Microstructural analysis, magnetic properties, magnetocaloric effect, and critical behaviors of Ni$_{0.6}$Cd$_{0.2}$Cu$_{0.2}$Fe$_2$O$_4$ ferrites prepared using the sol–gel method under different sintering temperatures

Noura Kouki,$^{ab}$ Sobhi Hcini, $^{cd}$* Michel Boudard, $^d$ Reema Aldawas $^a$ and Abdessalem Dhahri$^e$

This work focuses on the microstructural analysis, magnetic properties, magnetocaloric effect, and critical exponents of Ni$_{0.6}$Cd$_{0.2}$Cu$_{0.2}$Fe$_2$O$_4$ ferrites. These samples, denoted as S1000 and S1200, were prepared using the sol–gel method and sintered separately at 1000 °C and 1200 °C, respectively. XRD patterns confirmed the formation of cubic spinel structures and the Rietveld method was used to estimate the different structural parameters. The higher sintering temperature led to an increased lattice constant ($a$), crystallite size ($D$), magnetization ($M$), Curie temperature ($T_C$), and magnetic entropy change ($-\Delta S_M$) for samples that exhibited second-order ferromagnetic→paramagnetic (FM→PM) phase transitions. The magnetic entropy change at an applied magnetic field ($\mu_0H$) of 5 T, reaching maximum values of about 1.57–2.12 J kg$^{-1}$ K$^{-1}$, corresponding to relative cooling powers (RCPs) of 115 and 125 J kg$^{-1}$ for S1000 and S1200, respectively. Critical exponents ($\beta$, $\gamma$, and $\delta$) for samples around their $T_C$ values were studied by analyzing the $M(\mu_0H, T)$ isothermal magnetizations using different techniques and checked by analyzing the $-\Delta S_M$ vs. $\mu_0H$ curves. The estimated values of $\beta$ and $\gamma$ exponents (using the Kouvel–Fisher method) and $\delta$ exponent (from $M(T_C, \mu_0H)$ critical isotherms) were $\beta = 0.443 \pm 0.003$, $\gamma = 1.032 \pm 0.001$, and $\delta = 3.311 \pm 0.006$ for S1000, and $\beta = 0.403 \pm 0.008$, $\gamma = 1.073 \pm 0.016$, and $\delta = 3.650 \pm 0.005$ for S1200. Obviously, these critical exponents were affected by increased sintering temperature and their values were different to those predicted by standard theoretical models.

1. Introduction

Magnetic refrigeration (MR) is an alternative technology to refrigeration based on the conventional vapor cycle technique.\cite{1} Owing to its economic and environmental benefits, especially its energy efficiency, magnetic refrigeration is a promising technique for the future of refrigeration. Therefore, the equivalent of a conventional thermodynamic cycle can be realized magnetically with better energy efficiency and without greenhouse gases. Objectives in this area are the synthesis, optimization, and implementation of materials with high magnetocaloric power that are effective, economical, ecological, and practical for medium term implementation in pilot refrigeration systems. MR is based on the magnetocaloric effect (MCE), which is an intrinsic thermodynamic property of magnetic materials that causes a change in the temperature of the substance under the action of a magnetic field. Gadolinium (Gd) is a rare earth metal with a Curie temperature ($T_C$) close to room temperature. Gd is the only pure metallic magnetic material that has a high MCE around its Curie temperature ($T_C = 293$ K).\cite{2,3} Although Gd is the first choice for academic MR research near room temperature, its use is commercially limited owing to its high price ($\sim$4000 $\$/kg) and easy oxidation. In recent years, large MCEs have been found in other materials, such as perovskite manganites $\text{ABO}_3$ (ref. 4–8) and spinel ferrites Fe$_2$O$_4$, which have attracted increasing attention in MR research. The main advantages of these materials over Gd and GdSiGe alloys\cite{15,16} are low coercive forces, nonactive chemical properties (no oxidation), low costs, and high electric resistance (minimum Eddy current loss).\cite{16} Spinel ferrites have been studied less than perovskite materials in the field of MR. Accordingly, in this study, we have prepared ferrite samples with Ni$_{0.6}$Cd$_{0.2}$Cu$_{0.2}$Fe$_2$O$_4$ compositions using the sol–gel method at sintering temperatures of 1000 °C and 1200 °C. We
then investigated the effect of sintering temperature on their microstructural, magnetic, and magnetocaloric properties. Finally, a detailed study of critical exponents near their Curie temperatures ($T_C$) was conducted using various techniques, such as modified Arrott plots, the Kouvel–Fisher method, and critical isotherm analysis. Throughout this manuscript, we have used the abbreviations S1000 and S1200 to denote Ni$_{0.6}$Cd$_{0.2}$Cu$_{0.2}$Fe$_2$O$_4$ ferrites sintered at 1000 °C and 1200 °C, respectively.

