**PAPER**

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**Ab initio** study of the enantio-selective magnetic-field-induced second harmonic generation in chiral molecules

Antonio Rizzo,∗a G. L. J. A. Rikkenb and R. Mathevetb

We present a systematic _ab initio_ study of enantio-selective magnetic-field-induced second harmonic generation (MFISHG) on a set of chiral systems ((L)-alanine, (L)-arginine and (L)-cysteine, 3,4-dehydro-(L)-proline, (S)-α-phellandrene, (R, S)– and (S, S)-cystine disulphide; N-(4-nitrophenyl)-(S)-prolinol, N-(4-(2-nitrovinyl)phenyl)-(S)-prolinol, N-(4-tricyanovinyl)phenyl)-(S)-prolinol, (R)-BINOL, (S)-BINAM and 6-(M)-helicene). The needed electronic frequency dependent cubic response calculations are performed within a density functional theory (DFT) approach. A study of the dependence of the property on the choice of electron correlation, on one-electron basis set extension and on the choice of magnetic gauge origin is carried out on a prototype system (twisted oxygen peroxide). The magnetic gauge dependence analysis is extended also to the molecules of the set. An attempt to analyze the structure–property relationships is also made, based on the results obtained for biphenyl (in a frozen twisted conformation), for prolinol and for some of their derivatives. The strength of the effect is discussed, in order to establish its measurability with a proposed experimental setup.

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**1 Introduction**

Nonlinear spectroscopies1–5 are becoming increasingly popular as they are coming out from a small circle of aficionados to reach the communities of researchers working in fields as advanced and strategic as novel (bio)materials, sensors, renewable energy and medical diagnostics. This is due to the rapid advances in laser technology, responding to the growing needs for diverse spectroscopic tools, which could cover regions of spectra where linear spectroscopies are of little use due to lack of transparency of the sample. Multiphoton absorption, MPA, whose theoretical details were unveiled more than eighty years ago, in the early ages of quantum theory, by Gōppert-Mayer,6 and which became a spectroscopic technique after the development of laser sources some fifty years ago,7,8 is now a full blown area of research and technology, witnessing a unique explosion that in the electric dipole approximation, SHG is forbidden in isotropic media, like gases or liquids, and it can be observed only on surfaces, interfaces or locally oriented structures such as membranes21 or fibrils22 where centro-symmetry is permanently broken.

SHG is a special case of the more general process of Sum Frequency Generation,9–13,14 the non-resonant frequency doubling which has led to major advantages in microscopy,15 where the virtual absence of photooxidation and self-absorption phenomena and remarkable directionality of SHG have been exploited.16–20 The dominant light–matter interaction is through electric dipole coupling. Symmetry arguments can be used to show that in the electric dipole approximation, SHG is forbidden in isotropic media, like gases or liquids, and it can be observed only on surfaces, interfaces or locally oriented structures such as membranes21 or fibrils22 where centro-symmetry is permanently broken.16–20

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**Notes**

1 Consiglio Nazionale delle Ricerche – CNR, Istituto per i Processi Chimico-Fisici, UoS di Pisa, Area della Ricerca, Via G. Moruzzi 1, I-56124 Pisa, Italy. E-mail: rizzo@ipcf.cnr.it

2 Laboratoire National des Champs Magnétiques Intenses, UPR3228 CNRS/INSA/UJF/UPS, Toulouse & Grenoble, France

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lacking centro-symmetry, and therefore exhibiting optical activity in the presence of intense electromagnetic radiation. In contrast to other well known linear and nonlinear chiroptical phenomena, which need the involvement of magnetic dipole and/or electric quadrupole interaction to be rationalized, SFG relays entirely on electric dipole interactions.\(^9,10\) The effect disappears in racemic mixtures, and its intensity is proportional to (and it is therefore a measure of) the enantiomeric excess in a solution containing enantiomers.

A static electric field induces SHG in isotropic media, in what is known as Electric-Field-Induced SHG (EFISHG),\(^29\) a phenomenon exploited in electrooptic conversion\(^1\)–\(^3,30\) and in general in the study of materials. In a recent study,\(^31\) we have analyzed computationally the chiral response of optically active samples in EFISHG experimental setups where circularly polarized radiation is employed,\(^32\) estimating the circular intensity difference (CID) to be observed in yet to be performed experiments involving some representative proteinogenic amino acids. CID is related to what is commonly defined as circular dichroism, CD, the difference in molar extinction for left and right circularly polarized radiation,\(^11,13\) and it yields a tool for the study of stereochemistry and structure of biomolecular conformations.\(^34\)–\(^37\)

In this paper we will focus our attention on another possible nonlinear process, that is SHG induced by an external static magnetic field, labeled MFISHG, focusing on chiral media. Magnetic field induced SHG has been observed in several instances, in gases,\(^38\) surfaces,\(^39\) solids,\(^40,41\) films and nanoparticles,\(^42\) see also ref. 43 and the theoretical analysis by Kielich and Zawodny,\(^44\) and Manakov and co-workers.\(^45\) Magnetization induced SHG has also been reported,\(^46\) in films,\(^37\) and solids,\(^48,49\) Here we study the case where, as in SFG, the polarization measured after interaction of polarized radiation with a chiral sample, leading to frequency doubling, is different for the distinct enantiomers when the sample is subject to a static magnetic field. The phenomenon has been discussed by Georges H. Wagnière,\(^4,12\) and it is one element of the large variety of high-order optical effects which one can in principle detect in matter by exploiting the high intensity and high coherence of available laser sources. We will focus on the purely electronic form of MFISHG, neglecting orientational effects by the magnetic field. The expression of the macroscopic second harmonic generation is related to the microscopic contributions to the frequency dependent induced multipole moments, and in particular to an average taken over appropriate frequency dependent cubic response functions involving electric dipole and magnetic dipole perturbations. The observable is computed for a series of chiral reference systems ((\(L\))-alanine; (\(l\))-arginine; (\(l\))-cysteine; 3,4-dehydro-(\(l\))-proline; (\(S\))-\(\alpha\)-phellandrene; (\(R,S\))- and (\(S,S\))-cystine disulphide; \(N\)-(4-nitrophenyl)-(\(S\))-prolinol; \(N\)-(4-(2-nitrovinyl)-phenyl)-(\(S\))-prolinol; \(N\)-(4-tricyanovinyl-phenyl)-(\(S\))-prolinol; \(R\)-BINOL; (\(S\))-BINAM and 6-(\(M\))-helicene) with the aim of estimating the detectability of the optical effect. For further analysis of the structure–property relationships, and in particular of the effect of donor- and acceptor-substituents on reference molecular frames, calculations were performed on a chiral form of biphenyl, and its derivatives 4-amino-biphenyl, 4-nitro-biphenyl and 4-amino-4′-nitro-biphenyl. A density functional theory time-dependent approach\(^50,51\) has been employed for the response calculations, and a preliminary analysis of the dependence of the observable on some of the critical parameters of these high-order property calculations (electron correlation, choice of the functional, extension and quality of the basis set and, last but not least, dependence of the results on the choice of the magnetic gauge) is carried out on a prototype system, frozen twisted hydrogen peroxide. The dependence of the property on the choice of the origin of the magnetic gauge is also analyzed for the molecules of the set.

