion. Since all the absorption coefficients at 801 nm are comparable $0.07 \leq \epsilon_{801} \leq 0.15 \text{ M}^{-1} \text{ cm}^{-1}$ (i.e. $2.7 \times 10^{-22} \leq \sigma_{\text{Er}}(^4\text{I}_{15/2} \rightarrow ^4\text{I}_{9/2}) \leq 5.6 \times 10^{-22} \text{ cm}^2$, Fig. 6b and S19† and Table 2 column 2),²³ the upconverted

Table 2 Ground state absorption cross sections ($\sigma_{\text{Er}}^{0 \rightarrow 1}/\text{cm}^2$), rate constants ($k_{\text{Er}}^{j \rightarrow i}/\text{s}^{-1}$), upconversion quantum yields ($\phi_{\text{tot}}^{\text{up}}$, $\lambda_{\text{exc}} = 801 \text{ nm}$ and $P = 25 \text{ W cm}^{-2}$) and ESA efficiency (η_{ESA} , $\lambda_{\text{exc}} = 801 \text{ nm}$ and $P = 25 \text{ W cm}^{-2}$) for [GaErGa(bpb-bzimpy)₃]⁹⁺, [Er(L)₃]³⁺ (L = Et-bzimpy, Et-tpy, dpa-amide, dpa-ester) and [Er(dpa)₃]³⁻ in solution at 298 K. The excited state absorption cross sections ($\sigma_{\text{Er}}^{1 \rightarrow 2}/\text{cm}^2$) are deduced by using the upconversion mechanism depicted in Fig. 7b

Complexes	$\sigma_{\text{Er}}^{0 \rightarrow 1}{}^a$	$k_{\text{Er}}^{1 \rightarrow 0}/10^5{}^b$	$k_{\text{Er}}^{2 \rightarrow 0}/10^2{}^c$	$k_{\text{Er}}^{2 \rightarrow 1}/10^7{}^d$	$\phi_{\text{tot}}^{\text{up}}{}^e$	$\eta_{\text{ESA}}{}^f$	$\sigma_{\text{Er}}^{1 \rightarrow 2}{}^{a,g}$
[Er(Et-bzimpy) ₃] ³⁺	$4.6(2) \times 10^{-22}$ [0.125(6)]	1.80(2)	7.6(5)	2.5(1)	$1.7(2) \times 10^{-9}$	$5.6(7) \times 10^{-5}$	$10(1) \times 10^{-20}$ [26(3)]
[GaErGa(bpb-bzimpy) ₃] ⁹⁺	$2.7(1) \times 10^{-22}$ [0.074(4)]	2.48(2)	6.3(4)	2.5(1)	$1.7(2) \times 10^{-9}$	$6.8(9) \times 10^{-5}$	$17(2) \times 10^{-20}$ [43(6)]
[Er(Et-tpy) ₃] ³⁺	$5.6(3) \times 10^{-22}$ [0.146(7)]	5.32(6)	26.3(2.1)	2.5(1)	$5.5(6) \times 10^{-11}$	$5.2(7) \times 10^{-7}$	$2.8(4) \times 10^{-21}$ [0.7(1)]
[Er(dpa) ₃] ³⁻	$2.7(2) \times 10^{-22}$ [0.070(4)]	4.511(2)	10.2(7)	2.5(1)	$2.2(2) \times 10^{-10}$	$5.4(7) \times 10^{-6}$	$2.4(3) \times 10^{-20}$ [6.3(8)]
[Er(dpa-ester) ₃] ³⁺	$2.8(1) \times 10^{-22}$ [0.074(4)]	3.058(3)	9.9(5)	2.5(1)	$5.1(5) \times 10^{-10}$	$1.3(2) \times 10^{-5}$	$3.9(5) \times 10^{-20}$ [10(1)]
[Er(dpa-amide) ₃] ³⁺	$3.8(2) \times 10^{-22}$ [0.099(5)]	3.260(1)	12.3(9)	2.5(1)	$1.9(2) \times 10^{-10}$	$3.9(5) \times 10^{-6}$	$1.2(2) \times 10^{-20}$ [3.2(4)]

^a $\epsilon^{i \rightarrow j} = 2.6 \times 10^{20} \sigma^{i \rightarrow j}$ are given in $\text{M}^{-1} \text{cm}^{-1}$ between brackets.²³ ^b $k_{\text{Er}}^{1 \rightarrow 0} = 1/\tau_{\text{tot}}^{4\text{I}_{13/2}}$ from Table 1 column 7. ^c $k_{\text{Er}}^{2 \rightarrow 0} \approx 1/\tau_{\text{rad}}^{4\text{S}_{3/2}}$ (see text).

^d $k_{\text{Er}}^{2 \rightarrow 1} = 1/\tau_{\text{tot}}^{4\text{S}_{3/2}} = 1/\tau_{\text{rad}}^{4\text{S}_{3/2}}$ with $\tau_{\text{tot}}^{4\text{S}_{3/2}} = 40(2) \text{ ns}$ (Table 1, column 4). ^e $\lambda_{\text{exc}} = 801 \text{ nm}$ ($\tilde{\nu}_{\text{exc}} = 12\,284 \text{ cm}^{-1}$) and $P = 25 \text{ W cm}^{-2}$.

