

## PAPER

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## Extended layer-by-layer Madelung potential analysis of layered oxyhalide photocatalysts and other layered systems†

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A recent study on  $\text{Bi}_4\text{NbO}_8\text{Cl}$  and related Bi-based layered oxyhalide photocatalysts using a layer-by-layer Madelung potential ( $\text{L}^2\text{MP}$ ) analysis with a sphere model revealed that oxide anions in the fluorite-type slab of  $\text{Bi}_4\text{NbO}_8\text{Cl}$  are electrostatically destabilized by the 2nd nearest sublayer composed of the apical oxygen of the perovskite slab. Here we provide an extended  $\text{L}^2\text{MP}$  analysis of Bi-based layered oxyhalides using a modified (square-prism) model and apply it to other layered compounds. It is found that consideration of up to the 3rd nearest sublayer is necessary to account for the nearly equal valence band maximum between  $\text{Bi}_2\text{GdO}_4\text{X}$  and  $\text{Bi}_4\text{NbO}_8\text{Cl}$ . Additionally,  $\text{BiOX}$  and  $\text{Bi}_2\text{GdO}_4\text{X}$  show a distinct X-dependence of the Madelung site potential of oxide anions. The  $\text{L}^2\text{MP}$  analysis of  $\text{Bi}_2\text{WO}_6$ ,  $\text{Sr}_3\text{Sc}_2\text{Cu}_2\text{S}_2\text{O}_5$  and  $\text{Sr}_2\text{ScCuSO}_3$  revealed a sizable contribution of distant layers to their electrostatic potentials. This study demonstrates that this method can be applied to a wide range of layered materials and serves as a powerful guide toward finding composition and layer stacking sequences suitable for photocatalysis and other functions.

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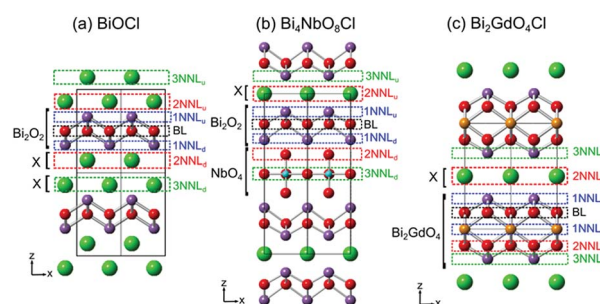
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## Introduction

There is an intimate relationship between the crystal structures of extended solids and the properties they exhibit. One of the key descriptors for ionic solids is the Madelung site potential, or the sum of the electrostatic potentials created at a specific crystallographic site by surrounding ions. The Madelung site potential was used to explain band formation in ionic solids such as  $\text{NaCl}$ ,  $\text{ZnS}$  and  $\text{SrTiO}_3$  in the 1960–70's,<sup>1–3</sup> and high- $T_c$  cuprate superconductors in the 1980–90's.<sup>4–6</sup> In recent years, there is a renewed interest in using the Madelung site potential, for example, valence skip fluctuation in  $\text{BaBiO}_3$ ,<sup>7</sup> the surface defect behaviour of metal oxides,<sup>8</sup> and the band location in eight  $\text{TiO}_2$  polymorphs.<sup>9,10</sup> Despite the long-range nature of Coulomb interactions, the difference in Madelung site potential is mainly explained by the local coordination environment and less attention appears to be paid to distant ions.<sup>4,9,10</sup>

Recently, we have developed a series of layered bismuth oxyhalide photocatalysts,  $\text{Bi}_4\text{NbO}_8\text{Cl}$ <sup>11</sup> and its derivatives,<sup>12–14</sup> as highly efficient visible-light responsive photocatalysts for water

splitting with extraordinary stability against self-oxidative deactivation by photo-generated holes. The photocatalytic stability of  $\text{Bi}_4\text{NbO}_8\text{Cl}$  and its narrow band gap (2.39 eV) are attributed to the presence of O 2p orbitals at the valence band maximum (VBM) rather than Cl 3p orbitals. The unusual upward shift of oxygen 2p bands is understood in terms of the Madelung site potential of anions<sup>15</sup> as well as the Bi lone pair effect.<sup>16</sup> In particular, Madelung site potential calculation revealed that the oxide anion in the fluorite-type blocks of  $\text{Bi}_4\text{NbO}_8\text{X}$  (X = Cl, Br) (Fig. 1b) and  $\text{Bi}_2\text{GdO}_4\text{X}$  (Fig. 1c) has a lower Madelung site potential in comparison with  $\text{BiOX}$  (Fig. 1a),



**Fig. 1** Crystal structures of (a)  $\text{BiOX}$  (X = Cl, Br, I), (b)  $\text{Bi}_4\text{NbO}_8\text{X}$  (X = Cl, Br) and (c)  $\text{Bi}_2\text{GdO}_4\text{X}$  (X = Cl, Br, I). The fluorite-derived basal layer (BL) and its neighboring sublayers ( $n$ -th nearest-neighbor layer;  $n\text{NNL}$ ) are indicated. Purple, red, yellow-green and blue spheres, respectively, represent Bi, O, halogen (X) and Nb atoms. The non-distorted structure is given for (b). For each, the unit cell is indicated by solid black lines.

