Electronic structure, magnetism and optical properties of orthorhombic GdFeO$_3$ from first principles

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Orthorhombic GdFeO$_3$ has attracted considerable attention in recent years because its magnetic structure is similar to that of the well-known BiFeO$_3$ material. Here, we investigate the electronic structure, magnetism and optical properties of orthorhombic GdFeO$_3$ in terms of density-functional-theory calculations. The modified Becke-Johnson (mBJ) exchange potential is adopted to improve on the description of the electronic structure. Our calculations show that the G-type antiferromagnetic (G-AFM ordering of Fe spins) phase of orthorhombic GdFeO$_3$ is stable compared to other magnetic phases. The semiconductor gap calculated with mBJ, substantially larger than that with GGA, is in good agreement with recent experimental values. Additionally, we also investigate the effects of spin–orbit coupling on the electronic structure, and calculate the complex dielectric functions and other optical functions of photon energy. The magnetic exchange interactions are also investigated, which gives a Néel temperature close to experimental observation. For confirming these mBJ results, we also study the electronic structure of rhombohedral (R3c) BiFeO$_3$ with mBJ, obtaining good consistency with experiment. These lead to a satisfactory theoretical understanding of the electronic structure, magnetism and optical properties of orthorhombic GdFeO$_3$ and can help elucidate the electronic structures and optical properties of other similar materials.

I. Introduction

Bismuth ferrite (BiFeO$_3$) is a representative of single-phase multiferroic materials, which displays antiferromagnetic order below $T_N \sim 643$ K and possesses a relatively high spontaneous electric polarization of 59.4 $\mu$C cm$^{-2}$ up to $T_N \sim 1100$ K.$^{1-3}$ Besides, BiFeO$_3$ is a perovskite oxide whose most stable phase is a rhombohedral distorted structure with space group R3c. As a magnetic material similar to BiFeO$_3$, orthorhombic distorted GdFeO$_3$, which has a Néel temperature 661 K and belongs to the perovskite rare-earth orthoferrites, has sparked substantial curiosity and stimulated relatively deep research.$^{4-7}$

The GdFeO$_3$ compound has a complex $H$–$T$ phase diagram and undergoes a plurality of magnetic phase transitions, accompanying the dramatic changes in electrical properties.$^b$ The orthorhombic distorted GdFeO$_3$ compound ($Pbnm$), with Gd$^{3+}$ ions at the center and Fe$^{3+}$ ions at the corners surrounded by oxygen octahedra, possesses weak ferromagnetism and ferroelectricity.$^{4,5}$ Under electric and magnetic fields, the ferroelectric polarization and magnetization of GdFeO$_3$ has been successfully brought under control for a variety of applications.$^a$ A spontaneous polarization of about 0.12 $\mu$C cm$^{-2}$ was obtained at 2 K$^c$, which is basically identical with the measured value in the perpendicular magnetic system.$^{10}$ According to the Bertaut notation, the spin structure of Fe$^{3+}$ is G$_A$F$_{2c}$.$^{11}$ Below $T^g_{Nd} = 2.5$ K, the magnetic order of Gd$^{3+}$ is antiferromagnetic along the $a$-axis, showing G$_x$ antiferromagnetic order and ferroelectric polarization characteristics.$^{4,12}$ The interaction between adjacent Fe$^{3+}$ and Gd$^{3+}$ layers induces ferroelectric polarization along the $c$-axis.$^{13}$ It was reported that the Fe spins in BiFeO$_3$ form a G-type antiferromagnetic (G-AFM) ordering, with the spins on the Fe$^{3+}$ ions being aligned antiferromagnetically along the [111] axis.$^{3}$ Interestingly, GdFeO$_3$ can show a ferromagnetism below 5 K$^4,14,15$ and at $T^F_{Nd} = 661$ K, Fe$^{3+}$ also forms G-AFM ordering in GdFeO$_3$ and shows a weak ferromagnetism due to the Dzyaloshinskii–Moriya interaction.$^{16-18}$ On the other hand, first-principles calculations suggested that the antiferromagnetic phase of the orthorhombic GdFeO$_3$ is more stable than the ferromagnetic phase,$^{19}$ and it was also pointed out that the transition from antiferromagnetic to paramagnetic ordering could occur at 670 K.$^6,7,9$

