Towards efficient discovery of green synthetic pathways with Monte Carlo tree search and reinforcement learning†

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Computer aided synthesis planning of synthetic pathways with green process conditions has become of increasing importance in organic chemistry, but the large search space inherent in synthesis planning and the difficulty in predicting reaction conditions make it a significant challenge. We introduce a new Monte Carlo Tree Search (MCTS) variant that promotes balance between exploration and exploitation across the synthesis space. Together with a value network trained from reinforcement learning and a solvent-prediction neural network, our algorithm is comparable to the best MCTS variant (PUCT, similar to Google’s Alpha Go) in finding valid synthesis pathways within a fixed searching time, and superior in identifying shorter routes with greener solvents under the same search conditions. In addition, with the same root compound visit count, our algorithm outperforms the PUCT MCTS by 16% in terms of determining successful routes. Overall the success rate is improved by 19.7% compared to the upper confidence bound applied to trees (UCT) MCTS method. Moreover, we improve 71.4% of the routes proposed by the PUCT MCTS variant in pathway length and choices of green solvents. The approach generally enables including Green Chemistry considerations in computer aided synthesis planning with potential applications in process development for fine chemicals or pharmaceuticals.

Introduction

The goal of computer aided synthesis planning (CASP) has traditionally been to identify synthesis pathways to desired compounds from buyable building-block chemicals through retrosynthesis.1–8 Retrosynthetic analysis is a procedure in which a target molecule is broken into precursors recursively until all precursors are commercially available (success) or the process reaches the termination criteria (failure).1–9,10 The idea of reaction codification and CASP was introduced in Vléduts’s visionary paper in 1963.11 Ever since the first logical retrosynthetic route planning based on a computer program demonstrated by Corey et al.12 in the 1960s, the field of CASP has been evolving rapidly along with the development of large comprehensive reaction databases.9 As examples, Synthia (formerly Chematica), has successfully designed synthesis routes for many medicinally important targets9,12,18 and Coley et al. has demonstrated that with expert chemistry input CASP synthetic routes can be translated successfully into recipes for execution on a robotic platform.17 Existing CASP programs have primarily focused on the success rate1 and the chemical diversity of routes,2,9 but rarely have taken Green Chemistry principles into considerations, e.g., routes with shortest possible length using less toxic and flammable solvents.18,19 However, it is becoming increasingly important to select routes that conform to green chemistry requirement in addition to finding synthetic pathways to buyable starting compounds.

One major challenge for an efficient CASP algorithm to include green chemistry considerations is limitations in chemical space search algorithms. The search algorithm is the core of synthesis planning as it determines the success rate and the route quality of the proposed synthesis pathways. Heuristic best first searches (BFS) or depth first searches (DFS) use hand-written heuristic functions to evaluate positions in the search tree as they explore the synthesis planning search space.1,5 However, given the large search space2–28 of chemical reactions, the searching efficiency of BFS/DFS is not high enough for good CASP performance.3 In addition, it is difficult to define strong heuristics for BFS/DFS CASP.1 To tackle this challenge, Segler et al.1 showed that Monte Carlo Tree Search (MCTS), a search scheme that demonstrated superhuman performance in playing the game of Go,21,22 is able to find buyable pathways with much higher success rate than traditional heuristic BFS.3 Segler
also innovatively demonstrated that it was possible to learn the rules of chemical synthesis for a specific domain by using data. Additionally, Kishimoto et al. demonstrated that the MCTS variant used by Segler significantly outperformed depth-first proof-number (DFPN) search in terms of success rate in the domain of retrosynthesis. More recently, Chen et al. introduced Retro*, a neural-based tree search method also demonstrating advanced performance.

The MCTS variant used by Segler is similar to Google’s Alpha Go algorithm. Though it has great search success rate, the method does not take green chemistry into account. Schreck et al. used another variant of MCTS, Upper Confidence bound applied to Trees (UCT), in a reinforcement learning approach to find synthesis pathways with as few buyable precursors as possible. However, the UCT method is prone to have a lower success rate in finding pathways than the Alpha Go-like PUCT (predictor + UCT) variant used by Segler. An efficient search scheme that maintains high search success rate and favors green chemistry routes is still missing.

