Hyperpositive non-linear effects: enantiodivergence and modelling†

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The chiral ligand N-methylephedrine (NME) was found to catalyse the addition of dimethylzinc to benzaldehyde in an enantiodivergent way, with a monomeric and a homochiral dimeric complex both catalysing the reaction at a steady state and giving opposite product enantiomers. A change in the sign of the enantiomeric product was thus possible by simply varying the catalyst loading or the ligand ee, giving rise to an enantiodivergent non-linear effect. Simulations using a mathematical model confirmed the possibility of such behaviour and showed that this can lead to situations where a reaction gives racemic products, although the system is composed only of highly enantioselective individual catalysts. Furthermore, depending on the dimer’s degree of participation in the catalytic conversion, enantiodivergence may or may not be observed experimentally, which raises questions about the possibility of enantiodivergence in other monomer/dimer-catalysed systems. Simulations of the reaction kinetics showed that the observed kinetic constant \( k_{obs} \) is highly dependent on user-controlled parameters, such as the catalyst concentration and the ligand ee, and may thus vary in a distinct way from one experimental setup to another. This unusual dependency of \( k_{obs} \) allowed us to confirm that a previously observed U-shaped catalyst order vs. catalyst loading-plot is linked to the simultaneous catalytic activity of both monomeric and dimeric complexes.

Introduction

Non-linear effects (NLEs) in asymmetric catalysis refer to cases in which the enantiomeric excess of the product does not scale linearly with the enantiomeric excess of the catalyst. The first examples and models of such behavioural differences between scalemic and enantiomerically pure catalysts were established by Kagan in 1986. Since then NLEs are considered as ubiquitous phenomena that provide additional information regarding the aggregation state of the catalyst or the formation of multi-ligand species (cf. also reviews and some recent examples). Not only being indicative of the catalytic system, NLEs also give clues to discussions on the origin of molecular homochirality in biology which is related to the origin of life.

Several models for NLEs have been described and discussed in the literature, all of them being the results of interactions between the enantiomers of the chiral catalyst thus generating diastereomeric perturbations of the entire system. A positive non-linear effect (i.e. asymmetric amplification, (+)-NLE) is essentially generated by the presence of a reservoir of racemic ideally catalytically inactive hetero-aggregate (meso), although pure homochiral aggregation can also lead to (+)-NLEs in certain cases. Amongst these models, Kagan established a hypothetical case wherein an unprecedented phenomenon could occur – that is, the chiral catalyst [would] be much more efficient when partially resolved than when enantiomerically pure. We recently have observed such a case, known as hyperpositive NLE, in the enantioselective addition of dialkylzincs to benzaldehyde when catalysed by the chiral N-benzy1ephedrine (NBE) ligand. Subsequent mechanistic investigations pointed towards a two-component catalysis where monomeric as well as homochiral dimeric catalysts are in equilibrium and in competition: both catalyse the reaction with different enantioselectivities, the dimeric catalyst being the less enantioselective one. Through the precipitation of a heterochiral aggregate, variation of the ligand ee leads to a change of the overall catalyst concentration and, therefore, to a change of the monomer–dimer equilibrium. This favours the more enantioselective monomeric catalyst at low ligand ee and gives rise to the hyperpositive non-linear effect (Fig. 2a, orange crosses). These findings challenge the widely applied Noyori model for asymmetric dialkylzinc additions, where only monomers are catalytically active, and shows how complex systems with concurrent catalytic cycles can emerge from a minimum of components.

In line with our studies on NLEs, we have explored additional ephedrine-based ligands in dialkylzinc addition reactions. The
results presented in this work show that $N$-methylephedrine ligand (NME) follows the same model as the related NBE ligand, with monomer and dimer both catalysing the reaction, the only difference being that no meso aggregate precipitates in the case of NME. However, the homochiral dimer catalyst not only yields the product with a lower ee than the monomer catalyst, it selectively gives the opposite enantiomeric product. This enantiodivergent behaviour allowed us to switch between the preference for either enantiomer of the product, just by varying the catalyst loading. Variation of the enantiomeric excess of the ligand also allowed switching the selectivity of the product formation, leading to an enantiodivergent non-linear effect. In addition, we developed a theoretical model giving a closed mathematical expression which takes into account the concomitant catalysis by monomeric and dimeric species. From this we simulated [product ee vs. catalyst concentration] and [product ee vs. ligand ee]-plots which allowed us to gain a better understanding of the two-component catalytic system. Furthermore, the model also allowed us to analyse the reaction kinetics. Simulations showed that the observed kinetic constant of the system $k_{\text{obs}}$ is indeed not unique for each catalytic system, but depends on user-controlled parameters such as the total catalyst concentration and the ligand ee. This has confirmed the hypotheses made earlier and allowed the simulation of a U-shaped catalyst order vs. catalyst-curve, which had been previously observed experimentally in the NBE-catalysed reaction. 