Despite a large number of experimental studies concerning the electromagnetic behavior of orthorhombic distorted GdFeO$_3$, the theoretical reports are extremely meagre and it is necessary to adopt suitable theoretical approaches to perform...
further study. Here, we investigate the electronic structure and magnetic and optical properties of orthorhombic GdFeO$_3$ through density functional theory (DFT) calculations. In order to better understand the electronic properties of GdFeO$_3$, we also investigate the electronic structure of BiFeO$_3$ for comparison. The remainder of this paper is organized as follows: computational details are described in the second section, main calculated results and analysis are presented in the third section, and finally, conclusions are presented in the fourth section.

II. Computational details

The full-potential linearized augmented plane wave method within the density-functional theory (DFT),$^{20,21}$ as implemented in the package Wien2k,$^{22}$ is utilized in our calculations. First, the popular generalized gradient approximation (GGA-PBE)$^{23}$ is adopted to optimize crystal structures and investigate electronic structures and magnetism. Because the standard semi-local GGA usually underestimates energy band gaps,$^{24}$ we use the mBJ approximation$^{25}$ for the exchange potential, taking the local density approximation (LDA)$^{26}$ to treat the correlation potential, as usual for improved description of electronic structures and optical properties. For the calculations of electronic structures, mBJ has been shown to significantly improve and accurately produce semiconductor gaps for sp semiconductors, wide-band-gap semiconductors, and transition-metal oxide semiconductors and insulators.$^{25,27-30}$ The electronic correlation has been effectively taken into account in this powerful mBJ approach.$^{25}$ Because the theoretical semiconductor gaps are improved, much better computational results can also be obtained for the optical properties. The full relativistic effects are calculated with Dirac equations for core states, and the scalar relativistic approximation is used for valence states.$^{31-32}$ We also take the spin–orbit coupling (SOC) into consideration. The cut-off energy is set to $-6 \text{ Ry}$ to separate core states from valance states. The $k$-mesh size in the first Brillouin zone is $11 \times 10 \times 7$ for GdFeO$_3$, and $10 \times 10 \times 10$ for BiFeO$_3$. We construct harmonic expansion up to $l_{\text{max}} = 10$, set $R_{\text{min}} \times K_{\text{max}} = 7$, and use magnitude of the largest vector $G_{\text{max}} = 12$ in charge density Fourier expansions. The radii of Gd, Bi, Fe and O atomic spheres are set to $2.27, 2.12, 1.99$ and $1.77 \text{ bohr}$, respectively. The self-consistent calculations are considered to be converged only when the integration of absolute charge-density difference per formula unit between the successive loops is less than $0.0001|e|$, where $e$ is the electron charge.

III. Results and discussion

A. Crystal structure

The orthorhombic GdFeO$_3$ system investigated here has space group $Pbnm$ (no. 62)$^{33,34}$ at low temperatures. The experimental lattice constants are $a = 5.349 \text{ Å}, b = 5.611 \text{ Å}$ and $c = 7.669 \text{ Å}$.$^{33}$ The crystal structure of orthorhombic GdFeO$_3$ is shown in Fig. 1(a). At first, we optimized the lattice parameters and ionic positions with GGA and LDA and the optimized lattice parameters are summarized in Table 1. Existing experimental results$^{33,34}$ are also presented for comparison. The GGA optimized volume $V$ is 2.0% larger than the experimental volume,$^{33}$ but the LDA optimized volume $V$ is 5.1% smaller. It can be clearly seen that the lattice constants and volume calculated with GGA are closer to the experimental data$^{33,34}$ than those with LDA. Therefore, the other properties are investigated on the basis of the GGA optimized results. The GGA-optimized atomic positions are summarized in Table 2. After internal structure optimization, the Gd atom occupies the $(0.9911, 0.0639, 0.25)$ site, the Fe atom the $(0, 0.5, 0)$ site, the O1 atom the $(0.7024, 0.4726, 0.25)$ site and the O2 atom the $(0.0927, 0.4726, 0.25)$ site in Wyckoff coordinates. These are consistent with the experimental orthorhombic structure.$^{33,34}$ Furthermore, the three optimized Gd–O bond lengths are $2.307, 2.347$ and $2.399 \text{ Å}$, which are slightly larger than the Fe–O bond lengths of $1.968, 1.974$ and $2.124 \text{ Å}$, respectively. These are in line with the relation of ionic radii, Gd$^{3+} >$ Fe$^{3+}$, and in accordance with the
Table 2 The atomic positions (x, y, z) optimized with GGA for GdFeO₃ in comparison with experimental values