In response property calculations involving magnetic dipole interactions the dependence of the results on the choice of the magnetic gauge origin in approximate calculations is often taken care using perturbation-dependent basis sets. Within our group, in particular, we usually resort to Gauge Including Atomic Orbitals (GIAO’s, also known as London Atomic orbitals, LAO’s)\(^52\). Although cubic response function related properties such as Cotton–Mouton\(^53\) and Jones\(^54\) birefringences have been studied using GIAO’s by resorting to the atomic orbital-based response theory allowing for the use of both time- and perturbation-dependent basis sets (OpenRSP approach) pioneered by Ruud and co-workers,\(^55\) those studies employed an independent particle approximation. To our knowledge no implementation of cubic response theory employing electron-correlated structure models is available to date. Note also that only the electronic contributions to the cubic response functions relevant for the observable in the MFISHG experiment are analyzed in this work. Vibrational contributions, which can in principle be non-negligible particularly for some of the molecules included in our set, are not included. Furthermore, our conclusions will apply to isolated, non-interacting molecules, since the effect of the environment is not accounted for.

The layout of the paper follows closely that of ref. 31. In Section 2 the theoretical background is laid down; the computational details are given in Section 3; the results are presented and discussed in Section 4; in Section 5 the experimental setup under construction at the Laboratoire dec Champs Magnétiqes Intenses is briefly described and the perspectives for experimental detection of MFISHG are discussed; Section 6 collects our conclusions.

2 Theory

2.1 Basic formulae

In this section, setting the theoretical background for the study, we will consider the possible symmetry allowed forms that the source terms in Maxwell’s equations, like the electric polarization \(P_{2\omega}\) or the magnetization \(M_{2\omega}\), can take, induced by the electric and magnetic field components of an electromagnetic plane wave at frequency \(\omega\), \(E_\omega\) and \(B_\omega\), respectively, in the presence of static electric or magnetic fields \(E_0\) and \(B_0\) respectively. In a purely electric dipole approximation, EFISHG is described by

\[
P_{2\omega} = Z_1(E_\omega E_\omega)E_\omega + Z_2(E_\omega, E_\omega)E_0
\]
with the material parameters $\chi_i$ representing second order hyperpolarizabilities. SHG induced by a static magnetic field is also symmetry allowed, through (to lowest order) the contributions

$$P_{2\omega} = \chi_1(B_0 \cdot B_0)E_{\omega} + \chi_2(E_0 \cdot B_0)B_{\omega}$$  \hspace{1cm} (2)

and

$$M_{2\omega} = \zeta_1(E_0 \cdot B_0)E_{\omega} + \zeta_2(E_0 \cdot E_0)B_{\omega}$$  \hspace{1cm} (3)

These effects (observable in principle in all media) involve more than one magnetic dipole interaction and therefore are very weak. They are generally neglected, but have been experimentally observed. If the medium is chiral, that is if $\chi_i^D = -\chi_i^L$, and we are looking for optical electric dipole interactions to generate MFISHG, the only symmetry allowed form is

$$P_{2\omega} = \psi_1^{D/L}(\frac{\partial E_0}{\partial t} \cdot B_0)E_{\omega} + \psi_2^{D/L}(\frac{\partial E_0}{\partial t} \cdot E_0)B_{\omega}$$  \hspace{1cm} (4)

which for a harmonic electromagnetic field can be written as

$$P_{2\omega} = i\omega \psi_1^{D/L}(E_0 \cdot B_0) + i\omega \psi_2^{D/L}(E_0 \cdot E_0)B_0$$  \hspace{1cm} (5)

The intuitive physical interpretation of eqn (1) is that the polarizability of the medium, induced by the external electric field, distorts its response to the alternating electric field, generating even harmonics. A similarly intuitive physical picture behind eqn (4) can be found by realizing that

$$\frac{\partial E_0}{\partial t} = \frac{1}{\chi} \frac{\partial P_0}{\partial t} = \frac{1}{\chi} I_0$$  \hspace{1cm} (6)

$I_0$ being the induced current density at optical frequencies and $\chi$ the electrical susceptibility. We can then write

$$P_{2\omega} = \psi_1^{D/L}(I_0 \cdot B_0)E_{\omega}$$  \hspace{1cm} (7)

The medium response is therefore different whether the induced current is parallel or anti-parallel to the external magnetic field. Such an effect has already been observed at low frequencies and was called electrical magneto-chiral anisotropy (eMChA). SHG induced by a static magnetic field in chiral media can therefore be described as the result of eMChA at optical frequencies.

A possible setup for the experiment discussed theoretically above is schematized in Fig. 1. A linearly polarized electromagnetic beam impinges in a fluid made by noninteracting, chiral molecular subject also to a static external magnetic induction field $B_0$, with a component aligned parallel to the direction of polarization of radiation. In the setup of Fig. 1, two photons of circular frequency $\omega$ are absorbed and a photon with circular frequency equal to $2\omega$ is emitted, in a process that we call, in analogy with its electric field a far more popular counterpart, MFISHG, magnetic field induced second harmonic generation. As recognized by Wagni`ere, the process, a particular example of the more general phenomenon of magnetic field induced sum frequency generation, involves the mixed electric and magnetic nonlinear optical response of the sample. In particular, focusing on third order effects, the average electric dipole moment $P_{2\omega}$, induced in a molecule by the interaction with the electromagnetic radiation in the static magnetic field, $P_{2\omega} = P^{(3)}(-2\omega; \omega, \omega, 0)$, a vector, can be written as

$$P^{(3)}(-2\omega; \omega, \omega, 0) = 2i\chi^{(3)}(-2\omega; \omega, \omega, 0)(E_0 \cdot B_0)E_{\omega}$$  \hspace{1cm} (8)