^f $\eta_{\text{ESA}} = \phi_{\text{tot}}^{\text{up}}/\phi_{\text{Er}}^{4\text{S}_{3/2}} = \phi_{\text{tot}}^{\text{up}}(k_{\text{Er}}^{2 \rightarrow 1} + k_{\text{Er}}^{2 \rightarrow 0})/k_{\text{Er}}^{1 \rightarrow 0}$ (see Fig. 3a). ^g Computed by using $k_{\text{Er}}^{\text{exc}(1 \rightarrow 2)} = \frac{\lambda_{\text{p}}}{hc} P \sigma_{\text{Er}}^{1 \rightarrow 2} = k_{\text{Er}}^{1 \rightarrow 0} \left(\frac{\eta_{\text{ESA}}}{1 - \eta_{\text{ESA}}} \right)$ (eqn (4)).

intensities monitored in solution at the same concentration (Fig. 7a) suggest the following decreasing order for the upconversion efficiencies: [Er(Et-bzimpy)₃]³⁺ > [GaErGa(bpb-bzimpy)₃]⁹⁺ > [Er(dpa-ester)₃]³⁺ > [Er(dpa-amide)₃]³⁺ ≈ [Er(Et-tpy)₃]³⁺ ≈ [Er(dpa)₃]³⁻. This trend is confirmed by the total upconversion quantum yields $\phi_{\text{tot}}^{\text{up}}$ determined in acetonitrile at room temperature with the help of the relative method using indocyanin green as a reference (Table 2, column 6; $\lambda_{\text{exc}} = 801 \text{ nm}$ and $P = 25 \text{ W cm}^{-2}$, see the Experimental section).⁴²

Having significantly improved both accuracy and reliability of the latter technique for measuring weak emitters thanks to the thorough procedures described by Charbonnière and co-workers²¹ and by Wurth *et al.*,^{42c} we ultimately found $\phi_{\text{tot}}^{\text{up}}$ ([Er(Et-bzimpy)₃]³⁺) = $1.7(2) \times 10^{-9}$ and $\phi_{\text{tot}}^{\text{up}}$ ([Er(Et-tpy)₃]³⁺) = $5.5(6) \times 10^{-11}$ for the upper and lower limits in these erbium complexes (Table 2, column 6; preliminary estimations in the 1.6×10^{-8} and 4.1×10^{-9} range).²² As expected for the ESA mechanism (Fig. 1 and 2), the upconversion quantum yields $\phi_{\text{tot}}^{\text{up}}$ (Table 2, column 6 for $\lambda_{\text{exc}} = 801 \text{ nm}$ and $P = 25 \text{ W cm}^{-2}$) and the associated ESA efficiencies $3.9(5) \times 10^{-6} \leq \eta_{\text{ESA}} = \phi_{\text{tot}}^{\text{up}}/\phi_{\text{Er}}^{4\text{S}_{3/2}} \leq 6.8(9) \times 10^{-5}$, Table 2, column 7) are found to be correlated with the increasing lifetimes of the intermediate Er(⁴I_{13/2}) excited level (Fig. S20†). Moreover, the unusual temperature dependence of the upconverted signals observed in these complexes (*i.e.* $I_{\text{Er}}^{\text{up}}$ increases with increasing temperature until reaching a maximum, Fig. S21 and S22†) is diagnostic for the operation of thermally-activated relaxation to reach the intermediate excited relays according to the upconversion mechanism proposed in Fig. 7b.²² The three-levels kinetic model depicted in Fig. 1 thus applies with $|0\rangle = \text{Er}(\text{I}_{15/2})$ corresponding to the ground state, $|1\rangle = \text{Er}(\text{I}_{13/2})$ being the intermediate excited relay (fed by fast internal conversion from ⁴I_{9/2}) and $|2\rangle = \text{Er}(\text{S}_{3/2})$ being the doubly excited emissive level. Since all the pertinent rate constants are at hand (Table 2, columns 3–5), the only unknown parameter $\sigma_{\text{Er}}^{1 \rightarrow 2} = \sigma_{\text{Er}}^{\text{I}_{13/2} \rightarrow \text{S}_{3/2}, \text{I}_{11/2} \rightarrow \text{I}_{13/2}}$ can be fitted (Table 2, column 8) to the experi-

mental ESA efficiencies η_{ESA} with eqn (4) (derived from eqn (1) and Fig. 2a)

$$\sigma_{\text{Er}}^{1 \rightarrow 2} = k_{\text{Er}}^{1 \rightarrow 0} \left(\frac{\eta_{\text{ESA}}}{1 - \eta_{\text{ESA}}} \right) \frac{hc}{\lambda_{\text{p}} P} \quad (4)$$