2. Experimental details

2.1. Synthetic process

Ni$_{0.6}$Cd$_{0.2}$Cu$_{0.2}$Fe$_2$O$_4$ ferrites were synthesized from [Ni(NO$_3$)$_2$·6H$_2$O], [Cd(NO$_3$)$_2$·4H$_2$O], [Cu(NO$_3$)$_2$·3H$_2$O] and [Fe(NO$_3$)$_3$·9H$_2$O] nitrates using the sol–gel method, the main steps of which are shown in Fig. 1. Stoichiometric amounts of nitrates were first dissolved in distilled water with regular stirring at 90°C.

Fig. 1  Schematic diagram representing the various synthesis steps for S1000 and S1200 samples using the sol–gel method.
90 °C on a hot plate. Controlled quantities of citric acid, used as a complexation agent, were added to the mixed solution. The solution pH was adjusted to about 7 by adding ammonia. Stoichiometric amounts of ethylene glycol, used as a polymerization agent, were then added to the solution. Heating and stirring were continued until gel formation after approximately 4 h. The gel was dried at 300 °C for 6 h to obtain a dry foam, which was then ground in a mortar and further dried at 500 °C for 12 h in air. The resulting powder was pressed into pellets (diameter, 10 mm; thickness, about 2 mm) and sintered at 700 °C for 12 h. Finally, the powder was ground and pressed again, and divided into two portions, which were sintered separately for 24 h at 1000 °C and 1200 °C, respectively.

2.2. Characterization

A PANalytical X’Pert Pro diffractometer using Ni-filtered CuKα radiation (λ = 1.5406 Å) was used to record X-ray diffraction patterns in the angular range of 20–80°. Rietveld analysis was using FullProf software was adopted for detailed structural characterization. The free surface morphologies of samples in pellet form were analyzed by scanning electron microscopy (SEM) using a Philips XL30 microscope under an accelerating voltage of 15 kV. A linear extraction magnetometer was used to record the temperature and magnetic field dependences of the magnetization. M(T) curves were measured in the temperature range 300 K ≤ T ≤ 750 K under a constant magnetic field (μ0H = 0.05 T), and M(μ0H, T) isotherms were taken at different temperatures near Tc in the magnetic field range 0 T ≤ μ0H ≤ 5 T.

3. Results and discussions

3.1. Microstructural analysis

3.1.1. XRD patterns. XRD patterns of S1000 and S1200 are shown in Fig. 2. Analysis of these patterns confirmed that both samples had cubic spinel structures with a Fd3m space group. Furthermore, no additional peaks related to phase impurities were detected, confirming the sample purity. As shown in the inset of Fig. 2, the XRD patterns showed a shift in the most intense peak (311) position to lower diffraction angles (2θ) with increasing sintering temperature. This indicated that the lattice constant changed with increasing sintering temperature. The compound structures were refined to Fd3m symmetry, accounting for the cation distributions (Cd0.2Fe2.8)[Ni0.6Cu0.2Fe1.2]O4. The distributions of Cd2+, Ni2+, Cu2+, and Fe3+ ions were proposed based on literature precedent. The atomic positions were fixed at 8a (1/8, 1/8, 1/8) for (Cd0.2Fe2.8)A cations, 16d (1/2, 1/2, 1/2) for [Ni0.6Cu0.2Fe1.2]B cations, and 32e (x, y, z) for O. Refinements of all parameters were continued until the value of the quality factor, χ2 (goodness of fit), was close to unity, which confirmed the goodness of the refinement (Fig. 3). The lattice constant (a), volume (V), cation–oxygen distances in A and B sites (R_A and R_B), average crystallite size (D), and intensity ratio (I_{220}/I_{222}) were obtained from Rietveld refinement, as summarized in Table 1. The results clearly indicated that the lattice parameters...
increased, while the $R_A$ and $R_B$ distances decreased, with increasing sintering temperature. This was in agreement with the results of previous studies.\textsuperscript{22,23} Furthermore, the obtained oxygen coordinate values were characteristic of a spinel-type structure.\textsuperscript{24,25} The average crystallite sizes of the samples were obtained from XRD peaks using the Scherrer equation: \textsuperscript{22}