The paper is organised as follows: we will (1) first present the experimental results obtained from the NME ligand, showing enantiodivergence in product ee ($ee_p$) vs. ligand ee ($ee_L$) and $ee_p$ vs. catalyst loading-plots, (2) then go on to a theoretical analysis where we present the mathematical model and discuss $ee_p$ vs. catalyst concentration- and $ee_p$ vs. $ee_L$-plots, (3) this is followed by a kinetic analysis of the system in which we investigate the relationship of $k_{\text{obs}}$ with the catalyst concentration and $ee_L$ and
simulate catalyst order vs. catalyst loading-plots. (4) Finally, we will discuss the impact of the simulations in a more general way and compare them with the experimental results.

Results & discussion

Nonlinear effect in the presence of N-methylephedrine as a chiral ligand

The NME and NBE ligands only differ by the N-methyl group being changed by an N-benzyl group. Therefore, we suspected that NME might generate diastereomeric and aggregation-induced perturbations in the same way as in Fig. 1 for the same reaction. The ligand has been previously applied in asymmetric alkylation using ZnEt2 as the reagent\textsuperscript{18-21} but, to the best of our knowledge, ZnMe\textsubscript{2} has not been investigated so far. Knowing that the NBE ligand displays a more pronounced NLE with ZnMe\textsubscript{2} than with ZnEt\textsubscript{2}\textsuperscript{15} we suspected that NME might show an NLE in conjunction with ZnMe\textsubscript{2}, even though it had been shown previously that there is no apparent NLE with ZnEt\textsubscript{2}\textsuperscript{22} Fig. 2a displays the correlation between the enantiomeric excess of the product (ee\textsubscript{P}) and the enantiomeric excess of the ligand (ee\textsubscript{L}) for the asymmetric addition of dimethylzinc to benzaldehyde using chiral (–)-NME (blue dots). Although its (1R,2S)-configuration is identical to the one of (–)-NBE, it surprisingly yielded mainly the S-product (P\textsubscript{S}) while (–)-NBE (orange crosses) gave the R-product (P\textsubscript{R}) under similar reaction conditions. The NME-dataset in Fig. 2a generates a negative non-linear effect ([(–)-NLE], albeit in an apparently inverted manner when compared to common representations of (–)-NLEs (this is due to ee\textsubscript{P} being defined here as the enantiomeric excess of P\textsubscript{R}, which leads to negative ee\textsubscript{P} values when P\textsubscript{S} is predominant).

In parallel, we conducted a screening of the catalyst loading using enantiopure (–)-NME (Fig. 2b, blue dots). While a catalytic loading of 20 mol\% (–)-NME gave the product in −11\% ee (P\textsubscript{S} being predominant), lowering the catalyst loading progressively displaced ee\textsubscript{P} into the positive scale, giving P\textsubscript{R} in +18\% ee at 2.5 mol\%. The shape of the curve was found similar to the one observed when using NBE (orange crosses), as well as the ee\textsubscript{P} increase from 20 mol\% to 2.5 mol\% (NME: +31\%; NBE: +33\%).

Such behaviour with enantiopure NME, similar to what was observed with the NBE ligand, suggests that both systems follow an analogous catalytic scheme, where both monomeric and dimeric complexes are catalytically active. However, in contrast to NBE the catalytic runs with scalenic NME ligand were homogeneous with no apparent precipitate in the reaction mixture. This may account for the observed differences in NLE curves in Fig. 2a. While the hyperpositive NLE with NBE ligand was caused by the precipitation of the RS-dimer (leaving only R and RR complexes in solution and allowing high asymmetric amplification), the (–)-NLE with NME ligand might be the result of the presence of all catalytic species in solution (including S, SS and the meso dimer RS).

Moreover, the observed negative ee\textsubscript{P} at high catalyst loading unambiguously indicates that the dimeric NME-catalyst is not only less enantioselective, it even yields mainly the opposite enantiomeric product compared to the monomeric catalyst. Catalysis in which both enantiomeric products may be obtained from the same catalyst enantiomer is called enantiodivergent catalysis. It has attracted a lot of attention as a means to access easily both product enantiomers. Over the last two decades, many examples have been reported in which slight changes of the catalyst (substrate, metal, counter-ion, etc.) or of the reaction conditions (solvent, temperature, additive, ligand-to-metal ratio) have inverted the stereochimistry of the product.\textsuperscript{25-28} In our case, the switch which allows toggling between both product enantiomers is the catalyst concentration, a factor which has not been discussed in this context so far.\textsuperscript{27}

These results made us wonder whether it is possible to switch the product enantiomer’s sign within an NLE curve, i.e. by varying ee\textsubscript{L} instead of the catalyst loading. Such “enantiodivergent non-linear effect” has already been observed in other catalytic systems,\textsuperscript{29-32} but their origin has never been studied; even the possible presence of two catalysts yielding opposite enantiomers has not been proposed.\textsuperscript{33} Therefore, we continued our studies on NME by performing a temperature screening of the catalysis with the enantiopure ligand (Fig. 3a, blue dots) or only 50\% ee\textsubscript{L} (orange triangles). At low temperature (0 °C), the product ee of the enantiopure ligand was well below the ee\textsubscript{P} of the scalemic sample, both being negative. Increasing the temperature increased ee\textsubscript{P} in both cases but not in the same manner: the difference between 100\% and 50\% ee\textsubscript{L} decreased progressively. At 40 °C the enantiopure ligand even surpassed the 50\% ee\textsubscript{L}-sample.