Translated into decadic molar absorption coefficients $0.7 \leq \epsilon^{1 \rightarrow 2} \leq 43 \text{ M}^{-1} \text{cm}^{-1}$ (Table 2, column 8), the excited state Er(²H_{11/2}, ⁴S_{3/2} ← ⁴I_{13/2}) absorptions appear to be two orders of magnitude more efficient than the ground state Er(⁴I_{9/2} ← ⁴I_{15/2}) absorption process and around one order of magnitude larger than the other Er(²S_{5/2} ← ⁴I_{15/2}) transitions recorded for the ground state absorption spectra of these complexes (Fig. 6a and S8, S9†). In this context, the [ErN₉] chromophores, produced by the binding of three bulky 2,6-bis(benzimidazol-2-yl)pyridine ligand strands possessing low-lying π^* orbitals in [Er(Et-bzimpy)₃]³⁺ and [GaErGa(bpb-bzimpy)₃]⁹⁺, give the most efficient excited state absorptions with $26 \leq \epsilon^{1 \rightarrow 2} \leq 43 \text{ M}^{-1} \text{cm}^{-1}$, which are at least one order of magnitude larger than those expected for standard intrashell f–f transitions. Recently, some non-negligible mixing of 4f-metal with ligand π orbitals have been demonstrated to significantly boost the efficiency of energy transfer processes in related europium tris-diketonate complexes,⁴³ and a similar mechanism might be responsible for this unexpected improvement for molecular upconversion. For testing this hypothesis, the oscillator strengths f_{ij} , which are proportional to the molar absorption coefficient ϵ ,⁴⁴ of the electric-dipole (ED), magnetic-dipole (MD) and electric-quadrupole (EQ) contributions to the ligand field Er(²S_{5/2} ← ⁴I_{15/2}) (Table S16†) and Er(²S_{5/2} ← ⁴I_{13/2}) (Table S17†) transitions intensities have been evaluated from SO-CASSCF calculations (see computational details in the ESI†).⁴⁵ As expected from the Judd–Ofelt theory,⁴⁴ the contributions of EQ transitions are negligible except for the hypersensitive Er(²H_{11/2} ← ⁴I_{15/2}) transition ($f_{\text{EQ}} = 6.2 \times 10^{-8}$), which possess a large Judd–Ofelt U⁽²⁾ matrix element.⁴⁶ The oscillator strengths of the two MD-allowed transitions Er(⁴I_{13/2} ← ⁴I_{15/2}) and Er(⁴I_{11/2} ← ⁴I_{13/2}), which obey the selection rules $\Delta S = 0$, $\Delta L = 0$ and $\Delta J = \pm 1$, and

Fig. 8 (a) Near-infrared downshifted $\text{Er}(^4\text{I}_{13/2} \rightarrow ^4\text{I}_{15/2})$ emission observed for $[\text{GaErGa}(\text{bpb-bzimpy})_3]$ (solid state, 298 K) upon laser excitation of the $\text{Er}(^4\text{I}_{9/2} \leftarrow ^4\text{I}_{15/2})$ transition at $\lambda_{\text{exc}} = 801 \text{ nm}$ ($\tilde{\nu}_{\text{exc}} = 12\,284 \text{ cm}^{-1}$) and for different incident pump intensities focused on a spot size of $\approx 0.07 \text{ cm}^2$. (b) Corresponding log–log plot of downshifted intensities I as a function of incident pump intensities P (in W cm^{-2}).

of $\text{Er}(^2\text{H}_{11/2} \leftarrow ^4\text{I}_{13/2})$ with only $\Delta J = \pm 1$, are found to compete with the forced ED transitions, the computed intensity of which roughly follow the trend reported for the aquo-ion.⁴⁶ Focusing on $\lambda_{\text{exc}} = 801 \text{ nm}$ ($\tilde{\nu}_{\text{exc}} = 12\,284 \text{ cm}^{-1}$) used in our studies, the absorption of the first photon is associated with the $\text{Er}(^4\text{I}_{9/2} \leftarrow ^4\text{I}_{15/2})$ transition (Fig. 6 and S9†). Its small experimental absorption coefficient ($\epsilon < 0.2 \text{ M}^{-1} \text{ cm}^{-1}$ in all studied complexes) is mirrored by the weak computed oscillator strengths $3.5 \leq f \leq 5.7 \times 10^{-8}$ (Table S16†) assigned to small Judd–Ofelt $U^{(6)}$ matrix elements, a curious trend which is characteristic for the transition to the highest-lying J -level in terms with S_{max} .⁴⁶ After relaxation to the intermediate $\text{Er}(^4\text{I}_{13/2})$ level, the second excitation process reaches $\text{Er}(^2\text{H}_{11/2})$, the energy of which ($18\,600\text{--}19\,400 \text{ cm}^{-1}$, Fig. S9†) matches well $\text{Er}(^4\text{I}_{13/2})$ ($6300\text{--}6800 \text{ cm}^{-1}$) + $\tilde{\nu}_{\text{exc}}$ ($12\,284 \text{ cm}^{-1}$) $\approx 18\,600\text{--}19\,000 \text{ cm}^{-1}$. Interestingly, the oscillator strength computed for the pertinent $\text{Er}(^2\text{H}_{11/2} \leftarrow ^4\text{I}_{13/2})$ excited state absorption ($f = 1.3 \times 10^{-5}$, Table S17†) is 2–3 orders of magnitude larger than that computed for the first $\text{Er}(^4\text{I}_{9/2} \leftarrow ^4\text{I}_{15/2})$ absorption process in agreement with the experimental ESA absorption coefficients (Table 2, column 8), which are 2–3 orders of magnitude larger than their GSA analogues (Table 2, column 2). However, the possibility that the non-emissive $\text{Er}(^4\text{I}_{9/2})$ and $\text{Er}(^4\text{I}_{11/2})$ excited state are indeed long-lived (*i.e.* $\tau_{\text{tot}}^{4\text{I}_{9/2}} \gg \tau_{\text{tot}}^{4\text{I}_{13/2}}$ or $\tau_{\text{tot}}^{4\text{I}_{11/2}} \gg \tau_{\text{tot}}^{4\text{I}_{13/2}}$) and may act as better relay than $\text{Er}(^4\text{I}_{13/2})$ for upconversion, cannot be ruled out without being explored prior to reach any conclusion (see Fig. 7b for the energy diagram).