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leading to the upward shift of the VBM. To further clarify the role and contribution of each layer, we subsequently developed a layer-by-layer Madelung potential ( $L^2MP$ ) analysis and found that the destabilization of the oxide anion in  $\text{Bi}_4\text{NbO}_8\text{X}$  and  $\text{Bi}_2\text{GdO}_4\text{X}$  is due to strong repulsive interactions from the oxide anions in the 2nd nearest-neighbor sublayer (2NNL; Fig. 1),<sup>15</sup> as shown in Fig. 2 (left) and S3.†

Given the long-range nature of Coulomb interactions, further sublayers may also affect the valence band structures of these Bi-based oxyhalides. However, the sphere model (Fig. 3a), where the electrostatic potentials of ions within  $R = 4 \text{ \AA}$  from the oxide anion in the fluorite-type layers were taken into consideration, cannot properly extract the contribution of the distant sublayers such as the 3NNL. In addition, the effect of halogens (X) on their Madelung potential has not been addressed, in spite of the fact that many isostructural (Cl, Br, and I) oxyhalides have different band gaps.

The aforementioned compounds are just a few examples of more than 100 oxyhalides with the  $\text{Bi}_2\text{O}_2$  fluorite layer.<sup>17</sup> Furthermore, there are a number of modular compounds whose structures are built from slabs of a parent motif interleaved with other layers. Available building blocks include anti-fluorite, rock salt and perovskite layers to name only a few. Such a structural diversity in layered materials has resulted in a wide range of functional properties including superconductivity in cuprates,<sup>18</sup> FeAs- and  $\text{BiS}_2$ -based compounds,<sup>19–21</sup> secondary batteries in layered rock-salt oxides,<sup>22</sup> and transparent conductivity in  $\text{Cu}_2\text{S}_2$ -based compounds.<sup>23,24</sup> We thus expect that Madelung analysis could be applied to other layered systems toward obtaining desired photocatalytic activities and other functions.

This study aims to address these issues by including a wider range of ions for  $\text{BiOX}$ ,  $\text{Bi}_4\text{NbO}_8\text{X}$  and  $\text{Bi}_2\text{GdO}_4\text{X}$  as well as other

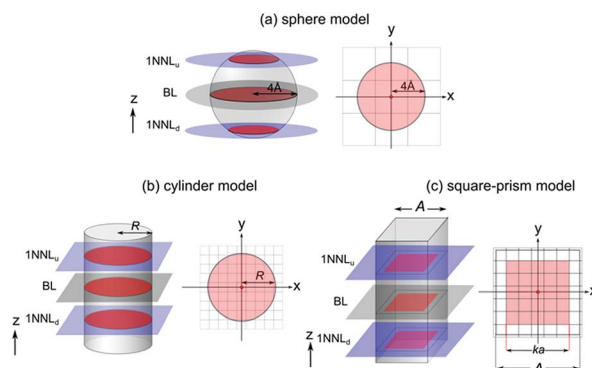


Fig. 3 Three models employed for calculating the electrostatic potential at the oxygen site in the BL from each sublayer: (a) sphere model,<sup>15</sup> (b) cylinder model, and (c) square-prism model. For each model, cations and anions within the red area are taken into consideration. On the right side of each panel, the unit cell is indicated in grey. The red area in the square-prism model shows that  $-ka/2 \leq x, y \leq ka/2$  (where  $k$  is an odd number satisfying  $ka \leq A \leq (k+1)a$ ). The oxide anion in question is placed at the center of (a) sphere, (b) circle, and (c) square.

classes of layered materials. Two alternate models are employed for layer-by-layer Madelung potential ( $L^2MP$ ) analysis by changing the size included in the calculation. This extended analysis revealed that a 3NN sublayer plays a crucial role in the valence band structures in  $\text{Bi}_2\text{GdO}_4\text{X}$  and  $\text{Bi}_4\text{NbO}_8\text{Cl}$ , while the origin of the X-dependence of the band gap is discussed. In addition,  $L^2MP$  analysis is also applied to  $\text{Bi}_2\text{WO}_6$ ,  $\text{Sr}_3\text{Sc}_2\text{Cu}_2\text{S}_2\text{O}_5$  and  $\text{Sr}_2\text{ScCuSO}_3$  to investigate the impact of distant sublayers on their Madelung potential.