A critical challenge for green chemistry CASP is the difficulty of evaluating chemical compounds and reactions in the search tree. Evaluation of compounds and reactions in the sense of green chemistry requires a fast-computing model to predict the 

- **Synthesis planning as a tree search problem**

The CASP problem can be viewed as a finite Markov Decision Process (MDP), defined by a state space $S$ composed of different states (i.e., compounds in retrosynthesis) $s$, an action space $A(s)$ defining the allowed retrosynthetic disconnection actions $a$ for any state (compound), and a transition function $T(s,a,s') : S \times A \times S \rightarrow [0,1]$ expressing the transition probability $p(s'|s,a)$ from state (compound) $s$ through action (disconnection rule, or template) $a$ to new state $s'$ (new compounds). A reward function $R(s,a,s') : S \times A \times S \rightarrow \mathbb{R}$ describes the reward taken by taking action $a$ from state $s$ and reaching new state $s'$, and finally a discount factor $\gamma \in [0,1]$ reflects a preference for shorter pathways. In the MDP, the policy function $p(a|s)$ is the probability distribution of the allowed actions $a \in A(s)$, and a value function is defined as $v^*(s) = \max_{a} \sum_{s'} T(s,a,s') [R(s,a,s') + \gamma v^*(s')]$, which can be tailored for different optimization purposes such as high success rate, low reaction cost and high process greenness. As mentioned above, we use templates to implement the retrosynthetic disconnection action, $a$.

The retrosynthesis process can be formulated as a tree search problem in an MDP environment. Given the large search width and depth of chemical retrosynthesis planning, the traditional depth/breadth first search is not efficient. Here a more efficient search scheme (MCTS, Fig. 1) is used. The detailed description of the MCTS process can be found in the Methods section. The core of MCTS is using an upper confidence bound (UCB) to prioritize the templates. Different UCB equations define different MCTS variants as shown in Table 1.
As shown in Table 1, in total, we study 6 MCTS variants: modified UCT with dynamic $c$ and value network (mUCT-dc-V), original UCT with value network (UCT-V), modified UCT without dynamic $c$ with value network (mUCT-V), PUCT$^{22,31}$ MCTS with value network (PUCT-V, is very similar to the MCTS used by Segler et al.$^{1}$), PUCT MCTS without value network (PUCT-bootstrapping), modified UCT with dynamic $c$ but without value network (mUCT-dc-bootstrapping).

**Training the value network using RL for efficient synthesis planning**

The MCTS process requires a reliable policy network and value network. The training process of the policy network in this work is described in the Methods, and it is very similar to the process used by Segler et al.$^{1,23,34}$ The training of the value network is one of the main points of this work. First of all, the value of a compound should reflect the “easiness” of synthesis based on the buyable catalog. Therefore defining the “easiness” and getting enough data for this standard is critical. At the same time, modifications to the MCTS algorithm are required to tolerate some inherent issue of the policy and value network and guarantee that synthesis planning is still successful even with a non-ideal policy/value network. To tackle these two challenges, we develop reinforcement learning (RL) and a modified UCT algorithm with dynamic $c$ tuning for the first time.

**RL and bootstrapping**

In order to maximize the success rate of the search, we consider the value of a state (or compound) as follows,$^{46}$

$$v^*(s) = \max_a \sum_s T(s, a, s') R(s, a, s') + \gamma v^*(s').$$

We only consider the deterministic case where $T(s, a, s') = 1$, if $s'$ is the result of retrosynthetic disconnection action $a$ (or template) taken by compound $s$, otherwise $T(s, a, s') = 0$; and we set $R(s, a, s')$ to 0 for all reactions. If $s'$ is buyable, $v^*(s') = 1$. Therefore

$$v^*(s) = \begin{cases} 
1 & \text{if } s \text{ is buyable} \\
\max_{a: A(s)} \gamma v^*(s') & \text{otherwise}
\end{cases}$$

Here $s'$ is the next state of $s$ so that $T(s, a, s') = 1$. $A(s)$ is the action space which defines all the allowable retrosynthetic disconnection actions for $s$. Note that the definition of $v^*(s)$ is recursive and depending on ergodic search in the action space, which is not possible. It is prohibitive to obtain the analytical solution of the exact value of $v^*(s)$. Instead, we can empirically approximate $v^*(s)$ by running MCTS on molecules randomly sampled from the chemical space. The resulting approximate of $v^*(s)$ through MCTS sampling is $z(s)$, and the output of the value network trained using $z(s)$ is defined as $v_0(s)$, so that $v_0(s) \approx z(s) \approx v^*(s)$.

Since one compound $s$ can be split into multiple compounds, defining $z(s)$ is complex. In order to bootstrap the process, we define the $z(s)$ value for each compound in the tree as shown in Fig. 2a. The $z(s)$ is defined similar to the Bellman

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The first three rows are UCT$^{22}$ type MCTS variants, and the last row is PUCT$^{22,31}$ type MCTSs. The modified UCT and the “dynamic $c$” tuning (mUCT-dc) in Table 1 are described in details in the Methods section. In short, we (1) modify the UCB equations of original UCT to a hybrid form that combines UCT and PUCT types of MCTS, in order to save the time required by the mandatory ergodic step in the initial step of searching by the original UCT, and (2) introduce a “dynamic $c$” trick to tune the value of the coefficient, $c$ (Table 1), in order to effectively force the tree expander to explore dynamically templates that are ranked low by the policy network to better balance exploration and exploitation than the original UCT and PUCT MCTS.