\begin{equation}
D = \frac{0.9\lambda}{\beta \cos(\theta)}
\end{equation}

where $\lambda$ is the X-ray wavelength, $\theta$ is the diffraction angle of the most intense peak (311), and $\beta$ is its full width at half maximum (FWHM). $\beta$ must be corrected according to the following formula:

\begin{equation}
\beta = \sqrt{\beta_x^2 - \beta_{\text{inst}}^2}
\end{equation}

where $\beta_x$ is the experimental FWHM and $\beta_{\text{inst}}$ is the FWHM of a standard silicon sample. The estimated values of average crystallite sizes are summarized in Table 1. The $D$ values clearly increased with increasing sintering temperature, in good agreement with previous reports.\textsuperscript{22,23} The intensity ratio ($I_{311}/I_{222}$) values are also shown in Table 1. The intensity of the (220) and (222) planes depends on the cation distribution in the tetrahedral (A) and octahedral sites (B), respectively.\textsuperscript{26} The calculated values increased with increasing sintering temperature, indicating that the degree of inversion\textsuperscript{†} was reduced with increasing sintering temperature.\textsuperscript{27}

\textsuperscript{†} Inversion is defined as the fraction of A sites occupied by Fe\textsuperscript{3+} cations and its value depends on the method of preparation and heat treatment effects.\textsuperscript{27}

### 3.1.2. SEM images

SEM micrographs of S1000 and S1200 are shown in Fig. 4. For both samples, the micrographs exhibited regular shaped grains, most with spherical morphology, while S1200 grains were significantly larger than those of S1000. Analysis of the SEM images using Image J software showed average particle sizes of about 0.180 and 0.412 $\mu$m for S1000 and S1200, respectively. Therefore, the SEM images confirmed the increase in particles size with increasing sintering temperature. The tendency of particles to join together and produce large particles with negligible porosity was evident in S1200. These observations were consistent with the XRD results, but the crystallite sizes estimated from XRD peaks were clearly smaller than the particle sizes obtained from SEM micrographs. This difference was attributed to the particles being composed of several crystallites, which caused internal strains or defects in the structure.\textsuperscript{3}

### 3.2. Magnetic properties

Variations in the magnetization ($M$) with temperature ($T$) under a magnetic field of 0.05 T for S1000 and S1200 samples are shown in Fig. 5. Clear ferromagnetic–paramagnetic (FM–PM) phase transitions were observed for both samples at their Curie temperatures ($T_C$). $T_C$ values were determined, accounting for the minimum values of $(dM/dT)$ vs. $T$ curves, which are given in the insets of Fig. 5. The $T_C$ values increased from 655 K to 680 K for S1000 and S1200, respectively. Furthermore, at the higher sintering temperature, the magnetization amplitude in the FM region also increased. The increments in both $M$ and $T_C$ values with increasing sintering temperature can be related to the

<table>
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<th>Samples</th>
<th>S1000</th>
<th>S1200</th>
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<td>$Fd\bar{3}m$</td>
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<td>Site symmetry</td>
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<tr>
<td>$\chi^2$ (%)</td>
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</table>

\textsuperscript{a} Numbers in parentheses are estimated standard deviations to the last significant digit. $b_{iso}$, the isotropic Debye–Waller factor. Structural parameters $a$, $V$, $R_A$, $R_B$, $D$, and $I_{220}/I_{222}$ are defined in the text. Agreement factors of profile $R_p$, weighted profile $R_{wp}$, and structure $R_B$, $\chi^2$, goodness of fit.
increase in crystallite size. Furthermore, according to the core/surface morphology, the spin arrangement on the surface is lower than that in the core. The surface is considered a magnetically dead layer, with the randomly disordered. Therefore, as the crystallite size increases with increasing sintering temperature, this nonmagnetic surface layer decreases and, consequently, the core/shell thickness ratio increases. This leads to the elimination of surface effects (broken exchange bonds, spin canting) and, therefore, an improvement in both $M$ and $T_C$ values was observed. This result can also be interpreted in another way by considering variation in the degree of inversion parameter. According to the XRD results, the degree of inversion was reduced with increasing sintering temperature. The decreased degree of inversion would cause the average exchange interaction between $\text{Fe}^{3+}$ ions in tetrahedral and octahedral sites to increase, which resulted in an increase in the $M$ and $T_C$ values. This explained why the as-prepared S1000 sample with a higher inversion degree had lower $M$ and $T_C$ values, while the S1200 sample with a low inversion degree had higher $M$ and $T_C$ values.