Looking for a long-lived intermediate excited state working as relay for ESA in molecular erbium complexes

Continuous laser excitation into the $\text{Er}(^4\text{I}_{9/2} \leftarrow ^4\text{I}_{15/2})$ transitions at $\lambda_{\text{exc}} = 801 \text{ nm}$ for $[\text{GaErGa}(\text{bpb-bzimpy})_3]^{9+}$, $[\text{Er}(\text{L})_3]^{3+}$ ($\text{L} = \text{Et-bzimpy}$, Et-tpy , dpa-amide , dpa-ester) and $[\text{Er}(\text{dpa})_3]^{3-}$ does not only induce the weak upconverted signals $\text{Er}(^2\text{H}_{11/2}, ^4\text{S}_{3/2} \rightarrow ^4\text{I}_{15/2})$ discussed above (Fig. 7a), but also downshifted $\text{Er}(^4\text{I}_{13/2} \rightarrow ^4\text{I}_{15/2})$ emissions at 1520 nm characterized

by linear $\log(I)\text{--}\log(P)$ plots with slopes close to one at low-to-medium intensity powers (Fig. 8 and Fig. S23, S24†).

We thus conclude that the detected emissive $\text{Er}(^4\text{I}_{13/2})$ level is fed by internal conversion from the initially non-emissive excited $\text{Er}(^4\text{I}_{9/2})$ level *via* internal conversion prior to emitting its characteristic NIR photons upon relaxing to the ground $\text{Er}(^4\text{I}_{15/2})$ state (see Fig. 7b). The time-dependent normalized population densities $N^{4\text{I}_{13/2}}(t)$ for the intermediate emissive $\text{Er}(^4\text{I}_{13/2})$ level thus follows a simplified sequence of two consecutive kinetic reactions $\text{Er}(^4\text{I}_{9/2}) \xrightarrow{k_1} \text{Er}(^4\text{I}_{13/2}) \xrightarrow{k_2} \text{Er}(^4\text{I}_{15/2})$ described in eqn (5) and (6), where $k_1 = 1/\tau_{\text{tot}}^{4\text{I}_{9/2}}$ and $k_2 = 1/\tau_{\text{tot}}^{4\text{I}_{13/2}}$.⁴⁷

$$k_1 \neq k_2 \Rightarrow N^{4\text{I}_{13/2}}(t) = N_0^{4\text{I}_{9/2}} \frac{k_1}{(k_2 - k_1)} [e^{-k_1 t} - e^{-k_2 t}] \quad (5)$$

$$k_1 = k_2 = k \Rightarrow N^{4\text{I}_{13/2}}(t) = N_0^{4\text{I}_{9/2}} \cdot kt \cdot e^{-kt} \quad (6)$$

When $k_1 \gg k_2$ (*i.e.* $\tau_{\text{tot}}^{4\text{I}_{9/2}} < \tau_{\text{tot}}^{4\text{I}_{13/2}}$), the time decay of the emissive $\text{Er}(^4\text{I}_{13/2})$ level approximately corresponds to a single exponential trace with its characteristic lifetime $1/k_2 = \tau_{\text{tot}}^{4\text{I}_{13/2}}$. When $k_1 \approx k_2$ (*i.e.* $\tau_{\text{tot}}^{4\text{I}_{9/2}} \approx \tau_{\text{tot}}^{4\text{I}_{13/2}}$), the time-dependence of the population density of the emissive $\text{Er}(^4\text{I}_{13/2})$ level corresponds to a two-phase process with a rising period controlled by $\tau_{\text{tot}}^{4\text{I}_{9/2}}$ (exponential in eqn (5) or linear in eqn (6)), followed by an exponential decay period controlled by $\tau_{\text{tot}}^{4\text{I}_{13/2}}$. Finally, $k_1 \ll k_2$ (*i.e.* $\tau_{\text{tot}}^{4\text{I}_{9/2}} > \tau_{\text{tot}}^{4\text{I}_{13/2}}$) results in a decay of the emissive $\text{Er}(^4\text{I}_{13/2})$ level showing a rough single exponential trace with a characteristic $\tau_{\text{tot}}^{4\text{I}_{9/2}} = 1/k_1$ lifetime reminiscent to that of the feeding $\text{Er}(^4\text{I}_{9/2})$ level. Pulsed-laser excitation into the $\text{Er}(^4\text{I}_{9/2} \leftarrow ^4\text{I}_{15/2})$ transition at $\lambda_{\text{exc}} = 805 \text{ nm}$ systematically leads to single exponential emission decays arising from the $\text{Er}(^4\text{I}_{13/2})$ level (Fig. S25 and S26†) with characteristic microsecond lifetimes, which exactly mirror those obtained upon ligand-centered excitation at 355 nm (Table 1, column 9) and therefore assigned to $\tau_{\text{tot}}^{4\text{I}_{13/2}}$. Similar results were obtained upon alternative excitation into the $\text{Er}(^4\text{I}_{11/2} \leftarrow ^4\text{I}_{15/2})$ transition at $\lambda_{\text{exc}} = 975 \text{ nm}$ (Table 1, column 10 and Fig. S27 and S28†), which confirms that the