## Methods

The electrostatic potential was calculated using the structural data reported in ref. 25–31. For  $\text{Bi}_4\text{NbO}_8\text{Cl}$  and  $\text{Bi}_2\text{WO}_6$ , non-distorted lattices were constructed, where the atomic coordinates of each atomic position were set to the average values of experimentally obtained structures, as shown in Tables S1 and S2.†<sup>25,31</sup> Structural optimization based on density functional theory (DFT) was performed for the hypothetical  $\text{Bi}_4\text{NbO}_8\text{Cl}$  structure using the Cambridge Serial Total Energy Package (CASTEP).<sup>32</sup> The energy was calculated using the PbEsol XC functional.<sup>33</sup> Cutoff energy and  $k$ -point grids were 630 eV and  $4 \times 4 \times 1$ . As illustrated in Fig. 3, the following two models were used to estimate the electrostatic potential generated at the oxygen site in the fluorite-type  $\text{Bi}_2\text{O}_2$  (or  $\text{Bi}_2\text{GdO}_4$ ) slab from each sublayer. The first model (cylinder model) took account of the electrostatic interactions between the oxide anion at the center of a cylinder with a radius of  $R$  (Fig. 3b) and all other atoms in the cylinder. Another model (square-prism model) considered the electrostatic interaction between the oxide anion at the center of a square-prism with a side length of  $A$  (Fig. 3c) and all other atoms within  $-ka/2 \leq x, y \leq ka/2$  ( $k$  is an odd number satisfying  $ka \leq A \leq (k+1)a$ ). The central oxygen was located at  $(x, y) = (0, 0)$ .

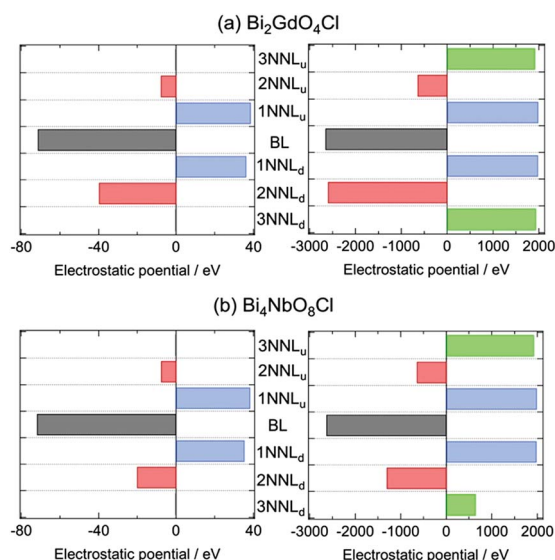


Fig. 2 Electrostatic potential at the oxygen site in the fluorite-type layer from adjacent sublayers of (a)  $\text{Bi}_2\text{GdO}_4\text{Cl}$  and (b)  $\text{Bi}_4\text{NbO}_8\text{Cl}$ . Calculation using (left) the sphere model<sup>15</sup> and (right) the square-prism model (this work) for  $A = 200 \text{ \AA}$  (Fig. 3). The non-distorted hypothetical structure was used for  $\text{Bi}_4\text{NbO}_8\text{Cl}$ .

In calculating the Madelung site potential at the oxygen site, we fixed the value of  $R$  (cylinder model) or  $A$  (square-prism model) and changed the  $z$ -axis range. It is important to employ an electrically neutral domain to achieve reasonable convergence when the calculation range expands. For this reason, we have defined the charge-neutral sublayer set for  $(m - 1)c \leq |z| < mc$  (where  $m = 1, 2, 3, \dots$ ) as  $U_1, U_2, U_3, \dots, U_m, \dots$  (see Fig. S4†), and calculated the Madelung potential as a function of  $m$ .

The Madelung site potential of  $\text{Bi}_4\text{NbO}_8\text{Cl}$  with the distortion-free model was calculated by the Fourier method incorporated into the VESTA program.<sup>34</sup> The radius of an ionic sphere and the reciprocal-space range were, respectively, set at 1.8 Å and  $3 \text{ Å}^{-1}$  after convergence of the Madelung site potentials with respect to these two parameters was checked.

## Results and discussion

We first applied the two models to  $\text{BiOCl}$  and checked how the Madelung potential at the oxide anion of the BL from adjacent sublayers evolves and is converged into the theoretical value when the lateral size included in the calculation increases. In the cylinder model, the electrostatic potential as a function of  $m$  is largely affected by  $R$  (Fig. 4a) and the electrostatic potential did not approach the value previously obtained by the Fourier method based on the calculation in reciprocal space, which is probably because the electrical neutrality is not guaranteed in this model.<sup>15,34</sup> In the square-prism model, in contrast, the electrostatic potential converged rapidly with  $A$  (Fig. 4b). Regardless of the choice of  $A$ , the electrostatic potential monotonically decreased with  $m$  and quickly converged to the expected value of 20.8 eV (Fig. 4b).