<table>
<thead>
<tr>
<th>Atom</th>
<th>Site</th>
<th>x (exp.33,34)</th>
<th>y (exp.33,34)</th>
<th>z (exp.33,34)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gd</td>
<td>4c</td>
<td>0.9911 (0.9844, 0.9846)</td>
<td>0.0639 (0.0628, 0.0626)</td>
<td>0.2500 (0.2500, 0.2500)</td>
</tr>
<tr>
<td>Fe</td>
<td>4b</td>
<td>0.0000 (0.0000, 0.0000)</td>
<td>0.5000 (0.5000, 0.5000)</td>
<td>0.0000 (0.0000, 0.0000)</td>
</tr>
<tr>
<td>O1</td>
<td>8d</td>
<td>0.7024 (0.6957, 0.6966)</td>
<td>0.3121 (0.3016, 0.3011)</td>
<td>0.0488 (0.0506, 0.0518)</td>
</tr>
<tr>
<td>O2</td>
<td>4c</td>
<td>0.0927 (0.1005, 0.1009)</td>
<td>0.4726 (0.4672, 0.4669)</td>
<td>0.2500 (0.2500, 0.2500)</td>
</tr>
</tbody>
</table>

Previously reported values. The distances between Gd³⁺ and Fe²⁺ ions are 3.136, 3.284 and 3.362 Å. The bond angles of Fe–O–Gd are 85.38° and 87.34°, deviating from the ideal values of 90°. These imply that the orthorhombic GdFeO₃ has undergone substantial structural distortion.

In addition, we have calculated the GGA total energies of four different magnetic ordering configurations: the ferromagnetic and three antiferromagnetic (AFM) ones. The three AFM structures are shown in Fig. 1(b–d) and are denoted by A-AFM, C-AFM and G-AFM, respectively. Taking the total energy of the lowest G-AFM structure as a reference, the other three energies are 0.54 eV, 0.31 eV and 0.048 eV, respectively. These imply that the orthorhombic GdFeO₃ has undergone substantial structural distortion.

B. Electronic structures

With the optimized crystal structure, we calculate with both GGA and mBJ potentials the spin-dependent energy band structures and the densities of states (DOSs) of orthorhombic GdFeO₃ between −6 eV and 4 eV. The two band structures are shown in Fig. 2. The conduction band bottom and the valence band top are located at the same S point in both of the band structures and indicate a direct gap for orthorhombic GdFeO₃. It can be seen that the GGA semiconductor gap is 0.61 eV [Fig. 2(a)], a little larger than the earlier first-principles result of 0.54 eV, but the mBJ-calculated semiconductor gap, 2.49 eV [Fig. 2(b)], is apparently larger than the GGA value and is in accordance with the experimental results (2.1–2.53 eV)

Very interestingly, there are many similar features between GdFeO₃ and BiFeO₃ as orthoferrite ABO₃ materials. As a typical multiferroic orthoferrite compound, however, BiFeO₃ shows the characteristics of an indirect band gap. For BiFeO₃, our GGA band gap of 0.965 eV is slightly lower than previous DFT values of 1.06 eV and 1.04 eV, but they are all too small to accurately describe the experimental values of 2.4 eV, 2.5 eV, and 2.74 eV. Our mBJ calculation produces a semiconductor gap of 2.354 eV for BiFeO₃ and it is in good agreement with the experimental values. In contrast, a band gap of 2.8 eV, obtained with a screened exchange potential, is too large in comparison with the experimental values. These results show that our mBJ gap of 2.49 eV is reasonable and should be accurate for orthorhombic GdFeO₃.