### 3.3. Magnetocaloric properties

Fig. 6 shows the $M(\mu_0H, T)$ isothermal magnetizations taken near the $T_C$ values of S1000 and S1200. The nature of the magnetic phase transitions of these samples was determined
using Banerjee’s criterion29 and $M^2$ vs. $\mu_0 H / M$ Arrott plots.30 Fig. 7 shows the Arrott plots of both samples. Analysis of these plots showed that they have positive slopes around the $T_C$, which confirmed that the magnetic phase transitions were second-order. Using $M(T, \mu_0 H)$ isotherms, the dependence of magnetic entropy change ($\Delta S_M$) on temperature ($T$) was calculated according to the following Maxwell relation:

$$\Delta S_M(T, \mu_0 H) = \sum \frac{M_i - M_{i+1}}{T_i - T_{i+1}} \Delta (\mu_0 H)$$  \hspace{1cm} (3)$$

where $\Delta T_{FWHM}$ is the fullwidth at 0.5$\Delta S_M^{\text{max}}$. The magnetic field dependences of $-\Delta S_M^{\text{max}}$ and RCP obtained for S1000 and S1200 are shown in Table 2 and Fig. 9. Both increased linearly with an increasing magnetic field. Furthermore, the increase in sintering temperature led to an increase in the $-\Delta S_M^{\text{max}}$ values from 1.57 J kg$^{-1}$ K$^{-1}$ for S1000 to 2.12 J kg$^{-1}$ K$^{-1}$ for S1200 (at $\mu_0 H = 5$ T). This variation in magnetic entropy change with increasing sintering temperature was in agreement with the results of previous studies.31,32 The estimated RCP values were 115 and 125 J kg$^{-1}$ for S1000 and S1200, respectively. These values were relatively high, indicating that the prepared samples are potential candidates for cooling power technology. Furthermore, the $-\Delta S_M^{\text{max}}$ and RCP values were comparable to those found at the same applied magnetic field in some other ferrite systems that are considered potential candidates for magnetic refrigeration.9,11,14

3.4. Critical behavior

3.4.1. Scaling. The critical behaviors of S1000 and S1200 near their Curie temperatures were analyzed by conducting a detailed study of their critical exponents ($\beta$, $\gamma$, and $\delta$). According to Stanley’s hypothesis,33 the exponents $\beta$ and $\gamma$ can

![Fig. 7](image_url) Arrott plots ($M^2$ vs. $\mu_0 H / M$) near the $T_C$ values of S1000 and S1200.

![Fig. 8](image_url) Temperature and magnetic field dependences of the magnetic entropy change for S1000 and S1200.
be estimated from the temperature dependence of spontaneous magnetization, $M_s$ (below $T_C$), and the initial magnetic susceptibility, $\chi_0$ (above $T_C$), respectively. Meanwhile, exponent $\delta$ can be deduced directly from the magnetic isotherm at $T = T_C$.

These three critical exponents can be determined using the following equations:  

$$M(T) = M_0(-\epsilon)^\delta \quad \text{(with } \epsilon < 0 \text{ and } T < T_C)$$  

$$\chi_0^{-1}(T) = \left(\frac{h_0}{M_0}\right)\epsilon^\nu \quad \text{(with } \epsilon > 0 \text{ and } T > T_C)$$

where $\epsilon$ is the reduced temperature ($\epsilon = (T - T_C)/T_C$), and $M_0$, $h_0$, and $D$ are the critical amplitudes. According to Stanley’s scaling hypothesis, the magnetic state equation in the critical region is: 

$$M = D(\mu_0 H)^{1/\delta} \quad \text{(with } \epsilon = 0 \text{ and } T = T_C)$$