Here $\chi^{(3)}(-2\omega; \omega, \omega, 0)$ is the third order molecular susceptibility tensor of rank four. The brackets indicate an isotropical average needed in fluids, yielding a scalar or, as in the specific case discussed here, a pseudoscalar. The three field vectors involved in the process are combined (a scalar product of two of them multiplied by the third) in order to yield a vector. The particular sequence of field vectors chosen in eqn (8) stands for any of the possible permutations. The third order susceptibility, formally represented by the label $\mathbf{m}^{\mu\mu}$ by Wagni`ere in ref. 4 and 12, after proper spatial averaging, carried out using the approach and expressions listed in ref. 4, 12 and 60, can be written as

$$\langle \chi^{(3)}(-2\omega; \omega, \omega, 0) \rangle = + \frac{1}{5} \gamma_{\omega,\omega,0} + \frac{1}{5} \gamma_{\omega,\omega,0} + \frac{1}{5} \gamma_{\omega,\omega,0} + \frac{1}{5} \gamma_{\omega,\omega,0} + \frac{1}{5} \gamma_{\omega,\omega,0} + \frac{1}{5} \gamma_{\omega,\omega,0} + \frac{1}{5} \gamma_{\omega,\omega,0} + \frac{1}{5} \gamma_{\omega,\omega,0} + \frac{1}{5} \gamma_{\omega,\omega,0}$$  \hspace{1cm} (9)

where

$$\gamma_{\omega,\omega,0} = \gamma_{\omega,\omega,0} - i \langle \mu_3 \mu_3 \mu_2 \mu_2 \rangle_{\omega,\omega,0}$$  \hspace{1cm} (10)

indicates a frequency dependent cubic response function involving the electric dipole operator $\mu$

$$\mu = \sum_j q_j r_j$$

and the magnetic dipole operator $m$

$$m = \sum_j \frac{q_j}{2M_j} (r_j \times p_j) = \sum_j \frac{q_j}{2M_j} l_j.$$  \hspace{1cm} (11)

Above the charge $q_j$, the spatial coordinate $r_j$, the masses $M_j$, the linear momentum $p_j = -i\hbar \nabla_j$ and the orbital angular momentum $l_j = (r_j \times p_j)$ of particle $j$ are introduced. The cubic response in eqn (10) is an axial (parity) odd (time reversibility) tensor of fourth order, and the effect, according to ref. 61, occurs only in a medium of non-centrosymmetric molecules. Moreover,
the following permutation symmetry relationships apply when all frequencies are far away from molecular resonances

\[
\zeta_{s,\beta,\gamma,\delta}(-2\omega_\alpha \omega_\beta,0,0) = \zeta_{s,\beta,\gamma,\delta}(-2\omega_\alpha \omega_\beta,0,0) = \zeta_{s,\beta,\gamma,\delta}(-2\omega_\alpha \omega_\beta,0,0)
\]

(12)

which imply

\[
\langle \chi^{(3)}(-2\omega_\alpha \omega_\beta,0,0) \rangle = \omega \psi_D^{DL} = \omega \psi_D^{DL}
\]

For a general discussion of nonlinear susceptibilities and their sum-over-state representation the reader should refer to the textbook ref. 2–5, see also ref. 62. Frequency dependent cubic response functions as those represented in eqn (10) can be obtained nowadays resorting to a fully analytical approach developed about thirty years ago,\textsuperscript{63,64} and that avoids the recourse to approximate sum-over-state schemes. The approach was applied to a wide range of electronic structure models, as SCF\textsuperscript{63} and MCSCF,\textsuperscript{66} Coupled Cluster\textsuperscript{67} and DFT.\textsuperscript{68,69} In this study we resort to the latter to obtain an estimate of the the polarization described by eqn (8).

### 2.2 Origin dependence

When the origin of the multipolar expansion is displaced from \(O\) to \(O'\), say \(O' = O + R\), which implies that the transformation \(r' = r - R\), the operators \(m\) and \(n\), in a neutral system, are transformed as follows \(\Delta Y = Y' - Y, \ i = x, y, z\) \textsuperscript{36}

\[
\Delta \mu_i = 0 \\
\Delta n_i = \frac{1}{2} \epsilon_{i \alpha \beta \gamma} R_{\alpha \beta \gamma} \nu^{0,0,0}_{\alpha,\beta,\gamma,\delta}
\]

(14)

where \(\nu^p\) is the velocity operator

\[
\nu^p = \sum_j q_j M_j P_j
\]

These relationships can be used to derive an expression for the change in the response function tensor elements of eqn (10)

\[
\zeta_{s,\beta,\gamma,\delta}(O') = \zeta_{s,\beta,\gamma,\delta}(O) + \frac{1}{2} \epsilon_{i \alpha \beta \gamma} R_{\alpha \beta \gamma} \nu^{0,0,0}_{\alpha,\beta,\gamma,\delta}
\]

(15)

where

\[
\nu^{0,0,0}_{\alpha,\beta,\gamma,\delta} = \nu_{\alpha,\beta,\gamma,\delta}(-2\omega_\alpha \omega_\beta,0,0) = \pm \frac{1}{2} \epsilon_{i \alpha \beta \gamma} \nu^{0,0,0}_{\alpha,\beta,\gamma,\delta}
\]

(16)

and \(\epsilon_{i \alpha \beta \gamma}\) is the alternating Levi-Civita tensor. In the limit a complete one-electron basis set, the hypervirial relationship

\[
|\mu_s, H'] = \mu_s^p
\]

(17)

holds, and we can exploit the equations of motions holding for exact cubic response functions,\textsuperscript{3,4,5}

\[
(\alpha_1 + \alpha_2 + \alpha_3 + 3i \epsilon) \langle \langle V_{\alpha_1} \nu^{0,0,0}_{\alpha_1,\alpha_2,\alpha_3}, V_{\alpha_2}, V_{\alpha_3} \rangle \rangle = \langle \langle \langle V_{\alpha_1}, H', V_{\alpha_2}, V_{\alpha_3} \rangle \rangle \rangle
\]

(18)

\[
+ \frac{1}{2} P_{\alpha_1} \langle \langle V_{\alpha_2}, V_{\alpha_3} \rangle \rangle \rangle = \langle \langle V_{\alpha_1}, H', V_{\alpha_2}, V_{\alpha_3} \rangle \rangle \rangle
\]

where \(V^p\) denotes the perturbation of circular frequency \(\omega\), \(H'\) is the Hamiltonian, \(P_{\alpha_1} \ldots \) indicates all permutations of the indices \(i, j, \ldots\). The combination of eqn (17) and (18) allows us to prove that, for the “exact” case, or (equivalently) in the limit of a complete one-electron basis set,

\[
\nu^{0,0,0}_{\alpha,\beta,\gamma,\delta} \to 0
\]

(19)

