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1 Introduction

Protons dissolve in many oxides¹ through hydration of oxygen vacancies, resulting in high temperature proton conduction with potentials for important applications such as in fuel cells and gas separation membranes. The relative dominance of protons and oxygen vacancies is determined by the *enthalpy* and *entropy* of the hydration reaction

$$H_2O(g) + O_O^{\times} + v_O^{\bullet \bullet} \leftrightarrows 2OH_O^{\bullet}, \tag{1}$$

with the corresponding equilibrium constant:

、2

$$K_{\rm Hydr} = \frac{\left(c_{\rm OH_{O}^{\bullet}}\right)^{2}}{c_{\rm v_{O}^{\bullet}} c_{\rm O_{O}^{\circ}} p_{\rm H_{2}O}} = \exp\left(\frac{\Delta_{\rm Hydr}S^{\circ}}{R}\right) \exp\left(-\frac{\Delta_{\rm Hydr}H^{\circ}}{RT}\right) \quad (2)$$

where c_i are the molar concentrations of protons, oxygen vacancies, and lattice oxygen species. The enthalpy of this reaction can be evaluated by both indirect² and direct measurements,³⁻⁵ or computationally, and correlates with various materials' parameters.⁵⁻⁹ The entropy on the other hand, has never been measured directly, and its nature and compositional dependence is in general poorly understood. We recently determined $\Delta_{Hydr}S^{\circ}$ of BaZrO₃ computationally,¹⁰ and revealed that its major contribution is the vibrational entropy of oxygen vacancies ($v_{0}^{\circ \circ}$), in addition to loss of water vapour.

On the relationship between chemical expansion and hydration thermodynamics of proton conducting perovskites

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In this contribution, we determine the compositional dependence of the chemical expansion and entropy of hydration of the proton conducting perovskites $BaZrO_3$, $BaSnO_3$, $BaCeO_3$, and $SrZrO_3$ by first principles phonon calculations. The calculations reveal that the cubic $BaZrO_3$ and $BaSnO_3$, which display the *least favourable* hydration enthalpies, -72 and -65 kJ mol⁻¹, respectively, exhibit the *most favourable* entropies, -108 and -132 J mol⁻¹ K⁻¹, respectively. The strong compositional dependency of the hydration entropy primarily originates from the entropy gain upon filling the oxygen vacancy, which is closely related to the chemical expansion coefficient of oxygen vacancies, and thus the chemical expansion upon hydration. The chemical expansion coefficient of oxygen vacancies is more negative for the cubic than the orthorhombic perovskites, leading to a considerably larger chemical expansion upon hydration of $BaZrO_3$ proton conducting electrolytes to some extent can be avoided, or reduced, by partial substitution of Zr by Ce.

Although the vibrational contribution from protons was found to be small, there is a significant configurational contribution due to their site degeneracy around the oxide ions in the perovskite structure. In this contribution, we continue our efforts to understand the entropy of hydration, by for the first time assessing its compositional dependence and relationship with the chemical expansion, computationally.

 $\Delta_{\rm Hydr}H^{\circ}$ has been shown to correlate with material specific parameters such as the difference in the electronegativity of the B- and A-site cations⁸ and the tolerance factor,⁹ which has been argued to reflect its dependence on the basicity, or partial charge density of the oxide ions.^{7,11} $\Delta_{\rm Hydr}S^{\circ}$, however, seemingly does not display any similar correlations,⁸ which partly may stem from the large scatter in the experimental range of values. Further, the compositional dependence of $\Delta_{\rm Hydr}S^{\circ}$ is less intuitive than that of $\Delta_{\rm Hydr}H^{\circ}$ as it is related to both the chemical expansion of the material upon v[•]_O and OH[•]_O formation, the bonding strengths of H⁺ and O²⁻, and the local structural relaxations induced by the two defects.

To assess the compositional and structural dependency of $\Delta_{\text{Hydr}}S^{\circ}$, we evaluate the vibrational formation entropies of $v_{0}^{\bullet \bullet}$ and OH_{0}^{\bullet} from first principles phonon calculations, and the configurational contribution originating from the site degeneracy of protons (and oxygen vacancies). We have chosen to focus on four structurally different proton conducting perovskites; BaZrO₃, BaSnO₃, BaCeO₃ and SrZrO₃. The two former take on a cubic structure (*Pm*3*m*), while the two latter adopt an orthorhombic structure (*Pnma*), thus allowing us to judge the effect of both symmetry and composition on $\Delta_{\text{Hydr}}S^{\circ}$. As shown



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Table 1 Selected experimentally determined hydration enthalpies $(\Delta_{Hydr} H^\circ)$ and entropies $(\Delta_{Hydr} S^\circ)$ of acceptor doped BaZrO_3, BaSnO_3, BaCeO_3 and SrZrO_3