Table 2  Magnetocaloric properties (at a magnetic field of 5 T) for S1000 and S1200 ferrites compared with those of other ferrite systems at the same applied magnetic field

| Samples                  | $T_C$ (K) | $\mu_0 H$ (T) | $|\Delta S_M^{max}|$ (J kg$^{-1}$ K$^{-1}$) | RCP (J kg$^{-1}$) | Ref.       |
|-------------------------|-----------|---------------|------------------------------------------|-------------------|------------|
| Ni$_{0.6}$Cd$_{0.2}$Cu$_{0.2}$Fe$_2$O$_4$ (1000 °C) | 655       | 5             | 1.57                                     | 115               | Present work |
| Ni$_{0.6}$Cd$_{0.2}$Cu$_{0.2}$Fe$_2$O$_4$ (1200 °C) | 680       | 5             | 2.12                                     | 125               | Present work |
| Zn$_{0.6}$Cu$_{0.4}$Fe$_2$O$_4$                        | 305       | 5             | 1.16                                     | 289               | 9          |
| Zn$_{0.6}$Ni$_{0.4}$Cu$_{0.2}$Fe$_2$O$_4$              | 565       | 5             | 1.41                                     | 141               | 9          |
| Zn$_{0.6}$Ni$_{0.4}$Cu$_{0.2}$Fe$_2$O$_4$              | 705       | 5             | 1.61                                     | 233               | 9          |
| Cu$_{0.6}$Zn$_{0.4}$Fe$_2$O$_4$                        | 373       | 5             | 1.77                                     | —                 | 11         |
| Cu$_{0.6}$Zn$_{0.4}$Fe$_2$O$_4$                        | 140       | 5             | 1.17                                     | —                 | 11         |
| Ni$_{0.6}$Mg$_{0.3}$Cu$_{0.3}$Fe$_2$O$_4$              | 690       | 5             | 1.56                                     | 136               | 14         |

Fig. 9  (a) Magnetic field dependence of maximum magnetic entropy change $-\Delta S_M^{max}$ and (b) relative cooling power values (RCP) of S1000 and S1200.

Fig. 10  Relative slope (RS) vs. temperature deduced from the four theoretical models for S1000 and S1200.
where $f_1$ and $f_2$ are regular analytical functions defined at the ($T > T_c$) and ($T < T_c$) regions, respectively. For a better estimation of $\beta$ and $\gamma$ critical exponents, this equation implies that $M(\varepsilon)^{-\beta}$ vs. $\mu_0H|\varepsilon|^{-(\beta+\gamma)}$ curves can be divided along two universal branches (one for $T < T_c$ and another for $T > T_c$).

### 3.4.2. Analysis

According to the mean field model, the Arrott plots ($M^2$ vs. $\mu_0H/M$) shown in Fig. 7 should give a series of straight lines in the high-field region at different temperatures around $T_c$, and the line at $T_c$ should cross the origin.\(^{33}\) As shown in Fig. 7, these conditions were not accurate, which indicated that the mean field model was invalid to describe the critical behaviors of S1000 and S1200. Therefore, to obtain the correct $\beta$ and $\gamma$ critical exponents for the samples, the $M(\mu_0H, T)$ isotherms were analyzed using the modified Arrott plots (MAP) by representing $M^{4/\beta}$ vs. $(\mu_0H/M)^{1/\gamma}$ curves\(^{34}\). Notably, the MAP gave more importance to data in the region of high magnetic fields compared to those in the weak-field region. Three theoretical models are generally applied to estimate $\beta$, $\gamma$, and $\delta$ exponents, as follows:\(^{35,36}\) (i) the 3D Heisenberg model with $\beta = 0.365$, $\gamma = 1.336$, and $\delta = 4.8$; (ii) the 3D Ising model with $\beta = 0.325$, $\gamma = 1.24$, and $\delta = 4.82$; and (iii) the tricritical mean-field model with $\beta = 0.25$, $\gamma = 1$, and $\delta = 5$. In our case, we first used these three models to construct MAPs (not presented in this work) for S1000 and S1200, and then, from each MAP, calculated the relative slope (RS), which is defined as:

$$RS = \frac{S(T)}{S(T_c)}$$

(9)