lifetimes of the non-emissive $\text{Er}({}^4\text{I}_{9/2})$ and $\text{Er}({}^4\text{I}_{11/2})$ levels are significantly shorter than $\tau_{\text{tot}}^{{}^4\text{I}_{13/2}}$. This leaves $\text{Er}({}^4\text{I}_{13/2})$ as the only available 'long-lived' intermediate relay for ESA in these complexes. It is worth noting here that the erbium-centered excitations at 801 nm ($\text{Er}({}^4\text{I}_{9/2} \leftarrow {}^4\text{I}_{15/2})$, Fig. 8b) or at 966 nm ($\text{Er}({}^4\text{I}_{11/2} \leftarrow {}^4\text{I}_{15/2})$, Fig. S29†) systematically exhibit convex $\log(I^{\text{down}}) - \log(P)$ plots with slopes of 1.0 only at low excitation powers for the downshifted $\text{Er}({}^4\text{I}_{13/2} \rightarrow {}^4\text{I}_{15/2})$. At high incident NIR power intensities, the two-photon ESA process is efficient enough (*via* $k_{\text{Er}}^{2 \rightarrow 1}$ illustrated in Fig. 1b) to compete with the direct internal $\text{Er}({}^4\text{I}_{9/2}) \rightarrow \text{Er}({}^4\text{I}_{13/2})$ (Fig. 7b) or $\text{Er}({}^4\text{I}_{11/2}) \rightarrow \text{Er}({}^4\text{I}_{13/2})$ conversions for feeding the emissive $\text{Er}({}^4\text{I}_{13/2})$ level.

Molecular light-upconversion operating in multi-center triple-helical chromium-erbium complexes

Having now a reliable model for ESA operating in mono-nuclear complexes, we finally re-considered the original proposal made a decade ago for justifying the molecular upconversion process detected in $[\text{CrErCr}(\text{bpb-bzimpy})_3]^{9+}$ and tentatively assigned to an ETU mechanism (Fig. 3 and 4).^{12,19} Direct excitation into the $\text{Cr}({}^2\text{T}_1 \leftarrow {}^4\text{A}_2)$ transition at $\lambda_{\text{exc}} = 718$ nm ($\tilde{\nu}_{\text{exc}} = 13986 \text{ cm}^{-1}$, Fig. 9a) in acetonitrile solution, where no Er-centered absorption occurs (Fig. 6a), indeed confirms the pioneerily reported¹² upconverted visible $\text{Er}({}^2\text{H}_{11/2} \rightarrow {}^4\text{I}_{15/2})$ and $\text{Er}({}^4\text{S}_{3/2} \rightarrow {}^4\text{I}_{15/2})$ emissions (Fig. 9b).

Combining the Er-centered rate constants measured for the $[\text{GaErGa}(\text{bpb-bzimpy})_3]^{9+}$ complex (Tables 1 and 2) with the Cr-centered rate constants extracted from previous detailed studies of a series of isostructural $[\text{MLnM}(\text{bpb-bzimpy})_3]^{9+}$ complexes ($\text{M}_1 = \text{Cr}, \text{Ga}; \text{Ln} = \text{Er}, \text{Y}; k_{\text{Cr}}^{1 \rightarrow 0} = \left(\tau_{\text{CrYCr}}^{{}^4\text{I}_{13/2}}\right)^{-1} = (296 \mu\text{s})^{-1}$, $W_{\text{S} \rightarrow \text{A}}^1 = 232 \text{ s}^{-1}$ at 293 K)¹² lead to the conclusion that $W_{\text{S} \rightarrow \text{A}}^2$ is the only missing parameter for computing the total upconversion quantum yield ($\phi_{\text{tot}}^{\text{up}}$, Fig. 4a) according to the ETU mechanism illustrated in Fig. 3b. The experimental upconversion quantum yields, determined in acetonitrile for $[\text{CrErCr}(\text{bpb-bzimpy})_3]^{9+}$ at room temperature with the help of the accurate relative method using indocyanin green as a reference, eventually amounts to $\phi_{\text{tot}}^{\text{up}} = 5.8(6) \times 10^{-8}$ ($\lambda_{\text{exc}} = 718$ nm and $P = 38.2 \text{ W cm}^{-2}$), a value which is one order of magnitude larger than that found for the ESA process occurring in $[\text{GaErGa}(\text{bpb-bzimpy})_3]^{9+}$ ($\phi_{\text{tot}}^{\text{up}} = 1.7(2) \times 10^{-9}$; $\lambda_{\text{exc}} = 801$ nm and $P = 25 \text{ W cm}^{-2}$). The latter result confirms the pioneering reports¹² claiming that, by using a tunable Ti-sapphire excitation laser, an upconverted signal could be detected only for ETU operating in $[\text{CrErCr}(\text{bpb-bzimpy})_3]^{9+}$,¹⁹ whereas no signal could be detected with the same setup for ESA in $[\text{GaErGa}(\text{bpb-bzimpy})_3]^{9+}$.