These results indicate that the square-prism model is more appropriate, so this model was applied to other Bi-containing oxyhalides (Fig. 5). The electrostatic potential for  $A = 200 \text{ Å}$  successfully converged to a value close to that obtained from the Fourier method: 21.9 eV for  $\text{BiOBr}$ , 23.7 eV for  $\text{BiOI}$  and 17.8 eV for  $\text{Bi}_2\text{GdO}_4\text{Cl}$ . As for  $\text{Bi}_4\text{NbO}_8\text{Cl}$ , we encountered a problem of conditional convergence<sup>35</sup> even for the square-prism model; due to the orthorhombic structure involving octahedral rotations

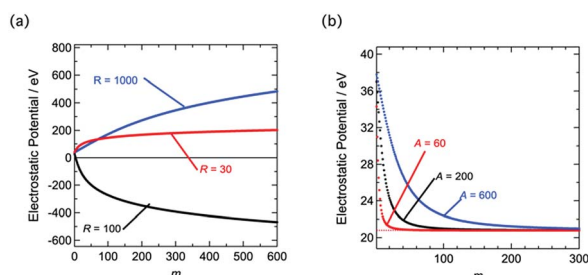


Fig. 4 The  $m$  dependence of the total electrostatic potential at the oxygen site of  $\text{BiOCl}$  calculated with (a) the cylinder model and (b) the square-prism model, for a fixed size of the circle ( $R = 30 \text{ Å}$ ,  $100 \text{ Å}$  and  $1000 \text{ Å}$ ) and square ( $A = 60 \text{ Å}$ ,  $200 \text{ Å}$  and  $600 \text{ Å}$ ). Madelung potential obtained by the Fourier method (corresponding to the infinite-size limit) is shown by the dotted red line in (b).

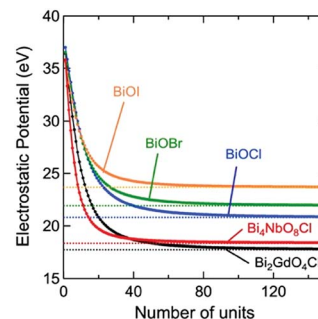


Fig. 5 Total electrostatic potential created at the oxygen site (in the fluorite-type layer) for  $\text{BiOX}$ ,  $\text{Bi}_2\text{GdO}_4\text{Cl}$  and  $\text{Bi}_4\text{NbO}_8\text{Cl}$  and plotted with respect to the number of  $U_m$ . Madelung site potentials of the oxygen site obtained by the Fourier method (implemented in the VESTA program<sup>34</sup>) are shown by the dotted lines. The Madelung site potentials of  $\text{BiOX}$  and  $\text{Bi}_2\text{GdO}_4\text{Cl}$  are taken from a previous report,<sup>15</sup> while a non-distorted structure was used for  $\text{Bi}_4\text{NbO}_8\text{Cl}$ .

(Fig. S2†), it was not possible to find an appropriate charge-neutral domain. Therefore, a non-distorted tetragonal structure ( $P4/mmm$  space group) was constructed. The difference in lattice parameters before and after optimization of this hypothetical structure is 1.23% (Tables S1 and S3†), which is within the margin of error of DFT calculation using the PBEsol XC functional. Using this hypothetical structure, we obtained a Madelung site potential of 18.3 eV by the Fourier method for the oxygen site in the fluorite-type  $\text{Bi}_2\text{O}_3$ . This is similar to the values for the actual structure (O1: 18.9 eV, O2: 18.2 eV, O3: 18.3 eV, and O4: 17.5 eV),<sup>15</sup> suggesting that the presence of octahedral tilting does not significantly influence the Madelung site potential. The electrostatic potential at the oxygen site using the non-distorted model converged well to the value obtained by the Fourier method (Fig. 5).

Fig. 6 shows the electrostatic potentials for  $\text{BiOCl}$  and  $\text{Bi}_2\text{YO}_4\text{Cl}$  from the BL,  $1\text{NNL}_u$  and  $2\text{NNL}_u$  plotted as a function of  $A$ . Because of common units (BL:  $[\text{O}_2]^{4-}$ ,  $1\text{NNL}$ :  $[\text{Bi}]^{3+}$  and  $2\text{NNL}$ :  $[\text{Cl}]^-$ ), the electrostatic potentials of each sublayer are almost the same. Moreover, the Madelung potential from each sublayer exhibits a linear dependence, with the slope ratio of BL :  $1\text{NNL}$  :  $2\text{NNL} = 4 : 3 : 1$  corresponding to the ratio of the formal charge in each sublayer:  $[\text{O}_2]^{4-}$ ,  $[\text{Bi}]^{3+}$  and  $[\text{Cl}]^-$  (see the ESI† for details).

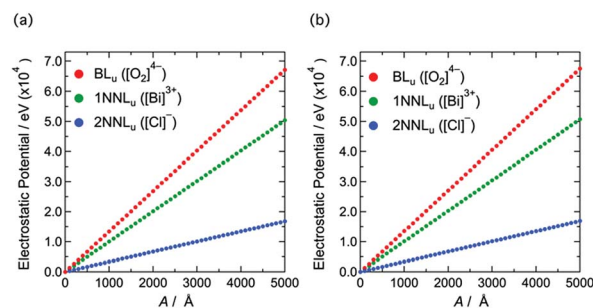


Fig. 6 The  $A$  dependence of the absolute value of electrostatic potential from the BL,  $1\text{NNL}_u$  and  $2\text{NNL}_u$  in (a)  $\text{BiOCl}$  and (b)  $\text{Bi}_2\text{GdO}_4\text{Cl}$  calculated using the square-prism model.