However, an interesting situation occurred at room temperature (20–25 °C): here, (–)-NME in 50\% ee gave positive ee\textsubscript{P}-values (i.e. R product), while the enantiopure ligand stayed negative (i.e. S product): this is nothing but the requirement for an enantiodivergent non-linear effect. The full NLE curve at room temperature (Fig. 3b, blue dots) confirmed this observation: the curve starts at 0\% ee\textsubscript{L} in the positive ee\textsubscript{P}-range, reaches a maximum, then falls down to cross the ee\textsubscript{P}-axis (at ee\textsubscript{L} of ca. 80\%) and ends up in the negative part of the ee\textsubscript{P}-scale. The use of (–)-NME (red squares) gave the appropriate mirror image of this curve. Thus, going to room temperature changes the (–)-NLE to a hyperpositive NLE, which in addition is also enantiodivergent.

Model studies: product ee vs. catalyst concentration (enantiopure system)

In the past, non-linear effects have been simulated and quantified using mathematical expressions of the considered models, such as the Kagan ML\textsubscript{m} models (catalysis by the aggregate)\textsuperscript{11} or the Noyori model (catalysis by the monomer, dimer acting as inactive reservoirs).\textsuperscript{13} These models have been further extended (electron-rich substrates,\textsuperscript{32} product inhibition\textsuperscript{14} for the Noyori model; monomers as reservoir species with catalytically active homochiral aggregates by Kagan\textsuperscript{14}). However, to the best of our knowledge there has been no attempt to unify both approaches and to consider both monomers and dimers being simultaneously catalytically active.\textsuperscript{14} In order to fill this
gap and to get a better understanding of hyperpositive and, in particular, enantiodivergent NLEs, we developed mathematical models which allow us to simulate eeP vs. catalyst concentration- and eeP vs. eeL-plots. We begin with Model I, which is based on an enantiopure system where an enantiopure ligand reacts with a metal to give monomeric and dimeric homochiral complexes (R and RR, respectively) both of which catalyse the reaction at different rates (k1 and k2) and with different enantioselectivities (ee1 and ee2), as shown in Fig. 4; [Cat tot] represents the total catalyst concentration. For the sake of simplicity, we assume that both R and RR-catalysts follow a similar mechanism with a rate law of type \( \frac{d[\text{Cat}]}{dt} = k_i[\text{Cat}][\text{Sub}][\text{Rea}] \) (with \( k_i \) and [Cat] being the respective rate constants and catalyst concentrations, [Sub] and [Rea] the substrate and reactant concentrations; all species are first-order). We also assume that the [RR]/[R]-ratio stays constant over the course of the reaction and depends only on the homochiral dimerization constant \( K_{\text{Homo}} \). This makes Model I reminiscent of Kagan’s MLn model, which also considers the ratio between different catalytic species to be constant over time. The case of a time-dependent [RR]/[R]-ratio will be discussed at the end of this study.

\[
\begin{align*}
\gamma &= \sqrt{1 + 8K_{\text{Homo}}[\text{Cat tot}]} - 1 - 4 \\
\text{eeP} &= \frac{ee_1 + \gamma k_2 ee_2}{1 + \gamma k_2 k_1} \\
\text{eeP} &= \frac{ee_1 + k_2 ee_2}{1 + k_2 k_1}
\end{align*}
\]
observed by varying [Cat\textsubscript{tot}], as long as ee\textsubscript{2} has a sufficiently low and negative value. This is further favoured by high values of \(K\textsubscript{Homo}\) (Fig. 5b) and \(k_2/k_1\) (Fig. 5c): the more the dimeric complex prevails over the monomer and the higher its relative activity, the more the curve becomes hyperbolic, pushing the point at which it crosses the [Cat\textsubscript{tot}]-axis (i.e. ee\textsubscript{P} = 0, labelled [Cat\textsubscript{tot}]) to lower [Cat\textsubscript{tot}]. [Cat\textsubscript{tot}] corresponds to an overall catalytic system where \(R\) and \(RR\) catalysts compensate each other to yield an overall racemic product – even if both give independently enantiopure products.