In approximate calculations, on the other hand, the third order susceptibility tensor at the displaced origin can be written as

\[
\langle \chi^{(3)}(-2\omega_\alpha \omega_\beta,0,0) \rangle(O') = \langle \chi^{(3)}(-2\omega_\alpha \omega_\beta,0,0) \rangle(O) + R \cdot S
\]

(20)

where

\[
S_X = -\frac{1}{15} \nu^{0,0,0}_{\alpha,\beta,\gamma,\delta} + \frac{1}{15} \nu^{0,0,0}_{\alpha,\beta,\gamma,\delta} - \frac{1}{30} \nu^{0,0,0}_{\alpha,\beta,\gamma,\delta} + \frac{1}{10} \nu^{0,0,0}_{\alpha,\beta,\gamma,\delta}
\]

(21)

\[
S_Y = -\frac{1}{15} \nu^{0,0,0}_{\alpha,\beta,\gamma,\delta} + \frac{1}{15} \nu^{0,0,0}_{\alpha,\beta,\gamma,\delta} - \frac{1}{30} \nu^{0,0,0}_{\alpha,\beta,\gamma,\delta} + \frac{1}{10} \nu^{0,0,0}_{\alpha,\beta,\gamma,\delta}
\]

(22)

\[
S_Z = -\frac{1}{15} \nu^{0,0,0}_{\alpha,\beta,\gamma,\delta} + \frac{1}{15} \nu^{0,0,0}_{\alpha,\beta,\gamma,\delta} - \frac{1}{30} \nu^{0,0,0}_{\alpha,\beta,\gamma,\delta} + \frac{1}{10} \nu^{0,0,0}_{\alpha,\beta,\gamma,\delta}
\]

(23)

### 3 Computational details

This exploratory study was carried out on a set of optically active molecular systems whose structures are shown in Fig. 2. The set includes: three proteinogenic amino acids ([L]-alanine, (I)-arginine and (L)-cysteine); five molecules—3,4-dehydro-(I)-proline, (S)-z-phellandrene, (R,S)- and (S,S)-cystine disulphide and N-(4-nitrophenyl)-(S)-prolinol, considered as typical prototypes of chiral systems with remarkable linear and nonlinear optical responses; two bi-aryls (1’-1-bi-2-R-naphthol—(R)-BINOL and (S)-1’-1-bi-2’-diamine—(S)-BINAM) and an helicene (6-(M)-helicene). In order to analyze the relationship between the structure and property, and in particular the effect of electron-donating and electron-withdrawing substituents on reference scaffolds, two non-commercially available derivatives of N-(4-nitrophenyl)-(S)-prolinol [N-(4’-2-nitrovinyl)-phenyl-(S)-prolinol and N-(4-tricyanovinyl-phenyl)-(S)-prolinol], and four model systems as a frozen twisted conformation of (R)-biphenyl and its (R)-4-amino-biphenyl, (R)-4-nitro-biphenyl
and (R)-4-amino-4'-nitro-biphenyl derivatives were also studied. Structures were optimized in the gas phase. For the three proteinogenic amino acids and for (R)-BINOL the geometries optimized at the B3LYP/6-31g(d,p) level already employed in ref. 31 and 74 were adopted. The structure of (R)-BINOL is taken from the study carried out by Sahnoun and co-workers.75 The structure of 6-(M)-helicene was the one optimized at the B3LYP/6-31G** level already employed in ref. 77. The geometrical parameters of all remaining molecules were optimized in this work at the B3LYP/aug-cc-pVTZ76,79 level using Gaussian.80

The calculation of the frequency dependent cubic response function tensor components was carried out for the isolated molecules at the TD-DFT level,50,51 employing the B3LYP functional70–72 with the aug-cc-pV5Z basis set and resorting, in addition to B3LYP, to four other popular functionals: BLYP,71,72,84 BHandHLYP,85 Cam-B3LYP,86–88 and SVWN5.89 BLYP71,72,84 combines Becke’s 1988 functional91—including both Slater’s exchange89 and correlations involving the gradient of the density—with both local and non-local terms of Lee, Yang, and Parr’s correlation functional.72,84 The Half-and-half Functional BHandHLYP86 employs an equal fraction of Hartree–Fock, Local Spin Density Approximation (LSDA)89 and Becke 88 exchange71 together with the LYP correlation contribution.72,84 CAM-B3LYP76–88 is a long range corrected version of B3LYP where the Coulomb-attenuating method was employed. SVWN589 couples the Slater exchange functional89 with the Volko, Wilk and Nussair correlation functional, and it is equivalent to LSDA.

4 Results and discussion of calculations

The results obtained for the third order susceptibility \(\langle \chi^{(3)}(-2\omega_{2}, \omega_{0}, 0) \rangle\) at the wavelength \(\lambda = 1064\) nm are shown in Table 1.

Among the chosen proteinogenic acids, (l)-cysteine yields the strongest response at \(\lambda = 1064\) nm, twice as strong as that of (l)-arginine and about 35% stronger than (l)-alanine (absolute values). For the latter, note that as we perform the calculation at shorter wavelengths, see Table 2, the third order susceptibility increases, with a strong enhancement, by a factor of twenty, at \(\lambda = 632.8\) nm (\(\approx 1.95\) eV, that is more than 1 eV lower than needed to approach the two-photon resonance with the first excited state, placed at \(\approx 5.96\) eV91). We will come back to the phenomenon of near resonant enhancement later on. Here we note that the observable appears to be, at least in the cases we analyzed, quite sensitive to the change in circular frequency.

Table 1 The third order susceptibility \(\langle \chi^{(3)}(-2\omega_{2}, \omega_{0}, 0) \rangle\).