C. Effects of spin–orbits coupling

The spin–orbit coupling (SOC) is important for electronic materials containing heavy atoms such as Gd and can lead to magnetocrystalline anisotropy. With the GGA + SOC method, we can calculate the total energy of orthorhombic GdFeO₃ by taking the SOC into account. Setting the magnetization in the [100], [010], [001], [110], [101], [011] and [111] directions, we obtain total energies of 2.6, 3.8, 0, 3.2, 8.7, 1.3 and 1.6 μeV, respectively. It is obvious that the lowest energy is along the
[001] direction. These results indicate that the easy magnetization axis (the most stable magnetic orientation) of orthorhombic GdFeO$_3$ is along the [001] axis.

In the easy axis, the total spin moment is precisely equivalent to 0 $\mu_B$ per formula unit without SOC for orthorhombic GdFeO$_3$. According to Hund’s rule, the cations Gd$^{3+}$ and Fe$^{3+}$ possess high spin values of $s = \frac{7}{2}$ and $s = \frac{5}{2}$, respectively, and the antiferromagnetic coupling makes the total spin moment equal to 0 $\mu_B$ per formula unit. Since part of the spin moments are located in the interstitial region, the spin moments of the individual Gd$^{3+}$ and Fe$^{3+}$ are 6.855 $\mu_B$ and 4.082 $\mu_B$, respectively. When taking the SOC into account, the spin moments of Gd$^{3+}$ and Fe$^{3+}$ are reduced to 6.811 $\mu_B$ and 4.080 $\mu_B$, respectively. The orbital moment of Fe 3d is 0.183 $\mu_B$, which has the same sign as the spin moment, and the orbital moment of Gd$^{3+}$ ion is 0.088 $\mu_B$.

The semiconductor gap $E_g$ is also investigated by using mBJ. The semiconductor gap becomes smaller (2.40 eV) when the SOC is taken into account. Fig. 4(a) shows the density of states of orthorhombic GdFeO$_3$ obtained with mBJ + SOC. Looking
magnetic orderings constructed by changing the Fe and Gd spins of the antiferromagnetic configuration to the ferromagnetic ordering, respectively. With the first principles total energies of the different magnetic configurations, we can determine the coupling constants of the effective Heisenberg model (eqn. (1)):

$$H = \sum_{ij} J_{ij} S_i \times S_j,$$

where $S_i$ is the spin operator at site $i$. Here, the summation is over spin pairs, and the spin exchange parameter $J_{ij}$ is limited to the nearest (Fe–Gd) and the next nearest (Fe–Fe and Gd–Gd) spin pairs. Although the absolute values of the magnetic moments in the spheres of Gd and Fe are 6.855 $\mu_B$ and 4.082 $\mu_B$, the Gd$^{3+}$ and Fe$^{3+}$ cations should theoretically contribute 7 $\mu_B$ and 5 $\mu_B$, respectively. We can assign spin values $s = 7/2$ and $s = 5/2$ to the Gd and Fe spins, respectively. Accordingly, there exists a relation between the magnetic energies $e_{ij}$ and exchange parameter $J_{ij} = J_{ij} S_i S_j$, where $S_i$ takes values of either $7/2$ or $5/2$. Taking the G-AFM ground state as a reference, the calculated total energies of the other three states are 159.9, 158.6 and 3.5 meV per formula unit, respectively. The total energy can be split into $E_0 + \sum e_{ij}$, where $E_0$ is defined to be independent of spin configuration. We obtain the following equations from the four magnetic structures (eqn (2)).

$$\begin{align*}
0 &= E_0 - 3e_{Fe-Fe} - 3e_{Gd-Gd} \\
159.9 &= E_0 + 8e_{Fe-Fe} + 3e_{Fe-Gd} + 3e_{Gd-Gd} \\
158.6 &= E_0 + 3e_{Fe-Fe} - 3e_{Gd-Gd} \\
3.5 &= E_0 - 3e_{Fe-Fe} + 3e_{Gd-Gd}
\end{align*}$$

From the above equations, we can calculate the $e_{ij}$ parameters, and then obtain exchange coupling parameters $J_{ij}$. The Gd–Gd and Fe–Gd spin exchange energies are much smaller than the Fe–Fe value (26.43 meV). As a result, the spin exchange parameters $J_{ij}$ are 0.03 meV between the nearest Fe–Gd pair, 4.23 meV between Fe–Fe, and 0.05 meV between Gd–Gd. It is clear that the Fe–Fe spin coupling is dominant over the other two.