**Model studies: product ee vs. ligand ee (scalemic system)**

In order to also simulate NLE curves, we expanded Model I to non-enantiopure ligands, as shown in Fig. 6. The resulting system (Model II) now includes the catalytic species \(S\) and \(SS\) (which, like their enantiomeric counterparts, are linked through \(K\textsubscript{Homo}\) and catalyse with the kinetic constants \(k_1\) and \(k_2\)) and also the heterochiral dimer RS, which is related to \(R\) and \(S\) through the dimerization constant \(K\textsubscript{Hetero}\) and may generate racemic products with a rate constant \(k_2^0\). Thus, a single scalemic ligand gives rise to 5 different catalytic species. To derive closed mathematical expressions, we followed the approach used by Noyori for the DAIB-model which consists in introducing \(a = [R] + [S]\) and \(\beta = [R][S]\) to simplify the equations. ee\textsubscript{P} and ee\textsubscript{L} are then given by eqn (3) and (4) as functions of \(a\) and \(\beta\). Since \(\beta\) is itself a function of \(a\) [cf. eqn (5)], ee\textsubscript{P} and ee\textsubscript{L} are linked

\[
[Cat\textsubscript{tot}] = [R] + 2[R][R] + [S] + 2[S][S] + 2[RS]
\]

\[
[M + L\textsubscript{R} + L\textsubscript{S}] \rightarrow [R] + [RR] + [S] + [SS] + [RS]
\]

(k\textsubscript{2}/k_1 was set to 1, ee\textsubscript{1} to 100% and \(K\textsubscript{Homo}\) to 33, which corresponds to the association constant of DAIB-ZnMe\textsubscript{2}. The graphs show that an enantiodivergent behaviour can indeed be

![Fig. 5 Simulation of the relationship between ee\textsubscript{P} and [Cat\textsubscript{tot}] according to eqn (1) and (2). The basic set of parameters is ee\textsubscript{1} = 100, ee\textsubscript{2} = -100, \(K\textsubscript{Homo}\) = 33 and k\textsubscript{2}/k_1 = 1. Each panel shows curves where one of the parameters has been varied: (a) ee\textsubscript{2}, (b) \(K\textsubscript{Homo}\) (c) k\textsubscript{2}/k_1. The product ee is defined as (\(P\_R - P\_S\))/(\(P\_R + P\_S\)).](image)

![Fig. 6 Schematic representation of Model II, which consists of monomeric (\(R, S\)) and both homo- (\(RR, SS\)) and heterochiral (\(RS\)) dimeric catalysts that all operate at a steady state and are linked through the equilibrium constants \(K\textsubscript{Homo}\) and \(K\textsubscript{Hetero}\). The catalysts are issued from the reaction of a metal salt (M) with a mixture of both ligand enantiomers (\(L\textsubscript{R} and L\textsubscript{S}\)) with an enantiomeric excess of ee\textsubscript{L}. They promote the reaction of a substrate (Sub) and a reactant (Rea) to form a chiral product with the overall enantiomeric excess ee\textsubscript{P}. The monomeric, the homochiral dimeric and heterochiral dimeric catalysts yield a product with a rate constant of \(k\_R, k\_RR,\) and \(k\_S, k\_SS\), respectively, and with an enantioselectivity of ee\textsubscript{P} and ee\textsubscript{2}\textsubscript{R} (and \(RR\)) or -ee\textsubscript{2} (\(S\) and \(SS\)). The \(RS\) dimer yields a racemic product (ee = 0).](image)
through α and depend only on the parameters $K_{\text{Homo}}, K_{\text{Hetero}}, k_1, k_2, k_2', e e_1, e e_2$ and $[\text{Cat}_{\text{tot}}]$. After defining these parameters, $e e_p$ vs. $e e_L$-datasets could be obtained by choosing appropriate values for α (cf. ESI Methods† for the details of the calculations and the general expressions for the upper and lower limits of α). For this study, we will focus on the cases where the NLE is hyperpositive and potentially enantiodivergent - that is with $K_{\text{Hetero}} > 2K_{\text{Homo}}$ which is, as in the Noyori model, a necessary condition to obtain (+)-NLEs – and $e e_1 > e e_2$.\(^{13}\)

Fig. 7 shows several cases computed from Model II. To simplify the discussion we have set $k_2' = 0$ in all simulations except in Fig. 7f.

$$e e_p = \frac{\sqrt{\alpha^2 - 4\beta(e e_1 + \alpha K_{\text{Homo}} k_2/k_1 e e_2)}}{\alpha + (\alpha^2 - 2\beta)K_{\text{Homo}} k_2/k_1 + \beta K_{\text{Hetero}} k_2/k_1}$$  \hspace{1cm} (3)

$$e e_L = \frac{\sqrt{\alpha^2 - 4\beta(1 + 2\alpha K_{\text{Homo}})}}{[\text{Cat}_{\text{tot}}]}$$  \hspace{1cm} (4)

Fig. 7 Simulation of NLEs with Model II, varying parameters (a) $K_{\text{Homo}}$, (b) $[\text{Cat}_{\text{tot}}]$, (c) $k_2/k_1$, (d) $e e_1$ and $e e_2$, (e) $K_{\text{Hetero}}$, and (f) $k_2'/k_1$. Fixed parameters: $[\text{Cat}_{\text{tot}}] = 0.11$, $k_2/k_1 = 1$, $e e_1 = 100$, $e e_2 = -100$ and $k_2'/k_1 = 0$ in all curves except where the corresponding parameter is varied; $K_{\text{Homo}} = 33$ (b, c, e), 100 (d) and 130 (f); $K_{\text{Hetero}} = 330000$ (a, d, f), 100 000 (b) and 33 000 (c). The product ee is defined as $(P_R - P_S)/(P_R + P_S)$. 