	$\Delta_{ m Hydr} H^{\circ} \ (m kJ \ m mol^{-1})$	$\Delta_{ m Hydr}S^{\circ}$ (J mol ⁻¹ K ⁻¹)		
BaZrO ₂				
$10 \text{ mol}\% \text{ Y}^6$	-80	-89		
BaSnO ₃				
5 mol% Y ¹²	-46	-95		
12.5 mol% Y ¹²	-50	-88		
12.5 mol% Y ¹³	-59	-97		
BaCeO ₃				
$10 \text{ mol}\% \text{ Y}^2$	-162	-167		
10 mol% Y ¹¹	-135	-141		
SrZrO ₃				
5 mol% Yb ¹⁴	-106	-127		

in Table 1, the experimental $\Delta_{Hydr}H^{\circ}$ and $\Delta_{Hydr}S^{\circ}$ for these perovskites cover a wide range of values, with the cubic BaZrO₃ and BaSnO₃ exhibiting the least exothermic $\Delta_{Hydr}H^{\circ}$, but also the least negative (most favourable) $\Delta_{Hydr}S^{\circ}$.

First principles phonon calculations allow us to determine both $\Delta_{Hydr} H^{\circ}$ and $\Delta_{Hydr} S^{\circ}$ as a function of temperature, and explore the individual contributions that constitute both thermodynamic parameters. The calculations are performed under constant volume and/or constant/zero pressure conditions, in order to separate contributions from local relaxations and chemical expansion to the vibrational entropies.

2 Thermodynamic formalism

2.1 Defect formation thermodynamics

The free energy of defect formation, $\Delta_F F_{defect}$ (or $\Delta_F G_{defect}$) is given by

$$\Delta_{\rm F} F_{\rm defect} = \Delta E_{\rm defect}^{\rm el} + \Delta_{\rm F} F_{\rm defect}^{\rm vib} + \sum_{1}^{\prime} \Delta n_i \mu_i(T, p) + q \left(\varepsilon_{\rm f} + \Delta \varepsilon\right)$$
(3)

where ΔE_{defect}^{el} is the total energy difference between the defective and perfect supercells, while $\Delta_F F_{defect}^{vib}$ is the vibrational (phonon) contribution to the formation energy. Further, Δn_i is the change in the number of atoms *i* with chemical potential μ_i , *q* is the effective charge of the defect, ε_f is the Fermi level and $\Delta \varepsilon$ aligns the core potentials of the perfect and the defective supercells to remedy shifts in the band edges due to the jellium background charge. The chemical potential of H₂O is given by

$$\mu_{\rm H_2O} = \mu_{\rm H_2O}^{\circ} + H_{\rm H_2O}^{\rm zp} + H_{\rm H_2O}^{\circ}(T) - TS_{\rm H_2O}^{\circ}(T) + k_{\rm B}T \ln\left(\frac{p_{\rm H_2O}}{p^{\circ}}\right)$$
(4)

where the zero-point energy $(H_{\rm H_2O}^{\rm zp})$, is obtained from the calculated vibrational frequency of a H₂O molecule, while $H_{\rm H,O}^{\circ}(T)$ and $S_{\rm H,O}^{\circ}(T)$ are taken from thermodynamic tables.¹⁵

The standard chemical potential, $\mu_{H_2O}^{\circ}$, is set to the calculated total electronic energy of the isolated H₂O molecule.

The contribution from phonons to the free energy, $F^{vib}(T)$ or $G^{vib}(T)$, and entropy, $S^{vib}(T)$, is in the harmonic approximation given by

$$F^{\text{vib}}(T) = \frac{1}{2} \sum_{\mathbf{q},s} h\nu(\mathbf{q},s) + k_{\text{B}}T \sum_{\mathbf{q},s} \ln\left(1 - \exp\left(-\frac{h\nu(\mathbf{q},s)}{k_{\text{B}}T}\right)\right)$$
(5)

$$S^{\text{vib}}(T) = -k_{\text{B}} \sum_{\mathbf{q},s} \ln\left(1 - \exp\left(-\frac{h\nu(\mathbf{q},s)}{k_{\text{B}}T}\right)\right) + \frac{1}{T} \sum_{\mathbf{q},s} \frac{h\nu(\mathbf{q},s)}{\exp\left(\frac{h\nu(\mathbf{q},s)}{k_{\text{B}}T}\right) - 1}$$
(6)

where $\nu(\mathbf{q},s)$ are the phonon frequencies throughout the **q**-space. Although the vibrational frequencies in principle should be evaluated at all **q**-points, the phonon spectra for the defective cells were only evaluated at the Γ -point of the defective supercells due to their large sizes. From eqn (6), the vibrational formation entropy of a defect is simply