In addition, instead of using a Monte Carlo roll-out$^{1,23,28,29}$ as the look-ahead mechanism in the “Evaluation” step, we use a value network to generate the value of a compound, similar to Alpha Go by Silver et al.$^{23}$ and envisioned by Segler et al.$^{1}$ If the compound is in our buyable catalog, the value will be set to 1 and overwrite the value given by the value network. The value network is trained through RL self-play. The self-play process (bootstrapping) and the definition of values are discussed in the following part. The value network trained in this way is therefore dependent on the buyable catalogue used.
Table 1  Summary of MCTS variants in this work. \( N(s,a) \) is the visit count of the state-action pair \((s,a)\). \( P(s,a) \) is the prior probability of the \((s,a)\) pair and the output of the policy network. \( \sum_{b} N(s,b) \) denotes the summation of \( N(s,b) \) over all allowable templates \( b \) available for state \( s \). \( v(s) \) is the value of \( s \) and the output of the value network. \( s,a \rightarrow s' \) indicates that \( s' \) is eventually reached after taking action \( a \) from position \( s \).

<table>
<thead>
<tr>
<th>MCTS variants</th>
<th>UCB equation</th>
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<tbody>
<tr>
<td>Original UCT</td>
<td>( UCB(s,a) = Q(s,a) + c\sqrt{\frac{2}{N(s,a)}} )</td>
</tr>
<tr>
<td>Modified UCT (mUCT)</td>
<td>( UCB(s,a) = Q(s,a) + c\sqrt{\frac{2}{1 + N(s,a)}} )</td>
</tr>
<tr>
<td>Modified UCT with dynamic ( c ) (mUCT-dc)</td>
<td>( UCB(s,a) = Q(s,a) + c\sqrt{\frac{2\ln N(s,a)}{1 + N(s,a)}} )</td>
</tr>
<tr>
<td>PUCT</td>
<td>( Q(s,a) = \frac{1}{\text{count}(s)} \sum_{s',a \rightarrow s} v(s') )</td>
</tr>
<tr>
<td>With value network</td>
<td>( Q(s,a) = \frac{1}{\text{count}(s)} \sum_{s',a \rightarrow s} z(s') )</td>
</tr>
</tbody>
</table>

The equation of \( v^*(s) \) with a discount factor of \( \gamma \). For a reaction \((s,a)\) pair in the tree, if all of its child compounds \((s')\) have a non-zero \( z \) value, then a \( Z(s,a) \) value for the reaction is defined as the average \( z \) value of its children, and the \( z \) value of the parent compound, \( s \), is defined as the maximum \( Z(s,a) \) value multiplied by \( \gamma \) among all actions available to \( s \) in the tree.

By initialization, the \( z \) values for the compounds are zero. Along the tree search process, the encountered compounds in the buyable catalog will be assigned a \( z(s) \) of 1, as shown in Fig. 2a. In the bootstrapping phase, no value network is used. The \( Q(s,a) \) value, which is important in UCB equations, is calculated using \( z(s) \) values in the bootstrapping phase as

Fig. 2  Bootstrapping process and the reinforcement learning process to train a value network. (a) The bottom-up propagation of \( z(s) \) value for bootstrapping. If the route is not from buyable precursors, the \( z \) value for all non-buyable compounds are zero (left). If the route is from buyable precursors, starting from the leaf buyable precursors \((z=1)\), the \( z \) value of a compound in the tree will be assigned as the average \( z \) value of the compounds’ immediate precursors times a discount factor \( \gamma \) (0 \(<\gamma \)< 1). If another route under the same compound generates higher \( z \) value than the current route, the \( z \) value of the compound will be updated to the larger value. Here circles denote compounds. Blue circles are compounds that are not commercially available and green ones are buyable compounds. The triangles denote the templates \( a \), through which compounds are transformed into corresponding precursors. (b) The RL process to train the value network. With the \( z \) value sampled in (b) from MCTS, a value network can be trained so that we can map \( v_0(s) \) to \( z(s) \).
shown in Table 1. Therefore in this process, a strong favor will be given to those actions which can lead to buyable products fast.

The $z(s)$ values are obtained retrospectively, i.e., they can only be calculated after running the MCTS. In order to guide the retrosynthesis, we need to evaluate the value prospectively, so we train a value function $v_0(s)$ that only takes molecular structure as an input to predict $z(s)$ (Fig. 2b). The value network can be further updated with more $z(s)$ data accumulate during the training of the RL algorithm. In this work, we will discuss the value network obtained by the Round 1 RL (trained with $z(s)$ values generated from bootstrapping MCTS) and the Round 2 RL (trained with $z(s)$ values generated from MCTS with the Round 1 RL value network). The bootstrapping MCTS variant used here is the mUCT-de-bootstrapping method. The details of the RL value network training can be found in the Methods section.