With the exchange constants $J_{ij}$ given, we can calculate the magnetic phase transition temperature, $T_N$. For this purpose, the quantum Green-function method is a good approach because it can produce accurate transition temperatures and reliable temperature dependence of magnetization or sublattice magnetization for a Heisenberg spin model. From our spin model, we aim to use this method to estimate the Néel temperature. If neglecting the much smaller Gd–Gd and Gd–Fe interaction energies, we can estimate the Néel temperature, $T_N = 605$ K, in terms of an analytical approach. Considering that we have not taken the SOC effect into account, this Néel temperature is very satisfactory compared to experimental values: 670 K7 and 661 K6.

### D. Exchange interactions

The magnetic exchange interactions on Gd$^{3+}$ and Fe$^{3+}$ can be investigated in terms of total energy calculations. We consider four different magnetic configurations, namely antiferromagnetic (ground state) and ferromagnetic ordering, and two other

### E. Optical properties

Optical spectroscopy analysis is a powerful tool to determine the energy band structure of a solid material. The complex dielectric function is directly related to the energy band...
structure of solids. For orthorhombic GdFeO₃, Fig. 5 shows the mBJ calculated curves of the complex dielectric function (the real and imaginary parts), absorption coefficient, reflectivity, energy loss function, refractive index, extinction coefficient, and optical conductivity as functions of the photon energy in the range of 0–9 eV. All the three polarization directions (E//x, y, and z) are considered.

The electronic polarizability of a material can be understood from the real part of the dielectric function \( \varepsilon_1(\omega) \) [Fig. 5(a)]. The static dielectric constant \( \varepsilon_1(0) \) along the three crystallographic directions is found to be 4.28 for E//x, 4.38 for E//y, and 4.20 for E//z. The optical spectra as a function of photon energy for orthorhombic GdFeO₃ calculated with mBJ: (a) real \( \varepsilon_1(\omega) \), (b) imaginary \( \varepsilon_2(\omega) \) parts of dielectric function, (c) absorption coefficient \( \alpha(\omega) \), (d) reflectivity \( R(\omega) \), (e) energy loss function \( L(\omega) \), (f) refractive index \( n(\omega) \), (g) extinction coefficient \( k(\omega) \), and (h) optical conductivity \( \sigma(\omega) \).

Fig. 5  Optical spectra as a function of photon energy for orthorhombic GdFeO₃ calculated with mBJ: (a) real \( \varepsilon_1(\omega) \), (b) imaginary \( \varepsilon_2(\omega) \) parts of dielectric function, (c) absorption coefficient \( \alpha(\omega) \), (d) reflectivity \( R(\omega) \), (e) energy loss function \( L(\omega) \), (f) refractive index \( n(\omega) \), (g) extinction coefficient \( k(\omega) \), and (h) optical conductivity \( \sigma(\omega) \).
The most prominent peak in the electronic structure of the well-known BiFeO₃ to support our mBJ-calculated results in the case of GdFeO₃. The magnetic potential is used for improving on the description of the electronic structures of GdFeO₃. Our calculated results show that mBJ exchange greatly improves the accuracy of the band gap value. The mBJ result accords well with the experimental value and overcomes the GGA underestimation of the band gap. Besides, the spin–orbit coupling is taken into account to determine the easy magnetic axis and investigate its effects on the electronic structure. We also calculate magnetic exchange constants and thereby achieve a good Néel temperature close to the experimental value. Finally, the optical properties are also investigated with mBJ. In addition, we also calculated the electronic structure of the well-known BiFeO₃ to support our mBJ-calculated results in the case of GdFeO₃. The magnetic similarity between these two perovskite oxide materials are very interesting. These calculated results should be useful to obtain further insight for GdFeO₃ and similar materials.

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References