$$\Delta_{\rm F} S_{\rm defect}^{\rm vib} = S_{\rm defect}^{\rm vib}(T) - S_{\rm bulk}^{\rm vib}(T), \tag{7}$$

and similar for $\Delta_F H_{defect}^{vib}$, and $\Delta_F F_{defect}^{vib}$ or $\Delta_F G_{defect}^{vib}$. Note that we are assuming a negligible change in the thermal expansion coefficient upon defect formation, such that $\Delta_F S_{defect}^{vib}$ and $\Delta_F G_{defect}^{vib}$ calculated by full volume relaxations represent zero/ constant pressure properties. We are currently also exploring anharmonic contributions to the defect formation entropies and thermal expansion of BaCeO₃ in more detail, which indicate that these contributions are of minor importance for the defect thermodynamics of BaCeO₃.

2.2 Hydration thermodynamics

The enthalpy and entropy of hydration according to eqn (1) can readily be determined from eqn (3)-(7):

$$\Delta_{\mathrm{Hydr}}H^{\circ} = 2\Delta_{\mathrm{F}}H^{\circ}_{\mathrm{OH}^{\bullet}_{\mathrm{O}}} - \Delta_{\mathrm{F}}H^{\circ}_{\mathrm{V}^{\bullet}_{\mathrm{O}}} = 2\left(\Delta E^{\mathrm{el}}_{\mathrm{OH}^{\bullet}_{\mathrm{O}}} + \Delta\varepsilon_{\mathrm{OH}^{\bullet}_{\mathrm{O}}} + \Delta_{\mathrm{F}}H^{\mathrm{vib}}_{\mathrm{OH}^{\bullet}_{\mathrm{O}}}\right) - \left(\Delta E^{\mathrm{el}}_{\mathrm{V}^{\bullet}_{\mathrm{O}}} + 2\Delta\varepsilon_{\mathrm{V}^{\bullet}_{\mathrm{O}}} + \Delta_{\mathrm{F}}H^{\mathrm{vib}}_{\mathrm{V}^{\bullet}_{\mathrm{O}}}\right) - \left(\mu^{\circ}_{\mathrm{H}_{2\mathrm{O}}} + H^{\mathrm{zp}}_{\mathrm{H}_{2\mathrm{O}}} + H^{\circ}_{\mathrm{H}_{2\mathrm{O}}}(T)\right)$$
(8)

$$\Delta_{\text{Hydr}} S^{\circ} = 2\Delta_{\text{F}} S^{\text{vib}}_{\text{OH}_{O}} - \Delta_{\text{F}} S^{\text{vib}}_{\text{v}^{\bullet}_{O}} - S^{\circ}_{\text{H}_{2}O}(T)$$
(9)

Upon experimental determination of $\Delta_{\text{Hydr}}S^{\circ}$ by curve fitting of eqn (2) to *e.g.* water uptake data from thermogravimetric measurements, it is usually assumed that the number of regular positions for OH₀[•] and v₀^{••} equal the number of oxide ions per formula unit.² For perovskites, OH₀[•] is normally found to take on a number of different configurations around each oxide ion, depending on the crystal structure. Further, in oxides where v₀^{••} preferentially forms on one of the several structurally distinct oxide ions, the number of regular positions for v₀^{••} is lower than the concentration of O lattice sites. The number of proton positions (N_{H}) around each oxide ion and the number of vacancy configurations (N_{v}) affect the chemical potential of OH₀[•] and $v_{O}^{\bullet\bullet}$, and thus K_{Hydr} . Not accounting for these configurations when evaluating defect thermodynamics by curve fitting results in an apparent hydration entropy, $\Delta_{Hydr}S_{app}^{\circ}$, which, in addition to the vibrational contributions, also includes configurational contributions:

$$\Delta_{\text{Hydr}}S_{\text{app}}^{\circ} = \Delta_{\text{Hydr}}S^{\circ} + \Delta_{\text{Hydr}}S_{\text{app}}^{\text{cont}}$$
$$= \Delta_{\text{Hydr}}S^{\circ} - 2R\sum_{i}^{N_{\text{H}}}p_{i,\text{H}}\ln(N_{\text{O}}p_{i,\text{H}}) + R\sum_{i}^{N_{\text{v}}}p_{i,\text{v}}\ln(N_{\text{O}}p_{i,\text{v}}) \quad (10)$$