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**Fig. 7** Simulation of NLEs with Model II, varying parameters (a) $K_{\text{Homo}}$, (b) $[\text{Cat}_{\text{tot}}]$, (c) $k_2/k_1$, (d) $e e_1$ and $e e_2$, (e) $K_{\text{Hetero}}$, and (f) $k_2'/k_1$. Fixed parameters: $[\text{Cat}_{\text{tot}}] = 0.11$, $k_2/k_1 = 1$, $e e_1 = 100$, $e e_2 = -100$ and $k_2'/k_1 = 0$ in all curves except where the corresponding parameter is varied; $K_{\text{Homo}} = 33$ (b, c, e), 100 (d) and 130 (f); $K_{\text{Hetero}} = 330000$ (a, d, f), 100 000 (b) and 33 000 (c). The product ee is defined as $(P_R - P_S)/(P_R + P_S)$. 

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\[
\beta = \frac{(\alpha + 2K_{\text{Homo}} \alpha^2 - [\text{Cat}_{\text{tot}}])}{4K_{\text{Homo}} - 2K_{\text{Hetero}}} \tag{5}
\]

**Influence of \(K_{\text{Homo}}\) [\(\text{Cat}_{\text{tot}}\), \(k_2/k_1\) and \(ee_{1/2}\).** A non-linear effect is hyperpositive as long as the highest product ee (labelled \(ee_{p}^{\text{max}}\)) is different from the \(ee_p\) for the enantiopure ligand (\(ee_{p}^{100}\)). In Model II, \(ee_{p}^{100}\) will be strongly dependent on \(K_{\text{Homo}}\) [\(\text{Cat}_{\text{tot}}\)] and \(k_2/k_1\) (Fig. 7a-c): higher the \(K_{\text{Homo}}\) [\(\text{Cat}_{\text{tot}}\)] or \(k_2/k_1\), lower the \(ee_{p}^{100}\). This is consistent with a higher proportion and a higher activity of the low \(ee_p\)-yielding RR-catalyst over its monomeric counterpart.

In all representations in Fig. 7, we selected conditions in which \(ee_{p}^{100}\) could be negative and where the NLE curve crosses the \(ee_{p}\)-axis, making it an enantiodivergent NLE. Lower the \(ee_{p}^{100}\), lower the crossing point at which \(ee_p = 0\) (\(ee_p^0\)). At this point, the outcomes of all catalysts compensate each other to yield a racemic product. \(ee_{p}^{100}\) diminishes as \(ee_{p}^{100}\) decreases; the maximum's \(ee_{p}\)-value (\(ee_{p}^{\text{max}}\)) is only slightly affected by \(K_{\text{Homo}}\) and, to a somewhat greater extent, by [\(\text{Cat}_{\text{tot}}\)] and \(k_2/k_1\). This is seen nicely if, for a given set of parameters, \(K_{\text{Homo}}\) is multiplied and [\(\text{Cat}_{\text{tot}}\)] divided by the same value: \(ee_{p}^{100}\) remains unchanged but \(ee_{p}^{\text{max}}\) and \(ee_{p}^{\text{max}}\) do not (cf. ESI Fig. 1†).

Fig. 7d shows the impact of \(ee_1\) and \(ee_2\) on \(ee_{p}^{100}\): lower the \(ee_2\), lower the \(ee_{p}^{100}\). This is also true for \(ee_1\); however, if both \(ee_1\) and \(ee_2\) are negative, the enantiodivergent NLE curve becomes a classic (+)-NLE for the \(S\)-product (\(P_s\), Fig. 7a, dashed curves); the same holds for the \(R\)-product (\(P_R\)) if \(ee_1 \geq ee_2\) and if both are positive. In a similar way, very high \(ee_1/2\) ((5)) additionally lead to a contraction of the curve between \(ee_0^{\text{Homo}}\) and \(ee_1\) (Fig. 7f) additional to a contraction of the curve between \(ee_0^{\text{Homo}}\) and \(ee_1\). As in Kagan’s ML2-model, a high activity of the meso catalyst leads to asymmetric depletion.

**Dynamic properties of models I and II**

Apart from simulating \(ee_p\) vs. \(ee_{p}\)-curves, NLE models have also been used to study the kinetic properties of the systems. This can be useful as some kinetic features may be characteristic for one or the other model, and thus be used as an additional probe to support the validity of a model for a specific catalytic reaction. Blackmond showed this in conjunction with the Kagan and Noyori models, and Mchieux and co-workers even built a toolset, using the different kinetic properties of both systems, which allows an easy distinction between monomer- and dimer-catalysed enantioselective systems (which is not necessarily possible on the basis of \(ee_p\) vs. \(ee_{p}\)-curves alone) and verified the origin of the non-linearity in the Noyori model by a kinetic system based on differential equations. This prompted us to extend our work on Models I and II to a kinetic study.

The mathematical expressions of the rate laws based on Models I and II turned out to correspond to a standard second-order rate law (eqn (6) and (7)), albeit with a more complex term for \(k_{\text{obs}}\) (cf. ESI Methods† for the calculation details). For aggregate-free catalysed reactions, with 1st order in catalyst, \(k_{\text{obs}}\) is the product of [\(\text{Cat}_{\text{tot}}\)] and the rate constant \(k_1\) (eqn (6)).