The sphere model, employed previously,<sup>15</sup> revealed that the Madelung site potential at the oxygen site in Bi<sub>2</sub>O<sub>2</sub> slabs of Bi<sub>4</sub>NbO<sub>8</sub>Cl (18.2 eV on average) is smaller than that of BiOCl (20.8 eV), which was attributed to the smaller electrostatic potential generated by the 2NNL<sub>d</sub> in Bi<sub>4</sub>NbO<sub>8</sub>Cl (−20.3 eV *vs.* −9.0 eV for BiOCl; Fig. S3†). However, when Bi<sub>4</sub>NbO<sub>8</sub>Cl and Bi<sub>2</sub>GdO<sub>4</sub>Cl are compared, the oxygen site in the latter is much more destabilized by the 2NNL<sub>d</sub> (−40.0 eV of Bi<sub>2</sub>GdO<sub>4</sub>Cl *vs.* −20.3 eV of Bi<sub>4</sub>NbO<sub>8</sub>Cl) as shown in Fig. 2. For this reason, the electrostatic potentials from several sublayers for Bi<sub>2</sub>GdO<sub>4</sub>Cl and hypothetical Bi<sub>4</sub>NbO<sub>8</sub>Cl were calculated using the square-prism model with  $A = 200$  Å (Fig. 2). It is seen that while the BL, 1NNL<sub>s</sub> and 2NNL<sub>u</sub> of the two compounds generate nearly the same electrostatic potential, the electrostatic potential of Bi<sub>2</sub>GdO<sub>4</sub>Cl from the 2NNL<sub>d</sub> is almost twice as large as that of Bi<sub>4</sub>NbO<sub>8</sub>Cl. Although the 2NNL<sub>d</sub> in both compounds has an oxygen square network, their densities are different, with the formal charges of [O<sub>2</sub>]<sup>4−</sup> in Bi<sub>2</sub>GdO<sub>4</sub>Cl and [O]<sup>2−</sup> in Bi<sub>4</sub>NbO<sub>8</sub>Cl. Importantly, this model shows the crucial contribution from the 3NNL. Since the 3NNL<sub>u</sub> of both compounds is composed of the [Bi]<sup>3+</sup> sublayer (Fig. 1), there is little difference in electrostatic potential. On the other hand, the 3NNL<sub>d</sub> of Bi<sub>4</sub>NbO<sub>8</sub>Cl consists of a Nb square network ( $d_{\text{Nb-Nb}} = 3.9$  Å =  $a$ ) and O square network ( $d_{\text{O-O}} = 2.7$  Å =  $1/\sqrt{2}a$ ), while that of Bi<sub>2</sub>GdO<sub>4</sub>Cl consists of a Bi square network ( $d_{\text{Bi-Bi}} = 3.9$  Å), giving electrostatic potentials of 662 eV and 1935 eV, respectively. These

values are again correlated with the formal charges of [NbO<sub>2</sub>]<sup>+</sup> and [Bi]<sup>3+</sup>. Accordingly, stabilization of Bi<sub>4</sub>NbO<sub>8</sub>Cl (*vs.* Bi<sub>2</sub>GdO<sub>4</sub>Cl) by the 2NNL<sub>d</sub> is compensated by destabilization by the 3NNL<sub>d</sub>, which accounts for the similar values of Madelung site potential in these compounds. The present results demonstrate that 3NNLs have a substantial impact on the Madelung potential at the oxygen site in the BL, thereby affecting the valence band structures.

Let us briefly discuss the influence of halogen anions on Madelung potential by comparing the isostructural series, BiOX or Bi<sub>2</sub>GdO<sub>4</sub>X (X = Cl, Br, I). The Madelung site potential of BiOX increases with increasing the ionic radius of X,  $r_X$ , whereas that of Bi<sub>2</sub>GdO<sub>4</sub>X hardly changes (Fig. S1†). This results from highly anisotropic lattice expansion (Table S4†);  $\Delta c/\Delta a$  is 16.2 for BiOX and 13.2 for Bi<sub>2</sub>GdO<sub>4</sub>X, where  $\Delta a$  and  $\Delta c$  are the differences in the lattice constant between X = Cl and I. The evolution of the interlayer distance is more pronounced for Bi-X and X-X (Fig. S6†). In particular, the X-X distance in BiOX (2NNLs and 3NNLs) remarkably changes with X. It is then likely that the predominant expansion electrostatically stabilizes the oxygen layer (BL), indicating that a double halogen layer can be used to tune the Madelung site potential.