 $N_{\rm O}$ is the number of oxide ions per formula unit, while $p_{i,\rm H}$ and $p_{i,\rm v}$ are the conditional probabilities for occupation of each configuration at an oxide ion by $OH_{\rm O}^{\bullet}$ and $v_{\rm O}^{\bullet\bullet}$, respectively. The general expression for the probability of occupation of a given configuration is for instance for the proton given by

$$p_{i,\mathrm{H}} = \frac{\exp(-E_i/k_\mathrm{B}T)}{\sum\limits_{i}^{N_\mathrm{H}} \exp(-E_i/k_\mathrm{B}T)}$$
(11)

where E_i are the relative energies of the different $N_{\rm H}$ configurations around the $N_{\rm O}$ oxide ions (3 for perovskites). It is important to note that $\Delta_{\rm Hydr}S_{\rm app}^{\rm conf}$ in eqn (10) only accounts for the configurational contributions stemming from distributing a proton or an oxygen vacancy over $N_{\rm H}$ and $N_{\rm v}$ configurations at one specific oxide ion, as the remaining contributions are accounted for through the mass action law (eqn (2)). At higher temperatures or in perovskites where the different $N_{\rm H}$ and $N_{\rm v}$ are degenerate, eqn (10) converges to

$$\Delta_{\text{Hydr}} S_{\text{app}}^{\circ} = \Delta_{\text{Hydr}} S^{\circ} + 2R \ln(N_{\text{H}})$$
(12)

3 Computational details

The first principles calculations are performed with the planewave Density Functional Theory (DFT) approach within the VASP (v.5.3.5) code,^{16,17} at the GGA-PBE¹⁸ and/or LDA¹⁹ level with a constant plane-wave cut-off energy of 500 eV. For the cubic BaZrO₃ and BaSnO₃, all defect calculations were performed with $3 \times 3 \times 3$ (135 atoms) supercell expansions, while $2 \times 2 \times 2$ supercells (160 atoms) were used for orthorhombic BaCeO₃ and SrZrO₃. Electronic integration was performed using a $2 \times 2 \times 2$ Monkhorst-Pack k-mesh over the supercell. Further, all calculations were performed using a reciprocal projection scheme, with ionic and electronic convergence criteria of 10^{-4} eV Å⁻¹ and 10^{-8} eV, respectively. The number of electrons in the supercell was adjusted to simulate the desired charge state of the defects, which was compensated by a homogeneous, opposite background charge. Finally, the defect calculations were performed both by fixing the volume to that of the defectfree bulk, or by relaxing both the supercell shape and its volume, in order to simulate constant volume, or zero/constant pressure conditions, respectively.

The phonon frequencies were calculated within the harmonic approximation using finite ionic displacements of ± 0.01 Å. Fourier transform and diagonalisation of the dynamical matrix were performed using the phonopy code.^{20,21} While BaZrO₃, BaCeO₃ and SrZrO₃ displayed no imaginary modes, BaSnO₃ exhibited imaginary modes at the *R*-point of the Brillouin zone with the GGA-PBE functional. With the LDA functional however, all modes of BaSnO₃ were found to be positive and this functional was therefore chosen to avoid erroneous contributions to the vibrational thermodynamics. All electronic contributions to the defect thermodynamics were, however, evaluated with the GGA-PBE functional – for consistency.

Table 2 shows the calculated lattice parameters and unit cell volumes of the four included perovskites. As is typically observed, GGA consistently overestimates the lattice parameters slightly, while the LDA parameters for BaSnO₃ are somewhat underestimated.

4 Results and discussion

4.1 Site degeneracy and configurational entropy

The orientation of the protonic defect around each oxide ion has previously been shown to depend on the symmetry of the structure, but also on the lattice constant/volume of the unit cell.²⁶ In BaZrO₃, BaSnO₃, BaCeO₃ and SrZrO₃, the proton takes on a position along the bisector of two O connecting lines and there are thus 4 proton positions per oxide ion.26-30 While these positions are degenerate in BaZrO₃ and BaSnO₃, they are nondegenerate in the orthorhombic BaCeO₃ and SrZrO₃. In the denser SrTiO₃, on the other hand, protons have been argued to take on a position along the edge of the TiO₆ polyhedra, resulting in 8 equivalent positions around each oxide ion.31,32 Our calculations are in general agreement with those of refs 26-30 showing that all included compositions exhibit proton positions at, or close to, the bisector of two oxygen connecting lines (see Fig. 1). The site is 4-fold degenerate in BaZrO₃ and BaSnO₃, while BaCeO₃ and SrZrO₃ display 4 unique minima around each of the O1 and O2 ions within 0.20 and 0.18 eV of

Table 2 Calculated lattice parameters and unit cell volumes (per f.u.) for BaZrO₃, BaSnO₃, BaCeO₃ and SrZrO₃. For BaSnO₃, the results refer to calculations with the LDA functional, while the GGA results are given in parentheses