According to Model I, where the monomer and dimer coexist and both of which are catalytically active, \(k_{\text{obs}}\) depends on the parameters previously discussed: the (now absolute instead of relative) rate constants \(k_1\) and \(k_2\), \(K_{\text{Homo}}\) and [\(\text{Cat}_{\text{tot}}\)] (eqn (9)). This holds for chiral, enantiopure catalysts as well as for achiral ones, where we consider only the system’s kinetic behaviour and not a possible product ee. Non-enantiopure catalytic systems following Model II additionally depend on \(ee_1\), \(k_2^*\) and \(K_{\text{Hetero}}\) (eqn (10)). For the sake of simplicity, we will consider here only the second-order rate law where the substrate and reactant concentrations are equal ([\(\text{Sub}\] = [\(\text{Real}\])], however the following discussion applies also to its more general form where both differ from each other.

**General second-order rate law** if [\(\text{Sub}\] = [\(\text{Real}\]):

\[
\frac{d[\text{Sub}]}{dt} = k_{\text{obs}}[\text{Sub}]^2 \tag{6}
\]

Integrated form of the second-order rate law:

\[
[\text{Sub}]_0 = \left(\frac{1}{[\text{Sub}]_0 + k_{\text{obs}}t}\right)^{-1} \tag{7}
\]

If only monomers exist:

\[
k_{\text{obs}} = k_1[\text{Cat}_{\text{tot}}] \tag{8}
\]

If monomers and dimers catalyse (Model I, achiral or enantiopure catalysts):

\[
k_{\text{obs}} = k_1 \sqrt{1 + 8K_{\text{Homo}}[\text{Cat}_{\text{tot}}] - 1} \frac{1}{4K_{\text{Homo}}} + k_2 \left(\sqrt{1 + 8K_{\text{Homo}}[\text{Cat}_{\text{tot}}] - 1}\right)^2 \frac{1}{16K_{\text{Homo}}} \tag{9}
\]

If monomers and dimers catalyse (Model II, chiral catalyst with any \(ee_1\)):

\[
k_{\text{obs}} = k_1 \alpha + k_2 K_{\text{Homo}} (\alpha^2 - 2\beta) + k_2 K_{\text{Hetero}} \beta \tag{10}
\]
\[ ee_L = \frac{\sqrt{\alpha^2 - 4\beta(1 + 2aK_{\text{Homo}})}}{[\text{Cat}_{\text{tot}}]} \]  

(4)

and

\[ \beta = \frac{(\alpha + 2K_{\text{Homo}}\alpha^2 - [\text{Cat}_{\text{tot}}])}{4K_{\text{Homo}} - 2K_{\text{Hetero}}} \]  

(5)

The interesting point here is that \([\text{Cat}_{\text{tot}}]\) and \(ee_L\) are user-controlled parameters, whose variation leads to singular changes in \(k_{\text{obs}}\). This can be seen in simulated \(k_{\text{obs}}/[\text{Cat}_{\text{tot}}]\) vs. \([\text{Cat}_{\text{tot}}]\)-plots: in the case of aggregate-free catalysed reactions (cf. eqn (8)) such a plot results in a flat line, whose y-intercept is equal to \(k_1\) (Fig. 8a, blue line). The same is observed for Model I (eqn (9)) in a special case that is when \(k_2/k_1 = 2\): the loss of monomeric catalyst upon increase of \([\text{Cat}_{\text{tot}}]\), because of dimeric aggregation, is then perfectly compensated by the higher activity of the dimer catalyst. Otherwise, an increase in \([\text{Cat}_{\text{tot}}]\) leads to a significant change of \(k_{\text{obs}}/[\text{Cat}_{\text{tot}}]\): it increases if \(k_2/k_1 > 2\) (grey dashed line) or decreases if \(k_2/k_1 < 2\) (orange dotted line), which is symptomatic of the changing \([\text{RR}]/[\text{R}]\)-ratio. At very low \([\text{Cat}_{\text{tot}}]\) the amount of \(\text{RR}\)-catalyst becomes negligible and \(k_{\text{obs}}/[\text{Cat}_{\text{tot}}]\) becomes equal to \(k_1\); on the other hand, \(k_{\text{obs}}/[\text{Cat}_{\text{tot}}] = 0.5k_2\) at very high \([\text{Cat}_{\text{tot}}]\) because of the prevalence of the dimeric catalyst. Thus, \(k_{\text{obs}}/[\text{Cat}_{\text{tot}}]\) varies over changing \([\text{Cat}_{\text{tot}}]\) even if \(R\) and RR catalyse with the same rate \((k_1 = k_2, \text{green dashed/dotted line})\). With non-enantiopure ligands \(k_{\text{obs}}/[\text{Cat}_{\text{tot}}]\) varies in a similar way to that in Fig. 7a and depends, in addition, also on \(k_2^*\) (cf. ESI Fig. 3† for a commented example with \(ee_L = 0\)).