<table>
<thead>
<tr>
<th>System</th>
<th>(\langle \chi^{(3)}(-2\omega_{2}, \omega_{0}, 0) \rangle)/a.u.</th>
</tr>
</thead>
<tbody>
<tr>
<td>(l)-Alanine</td>
<td>−0.3767</td>
</tr>
<tr>
<td>(l)-Cysteine</td>
<td>−0.5203</td>
</tr>
<tr>
<td>(l)-Arginine</td>
<td>0.2748</td>
</tr>
<tr>
<td>3,4-Dehydro-(l)-proline</td>
<td>1.893</td>
</tr>
<tr>
<td>(S)-z-Phellandrene</td>
<td>−7.387</td>
</tr>
<tr>
<td>(S,S)-Cystine disulfide</td>
<td>−2.597</td>
</tr>
<tr>
<td>(S,S)-Cystine disulfide</td>
<td>7.740</td>
</tr>
<tr>
<td>N-(Nitrophenyl)-(S)-prolinol</td>
<td>13.20</td>
</tr>
<tr>
<td>N-(4-(2-Nitrovinyl)-phenyl)-(S)-prolinol</td>
<td>−38.76</td>
</tr>
<tr>
<td>(R)-BINOL</td>
<td>−16.61a</td>
</tr>
<tr>
<td>(S)-BINAM</td>
<td>23.82</td>
</tr>
<tr>
<td>6-(M)-Helicene</td>
<td>96.49</td>
</tr>
<tr>
<td>(R)-Biphenyl</td>
<td>5.675</td>
</tr>
<tr>
<td>(R)-4-Amino-biphenyl</td>
<td>7.849</td>
</tr>
<tr>
<td>(R)-4-Nitro-biphenyl</td>
<td>−20.54</td>
</tr>
<tr>
<td>(R)-4-Amino-4'-nitro-biphenyl</td>
<td>−135.2</td>
</tr>
</tbody>
</table>

\[a\text{ At } \lambda = 780\text{ nm, }\langle \chi^{(3)}(-2\omega_{2}, \omega_{0}, 0) \rangle = −120.0\text{ a.u. At } \lambda = 632.8\text{ nm, which in our case means less than 5 nm away from the 2-photon intermediate state resonance with the third excited state, }\langle \chi^{(3)}(-2\omega_{2}, \omega_{0}, 0) \rangle = −370.3\text{ a.u.}\]
The response of the five next molecules in Table 1 is definitely stronger than that of the proteinogenic amino acids, in particular for N-(nitrophenyl)-(S)-prolinol. The latter behaves similarly to the two bi-aryls, (R)-BINOL and (S)-BINAM, exhibiting rather sizable third order susceptibilities of ≈17 a.u. and ≈24 a.u. absolute values, respectively. With a wavelength of 780 nm the response of (R)-BINOL is enhanced by an order of magnitude ($\chi^{(3)}(-2\omega_{0},\omega_{0},0)$) ≈ −0.3767 relative to the amino-derivative, down to ≈ 0.03 eV, with respect to the third excited state). Again, we will get back briefly on this evidence below. Note also that the two diastereoisomers of cystine disulfide, (R,S) and (S,S), have a significantly different response, both in terms of sign and absolute value.

The case of biphenyl and its three derivatives is very instructive. The frozen conformation of biphenyl has the two phenyl rings at a dihedral angle of ≈ 38.7°, reduced to ≈ 37.8° in the amino-derivative, ≈ 37.4° in the nitro-derivative, and down to ≈ 33.9° in the nitro-amino-derivative. Adding donor or acceptor substituents in the para position to the reference structure enhances the property, only slightly when the substituent is the donor amino group, quite substantially when an acceptor nitro group is attached. In this case the third order susceptibility increases by a factor of more than 3.6 in absolute value, and it changes sign with respect to biphenyl. The real enhancement of the effect is seen nevertheless when the systems assume a D–B–A configuration with the donor and the acceptor placed in positions 4 and 4′, respectively. The third order susceptibility ($\chi^{(3)}(-2\omega_{0},\omega_{0},0)$) of (R)-4-amino-4′-nitro-biphenyl has the largest value of the set in Table 1, −135.2 a.u., suggesting that push–pull chromophores might be good candidates for intense MFIHG. The MFISH of (R)-4-amino-4′-nitro-biphenyl is indeed stronger than that of 6-(M)-helicene, the best inherently chiral system in Table 1 with its ($\chi^{(3)}(-2\omega_{0},\omega_{0},0)$) of ≈ 96 a.u. Note that these values still yield induced polarizations which are still below the current threshold for detection, see Section 5 below.

It is well known that linear and nonlinear chiroptical properties of bi-aryls can vary substantially for small changes in the dihedral angle between the aryls. In order to estimate the dependence on the MFISHG of biphenyl and its three derivatives in Table 1 on (small) changes in the dihedral angle, calculations have been carried out on the optimized geometry structures where the dihedral angle was fixed to 35°, without further optimization of the remaining parameters. Results in

Table 2 The wavelength dependence of the third order susceptibility ($\chi^{(3)}(-2\omega_{0},\omega_{0},0)$) for (L)-alanine. B3LYP/aug-cc-pVTZ

<table>
<thead>
<tr>
<th>λ (nm)</th>
<th>$\hbar\omega$ (eV)</th>
<th>($\chi^{(3)}(-2\omega_{0},\omega_{0},0)$) (a.u.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1064</td>
<td>1.165</td>
<td>−0.3767</td>
</tr>
<tr>
<td>780.0</td>
<td>1.590</td>
<td>−1.791</td>
</tr>
<tr>
<td>632.8</td>
<td>1.959</td>
<td>−7.820</td>
</tr>
</tbody>
</table>

The hints given by the analysis of the biphenyl family prompted us to further explore the possibility to establish some reasonable structure/property relationships, starting in particular from the structure of N-(nitrophenyl)-(S)-prolinol (the molecule in our set with the highest response without a bi-arly or helical structure) and modifying it by inserting a vinyl group between the aromatic ring and the nitro substituent (N-(4-(2-nitrovinyl)phenyl)-(S)-prolinol) or the same vinyl group with three CN substituents (N-(4-tricyanovinyl-phenyl)-(S)-prolinol). If the 4-(2-nitrovinyl) derivative yields an averaged third order susceptibility halved with respect to the starting N-(nitrophenyl)-(S)-prolinol, the 4-tricyanovinyl-phenyl derivative, with its three strong electron-withdrawing groups, exhibits a MFISHG strongly enhanced (more than three times larger and of opposite sign) with respect to that of the progenitor.

4.1 Frequency dependence and near resonant enhancement of MFISHG

We have noted above the quite remarkable dependence of the MFISHG of (L)-alanine and (R)-BINOL on the circular frequency of electromagnetic radiation, and we discussed the possibility of enhancing the response of the molecule by exploiting near resonant conditions in an experiment. Fig. 3 gives a graphical...
view of the distribution of the lowest four excited states of the molecules included in an analysis, as obtained at the B3LYP/aug-cc-pVTZ level, and it shows where the one-photon and two-photon resonances lie in the energy spectrum when the three laser wavelengths (1064 nm, 780 nm and 632.8 nm) are considered. The seventeen systems listed in Table 1 are identified in the figure by acronyms, and are arranged in the same order as in the table. The lowest excited state of \( \{N-(4\text{-tricyanovinyl-phenyl})-(S)-(prolinol)\} \) (TCVPP in the figure) is placed at 2.87 eV \( (\lambda = 431.6 \text{ nm}) \), and it is 100 nm off the two-photon resonance at an electromagnetic field wavelength of 1064 nm. This near resonance has most likely a relevant effect on the quite strong resonance of \( \{N-(4\text{-tricyanovinyl-phenyl})-(S)-(prolinol)\} \) at \( 397 \text{ nm} \). The two-photon resonances corresponding to the same biaryl when going from one-photon to two-photon theory, as introduced by Norman and co-workers \(^{94,95}\) and later recently employed in a study of two-photon absorption involving one of the present authors.\(^{97}\) To our knowledge, an appropriate damped response code that could handle cubic response properties is still missing.