Performance of the novel MCTS variant and RL value network in finding successful synthetic routes

The performance of different MCTS variants is shown in Fig. 3. We conduct two types of experiments: tree expansion with a fixed expansion time of 30 s (Fig. 3a); and tree expansion with a fixed root visit count (Fig. 3b), which is independent of computers used and thus more reproducible.

Among all the MCTS variants, for our test set of 1000 compounds, the mUCT-de-V and PUCT-V algorithms outperform the other variants with a fixed expansion time of 30 s (Fig. 3a). Especially when compared to the original UCT method (UCT-V), the mUCT-de-V outperforms by 19.7% with the same value network and same initialization of the $c$ value. This is a result of the active exploration promoted by the dynamic $c$ tuning. Without the dynamic $c$, the performance of the modified UCT without dynamic $c$ (mUCT-V) and the original UCT method (UCT-V) are very close. In addition, mUCT-de-V is able to find synthesis pathways in 30 s for many target compounds that cannot be solved by PUCT-V method, providing a new effective approach to finding synthetic pathways connecting targets to buyable chemicals (cf. Fig. 4).

Fig. 4 shows the probability $P(s,a)$ of each step $(s,a)$ given by the policy network together with the ranking of the template among the top 50 templates suggested by the policy network. The success of the mUCT-de-V method is attributed to templates with small $P(s,a)$ and low ranking can still be effectively explored due to the dynamic tuning of the exploration coefficient $c$ in the UCB equation. On the contrary, PUCT-V tree expander is trapped by the high ranking templates that eventually are insufficient in forming a valid synthetic route, as a result of a more greedy strategy in the definition of its UCB equation.

Traditionally, since PUCT utilizes the information of the prior probabilities of different templates explicitly, it does not need to compute the logarithmic function (Table 1) and will search more quickly. As a result, the PUCT algorithm outperforms the original UCT method. However, with the dynamic $c$ to promote active exploration, the mUCT-de-V method even outperforms the PUCT algorithm in both test sets and training sets. In addition, since the policy network trained on the Reaxys database focuses only on single step transformations and does not necessarily reflect the best prior probability $P(s,a)$ that helps maximize the success rate of multi-step retrosynthesis search, it may mislead the PUCT algorithm. In contrast, UCT type MCTS variants do not explicitly include the $P(s,a)$ into the equation for UCB (Table 1). Instead, they only use the results of the policy network to shrink the width of the searching. Therefore modified UCT with dynamic $c$ trick (mUCT-de-V) is more robust against the imperfect policy network quality and the forced exploration guarantees broader selection of templates and various synthesis routes, which will lead to greener pathways as we will show in the following parts.

When the termination criteria of tree expansion is switched to a fixed root visit count, the advantage of mUCT-de-V algorithm is even more obvious (Fig. 3b). The power of dynamic $c$ makes the modified UCT method outperform the PUCT-V.
Fig. 4  Examples of chemical routes that mUCT-dc-V method can solve within 30 s while PUCT-V cannot. The value of the $P(s,a)$ given by the policy network and the ranking of the template among the top 50 templates are given. The unique advantage of mUCT-dc-V method is that the $P(s,a)$ value is not explicitly used, therefore even if the $P(s,a)$ value is extremely small as a result of the imperfect policy network, the valid template will still be explored by the tree expander, which is not the case in PUCT-V method. The value network here is Round 1 RL value network. The policy network used by both MCTS variants are the same. The restrictions for both MCTS variants are the same: top 50 templates given by policy network are considered, maximum depth is 10, and minimum plausibility is 0.75 (see Methods). The affected functional groups in each step are marked in blue. The buyable compounds are framed in green.
method by $\sim$16%. Since the UCT method requires the computation of the logarithmic function and forces exploration of low ranked templates, UCT searches much slower than the PUCT method. With 30 s as time limit, UCT actually visits the root much less than the PUCT method. However with the dynamic $c$ trick, our novel UCT is able to search more efficiently and have a slightly higher success rate compared to PUCT. When both methods are given the same root visit count limit, the mUCT-dc-V method largely outperforms the PUCT method due to the more efficient searching.

The value network used in Fig. 3 is the value network from Round 1 RL. The results of Round 2 RL value network can be found in Table S1 in the ESI† and are not significantly different from the results of Round 1 RL value network. The $c$ value is 0.1 for UCT type MCTS variants, and is set to 1 for PUCT type of MCTS variants. The effect of different $c$ values is studied in Table 2, and it does not significantly affect the performance. Here the values for buyable compounds are set to be 1 during the tree search regardless of the prediction of the value network. The results of the tree expansion without the prior knowledge of buyable compounds during value setting is shown in Fig. S1 and Table S2 in the ESI† In general, the performance is much worse than the case in Fig. 3, implying that the MCTS tree expander favors pathways with strong incentive such as high weight for buyable compounds.