Now that we possess a tool (square-prism model) that can evaluate the impact of distant sublayers, we are ready to apply the square-prism model to other layered compounds. This model was applied to Sr<sub>3</sub>Sc<sub>2</sub>Cu<sub>2</sub>S<sub>2</sub>O<sub>5</sub> and Sr<sub>2</sub>ScCuSO<sub>3</sub> with the antifluorite Cu<sub>2</sub>S<sub>2</sub> layer as shown in Fig. 7a. They show excitonic luminescence properties, which can be tuned by the composition and stacking pattern.<sup>30</sup> Sr<sub>3</sub>Sc<sub>2</sub>Cu<sub>2</sub>S<sub>2</sub>O<sub>5</sub> is also a promising p-type transparent conductor due to its high conductivity and a large band gap.<sup>24</sup> The Madelung potential at the sulfur site (BL) for Sr<sub>3</sub>Sc<sub>2</sub>Cu<sub>2</sub>S<sub>2</sub>O<sub>5</sub> is 17.0 eV, which is smaller than 18.2 eV for Sr<sub>2</sub>ScCuSO<sub>3</sub>. As shown in Fig. 7b, sublayers up to 3NNLs and 4NNL<sub>u</sub> are common to the two compounds and thus give nearly the same electrostatic potential. The smaller Madelung potential of Sr<sub>3</sub>Sc<sub>2</sub>Cu<sub>2</sub>S<sub>2</sub>O<sub>5</sub> can be attributed to the smaller electrostatic potential from the 4NNL<sub>d</sub> sublayer in Sr<sub>3</sub>Sc<sub>2</sub>Cu<sub>2</sub>S<sub>2</sub>O<sub>5</sub>.

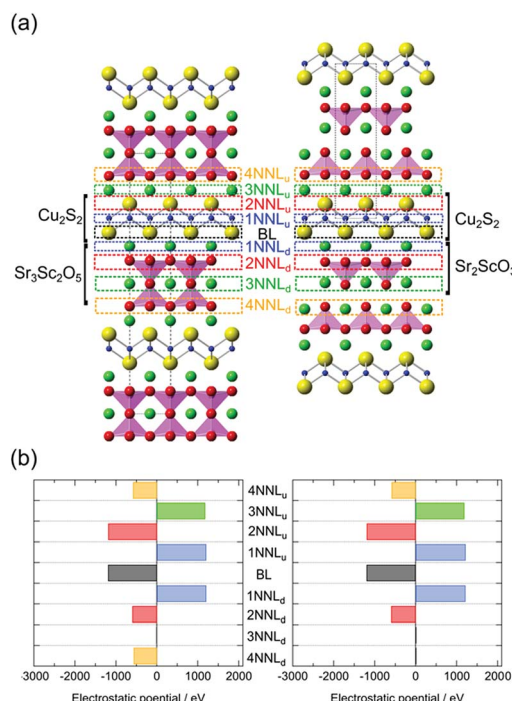


Fig. 7 (a) Crystal structures of Sr<sub>3</sub>Sc<sub>2</sub>Cu<sub>2</sub>S<sub>2</sub>O<sub>5</sub> (left) and Sr<sub>2</sub>ScCuSO<sub>3</sub> (right) with antifluorite-derived Cu<sub>2</sub>S<sub>2</sub> layers. The S layer and its neighboring sublayers ( $n$ -th nearest-neighbor layer;  $n$ NNL) are indicated. Blue, yellow, green and red spheres, respectively, represent Cu, S, Sr and O atoms, and ScO<sub>5</sub> pyramids are shown in pink. (b) Electrostatic potential at the sulfur site in the Cu<sub>2</sub>S<sub>2</sub> layer from adjacent sublayers calculated using the square-prism model with  $A = 200$  Å.

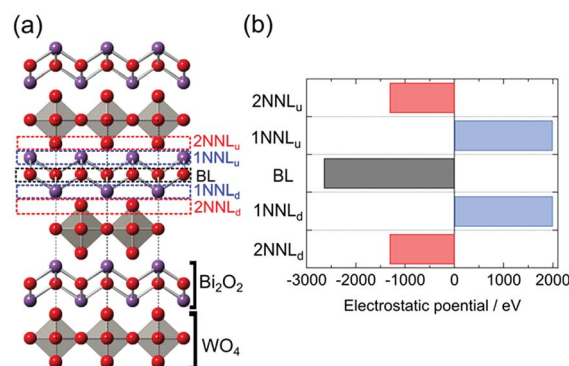


Fig. 8 (a) The non-distorted structure of Bi<sub>2</sub>WO<sub>6</sub>. The fluorite-derived basal layer (BL) and its neighboring sublayers ( $n$ -th nearest-neighbor layer;  $n$ NNL) are indicated. Purple and red spheres represent Bi and O atoms, respectively. (b) Electrostatic potential at the oxygen site of Bi<sub>2</sub>WO<sub>6</sub> in the fluorite-type (BL) layer from adjacent sublayers calculated using the square-prism model with  $A = 200$  Å.

composed of  $[\text{ScO}_2]^-$  compared with that of the  $4\text{NNL}_d$  in  $\text{Sr}_2\text{-ScCuSO}_3$  composed of  $[\text{SrO}]^0$  sublayers.