	BaZrO ₃		BaSnO ₃		BaCeO ₃		SrZrO ₃	
	DFT	Exp. ²²	DFT	Exp. ²³	DFT	Exp. ²⁴	DFT	Exp. ²⁵
a (Å)	4.236	4.19	4.096 (4.188)	4.118	6.289	6.235	5.859	5.815
b (Å)	_		_ ` `		8.865	8.781	8.254	8.196
c (Å)	_		_		6.286	6.212	5.819	5.786
$V(Å^3)$	75.96	73.6	68.74 (73.44)	69.83	87.62	85.03	70.83	68.94



Fig. 1 Proton positions in (left) BaZrO₃ and (right) SrZrO₃ with *b* as the apical axis. For SrZrO₃ we also report the stability (in eV) of the different configurations relative to the most stable configuration around the O1 ion.

each other, respectively (cf. Fig. 1 for SrZrO₃). In BaCeO₃ and $SrZrO_3$, we also find that $v_0^{\bullet\bullet}$ is favoured by merely 70 and 60 meV, for BaCeO₃ and SrZrO₃, respectively, at the O1 compared to the O2 site. As shown in Fig. 2, the non-degenerate proton and oxygen vacancy configurations in SrZrO₃ and BaCeO₃ render the configurational contribution to the hydration entropy temperature dependent. At the lowest temperatures, the contribution is actually negative due to the preference for occupation of the O1 site by oxygen vacancies. However, due to the small energy difference between the various proton and oxygen vacancy configurations, the configurational contribution approaches that for BaZrO₃ even at \sim 600 K. Both SrZrO₃ and BaCeO3 also undergo several phase transitions upon heating, which may affect the relative stability of the different proton and vacancy configurations. Hence, the configurational contribution to $\Delta_{\text{Hvdr}}S^{\circ}$ is approximately $2R \ln(4)$ (23 J K⁻¹



Fig. 2 Configurational contribution to $\Delta_{Hydr}S^{\circ}$ when neglecting the site degeneracy of protons and oxygen vacancies ($\Delta_{Hydr}S^{conf}_{app}$) of BaZrO₃ and SrZrO₃ as a function of temperature according to eqn (10).

mol⁻¹) within the majority of the experimental temperature window (600–1100 K) for all included perovskites. Any compositional dependence of $\Delta_{Hydr}S^{\circ}$, in contrast to the conclusions of Tauer *et al.*,³³ is therefore of a purely vibrational nature, and will be the focus in the remaining parts of this contribution.

4.2 Chemical expansion

In our previous contribution on $BaZrO_3$,¹⁰ the formation volume of a defect was shown to have major implications for its formation entropy under zero/constant pressure conditions. In addition, the formation volume determines the chemical expansion coefficient of defects, and thus also the chemical expansion of the material upon *e.g.* hydration (eqn (1)).

The normalised volumetric chemical expansion coefficient, ε_i , of a defect *i* is here taken as:³⁴

$$\varepsilon_i = \frac{1}{\delta} \left(\frac{V_i}{V_0} - 1 \right) \tag{13}$$

where V_i and V_0 are the volumes of the defective and perfect supercells, respectively, while δ is the defect concentration in mole fractions (1/27 for BaZrO₃ and BaSnO₃, and 1/32 for BaCeO₃ and SrZrO₃). Correspondingly, the chemical expansion upon hydration (eqn (1)) may be quantified through

$$\varepsilon_{\rm Hydr} = 2\varepsilon_{\rm OH_0^{\bullet}} - \varepsilon_{\rm v_0^{\bullet\bullet}} \tag{14}$$

where $\varepsilon_{OH_O^{\bullet}}$ and $\varepsilon_{v_O^{\bullet}}$ are the chemical expansion coefficients of OH_O^{\bullet} and $v_O^{\bullet\bullet}$, respectively. ε_{Hydr} is thus given as the relative expansion upon hydration of 1 mole of $v_O^{\bullet\bullet}$, and with respect to the volume of the *defect-free lattice*. Experimentally, however, chemical expansion coefficients are taken as the relative volume of the hydrated and dry materials – *i.e.* relative to the *defective lattice*. Experimental ε_{Hydr} will therefore appear somewhat larger than those determined computationally in this work and in *e.g.* ref. 34.