However, the upconversion quantum yield for the ETU mechanism (Fig. 4b with $\lambda_{\text{exc}} = 718$ nm and $P = 38.2 \text{ W cm}^{-2}$) is predicted to be $\phi_{\text{tot}}^{\text{up}}(\text{ETU}) = 2.5 \times 10^{-14}$ with the reasonable assumption that $W_{\text{Cr} \rightarrow \text{Er}}^2 = W_{\text{Cr} \rightarrow \text{Er}}^1 232 \text{ s}^{-1}$. It can be expanded to $\phi_{\text{tot}}^{\text{up}}(\text{ETU}) = 2.0 \times 10^{-11}$ upon suspicious saturation $W_{\text{Cr} \rightarrow \text{Er}}^2 \geq 10^6 \text{ s}^{-1}$ while $W_{\text{Cr} \rightarrow \text{Er}}^1 = 232 \text{ s}^{-1}$. In consequence, whatever the magnitude of $W_{\text{Cr} \rightarrow \text{Er}}^2$, the computed $\phi_{\text{tot}}^{\text{up}}(\text{ETU})$ are at least three orders of magnitude smaller than the experimental value. The situation becomes much less critical if one considers that the

Fig. 9 (a) NIR absorption spectrum of $[\text{CrErCr}(\text{bpb-bzimpy})_3]^{9+}$ (0.01 M in acetonitrile at 298 K) showing Cr-centered transitions with decadic molar absorption coefficient ϵ per chromium center and (b) upconverted visible $\text{Er}({}^2\text{H}_{11/2} \rightarrow {}^4\text{I}_{15/2})$ and $\text{Er}({}^4\text{S}_{3/2} \rightarrow {}^4\text{I}_{15/2})$ emissions recorded upon laser excitation of the $\text{Cr}({}^2\text{T}_1 \leftarrow {}^4\text{A}_2)$ transition at $\lambda_{\text{exc}} = 718$ nm ($\tilde{\nu}_{\text{exc}} = 13927 \text{ cm}^{-1}$) and using incident pump intensity $P = 38.2 \text{ W cm}^{-2}$ in acetonitrile solution at 298 K ($c \sim 10 \text{ mM}$). The blank (red curve) was recorded from pure acetonitrile solvent using the same incident pump intensity.

absorption of the second photon at 718 nm (13927 cm^{-1}) may be performed either (inefficiently) by a chromium sensitizer ($k_{\text{Cr}}^{\text{exc}(0 \rightarrow 1)}$ highlighted in red in Fig. 10a) followed by the second energy transfer of magnitude $W_{\text{Cr} \rightarrow \text{Er}}^2$ according to the ETU mechanism or (efficiently) by the erbium cation in its 'long-lived' intermediate $\text{Er}({}^4\text{I}_{13/2})$ excited state *via* the ESA mechanism $6500(300) + 13927 \approx 20400(300) \text{ cm}^{-1}$ to reach either the highest crystal field sublevels of the $\text{Er}({}^2\text{H}_{11/2})$ manifold or the lowest sublevels of the $\text{Er}({}^4\text{F}_{7/2})$ manifold ($k_{\text{Er}}^{\text{exc}(1 \rightarrow 2)}$ highlighted in blue in Fig. 10a).

Introducing $\epsilon_{\text{Er}}^{1 \rightarrow 2} = 50 \text{ M}^{-1} \text{ cm}^{-1}$ ($\lambda_{\text{exc}} = 718$ nm), inspired by $\epsilon_{\text{Er}}^{1 \rightarrow 2} = 43(6) \text{ M}^{-1} \text{ cm}^{-1}$ ($\lambda_{\text{exc}} = 801$ nm) deduced for ESA operating in $[\text{GaErGa}(\text{bpb-bzimpy})_3]^{9+}$, into the adapted equation (Fig. 10b and S30†) gives $\phi_{\text{tot}}^{\text{up}}(\text{ETU}) = 1.9 \times 10^{-10}$, which results in a noticeable gain of four orders of magnitude whatever the value for the second $\text{Cr} \rightarrow \text{Er}$ energy transfer rate constant

Fig. 10 (a) Energy scheme for the upconversion mechanism operating in [CrErCr(bpb-bzimpy)₃]⁹⁺ upon Cr(²T₁ ← ⁴A₂) excitation at λ_{exc} = 718 nm (ν̄_{exc} = 13 927 cm⁻¹) and (b) associated modeling of the upconversion quantum yield (ϕ_{tot}^{up}) obtained under steady-state (S-S) excitation. The additional competitive Er-centered ESA mechanism is highlighted in blue. Full upward arrows = photonic excitation, full downward arrows = photonic emission, dashed downward arrow = global (radiative + non-radiative) relaxation processes, dotted lines = energy transfer processes, wavy lines: non-radiative internal conversions.