For the best theoretical model, the RS values should be close to unity, mainly because the MAPs are series of parallel isotherms.\(^{37}\) Fig. 10 shows the values of RS vs. $T$ corresponding to each model. For both samples, the RS values were clearly higher than unity. This suggested that the theoretical models were not useful for estimating the critical exponents of the prepared samples. Therefore, we concluded that the samples had unconventional critical exponents. This motivated us to construct new MAPs. We attempted to find new critical exponent values until we obtained parallel isotherms at around $T_c$, and the isotherm of $M^{4/\beta}$ vs. $(\mu_0H/M)^{1/\gamma}$ at $T_c$ crossed the origin. These conditions were reached with values of $\beta = 0.444$ and $\gamma = 1.031$ for S1000, and $\beta = 0.408$ and $\gamma = 1.075$ for S1200 (Fig. 11). Linear extrapolation of the experimental data in Fig. 11 allowed determination of the spontaneous magnetization, $M_s$, (below $T_c$), by intersection with the $M^{4/\beta}$ axis and the initial magnetic susceptibility values, $\chi_0^{-1}$ (above $T_c$), by intersection with the $(\mu_0H/M)^{1/\gamma}$ axis. The $M_s(T)$ and $\chi_0^{-1}(T)$ data were then adjusted using the eqn (5) and (6), respectively. During this adjustment, new values of $\beta$ and $\gamma$ were obtained simultaneously. The $T_c$...
value associated with each curve was also determined. After several adjustment tests, the values of $\beta$, $\gamma$, and $T_c$ converged toward their optimal values. As shown in Fig. 12, the theoretical and experimental curves were in good agreement. The obtained values shown in Fig. 12 also agreed well with those presented in Fig. 11. The $\beta$, $\gamma$, and $T_c$ values were also determined more precisely using the Kouvel–Fisher method using the following two equations:\textsuperscript{38,39}

$$M_s(T) \left(\frac{dM_s(T)}{dT}\right)^{-1} = \frac{(T - T_c)}{\beta}$$

(10)

$$\chi_0^{-1}(T) \left(\frac{d\chi_0^{-1}(T)}{dT}\right)^{-1} = \frac{(T - T_c)}{\gamma}$$

(11)

Linear adjustment of $M_s(T) \left(\frac{dM_s(T)}{dT}\right)^{-1}$ vs. $T$ gave a slope equal to $1/\beta$, while the linear adjustment of $\chi_0^{-1}(T) \left(\frac{d\chi_0^{-1}(T)}{dT}\right)^{-1}$ vs. $T$ gave a slope equal to $1/\gamma$. These curves are shown in Fig. 13. The red lines represent the best adjustments using eqn (10) and (11). The obtained values of $\beta$, $\gamma$, and $T_c$ parameters shown in Fig. 13 were consistent with those used in Fig. 11. The variation in the critical isotherm $M(T_c, \mu_0 H)$ can be described by a power law characterized by the critical exponent $\delta$. This exponent can be obtained by adjusting $M(T_c, \mu_0 H)$ curve in log–log scale using eqn (7), as shown in Fig. 14. From this adjustment, it was possible to determine the slope of the curve ($1/\delta$) and consequently the values of $\delta$ (see Fig. 14). The three critical exponents, $\beta$, $\gamma$, and $\delta$, determined previously can be related, according to statistical theory, to the Widom relation:\textsuperscript{40}

$$\delta = 1 + \frac{\gamma}{\beta}$$

(12)

By replacing $\beta$ and $\gamma$ in this equation with the values shown in Fig. 12 and 13, eqn (12) gave $\delta$ values of 3.331 and 3.329 for S1000, and 3.638 and 3.662 for S1200. These values were very close to those estimated from the critical isotherm at $T_c$, as shown in Fig. 14 ($\delta = 3.311$ and 3.650 for S1000 and S1200, respectively). These results implied that the values of $\beta$, $\gamma$, and $\delta$ were well determined. Variation in the term $(M|^{1/\delta})$ vs. $(\mu_0 H|^{(1/\delta-\gamma)})$ according to eqn (8) is plotted in Fig. 15 using the $\beta$ and $\gamma$ values previously obtained by the Kouvel–Fisher method for some temperatures below and above the $T_c$. Fig. 15 shows the superposition of all curves along two separate branches for $(T < T_c)$ and $(T > T_c)$, respectively, which suggested that the obtained values of critical exponents and those for $T_c$.
were reasonably precise. The insets of Fig. 15 also show the variation in these curves on the log–log scale. A linear superposition was observed at higher magnetic fields in the curves in the two regions (below and above $T_C$), while the setting of the curves to the log–log scale became especially bad in the weak fields (some bifurcations of the curves were observed in different directions). This confirmed that this scale theory gives much more importance to data in higher fields than those in weak fields.