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Table 3 Linear (ε_a , ε_b , ε_c) and volumetric (ε_i) chemical expansion coefficients, formation volumes ($\Delta_F V_i$), of OH_O[•] and v_O[•], and chemical expansion coefficient upon hydration (ε_{Hydr}) for the included perovskites. The v_O[•] in BaZrO₃ and BaSnO₃ is aligned with the Zr(Sn)-v_O[•]-Zr(Sn) axis parallel to the a-axis, while in BaCeO₃ and SrZrO₃ it is parallel to the b-axis. Note that the calculations on BaSnO₃ are performed with the LDA functional

	BaZrO ₃		BaSnO ₃		BaCeO ₃		SrZrO ₃		
	OH_O^{\bullet}	v _o	OH_0^{\bullet}	v _O	OH_O^{\bullet}	v _O	OH_O^{\bullet}	v₀••	
€ _{i,a}	0.0158	-0.0496	0.007	-0.0379	-0.0475	-0.1111	-0.0463	-0.1176	
Ei,b	-0.0566	-0.0952	-0.0540	-0.0973	-0.0429	-0.0688	-0.0110	-0.0323	
Ei,c	-0.0178	-0.0952	-0.0138	-0.0973	0.0238	-0.0454	-0.0116	-0.0560	
$\Delta_{ m F} V_i$	-4.44 Å 3	-18.1 Å ³	-4.14 Å 3	-15.9 Å^3	-5.86 Å^3	-19.7 Å 3	-4.87 Å 3	-14.6 Å^{3}	
ε_i	-0.0587	-0.239	-0.060	-0.232	-0.067	-0.225	-0.069	-0.206	
8 _{Hvdr}	0.122		0.111		0.0911		0.068		
	0.160 (In doped), ³⁷		_		0.030–0.099 (Yb doped), ³⁹		_		
ε_{Hvdr}^{Exp}	0.176 (Y	doped), ⁶			0.151 (Y doped) ³⁸				
	0.115-0.227 (Y doped) ³⁸					• /			

Table 3 shows the calculated chemical expansion coefficients and formation volumes of OH_{O}^{\bullet} and $v_{O}^{\bullet\bullet}$ (both linear and volumetric), along with the chemical expansion upon hydration. All expansion coefficients are given per mole of formed defect. Both defects lead to anisotropic relaxations of the supercell for the four perovskites, even for the cubic BaZrO₃ and BaSnO₃, where $v_0^{\bullet \bullet}$ leads to a tetragonal cell, in line with experimental reports.7,35,36 For all compositions, both defects display negative formation volumes and are thus smaller than the native oxide ions, with $v_0^{\bullet \bullet}$ leading to a considerably larger contraction than OH_{O}^{\bullet} . $\varepsilon_{v_{O}^{\bullet}}$ increases (less negative) notably in the order BaZrO₃ \rightarrow BaSnO₃ \rightarrow BaCeO₃ \rightarrow SrZrO₃, while $\varepsilon_{OH_0^{\bullet}}$ follows the opposite trend. The calculated $\epsilon_{\rm Hydr}$ are comparable with the experimentally determined values, although the experimental values vary considerably with the dopant concentration, and the adopted method (which most likely originates from uncertain proton concentrations). Hence, the compositional dependence of ε_{Hydr} is somewhat unclear from the experimental values. Our



Fig. 3 $\Delta_{FS_{vo}^{hb}}$ of SrZrO₃ and BaZrO₃ calculated under constant volume or zero/constant pressure (*i.e.*, relaxed volume) conditions.

calculated $\varepsilon_{\text{Hydr}}$ on the other hand decreases notably in the order $\text{BaZrO}_3 \rightarrow \text{BaSnO}_3 \rightarrow \text{BaCeO}_3 \rightarrow \text{SrZrO}_3$, which stems from both the decreasing $\varepsilon_{\text{OH}_0^*}$ and the increasing $\varepsilon_{\text{V}_0^*}$. Interestingly, these results therefore indicate that issues associated with the chemical expansion of BaZrO_3 electrolytes upon hydration may for instance be reduced by adopting BaZrO_3 -BaCeO₃ solid solutions.

4.3 Defect entropies

Fig. 3 shows the vibrational contribution to the formation entropy $(\Delta_F S_{V_O}^{vib})$ of $v_O^{\bullet\bullet}$ in SrZrO₃ and BaZrO₃ calculated under constant volume and zero/constant pressure conditions. For both compounds, $\Delta_F S_{V_O}^{vib}$ is negative, and significantly more negative under zero/constant pressure than under constant volume conditions – which stems from a blue shift of the phonon spectrum due to the negative formation volume of $v_O^{\bullet\bullet}$.¹⁰ Further, the effect is more noticeable for BaZrO₃ than for SrZrO₃, in line with the more negative formation volume of $v_O^{\bullet\bullet}$ in BaZrO₃. Similar effects of volume relaxation on $\Delta_F S_{V_O}^{vib}$ were also found for BaSnO₃ and BaCeO₃.