(100 ≤ W_{Cr→Er}² ≤ 10⁶ s⁻¹). The remaining gap by a factor 100 with respect to the experimental quantum yield ϕ_{tot}^{up}(ETU) = 5.8 (6) × 10⁻⁸ is difficult to unambiguously assign, but it could be related to some improved intrinsic erbium-centered quantum yield ϕ_{Er}^{4S_{3/2}} = k_{rad}^{4S_{3/2}}/k_{tot}^{4S_{3/2}} = τ_{tot}^{4S_{3/2}}/τ_{rad}^{4S_{3/2}} in going from [GaErGa(bpb-bzimpy)₃]⁹⁺ (ϕ_{Er}^{4S_{3/2}} = 2.5 × 10⁻⁵) to [CrErCr(bpb-bzimpy)₃]⁹⁺ where minor mixing with low-lying Cr-based LMCT states may severely reduce τ_{rad}^{4S_{3/2}}.⁴³ We conclude that the main upconversion mechanism operating in [CrErCr(bpb-bzimpy)₃]⁹⁺ starts with an initial Cr(²T₁ ← ⁴A₂) excitation (718 nm), followed by fast internal conversion to reach the Cr(²E) level, from which a Cr(²E)-to-Er(⁴I_{9/2}) energy transfer occurs (W_{Cr→Er}¹). The major pathway for superexcitation is associated with an efficient Er(²H_{11/2}, ⁴F_{7/2} ← ⁴I_{13/2}) absorption of the second photon at 718 nm, followed by internal conversion to Er(⁴S_{3/2}) and ultimate green Er(⁴S_{3/2} → ⁴I_{15/2}) photoluminescence. The experimental upconversion quantum yield of ϕ_{tot}^{up}(ETU) = 5.3(5) × 10⁻⁸ obtained experimentally for the

dinuclear analogue [CrEr(pb-bzimpy)₃](CF₃SO₃)₆ in the same conditions (acetonitrile, 298 K, λ_{exc} = 718 nm and P = 38.2 W cm⁻²) is a very strong support for the proposed mixed ETU/ESA mechanism since a pure ETU mechanism should be accompanied by a decrease of the upconverted emission by a factor 10²–10³ in going from CrErCr to CrEr due to the removal of the contribution provided by the concerted Cr-centered ETU mechanism in going from SAS = CrErCr to SA = CrEr systems.^{13,20b}

Comparison with ETU/ESA mechanisms operating in ionic solids and in nanoparticles doped with Cr/Er is rather tricky because the low-field [CrO₆] chromophores found in these oxides are rarely used as sensitizers for upconversion.^{9g,48–50} In most studies dealing with Cr/Er mixtures, the elected solid garnet is co-doped with Cr³⁺, Er³⁺ and Yb³⁺, where Yb³⁺ is used as a near-infrared sensitizer (via its Yb(²F_{5/2} ← ²F_{7/2}) transition at 980 nm).^{9g} In absence of Yb³⁺, Er³⁺ is itself usually exploited as the sensitizer (via its Er(⁴F_{7/2} ← ⁴I_{15/2}) transition at 488 nm

(ref. 48) or its $\text{Er}({}^4\text{I}_J \leftarrow {}^4\text{I}_{15/2})$ transitions in the near-infrared range ($J = 15/2, 13/2, 11/2$ and $9/2$),⁴⁹ whereas Cr^{3+} contributes to improve the upconversion properties by working as a relay *via* energy transfers from the erbium centers. The photoluminescent properties of $\text{Cr} : \text{Cr} : \text{GGG}$ (GGG = gadolinium gallium garnet) represent an exception since direct Cr-centered excitation at 633 nm into the $\text{Cr}({}^4\text{T}_2 \leftarrow {}^4\text{A}_2)$ transition is followed by $\text{Cr}({}^4\text{T}_2) \rightarrow \text{Er}({}^4\text{I}_{9/2})$ energy transfer and subsequent multi-erbium cross-relaxation processes, which eventually promote a dual $\text{Er}({}^4\text{I}_{11/2} \rightarrow {}^4\text{I}_{13/2})$ at 2800 nm and $\text{Er}({}^4\text{I}_{13/2} \rightarrow {}^4\text{I}_{15/2})$ at 1600 nm emission.⁵⁰