The influence of the critical exponents on the MCE can be demonstrated by analyzing the variation in the magnetic entropy change with magnetic field according to following power law:  

$$|\Delta S^\text{max}_M| = a(\mu_0 H)^n$$  

(13)

where $a$ is a constant and $n$ is an exponent that depends on the magnetic state of the samples. In this particular case, at $T = T_C$, the $n$ exponent becomes independent of the magnetic field. Therefore:

$$n(T_C) = 1 + \frac{\beta - 1}{\beta + \gamma}$$  

(14)

where $\beta$ and $\gamma$ are the critical exponents. By multiplying the Widom relation by $\beta$, we can write $\beta \delta = (\beta + \gamma)$, and consequently eqn (14) can be rewritten as:

$$n = 1 + \frac{1}{\beta} \left(1 - \frac{1}{\beta}ight)$$  

(15)

By considering the values of $\beta$ and $\gamma$ obtained from the Kouvel–Fisher method, and the values of $\delta$ obtained from the $M(T_C, H)$ critical isotherms, the $n$ values calculated from the previous relations were:

From eqn (14):

$$\begin{align*}
  n &= 622 \text{ for sample sintered at 1000 °C} \\
  n &= 0.596 \text{ for samples sintered at 1200 °C}
\end{align*}$$

From eqn (15):

$$\begin{align*}
  n &= 0.620 \text{ for sample sintered at 1000 °C} \\
  n &= 0.594 \text{ for sample sintered at 1200 °C}
\end{align*}$$

The $n$ values can also be estimated by fitting the $|\Delta S^\text{max}_M|$ vs. $\mu_0 H$ curve at $T_C$ according to eqn (13), as shown in Fig. 16. The fitting curves gave $n$ values of 0.626 and 0.601 for S1000 and S1200, respectively. These values were in good agreement with those obtained from eqn (14) and (15). This suggested that the critical exponents obtained for the studied samples were reliable. The obtained values of the critical exponents of S1000 and S1200 (present work) are compared with the predicted values of the four theoretical models in Table 3.  

$\text{We found that}$ $\beta$ and $\delta$ exponents lay between the mean field model and the 3D Heisenberg model, while the $\delta$ values were close to that of the mean field model. However, as the sintering temperature increased, the critical exponent $\beta$ decreased, while $\gamma$ and $\delta$ exponents increased. The decreasing $\beta$ exponent reflected a faster growth of the ordered moment with increasing sintering temperature. However, the deviation of this exponent from the mean field model was attributed to the presence of an inhomogeneous magnetic state in the samples.$

![Fig. 15](Image)

**Fig. 15** Scaling plots for S1000 and S1200 below and above $T_C$ values using $\beta$ and $\gamma$ estimated using the Kouvel–Fisher method. Insets show plots in the log–log scale.

![Fig. 16](Image)

**Fig. 16** Magnetic field dependence of $|\Delta S^\text{max}_M|$ at $T_C$ fitted to the power law $|\Delta S^\text{max}_M| = a(\mu_0 H)^n$ for S1000 and S1200.
4. Conclusion
The sol–gel method was used to prepare Ni$_{0.6}$Cd$_{0.4}$Cu$_{0.2}$Fe$_2$O$_4$ ferrites at sintering temperatures of 1000 °C and 1200 °C. XRD patterns and Rietveld refinements confirmed that the samples had cubic spinel type structures. Magnetic measurements showed that the samples presented second-order FM–PM phase transitions with increasing magnetization amplitude and Curie temperature when the sintering temperature was increased. The values of maximum magnetic entropy change and relative cooling power were relatively high and comparable to those of some other ferrite systems considered potential candidates for magnetic refrigeration. This showed that the prepared samples were useful for cooling power technology. Critical exponents $\beta$, $\gamma$, and $\delta$ were determined using different methods. These exponents belonged to a different universality class, with exponent $\beta$ decreasing and exponents $\gamma$ and $\delta$ increasing with increasing sintering temperature.

Conflicts of interest
There are no conflicts to declare.

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