### 4.2 Magnetic gauge origin dependence

In order to estimate the dependence of the property under analysis in this study on the choice of the magnetic field gauge we computed the vector \( S \), defined in eqn (20), see also eqn (21) to (23), at the B3LYP/aug-cc-pVTZ level, for \( \lambda = 1064 \text{ nm} \), for the seventeen molecules of our set. Table 4 collects the results.

In order to give a rough estimate of the effect of a change in the magnetic gauge origin on a given molecule, we use the percentage change (in absolute value) of the MFISHG listed in Table 1 upon displacement of the magnetic gauge origin by a vector \( R = (10., 10., 10.) \text{ a.u.} \) (last column in Table 4). In most cases such displacement changes the property by less than 10%. Exceptions are the proteinogenic acids, exhibiting a relatively weak MFISHG, which can change by up to more than 30% upon the given change of origin, and the two derivatives of \( N-(nitrophenyl)-(prolinol) \), where \( N-(4\text{-}2\text{-nitro-phenyl})-(S)-(prolinol) \) (in particular) has a strong response to the displacement of origin, by \( \approx 68\% \), mostly due to the large value of \( S_y \) The y axis goes perpendicular to the plane defined by the pyrrolidine nitrogen and its two carbon neighbors and the (slightly tilted) plane of the phenyl ring and its nitrovinyl substituent. It is therefore nearly aligned to the C–C bond connecting the methanol group and the five membered ring. The MFISHG of \( N-(4\text{-tricyanovinyl-phenyl})-(S)-(prolinol) \) is also quite sensitive to the change in the magnetic gauge origin, with quite large contributions coming again from \( S_y \). The results in this section, if compared to the conclusions of the study on the model study on frozen twisted hydrogen peroxide given in the Appendix, show that the dependence of the spatially averaged third order susceptibility contributing to MFISHG on the choice of the magnetic gauge origin is less severe on large, extended molecules than on small systems.

<table>
<thead>
<tr>
<th>System</th>
<th>( S_x )</th>
<th>( S_y )</th>
<th>( S_z )</th>
<th>Perc.</th>
</tr>
</thead>
<tbody>
<tr>
<td>(L)-Alanine</td>
<td>–1.0065</td>
<td>0.4232</td>
<td>–0.6289</td>
<td>32.2</td>
</tr>
<tr>
<td>(L)-Cysteine</td>
<td>0.1186</td>
<td>1.9041</td>
<td>–0.8274</td>
<td>–23.0</td>
</tr>
<tr>
<td>(L)-Arginine</td>
<td>–0.0242</td>
<td>0.3184</td>
<td>–1.1422</td>
<td>–30.9</td>
</tr>
<tr>
<td>3,4-Dehydro-(L)-(proline)</td>
<td>3.4266</td>
<td>0.1222</td>
<td>–2.2863</td>
<td>6.7</td>
</tr>
<tr>
<td>(B)-( \alpha )-(Phendralene)</td>
<td>3.4266</td>
<td>0.1222</td>
<td>–2.2863</td>
<td>–1.7</td>
</tr>
<tr>
<td>(R)-( \alpha )-(Phendralene)</td>
<td>–1.3570</td>
<td>–1.2926</td>
<td>–0.4861</td>
<td>12.1</td>
</tr>
<tr>
<td>(S)-( \alpha )-(Cystine disulfide)</td>
<td>3.1612</td>
<td>2.4226</td>
<td>2.3542</td>
<td>10.1</td>
</tr>
<tr>
<td>(R)-( \alpha )-(Cystine disulfide)</td>
<td>–2.8542</td>
<td>10.5977</td>
<td>6.5689</td>
<td>10.8</td>
</tr>
<tr>
<td>(R)-BINO</td>
<td>0.0000</td>
<td>0.0000</td>
<td>–6.4878</td>
<td>3.9</td>
</tr>
<tr>
<td>(S)-BINAM</td>
<td>5.1076</td>
<td>–2.3164</td>
<td>4.4359</td>
<td>3.0</td>
</tr>
<tr>
<td>6-(M)-Helicene</td>
<td>–0.0026</td>
<td>–26.3366</td>
<td>0.0059</td>
<td>–2.7</td>
</tr>
<tr>
<td>(R)-Biphenyl</td>
<td>0.0017</td>
<td>–0.0075</td>
<td>0.0048</td>
<td>0.0</td>
</tr>
<tr>
<td>(R)-4-Amino-(2-Nitro-biphenyl)</td>
<td>–0.2180</td>
<td>10.9710</td>
<td>15.3791</td>
<td>–12.7</td>
</tr>
<tr>
<td>(R)-4-Nitro-biphenyl</td>
<td>–2.9692</td>
<td>–0.0031</td>
<td>–0.0028</td>
<td>–3.8</td>
</tr>
<tr>
<td>(R)-4-Amino-4’-nitro-biphenyl</td>
<td>–5.6079</td>
<td>–6.8277</td>
<td>34.2538</td>
<td>–1.6</td>
</tr>
</tbody>
</table>

| Table 4 | The \( S \) vector (units \( 10^2 \times \text{ a.u.} \), \( \lambda = 1064 \text{ nm} \). B3LYP/aug-cc-pVTZ. In the last column, the percentage change of the property (cf. Table 1) for a change in the origin through the vector \( R = (10., 10., 10.) \text{ a.u.} \) is given. |

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5 Experimental perspectives

The relation between the dipole moment in SI units and the third order susceptibility in atomic units is\(^{98}\)

\[
p^{(3)}(-2\omega_\alpha\omega_\beta,0) [\text{C m}] = 1.36411 \times 10^{-58} \times \langle \chi^{(3)}(-2\omega_\alpha\omega_\beta,0) \rangle \]
\[
[\text{a.u.}] \times (E [\text{V m}^{-1}])^2 \times (B [\text{T}])
\]