$\text{Bi}_2\text{WO}_6$ , a visible-light responsive photocatalyst,<sup>36</sup> has a layer stacking similar to that of  $\text{Bi}_4\text{NbO}_8\text{Cl}$  with fluorite-type  $\text{Bi}_2\text{O}_2$  and perovskite-type  $\text{WO}_4$  layers, but the halogen layer is absent (Fig. 8a). The Madelung potential at the oxygen site in the  $\text{Bi}_2\text{O}_2$  layer is 14.9 eV on average, which is much smaller than that of  $\text{Bi}_4\text{NbO}_8\text{Cl}$  (18.2 eV). The  $\text{L}^2\text{MP}$  analysis using a hypothetical non-distorted lattice (Fig. 8b) shows that the difference arises from a smaller electrostatic potential from the  $[\text{O}]^{2-}$  sublayer in the  $2\text{NNL}_u$  (vs. the  $[\text{Cl}]^-$  sublayer in  $\text{Bi}_4\text{NbO}_8\text{Cl}$ ). Thus, one expects a lower VBM for  $\text{Bi}_2\text{WO}_6$ . However, the experimentally obtained VBM of  $\text{Bi}_2\text{WO}_6$  of 2.68 V (vs. SHE at pH 2.0) is much lower than that of  $\text{Bi}_4\text{NbO}_8\text{Cl}$  of 2.27 V.<sup>12</sup> Such a discrepancy implies that there are other factors, such as the hybridization effect between Bi 6s orbitals and O 2p orbitals, which has been shown to be quite strong in  $\text{Bi}_4\text{NbO}_8\text{Cl}$ .<sup>16</sup>

## Conclusions

We investigate the influence of distant sublayers on Madelung site potential by developing an alternate model for  $\text{L}^2\text{MP}$  analysis and applying it to  $\text{Bi}_2\text{O}_2$ -based oxyhalide photocatalysts ( $\text{BiOX}$ ,  $\text{Bi}_2\text{GdO}_4\text{X}$  and  $\text{Bi}_4\text{NbO}_8\text{X}$ ) and other layered materials. Among the two models examined, the square-prism model shows better convergence. A significant contribution of electrostatic potential from the  $3\text{NNL}$  explains nearly the same Madelung site potential between  $\text{Bi}_2\text{GdO}_4\text{Cl}$  and  $\text{Bi}_4\text{NbO}_8\text{Cl}$ . In addition, a strong X dependence of the Madelung site potential is observed in  $\text{BiOX}$ . The  $\text{L}^2\text{MP}$  analysis of  $\text{Bi}_2\text{WO}_6$ ,  $\text{Sr}_3\text{Sc}_2\text{Cu}_2\text{-S}_2\text{O}_5$  and  $\text{Sr}_2\text{ScCu}_5\text{O}_3$  further revealed a distant layer contribution to the Madelung site potential. Given the enormous variations of composition and layer stacking sequences in layered oxyhalides and more generally mixed-anion compounds,<sup>37</sup>  $\text{L}^2\text{MP}$  analysis offers large potential for the prediction and tuning of desired valence band structures for photocatalytic applications and other functions, possibly with the aid of a machine learning approach.

## Conflicts of interest

The authors declare no competing financial interest.