Fig. 4 shows $\Delta_F S_{\text{OH}_0}^{\text{vib}}$ and $\Delta_F S_{\text{v}_0}^{\text{vib}}$ for the four included perovskites, calculated under zero/constant pressure conditions. For all compositions, $\Delta_F S_{\text{v}_0}^{\text{vib}}$ is significantly more negative than $\Delta_F S_{\text{OH}_0}^{\text{vib}}$, and also shows a stronger compositional dependence. While all $\Delta_F S_{\text{OH}_0}^{\text{vib}}$ fall within 15 J mol⁻¹ K⁻¹ of each other throughout the temperature interval, $\Delta_F S_{\text{v}_0}^{\text{vib}}$ differs by as much as 80 J mol⁻¹ K⁻¹ at 1000 K between BaZrO₃ and SrZrO₃. The compositional variation of $\Delta_F S_{\text{v}_0}^{\text{vib}}$ to some extent follows $\varepsilon_{\text{v}_0}^{\text{o}}$, with the two cubic perovskites, BaZrO₃ and BaSnO₃, displaying significantly more negative entropies than the orthorhombic SrZrO₃ and BaCeO₃. Hence, the compositional dependence of $\Delta_F S_{\text{v}_0}^{\text{vib}}$ partly stems from the changes in $\varepsilon_{\text{v}_0}^{\text{o}}$ through the series, and partly from varying local structural relaxations around the defect (as reflected by the compositional dependence of $\Delta_F S_{\text{v}_0}^{\text{vib}}$ under constant volume conditions, see Fig. 3).

4.4 Hydration thermodynamics

Fig. 5 compares the calculated enthalpies (a) and entropies (b) of hydration according to eqn (1) for $BaZrO_3$, $BaSnO_3$, $SrZrO_3$ and $BaCeO_3$. Interestingly, the vibrational contribution to



Fig. 4 Calculated $\Delta_F S_{OH_{\odot}}^{vib}$ and $\Delta_F S_{v_{\odot}}^{vib}$ for the four included perovskites calculated under zero/constant pressure conditions.



Fig. 5 Calculated (a) enthalpy and (b) entropy of hydration of $BaZrO_3$, $BaSnO_3$, $SrZrO_3$ and $BaCeO_3$.

Table 4	Temperature	averaged	standard	hydration	enthalpy	and
entropy of	of BaZrO ₃ , BaSr	10 ₃ , SrZrO	3 and BaC	eO ₃ . For co	onsistency	with
experimental values, a constant configurational contribution of 23 J						
mol $^{-1}$ K $^{-1}$ to $\Delta_{Hydr} \overline{S}^{\circ}$ has been included for all compositions						

	$\Delta_{ m Hydr}ar{H}^{\circ}\ (m kJ\ mol^{-1})$	$\Delta_{ m Hydr}ar{ar{S}}^{\circ}_{ m app}\ \left({ m J}\ { m mol}^{-1}\ { m K}^{-1} ight)$		
BaZrO ₃	-65	-84		
BaSnO ₃	-60	-109		
SrZrO ₃	-97	-141		
BaCeO ₃	-122	-171		

i.e. including the configurational contribution according to eqn (11) (23 J mol⁻¹ K⁻¹ for all compositions).

The $\Delta_{\text{Hvdr}} \overline{H}^{\circ}$ values are, as expected, very close to the low temperature $\Delta_{\text{Hydr}}H^{\circ}$ in Fig. 5, and are in reasonable agreement with the experimental ranges of enthalpies for all compositions in Table 1. The calculated values also reflect the experimentally observed compositional dependence, with the two cubic perovskites displaying significantly less exothermic enthalpies than the two orthorhombic members. Also the calculated $\Delta_{\text{Hvdr}}\bar{S}^{\circ}$ values are in good agreement with the experimental values in Table 1. The entropies display a distinct compositional dependence, and are in general less negative for the cubic BaZrO₃ and BaSnO₃ than for the orthorhombic SrZrO₃ and BaCeO₃. The less negative $\Delta_{Hvdr}S^{\circ}$ of BaZrO₃ and BaSnO₃ stabilises protons compared to in the orthorhombic BaCeO₃ and SrZrO₃, in line with Kreuer's remark that the favourable proton transport properties of BaZrO3 stems from entropic stabilisation of protons.⁶ As noted in Fig. 3, $\Delta_F S_{v_1}^{vib}$ is the dominant vibrational contribution to $\Delta_{Hydr}S^{\circ}$, and also shows a stronger compositional dependency than $\Delta_F S_{OH_{2}}^{vib}$. Hence, one may argue that the entropic stabilisation of protons in BaZrO₃/BaSnO₃ is, more precisely, a result of entropic *destabilisation* of $v_0^{\bullet\bullet}$.