Above we specify within square brackets the units for the associated quantity. In SI units, bulk SHG is usually described by

\[
P_{\text{SHG}}(-2\omega_\alpha\omega_\beta,0) \equiv \varepsilon_0 \mathbf{d}: \mathbf{E}_c \mathbf{E}_\alpha
\]

where \(\mathbf{d}\) is the SHG tensor (units m V\(^{-1}\)). We define similarly the MFISHG tensor \(\mathbf{d}\)

\[
P_{\text{MFISHG}}(-2\omega_\alpha\omega_\beta,0) \equiv \varepsilon_0 \mathbf{d} : \mathbf{B}_c \mathbf{E}_\alpha \mathbf{E}_\beta
\]

(\text{units mV}^{-1} \text{T}^{-1}). Combining eqn (25) and (26) leads to

\[
\mathbf{d} = 1.54064 \times 10^{-47}/\langle \chi^{(3)} \rangle N
\]

where \(N\) is the molecular density (units m\(^{-3}\)). In order to determine the measurability of MFISHG, we should compare \(\mathbf{d}\) with the detection limits for the SHG tensor components \(\mathbf{d}\). If we deliberately focus on non-resonant conditions, and consider the calculated result for (R)-BINOL at \(\lambda = 780\) nm, at a concentration of 10 molar, and a magnetic field of 30 T, we find \(\mathbf{d}_B = 0.55 \text{ fm V}^{-1}\). Experimental values of \(\mathbf{d} = 3.3 \text{ fm V}^{-1}\) have been reported (e.g. \(d_{12}\) of potassium pentaborate),\(^{99}\) implying that a dedicated effort is necessary to observe MFISHG.

The MFISHG electric field \(E_{\text{MFISHG}}(2\omega_\alpha)\) is proportional to \(\mathbf{d}\). As \(\mathbf{d}^\dagger = -\mathbf{d}\), electric fields from samples with opposite handedness are opposite, but the measured intensities, proportional to \(E_{\text{MFISHG}}^2\), are equal. To recover the opposite signs of the MFISHG electric fields of the two enantiomers, one can let this MFISHG optical wave interfere with a SHG electric wave generated in a separate SHG crystal positioned in the fundamental beam just behind the chiral sample being studied. The amplitude of the summed SHG wave will become dependent on the handedness of the sample. The setup under construction for the experimental observation of MFISHG is shown in Fig. 4. It consists of a Chameleon Ultra II Ti-sapphire oscillator (Coherent, Inc.) with tunable wavelength between 680 nm and 1080 nm delivering 140 fs pulse at a repetition frequency of 80 MHz. At optimal wavelengths (780–800 nm) average power is above 3.5 W which corresponds to a peak power above 300 kW. Assuming slight focusing down to a beam waist of 50 μm which corresponds to a 10 mm Rayleigh range, the maximum electric field is on the order of 240 MV m\(^{-1}\) which exceeds the dielectric strength of most organic solvents.

The magnetic field will be generated by a split coil magnet inspired by ref. 100 generating slightly under damped pulses (\(Q \sim 3\)–4 ideally suited for better signal extraction), in a typical (5 mm\(^3\)) volume with 1 ms oscillation time and 40 T maximum magnetic field symmetries, giving access to information on molecular high order susceptibilities not involved in other linear and nonlinear chiroptical spectroscopies. It is on the other hand worth mentioning that oriented chiral assemblies should in principle show SHG due to magnetic alignment, and there the effect might be stronger than estimated for our contribution, which arises in isotropic samples due to electronic rearrangements.

6 Conclusions

The observable (the MFISHG third order susceptibility) is, as already seen for other high order frequency dependent properties arising from the interaction of molecules with electric and magnetic multipoles, computationally highly demanding. It shows a non-negligible dependence on all the variables involved, particularly relevant when it comes to electron correlation, the choice of the form of the functional in the TD-DFT approximation and the choice of the magnetic gauge. Further analysis is needed, and the exploitation of novel developments in the area of computational chemistry, as the use of sophisticated electron correlated structure models (as coupled cluster in its CC2 declination, which can nowadays be employed for robust predictions of mixed electric and magnetic properties, see for example ref. 101), and gauge origin invariant approaches, or of the forthcoming new high response methodologies, exploiting linear scaling techniques, or within the quantum mechanics/molecular mechanics framework. Also, the effect of the environment and of molecular vibrations should be taken into account, which further complicates the picture. The results obtained for MFISHG suggest that this effect is at the limits of measurability. A systematic study of structure–property relationships may identify molecules exhibiting a MFISHG effect falling within current experimental reach.

The MFISHG effect, that can appear in electrically conducting materials, such as ionic solutions, conducting polymers and others, is predicted to be very small, at the limit of detectability indeed. As such, it cannot be suggested as a tool for characterization purposes in the field of chirooptics. Nevertheless, it illustrates a fundamental interplay between molecular and magnetic field symmetries, giving access to information on molecular high order susceptibilities not involved in other linear and nonlinear chiroptical spectroscopies. It is on the other hand worth mentioning that oriented chiral assemblies should in principle show SHG due to magnetic alignment, and there the effect might be stronger than estimated for our contribution, which arises in isotropic samples due to electronic rearrangements.
Macroscopic orientation of the sample could therefore strongly enhance the MFISHG chiroptical effect.

Appendix: twisted frozen hydrogen peroxide

Table 5 allows to analyze the dependence of third order susceptibility $\chi^{(3)}(2\omega_0,\omega_0,0)$ on the extension and quality of the basis set for hydrogen peroxide frozen at a geometry where the HOOH angle is of 120°. Results are shown both for HF-SCF and DFT structural models. It is noted that the effect of electron correlation, as deduced from a comparison of HF-SCF and DFT/B3LYP results, appears to be relevant, with correlation enhancing the response by a factor, rather constant throughout Table 5, of about three. As far as the convergence of the results with the extension and quality of the basis set, double augmentation appears to make a notable difference, with respect to non-augmented or singly augmented sets. Going beyond double augmentation has a much lower effect on the convergence patterns. As far as the cardinal number is concerned, the results pretty close to those obtained with the largest (d-aug-cc-pV5Z) basis set are already obtained at double-zeta level (d-aug-cc-pVDZ results are within 1.5% of those obtained with the d-aug-cc-pV5Z basis set).

Table 6 shows the results obtained using five quite popular density functionals on our frozen twisted hydrogen peroxide. The strong dependence of the results on the choice of the functional is to be noted. BLYP, the functional yielding the largest value of the observable among the set, doubles the value predicted by B3LYP. Cam-B3LYP and BHandHLYP give reasonably close susceptibilities, still three to four times weaker than that predicted by BLYP. The SVWN5 value is about 30% larger than that yielded by B3LYP.