**Incorporating solvent greenness into synthesis planning**

With a high success rate supported by the mUCT-dc-V method, MCTS is also able to steer the synthesis planning towards greener processing routes. Here, we focus on using MCTS, RL and mUCT-dc-V to find synthesis routes with fewer steps and greener solvents. Although we focus on the solvent greenness and the length of the synthetic routes, the same method can be extended to other green chemistry considerations, such as mild reaction temperatures and economical catalysts.34

We assign different scores to different solvents in our solvent library based on biosafety and flammability as suggested by Byrne et al.44 “Green” solvents, such as ethanol and water, are assigned the score of 1. “Mediocre” solvents, such as methyl isobutyl ketone (MIBK) and toluene, are given the score of 0. “Non-green” solvents, such as tetrachloromethane and 1,2-dimethoxyethane, are allotted a negative score of $-1$ (see Fig. 5a). Gao et al.44 proposed a neural network model that can predict suitable solvents with a probability distribution when given a target reaction. Based on this result, we define the reaction solvent score as the weighted average of the suggested solvents by the solvent model (Fig. 5b). The solvent model

![Fig. 5 Using MCTS to find short synthesis pathways using green solvents. (a) Assigning scores for solvents in the solvent database. (b) Using the prediction of the solvent prediction model to define the reaction solvent score. The suggested top three solvents are shown with the probabilities listed and solvent scores in parentheses. The reaction solvent score (RSS) is defined as the weighted average of the top three solvent scores. (c) Converting the reactions solvent score to reaction solvent penalty (R solvent penalty), then defining the compound solvent score using the R solvent penalty. The compound solvent score (CSS) is defined as the maximal cumulated RSS in a valid pathway. The greenest route for a compound is the path which lead to the CSS. CSS is a function of tree expander and the root compound. It is essential to optimize the tree expander so that CSS can be optimized.](https://www.chemsoc.org/content/11/34/10965)

![Fig. 6 The greenness of the synthetic routes (compound solvent score (CSS) of the root compound) generated by mUCT-dc-V when compared with PUCT-V as baseline method. Both algorithms use Round 1 RL value network and the tree expansion is restricted within 30 s. 71.4% of the cases show higher root CSS generated by mUCT-dc-V than by PUCT-V.](https://www.chemsoc.org/content/11/34/10965)

### Table 2  Study of the effect of $c$ value on the success rate for UCT type MCTS variants.

<table>
<thead>
<tr>
<th>MCTS variants</th>
<th>$c$ value</th>
<th>Test set</th>
<th>Training set</th>
</tr>
</thead>
<tbody>
<tr>
<td>UCT-V</td>
<td>$c = 0.1$</td>
<td>0.548</td>
<td>0.584</td>
</tr>
<tr>
<td></td>
<td>$c = 1$</td>
<td>0.505</td>
<td>0.552</td>
</tr>
<tr>
<td>mUCT-V</td>
<td>$c = 0.1$</td>
<td>0.533</td>
<td>0.570</td>
</tr>
<tr>
<td></td>
<td>$c = \sqrt{2}$</td>
<td>0.564</td>
<td>0.602</td>
</tr>
</tbody>
</table>
Fig. 7  Case study of the greenest routes generated by PUCT-V and mUCT-dc-V algorithms. The root CSS is the compound solvent score (CSS) of the root compound, which reflects the overall greenness of the best route in the tree. Typically PUCT-V algorithm generates much longer synthetic routes with resultant accumulative reaction penalties, or CSS, much more negative than the routes generated by mUCT-dc-V algorithm. Orange framed compounds are the most probable solvent suggested by the solvent prediction network. Note that the reaction solvent score (RSS) is the weighted average of the top three solvents suggested, therefore even the top 1 solvents are the same for two reactions, their RSS'es may vary, and therefore their reaction penalties may vary. Green framed compounds are commercially available compounds. The affected functional groups in each step are marked in blue.
MCTS can be misled by the policy network. On the contrary, the
Therefore, with an inaccurate prior probability value, the PUCT
relative ranking given by the policy network is meaningful.
Evidently, the policy network trained on the Reaxys database does not neces-
sarily require a higher prior probability to select templates with higher prior probability. However, the
output of the policy network

\[
\text{CSS}(\text{root compound}, \text{tree expander}) = \max_{\text{path}} \left( \sum_{R \in \text{path}} (\text{RSP}(R)) \right)
\]

The optimization problem to maximize the route greenness boils down to the problem of finding the greenest valid route
argmax_{\text{path}} (\sum_{R \in \text{path}} (R \text{ penalty in the path})) for the root compound. We use the CSS of the root compound as the greenness of the synthetic route in this work. Moreover, the overall goal is to optimize the compound solvent scores of the root compounds (or route greenness) by developing efficient MCTS tree expanders.