The other user-controlled parameter, \(ee_L\), also influences \(k_{\text{obs}}\) (at constant \([\text{Cat}_{\text{tot}}]\)) as seen in \(k_{\text{obs}}\) vs. \(ee_L\) plots (Fig. 8b, computed from eqn (10)). \(k_{\text{obs}}\) is constant if \(k_2/k_1 = k_2^*/k_1 = 2\) (blue full line). The case of \(k_2^*/k_1\)-values higher than 2 leads to an increase in \(k_{\text{obs}}\) especially at high \(ee_L\), where the concentration of the homochiral dimers is also higher, and results in a positive slope (orange dotted line). On the other hand, increasing \(k_2^*/k_1\) gives a negative slope as it affects \(k_{\text{obs}}\), mostly at low \(ee_L\), where the proportion of RS-dimers is highest (grey dashed line). A simultaneous increase of \(k_2^*/k_1\) and \(k_2/k_1\) by the same amount also yields a negative slope if \(K_{\text{Hetero}} > 2K_{\text{Homo}}\) (green dashed/dotted line). If the \(K_{\text{Hetero}}/K_{\text{Homo}}\)-relationship is inverted then a positive slope is obtained, cf. ESI Fig. 4†.

In our previous study, we also determined the catalyst order \(c\) of the NBE-catalysed reaction when considering the system to follow the rate law \(-d[\text{Sub}]/dt = k([\text{Cat}_{\text{tot}}])[\text{Sub}][\text{Rea}]\). For this, we had determined the catalyst order \(c\) using Variable Time-Normalised Analysis (VTNA) of rate profiles obtained from enantiopure NBE at different catalyst loadings. Plotting catalyst order \(c\) vs. catalyst loading gave an unusual U-shaped plot which we postulated to originate in a \([\text{Cat}_{\text{tot}}]\)-induced change of \(k_{\text{obs}}\). The present kinetic model now allows us to verify this assumption, since Fig. 8a shows that \(k_{\text{obs}}\) does indeed change with varying \([\text{Cat}_{\text{tot}}]\). To this end, we generated sets of rate profiles from eqn (7) and (9) by varying \([\text{Cat}_{\text{tot}}]\) and leaving all other parameters unchanged. Then, \(c\) was determined from two different rate profiles at a time using VTNA; the results are shown in Fig. 9.
in Fig. 9. In concordance with the previous discussion, $k_2/k_1 = 2$ gives a constant $c$ value while varying the catalyst loading, since $k_{obs}$ is constant in that special case. $k_2/k_1 < 2$ gives a U-shaped curve, here with a minimum at $c \approx 0.7$ (blue dots) and similar to the experimental one obtained from NBE. On the other hand $k_2/k_1 > 2$ gives a bell-shaped curve with a maximum at $c \approx 1.5$ (grey triangles). Increasing $K_{Homo}$ displaces the curve’s extremum to lower catalyst loading values (green crosses).

**Discussion**

Models I and II, although kept conceptually simple, allow us to gain a better understanding of several aspects of the NME- and NBE-catalysed reactions, which we will discuss in this section. Globally, the shape of the experimental curves ($ee_P$ vs. catalyst loading as well as $ee_P$ vs. $ee_L$, Fig. 2 and 3) corresponds well with the computed curves in Fig. 5 and 7. Previously published graphs from reactions with NBE and ZnEt$_2$ as the dialkylzinc reagent,\(^\text{19}\) giving a much weaker hyperpositive NLE than ZnMe$_2$, also concord with the simulated ones in this study. The simulations have also given insights into the differences between the NME- and NBE-based catalysts: the overall lower $ee_P$-values with NME may be due to lower $ee_L$ or $ee_H$ values, as well as to higher $K_{Homo}$ and/or $k_2/k_1$. The slight hyperpositive (and enantiodivergent) NLE with NME at room temperature (Fig. 3b) may arise from $K_{Hetero}$ being only slightly higher than $2K_{Homo}$ since pronounced hyperpositive NLEs were simulated only when $K_{Hetero}$ and $K_{Homo}$ are highly different (cf. Fig. 7e). At 0 °C (Fig. 2a), NME even generates a ($-$)-NLE which precludes that upon cooling, $K_{Hetero}$ falls below the value of $2K_{Homo}$.