Conclusions

When Reinhard and Güdel concluded in 2002 that ‘*there is no chance to induce and observe upconversion luminescence in $[\text{Ln}(\text{dpa})_3]^{3-}$ molecular compounds*’,¹⁸ (understood that only reasonable incident intensity powers are considered)^{25,26} their completely pertinent reasoning was based on (i) the observation of intermediate $\text{Ln}({}^{2S+1}L_J)$ levels with only sub-microsecond lifetimes in $\text{Na}_3[\text{Ln}(\text{dpa})_3] \cdot 13\text{H}_2\text{O}$ and (ii) the reasonable hypothesis that all f-f absorptions possess cross sections within the 10^{-24} to 10^{-22} cm^2 range. As synthetic chemists, it was rather obvious to find a way to remove the unfavorable high-energy water oscillators which limit the $\text{Er}({}^4\text{I}_{13/2})$ lifetime in $\text{Na}_3[\text{Ln}(\text{dpa})_3] \cdot 13\text{H}_2\text{O}$ with the preparation of $(\text{NHET}_3)_5[\text{Er}(\text{dpa})_3](\text{CF}_3\text{SO}_3)_2$ (**1**). The latter complex can be directly used in the solid state, but it also gives water-free $[\text{Er}(\text{dpa})_3]^{3-}$ anions in acetonitrile and displays $\text{Er}({}^4\text{I}_{13/2})$ excited lifetimes reaching a few microseconds at room temperature as found in closely related nine-coordinate $\text{Er}(\text{III})$ complexes fitted with more sophisticated organic ligands in $[\text{GaErGa}(\text{bpb-bzimpy})_3]^{9+}$ and $[\text{Er}(\text{L})_3]^{3+}$ ($\text{L} = \text{Et-bzimpy}, \text{Et-tpy}, \text{tpy}, \text{dpa-amide}, \text{dpa-ester}$). With this in mind, the predicted upconversion quantum yields produced under reasonable excitation intensity powers ($1\text{--}30 \text{ W cm}^{-2}$) for the ESA mechanism (Fig. 1) should not exceed $\phi_{\text{tot}}^{\text{up}} = 10^{-11}$ (Fig. 2b). However, our experimental data, that we believe to be as accurate as possible, point to $5 \times 10^{-11} \leq \phi_{\text{tot}}^{\text{up}} \leq 2 \times 10^{-9}$ (Table 2). After a thorough and unfruitful look for the existence of alternative long-lived non-emissive excited relays, we conclude that the only acceptable explanation relies on unusually large Er-based cross sections for the excited-state absorption in the $2 \times 10^{-21} \leq \sigma_{\text{Er}}^{1 \rightarrow 2} \leq 2 \times 10^{-19} \text{ cm}^2$ (i.e. $1 \leq \epsilon_{\text{Er}}^{1 \rightarrow 2} \leq 50 \text{ M}^{-1} \text{ cm}^{-1}$). This phenomenon also plays a crucial role in boosting the apparent ETU mechanism initially assigned to $[\text{CrErCr}(\text{bpb-bzimpy})_3]^{9+}$ where $\text{Cr}(\text{III})$ act as sensitizers for $\text{Er}(\text{III})$. Interestingly, chromophores for which the excited-state absorption (ESA) cross section is larger than the ground state absorption (GSA) in a target spectral region may display reverse saturable absorption (RSA) which finds applications in optical limiting devices.⁵¹ This behavior is observed in many organic molecules, including metal-containing porphyrin and phthalocyanine chromophores,⁵² but might be extended to metallosupramolecular assemblies with 4f-block cations. Finally, the decomposition of the total upconversion

quantum yields $\phi_{\text{tot}}^{\text{up}} = \eta_{\text{ESA}}\phi_{\text{Er}}^{4\text{S}_{3/2}}$ or $\phi_{\text{tot}}^{\text{up}} = \eta_{\text{ETU}}\phi_{\text{Er}}^{4\text{S}_{3/2}}$ demonstrate that the intrinsic quantum yield $\phi_{\text{Er}}^{4\text{S}_{3/2}}$, which is very low for the molecular erbium complexes ($2.5 \times 10^{-5} \leq \phi_{\text{Er}}^{4\text{S}_{3/2}} \leq 1 \times 10^{-4}$, Table 1), represents a crucial parameter to be optimized since it counts for approximately 50–70% of $\phi_{\text{tot}}^{\text{up}}$. In this context, Charbonnière and coworkers cleverly exploited $\text{Tb}(\text{III})$ as an alternative activator for molecular upconversion, since its associated rough intrinsic quantum yield of $\phi_{\text{Tb}}^{5\text{D}_4} = \tau_{\text{tot}}^{5\text{D}_4} / \tau_{\text{rad}}^{5\text{D}_4} = (170 \times 10^{-6} / 10^{-3}) = 0.17$,⁵³ estimated for soluble heteronuclear Yb/Tb assemblies, may overcome the limited efficiency of the associated cooperative upconversion mechanism (η_{CU}). This approach lead to the currently largest reported molecular light upconversion quantum yield of $\phi_{\text{tot}}^{\text{up}} = \eta_{\text{CU}}\phi_{\text{Tb}}^{5\text{D}_4} = 10^{-7}$ (CD_3OD , 298 K, $\lambda_{\text{exc}} = 980 \text{ nm}$ and $P = 2.86 \text{ W cm}^{-2}$).^{21c} Further efforts should be now focused on theoretical justifications for the unusually large absorption cross sections found for the excited state absorption occurring in molecular erbium complexes.

Author contributions

Conceptualization, B.G., H.B, H.N. and C.P.; methodology and practical chemical and spectroscopic studies, B.G., I.T., J.-R.J. and A.F.; crystallography, L.G. and C.B.; theoretical calculations H.B. (crystal-field and electronic transitions) and C.P. (kinetics); writing draft report, B.G., I.T. and H.B.; writing of ms and editing C.P. and A.F.; project administration and funding acquisition C.P.

Conflicts of interest

They are no conflict to declare.

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