**Performance of finding routes leading to high compound solvent scores**

The modified UCT method with dynamic \(c\) tuning (mUCT-dc-V) significantly outperforms the PUCT MCTS (PUCT-V) method in terms of finding greener synthesis routes, when using the same value network (Round 1 RL value network) and policy network. We run the MCTS tree expansion on 2000 randomly selected testing compounds for 30 s. Among the root compounds that are solvable for both MCTS variants, 74% of them demonstrated CSS(mUCT-dc-V) \(>\) CSS(PUCT-V) in 30 s MCTS tree expansion, and 71.4% of the compounds are improved by the mUCT-dc-V method (Fig. 6). Fig. 7 shows examples of the compounds that are solvable to both MCTS variants but show quite different solvent greenness in resulting synthetic routes. The key is that PUCT algorithm always tends to find long synthetic pathways without fully exploring the template space.

The reason for mUCT-dc-V generating greener synthetic routes lies in the intrinsic difference of the PUCT MCTS and the modified UCT with dynamic \(c\) MCTS. In the UCB equation of PUCT-V (Table 1), the output of the policy network \(P(s,a)\) is explicitly used in the exploration term to steer the tree expander to select templates with higher prior probability. However, the policy network trained on the Reaxys database does not necessarily reflect the true value of the prior probability, although the relative ranking given by the policy network is meaningful. Therefore, with an inaccurate prior probability value, the PUCT MCTS can be misled by the policy network. On the contrary, the forced exploration by the dynamic \(c\) tuning in the modified UCT

MCTS provides a broader vision to explore templates that are underestimated by the policy network, since the value of the prior probability is not used explicitly in the UCB equation here but only the ranking is used. In addition, since \(P(s,a)\) for many low ranked templates is extremely small as a result of the imperfect policy network, the templates are not effectively explored by the PUCT MCTS. As a consequence, the PUCT MCTS tree expander will prefer to exploring edges that policy network prefers instead of exploring templates that might be ranked lower but can eventually yield shorter routes.

We also trained a value network using the CSS values generated from MCTS experiments. The route greenness is worse than the value networks only considering synthetic “easiness” (i.e. Round 1 RL value network). The main reason is that the CSS value based value network sacrifices the success rate of the synthesis planning. Consequently with deficient routes to choose from, the CSS value network is worse at finding green pathways. The loss–iteration curve of the CSS value network can be found in Fig. S2.† The comparison of the performances of CSS value network and the Round 1 RL value network can be found in Table S3.†

We have also compared the performance of PUCT-bootstrapping method with the mUCT-dc-V with Round 1 RL value network. The novel UCT method again defeats the PUCT-bootstrapping algorithm in terms of the solvent greenness of the generated synthetic routes. The performance can be found in Table S4 in the SI.†

**Conclusions**

We developed a new MCTS variant, modified UCT with dynamic \(c\) tuning, leading to a much improved success rate in finding valid synthesis pathways compared with the original UCT method by using the new method with a RL trained value network as a look-ahead mechanism. The success rate of the new MCTS variant is comparable to the Alpha Go-like PUCT MCTS algorithm. With a fixed root visit count, the MCTS variant significantly outperforms the PUCT MCTS, meaning its search is more visit-count efficient. Furthermore, when considering the route greenness, our MCTS variant is able to beat the PUCT algorithm with 71.4% pathways improved. The new MCTS is successful in generating much shorter pathways since it is not misled by the imperfect policy network thanks to the forced exploration promoted by the dynamic \(c\) tuning. Our work paves the way towards design of shorter synthesis pathways for organic molecules with greener processes. Furthermore, our modified algorithm allows the user to incorporate additional reaction-level of pathway-level penalties to bias exploration besides the considerations in green chemistry.

**Methods**

**Details of synthesis planning as a tree search problem**

Starting from the target molecule, a deep neural network (policy network) outputs a probability distribution \(p(a|s) = P(s,a)\) over the allowed deconstruction actions \(a \in A(s)\) (the reaction templates). The policy network is used to rank the possibilities
associated with different templates. We use the top 50 templates suggested by the policy network in the experiments of this article. In the initial round, the template with the highest ranking is visited first. In the regular round, the upper confidence bound (UCB) of each edge (state-action pair, \((s,a)\), i.e. the compound-template pair) of the tree is calculated using the UCB equations shown in Table 1. In the UCT type MCTS (the first three rows of Table 1), the UCB equations are not explicitly related to the output of the policy network, except that the template is ranked by the policy network, and the templates are visited in the ranked order. However, in the Alpha Go-like PUCT MCTS (PUCT in Table 1), the \(P(s,a)\) is explicitly incorporated in the exploration term, steering a stronger favor towards exploiting highly probable actions/templates.