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## References

- 1 P. A. Cox, *The Electronic Structure and Chemistry of Solids*, Oxford University Press, Oxford, United Kingdom, 1986.
- 2 A. H. Kahn and A. J. Leyendecker, *Phys. Rev.*, 1964, **135**, A1321–A1325.
- 3 J. D. Levine and S. Freeman, *Phys. Rev. B: Solid State*, 1970, **2**, 3255–3272.
- 4 J. B. Torrance and R. M. Metzger, *Phys. Rev. Lett.*, 1989, **63**, 1515–1518.
- 5 Y. Ohta, T. Tohyama and S. Maekawa, *Phys. Rev. Lett.*, 1991, **66**, 1228–1231.
- 6 Y. Ohta, T. Tohyama and S. Maekawa, *Phys. Rev. B: Condens. Matter Mater. Phys.*, 1991, **43**, 2968–2982.
- 7 I. Hase and T. Yanagisawa, *Phys. Rev. B: Condens. Matter Mater. Phys.*, 2007, **76**, 174103.
- 8 A. Walsh, *Appl. Phys. Lett.*, 2011, **98**, 261910.
- 9 D. O. Scanlon, C. W. Dunnill, J. Buckeridge, S. A. Shevlin, A. J. Logsdail, S. M. Woodley, C. R. Catlow, M. J. Powell, R. G. Palgrave, I. P. Parkin, G. W. Watson, T. W. Keal, P. Sherwood, A. Walsh and A. A. Sokol, *Nat. Mater.*, 2013, **12**, 798–801.
- 10 J. Buckeridge, K. T. Butler, C. R. A. Catlow, A. J. Logsdail, D. O. Scanlon, S. A. Shevlin, S. M. Woodley, A. A. Sokol and A. Walsh, *Chem. Mater.*, 2015, **27**, 3844–3851.
- 11 H. Fujito, H. Kunioku, D. Kato, H. Suzuki, M. Higashi, H. Kageyama and R. Abe, *J. Am. Chem. Soc.*, 2016, **138**, 2082–2085.
- 12 H. Kunioku, M. Higashi, C. Tassel, D. Kato, O. Tomita, H. Kageyama and R. Abe, *Chem. Lett.*, 2017, **46**, 583–586.
- 13 H. Suzuki, H. Kunioku, M. Higashi, O. Tomita, D. Kato, H. Kageyama and R. Abe, *Chem. Mater.*, 2018, **30**, 5862–5869.
- 14 A. Nakada, A. Saeki, M. Higashi, H. Kageyama and R. Abe, *J. Mater. Chem. A*, 2018, **6**, 10909–10917.
- 15 D. Kato, K. Hongo, R. Maezono, M. Higashi, H. Kunioku, M. Yabuuchi, H. Suzuki, H. Okajima, C. Zhong, K. Nakano, R. Abe and H. Kageyama, *J. Am. Chem. Soc.*, 2017, **139**, 18725–18731.
- 16 H. Kunioku, M. Higashi, O. Tomita, M. Yabuuchi, D. Kato, H. Fujito, H. Kageyama and R. Abe, *J. Mater. Chem. A*, 2018, **6**, 3100–3107.
- 17 D. O. Charkin, *Russ. J. Inorg. Chem.*, 2008, **53**, 1977–1996.
- 18 Y. Tokura and T. Arima, *Jpn. J. Appl. Phys., Part 1*, 1990, **29**, 2388–2402.
- 19 Y. Kamihara, T. Watanabe, M. Hirano and H. Hosono, *J. Am. Chem. Soc.*, 2008, **130**, 3296–3297.
- 20 K. Tanabe and H. Hosono, *Jpn. J. Appl. Phys.*, 2012, **51**, 010005.
- 21 Y. Mizuguchi, *J. Phys. Soc. Jpn.*, 2019, **88**, 041001.
- 22 J. B. Goodenough and K. S. Park, *J. Am. Chem. Soc.*, 2013, **135**, 1167–1176.
- 23 K. Ueda, S. Inoue, S. Hirose, H. Kawazoe and H. Hosono, *Appl. Phys. Lett.*, 2000, **77**, 2701–2703.
- 24 M. L. Liu, L. B. Wu, F. Q. Huang, L. D. Chen and I. W. Chen, *J. Appl. Phys.*, 2007, **102**, 116108.
- 25 A. M. Kusainova, S. Y. Stefanovich, V. A. Dolgikh, A. V. Mosunov, C. H. Hervoches and P. Lightfoot, *J. Mater. Chem.*, 2001, **11**, 1141–1145.
- 26 M. Schmidt, H. Oppermann, C. Hennig, R. W. Henn, E. Gmelin, N. Soger and M. Binnewies, *Z. Anorg. Allg. Chem.*, 2000, **626**, 125–135.
- 27 K. G. Keramidas, G. P. Voutsas and P. I. Rentzeperis, *Z. Kristallogr.-Cryst. Mater.*, 1993, **205**, 35–40.

- 28 J. Ketterer and V. Kramer, *Acta Crystallogr., Sect. C: Cryst. Struct. Commun.*, 1986, **42**, 1098–1099.
- 29 K. Ottschi, H. Ogino, J. Shimoyama and K. Kishio, *J. Low Temp. Phys.*, 1999, **117**, 729–733.
- 30 H. Ogino, J. Shimoyama, K. Kishio, Y. Katsura, M. Tsuboi, K. Yamanoi, M. Cadatal-Raduban, T. Nakazato, T. Shimizu and N. Sarukura, *Appl. Phys. Lett.*, 2012, **101**, 191901.
- 31 H. Okudera, Y. Sakai, K. Yamagata and H. Takeda, *Acta Crystallogr., Sect. B: Struct. Sci., Cryst. Eng. Mater.*, 2018, **74**, 295–303.
- 32 S. J. Clark, M. D. Segall, C. J. Pickard, P. J. Hasnip, M. J. Probert, K. Refson and M. C. Payne, *Z. Kristallogr.–Cryst. Mater.*, 2005, **220**, 567–570.
- 33 J. Perdew, A. Ruzsinszky, G. Csonka, O. Vydrov, G. Scuseria, L. Constantin, X. Zhou and K. Burke, *Phys. Rev. Lett.*, 2008, **100**, 136406.
- 34 K. Momma and F. Izumi, *J. Appl. Crystallogr.*, 2011, **44**, 1272–1276.
- 35 H. M. Eyring, *Phys. Rev.*, 1932, **39**, 675–687.
- 36 A. Kudo and S. Hijii, *Chem. Lett.*, 1999, **28**, 1103–1104.
- 37 H. Kageyama, K. Hayashi, K. Maeda, J. P. Attfield, Z. Hiroi, J. M. Rondinelli and K. R. Poeppelmeier, *Nat. Commun.*, 2018, **9**, 772.