Turning to the hydration enthalpy, the strong compositional dependency (cf. Table 4) is generally in line with experimental observations; orthorhombic perovskites exhibit more exothermic values than the cubic, i.e. it becomes more exothermic with a decreasing electronegativity difference between the B- and A-site cations, but also with a decreasing Goldschmidt tolerance factor.^{8,9} These correlations are usually attributed to the basicity/ionicity of the oxide, which affects the bonding strength of $v_0^{\bullet\bullet}$ and OH_0^{\bullet} ; the oxide with the most exothermic hydration enthalpies exhibits the most stable protonic defects, and the most unstable oxygen vacancies (i.e. most stable oxide ions). There is further a striking correlation between both the calculated and experimental $\Delta_{Hydr} \bar{H}^{\circ}$ and $\Delta_{\text{Hvdr}}\bar{S}^{\circ}$ values, with the compound displaying the most exothermic $\Delta_{\text{Hydr}} \overline{H}^{\circ}$ (BaCeO₃) also displaying the most negative (least favorable) $\Delta_{Hydr}\bar{S}^{\circ}$. The correlation between $\Delta_{Hydr}\bar{H}^{\circ}$ and $\Delta_{Hydr}\bar{S}^\circ$ is far from intuitive since $\Delta_{Hydr}\bar{S}^\circ$ depends on both $\Delta_F S_{OH_{\circ}}^{vib}$ and $\Delta_F S_{v_{\circ}}^{vib}$, neither of which correlate directly with either $\varepsilon_{v_0^{\bullet\bullet}}$, $\varepsilon_{OH_0^{\bullet}}$ or ε_{Hydr} , or the volume of the unit cells, as one might expect. The contributions to $\Delta_F S_{OH_{\circ}}^{vib}$ and $\Delta_F S_{v_{\circ}}^{vib}$ are related to both the vibrational frequency of protons and oxide ions, local relaxations upon protonation and filling of the $v_0^{\bullet\bullet}$, in addition to the chemical expansion of the unit cell upon

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hydration. These properties can again be related to the size of the cations in the perovskite structure (which determines the unit cell volume and symmetry), and their electronegativity (which determines the interatomic bonding strengths), which – somewhat speculatively – explains the observed correlation between $\Delta_{Hydr}\bar{H}^{\circ}$ and $\Delta_{Hydr}\bar{S}^{\circ}$. Conclusively, the oxide exhibiting the most stable protons and least stable vacancies enthalpywise, also exhibits weaker entropic destabilisation of vacancies, and thus the least favourable entropies of hydration.

5 Conclusions

We have in this work assessed the thermodynamics of hydration, with emphasis on vibrational contributions, and the chemical expansion of the four proton conducting perovskites BaZrO₃, BaSnO₃, BaCeO₃ and SrZrO₃ from first principles phonon calculations. The calculations show that both OH₀ and $v_0^{\bullet\bullet}$ are smaller than the native oxide ions in all materials. Further, $v_0^{\bullet\bullet}$ is significantly smaller than the proton and is therefore the main contributor to the chemical expansion of the materials upon hydration. The chemical expansion shows a distinct compositional dependence, and decreases in the order $BaZrO_3 \rightarrow BaSnO_3 \rightarrow BaCeO_3 \rightarrow SrZrO_3$. As such, proton conducting electrolytes composed of BaZrO3-BaCeO3 or BaZrO₃-SrZrO₃ solid solutions should be less prone to issues stemming from chemical expansion upon hydration than pure BaZrO₃ electrolytes. There is further a significant contribution from phonons to the formation entropy of both OH_0^{\bullet} and $v_0^{\bullet\bullet}$. The contribution is most pronounced for $v_0^{\bullet\bullet}$, and negative for all compositions, and becomes less negative in the order $BaZrO_3 \rightarrow BaSnO_3 \rightarrow BaCeO_3 \rightarrow SrZrO_3$, *i.e.* with increasing (less negative) chemical expansion of $v_0^{\bullet\bullet}$. Both the calculated $\Delta_{\text{Hydr}}H^{\circ}$ and $\Delta_{\text{Hydr}}S^{\circ}$ are in good agreement with the experimental ranges of values, and display a distinct compositional dependence. $\Delta_{Hvdr}S^{\circ}$ is in general less negative (more favourable) for the cubic BaZrO₃ and BaSnO₃ than the orthorhombic members, reflecting its dependence on the vibrational formation entropy, and again the chemical expansion, of $v_0^{\bullet\bullet}$. Interestingly, there is a general correlation between $\Delta_{Hvdr}H^{\circ}$ and $\Delta_{\text{Hvdr}}S^{\circ}$; the two cubic members display the least exothermic $\Delta_{\text{Hvdr}}H^{\circ}$, but also the least negative (and thus most favourable) $\Delta_{\text{Hvdr}}S^{\circ}$ – an effect which may stem from both parameters being related to the bonding strengths of O^{2-} and H^+ .

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