The most promising template selected for each compound \(s\) is the template with the highest UCB(\(s,a\)) value. This template selection process takes place recursively in the tree until a leaf node (nodes without a child along the edge of the most promising template) is encountered. The tree expander will expand the leaf node by applying the selected template to it. In this way, new leaf nodes (new precursors) are added to the tree. Once the new leaf nodes are generated, a second neural network, value network, is used to evaluate the novel compounds. We check the commercial availability of the leaf nodes and once the leaf nodes are found to be in our catalogue of purchasable compounds, the value of this compound is set to 1 (overwriting the node’s \(Q\) value).

A key issue of the modified UCB1 (see the ESI†) and the original UCB1 in MCTS is that the tree expander may tend to visit the known best action too many times and it does not sample other options enough, when the parameter \(c\) is not appropriate. Additionally, when the value network cannot evaluate the states perfectly, it is important to explore the actions efficiently in a dynamic way. Therefore, we proposed the dynamic \(c\) trick to solve these challenges, and as shown in the main text, it significantly improves performance of the MCTS algorithm.

First of all, we have a policy network that ranks the probability to apply each template (or action). Therefore we always start from the highest ranked template and then visit the second ranked and so on. The event we want to investigate is the case when the tree expander switches from the highest ranked template to the second highest ranked template when using modified UCB1 equation.

Suppose that in the last round, the expanded template \(\text{action } a\) has the UCB\(_{\text{expanded}} = Q(s,a) + c \frac{2 \ln \left( \sum_b N(s,b) \right)}{1 + N(s,a)}\) and the next unexpanded template has the UCB\(_{\text{unexpanded}} = c \frac{2 \ln \left( \sum_b N(s,b) \right)}{1 + N(s,a)}\). the unexpanded template will not be visited in the current round unless

\[
\text{UCB}_{\text{unexpanded}} = c \frac{2 \ln \left( \sum_b N(s,b) \right)}{1 + N(s,a)} > \text{UCB}_{\text{expanded}} = Q + c \frac{2 \ln \left( \sum_b N(s,b) \right)}{1 + N(s,a)}
\]

Specifically, if the switching is between the highest ranked template and the second highest ranked template, \(\sum_b N(s,b) = N(s,a)\), since only the highest ranked template \(a\) is visited. Therefore the minimum \(N(s,a)\) required for the highest ranked template \(a\) can be solved from

\[
\min N(s,a) = \frac{2 \ln \left( \sum_b N(s,b) \right)}{(c + 1) N(s,a)}
\]

**The modified UCT with a dynamic \(c\) tuning**

The UCB equation of the original UCT\(^{28,29}\) MCTS (Table 1) is usually called the UCB1 (ref. 30 and 31) equation. In this work, a dynamic \(c\) tuning was used on the basis of the modified UCB equation for UCT algorithm\(^{28,29}\) (or modified UCB1 (ref. 30) equation) (Table 1). The analysis of the modified UCB1 equation can be found in the ESI†. The dynamic \(c\) trick is to: (1) enhance the sampling rate and dynamically tune the exploration rate, (2) to balance the imperfect policy network.
The data $z(s)$ values are collected from 40 s tree expansion (mUCT-dc-bootstrapping) for 112 000 product molecules randomly selected from the Reaxys database.

### Table 3

<table>
<thead>
<tr>
<th>$Q/c$ value</th>
<th>$\min N(s,a)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>3879</td>
</tr>
<tr>
<td>5</td>
<td>281326</td>
</tr>
<tr>
<td>9</td>
<td>$3.88 \times 10^{17}$</td>
</tr>
</tbody>
</table>

The dynamic method to decide the value of $c$. We define $c$ as half of the current max $Q(s,b)$ value during the tree expansion process during which the visit count of the compound $s$ increases.

\[
c \sqrt{2 \ln(N(s,a))} > Q + \sqrt{\frac{2 \ln(N(s,a))}{1 + N(s,a)}}, \quad \text{since} \ c > 0,
\]

\[
\sqrt{2 \ln(N(s,a))(1 - \frac{1}{\sqrt{1 + N(s,a)}})} > \frac{Q}{c}.
\]

Therefore the minimum required visit time $N(s,a)$ depends on the ratio of $Q(s,a)$ and $c$. The solution of this inequality vs. the value of $Q/c$ is shown in Fig. 8 and Table 3. As $Q/c$ value increases, the minimum required $N(s,a)$ increases very rapidly. Therefore if we don’t choose $c$ carefully, the tree expander will be searching the highest ranked template almost forever. If the highest ranked template cannot lead to a buyable pathway, the tree expander is “trapped” in this suboptimal branch. The phenomena shown in Fig. 8 and Table 3 also comport with the idea that $c$ is promoting “exploration” in the sense that the extent to which $c$ is promoting exploration depends on its relative value to $Q$.