The discussion of the dependence of the results on the choice of the origin of the multipolar expansion, or, to be more precise, on the origin of the magnetic gauge, is based on the

Table 5 Dependence of the third order susceptibility $\chi^{(3)}(2\omega_0,\omega_0,0)$ of twisted hydrogen peroxide on the extension and quality of the basis set, both for HF-SCF and DFT structure models. $\lambda = 1064$ nm

<table>
<thead>
<tr>
<th>System</th>
<th>HFSCF</th>
<th>DFT/B3LYP</th>
</tr>
</thead>
<tbody>
<tr>
<td>aug-cc-pVDZ</td>
<td>0.0937</td>
<td>0.2087</td>
</tr>
<tr>
<td>d-aug-cc-pVDZ</td>
<td>0.1047</td>
<td>0.3099</td>
</tr>
<tr>
<td>t-aug-cc-pVDZ</td>
<td>0.1061</td>
<td>0.3183</td>
</tr>
<tr>
<td>q-aug-cc-pVDZ</td>
<td>0.1060</td>
<td>0.3158</td>
</tr>
<tr>
<td>aug-cc-pVTZ</td>
<td>0.0944</td>
<td>0.2442</td>
</tr>
<tr>
<td>d-aug-cc-pVTZ</td>
<td>0.1011</td>
<td>0.3049</td>
</tr>
<tr>
<td>t-aug-cc-pVTZ</td>
<td>0.1020</td>
<td>0.3088</td>
</tr>
<tr>
<td>q-aug-cc-pVTZ</td>
<td>0.1021</td>
<td>0.3109</td>
</tr>
<tr>
<td>aug-cc-pVQZ</td>
<td>0.0987</td>
<td>0.2831</td>
</tr>
<tr>
<td>d-aug-cc-pVQZ</td>
<td>0.1015</td>
<td>0.3075</td>
</tr>
<tr>
<td>t-aug-cc-pVQZ</td>
<td>0.1013</td>
<td>0.3081</td>
</tr>
<tr>
<td>q-aug-cc-pVQZ</td>
<td>0.1013</td>
<td>0.3078</td>
</tr>
<tr>
<td>aug-cc-pV5Z</td>
<td>0.1001</td>
<td>0.2977</td>
</tr>
<tr>
<td>d-aug-cc-pV5Z</td>
<td>0.1012</td>
<td>0.3075</td>
</tr>
</tbody>
</table>

Table 6 The third order susceptibility $\chi^{(3)}(2\omega_0,\omega_0,0)$ of twisted hydrogen peroxide computed with the d-aug-cc-pV5Z basis set. $\lambda = 1064$ nm

<table>
<thead>
<tr>
<th>System</th>
<th>$\chi^{(3)}(2\omega_0,\omega_0,0)/$a.u.</th>
</tr>
</thead>
<tbody>
<tr>
<td>B3LYP</td>
<td>0.3075</td>
</tr>
<tr>
<td>BLYP</td>
<td>0.6325</td>
</tr>
<tr>
<td>Cam-B3LYP</td>
<td>0.2221</td>
</tr>
<tr>
<td>BHandHLYP</td>
<td>0.1629</td>
</tr>
<tr>
<td>SVWN5</td>
<td>0.4006</td>
</tr>
</tbody>
</table>

Table 7 The dependence of the extension of the basis set for the $S$ vector, containing the molecular contribution to the correction on the third order susceptibility when the origin of the magnetic gauge is displaced for the twisted form of hydrogen peroxide (a.u.). Geometry as shown. See eqn (20) and text for further information.

<table>
<thead>
<tr>
<th>System</th>
<th>HFSCF</th>
<th>DFT/B3LYP</th>
</tr>
</thead>
<tbody>
<tr>
<td>aug-cc-pVDZ</td>
<td>$-3.6353$</td>
<td>$-5.9840$</td>
</tr>
<tr>
<td>d-aug-cc-pVDZ</td>
<td>$-0.0593$</td>
<td>$-2.4548$</td>
</tr>
<tr>
<td>t-aug-cc-pVDZ</td>
<td>$-0.0477$</td>
<td>$-6.6516$</td>
</tr>
<tr>
<td>q-aug-cc-pVDZ</td>
<td>$0.0975$</td>
<td>$-4.5901$</td>
</tr>
<tr>
<td>aug-cc-pVTZ</td>
<td>$-2.8531$</td>
<td>$-7.4653$</td>
</tr>
<tr>
<td>d-aug-cc-pVTZ</td>
<td>$0.1361$</td>
<td>$0.3887$</td>
</tr>
<tr>
<td>t-aug-cc-pVTZ</td>
<td>$-0.0221$</td>
<td>$-0.1184$</td>
</tr>
<tr>
<td>q-aug-cc-pVTZ</td>
<td>$-0.0092$</td>
<td>$0.1219$</td>
</tr>
<tr>
<td>aug-cc-pVQZ</td>
<td>$-1.4801$</td>
<td>$-7.7739$</td>
</tr>
<tr>
<td>d-aug-cc-pVQZ</td>
<td>$0.0686$</td>
<td>$0.3579$</td>
</tr>
<tr>
<td>t-aug-cc-pVQZ</td>
<td>$0.0181$</td>
<td>$-0.0368$</td>
</tr>
<tr>
<td>q-aug-cc-pVQZ</td>
<td>$0.0159$</td>
<td>$-0.0241$</td>
</tr>
<tr>
<td>aug-cc-pV5Z</td>
<td>$-0.7542$</td>
<td>$-4.1557$</td>
</tr>
<tr>
<td>d-aug-cc-pV5Z</td>
<td>$0.0285$</td>
<td>$-0.0492$</td>
</tr>
</tbody>
</table>

evidence shown in Fig. 5, see also Table 7. In the limit of a complete one-electron basis set, $S_\lambda$ should vanish, exactly for the HF-SCF case. To start seeing some trend of convergence once again we need to resort at least to doubly augmented correlation consistent sets, although as the cardinal number increases,
the correction needed when using augmented sets decreases steadily. Once at the doubly augmented level though the correction drops to small values, and yet convergence towards zero is rather slow and not necessarily monotonous. The pattern is even more complicate for B3LYP. Here the hypervirial relationship kicks in more slowly, and it is not safe to speak of convergence towards a vanishing correction $S$ within the range of basis sets employed in our analysis.

**Acknowledgements**

A grant of computer time from the Norwegian Supercomputing Program is gratefully acknowledged.

**References**


81 Dalton, a molecular electronic structure program, Release 2.0, 2005, see http://www.kjemi.uio.no/software/dalton/dalton.html.