In the case of NBE (Fig. 2a, orange crosses), the strong hyperpositive curve arises from the precipitation of the heterochiral dimer, which has an effect similar to an increase of $K_{Hetero}$. The simulations in Fig. 7 also reveal that, even if the dimeric catalyst gives the opposite product enantiomer in its pure form ($ee_L = -100\%$), the NLE does not need to be enantiodivergent as long as the dimer’s participation in the catalytic process is sufficiently low (due to low $K_{Homo}$[Cat$_{tot}$] or $k_2/k_1$, as seen in Fig. 7a–d). Therefore, dimeric NBE-ZnMe$_2$ might yield the opposite enantiomer as well, even though its NLE and $ee_P$ vs. catalyst loading-plot (Fig. 2, orange crosses) don’t show direct evidence for that. This possibility is particularly interesting as it raises questions whether enantiodivergence between monomeric and dimeric catalysts might be a systematic phenomenon. There are various cases of 2 : 1 ligand-to-metal complexes which have been reported to yield the other product enantiomer than their 1 : 1 counterparts.\(^\text{43–51}\) Seebach’s TADDL-Ti-complexes being the first (but largely unnoticed) ones.\(^\text{46–48}\) 2 : 1 complexes can be considered as analogues of dimeric complexes, both are even equivalent in Kagan’s ML$_2$ model.\(^\text{11}\)

Also, the models suggest that it is possible to obtain racemic products when working at a certain catalyst concentration ([Cat$_{tot}$], cf. Fig. 5) or ligand ee ($ee_L$, cf. Fig. 7), although the system consists of only highly enantioselective catalytic species. This is well exemplified with the results from NME-catalysed reactions at room temperature (Fig. 3b): the only 2% $ee_P$ yielded by the enantiopure catalyst may let one think that NME is a totally nonenantioselective catalyst in this reaction, which is definitely not the case.

Furthermore, the kinetic models show that the evolution in the ratios between the different catalytic species, when changing reaction parameters like [Cat$_{tot}$] or $ee_L$, results in a variation of the rate constant $k_{obs}$. This allowed us to confirm our previous assumption that a U-shaped curve for the catalyst order $c$ vs. catalyst loading plot is consistent with Model I, assuming the monomer has more than half the catalytic activity of the dimer ($k_2/k_1 < 2$). However, this feature also shows the limitations of Model I and II: the simulated U-shaped $c$ vs. catalyst loading-curve (Fig. 9) spans over several orders of magnitude of catalyst loading, whereas the experimental one obtained from NBE spans only from 20 to 2.5 mol%.\(^\text{15}\) For second order kinetic laws, plotting 1/[Sub]$_2$ vs. time gives a linear graph, however the corresponding experimental plots from NBE- and NME-catalysed reactions are mostly curved (cf. ESI Fig. 5f). This shows that $k_{obs}$ is not constant over time, most probably because of a change of the catalysts’ concentrations (and their respective ratios) as the reaction goes on. It is known for DAIB-catalysed reactions that the reactants and, in certain cases, the reaction products have an influence on the catalyst composition, which results in altered reaction rates and non-constant $ee_P$ over time.\(^\text{14,15}\) Indeed, an increase of $ee_P$ over the course of the reaction was observed in the NBE-catalysed reaction.\(^\text{15}\) Therefore, further studies (and quantitative analyses in particular) of NME- and NBE-catalysed dialkylzinc additions will require more complex models, which include the concentrations of the starting materials and of the products and their possible influence on the catalyst distribution.

**Conclusion**

In summary, we have found an enantiodivergent behaviour in NME-catalysed reactions which may be explained by the monomer–dimer catalytic model, known for the parent ligand NBE. Simulation allowed us to confirm this and to get further insights into the model, such as the dependence of the observed kinetic constant $k_{obs}$ on the catalyst concentration and on the enantiomeric excess of the ligand. We expect this may serve as a basis for further research to understand past and future cases of enantiodivergent non-linear effects, as well as of asymmetric catalytic systems in general. Further work in our group will aim at the elucidation and the quantification of the different parameters governing reactions catalysed by ephedrine-derivatives, at the extension of the models in order to include the influence of reactants and products and at the investigation of aggregation-induced complex catalytic systems in a general way.

**Data availability**

Synthetic and mathematical procedures, as well as additional data and chromatograms for every experimental catalytic run can be found in the ESI.\(^\text{†}\) Detailed experimental data (reactant quantities, reaction conditions, raw and treated results for all catalytic runs, including reactions with NBE) and calculated data for all simulated graphs in this paper can be found in the
respective excel files (SI_experimental_Data.xlsx, SI_Simulated_NLE.xlsx, SI_kinetic_model.xlsx).

Author contributions
Y. G. performed the synthetic experiments and developed the catalytic models, along with their mathematical expressions. A. M.-F. and T. A. participated in data analyses. S. B.-L. conceptualized and supervised the study and wrote the manuscript with Y. G.

Conflicts of interest
There are no conflicts to declare.

Acknowledgements
This work was funded by the French National Research Agency (ANR) through the Programme d’Investissement d’Avenir under contract ANR-11-LABX-0058_NIE within the Investissement d’Avenir program ANR-10-IDEX-0002-02.

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27 The effect of catalyst concentration has been studied in one case, however it was the ligand-to-metal ratio which turned out to be the key factor to enantiodivergence there (cf. ref. 45).


31 Only Kagan mentioned this briefly when discussing the general MLn model (cf. ref. 11).


34 Noble-Terán and co-workers discussed that possibility, but built their kinetic models for the two extreme cases were either only monomers or dimers catalyse, cf. ref. 39.

35 $K_{\text{homo}}$ in Models I and II should not be confused with Noyori’s $K_{\text{homo}}$, which was defined as a dissociation constant (cf. ref. 13).


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