In practice, here we choose $Q/c = 2$ as the standard. Therefore the first template switch will happen when the first template has been visited for 24 times. If we set $c = \text{current max}(Q(s,b))/2$ for all visited $Q(s,a)$ values and we assume random variables $Q(s,b) \in (0,1)$ (this is the requirement of Chernoff–Hoeffding bound\textsuperscript{10} and is the actual case in this paper) are not changing with tree expansion, the average and maximal visit count for each template before all templates are visited can be calculated as shown in Fig. 9 (a and b) using $10^4$ random simulations. In fact, this trick allows visiting all the templates with decaying visit times according to their ranking given by the policy network.

One issue for the aforementioned strategy is, the $Q(s,b)$ values are actually changing as the tree expands. Therefore we propose a method to tune the $c$ on the go. The idea is shown in Fig. 10. The value of $c$ is tuned as current observed max $Q/2$ as the tree expands. The benefit of this promoted exploration of low ranked templates can be seen in Fig. 4, where low ranked templates lead to successful synthetic routes while PUCT-V method is trapped in the high ranked templates which eventually are insufficient in forming a valid synthetic route.
here is set to be 0.7 to enhance the differentiation of synthesis “easiness”. We again choose \( z(s) \) as the \( \max(z(s)) \) trees where compound \( s \) appears. Then after adding \( z(s) = 1 \) for the buyable compounds encountered in the 62,000 trees, we get 321,395 \( z(s) \) values. We split them with 8 : 2 ratio and get a training set of 257,116 compounds and a test set of 64,279 compounds.

The training of value networks based on compound solvent scores

In the Round 2 RL, we also collected the CSS values generated from the tree expansion of the 62,000 compounds. In total, we collected CSS(s) value for 282,398 compounds successfully solved in the tree expansion. Then we split them with 8 : 2 ratio and get a training set of 225,918 compounds and a test set of 56,480 compounds.

The training of the policy network and the extraction of templates

The training of the policy network is similar to the work by Segler et al.\textsuperscript{13}. With an input of the Morgan fingerprint, the policy network gives the probability distribution \( p(a|s) = P(s,a) \) over the allowed actions (templates) \( a \in A(s) \). The policy network is trained on 5.4 million reaction examples from Reays. The training and validation details and the model architecture can be found on Github.\textsuperscript{35} The extraction of templates is described in our previous work.\textsuperscript{34}

RSS calculation

We used the same reaction condition prediction model as described in Gao et al.\textsuperscript{14} The model predicts catalysts, solvents and reagents in a sequential manner. Since we only consider solvent greenness, we only use part of the model. Given a reaction, the model first predicts a catalyst, and combine the catalyst and reaction information to predict a solvent. We took the softmax scores for the top 3 solvent predictions and calculated a weighted sum as the RSS, as follows:

\[
\text{RSS} = \frac{\sum_{i=1}^{3} nn(s_i) \times \text{greenness}(s_i)}{\sum_{i=1}^{3} nn(s_i)}
\]

where \( nn(s_i) \) is the softmax score given by the neural network model for solvent prediction. \( \text{greenness}(s_i) \) is the greenness score of the solvents, which is defined in the way shown in Fig. 5. The definition of green, mediocre and non-green solvents is as reported by Byrne et al.\textsuperscript{41}

Training set and test set of the MCTS success rate experiments and greenness tests

The training set in Fig. 3 is the training set of the 112,000 compounds used in the bootstrapping phase. We randomly choose 1000 compounds from the training set of the value network training for test purpose of MCTS success rate experiment. The 1000 compounds from the training set are the same for all MCTS variants in Fig. 3.
The test set for the MCTS experiments in Fig. 3 are different from the test set of the value network validation. We choose the compounds in the Reaxys data base that are not in the training sets used in training the value networks, and are not in the “training set” in MCTS success rate experiments. There are in total 1 673 879 compounds in Reaxys that are not seen in the previous value network training process, and we use these compounds as the test set for MCTS success rate testing. In the MCTS success rate experiments shown in Fig. 3, we choose 1000 compounds from the test set randomly. The 1000 tested compounds from the test set are the same for all MCTS variants in Fig. 3.

The test set of the greenness tests in Fig. 6 is randomly selected 2000 compounds from the test set, and Tables S3 and S4† is randomly selected 500 compounds from the 1000 tested compounds used in the success rate experiments.

Conflicts of interest

There are no conflicts to declare.

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Notes and references

27 J. S. Schreck, https://github.com/jsschreck/retroRL.