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# Comprehending the quadruple bonding conundrum in $C_2$ from excited state potential energy curves†

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The question of quadruple bonding in  $C_2$  has emerged as a hot button issue, with opinions sharply divided between the practitioners of Valence Bond (VB) and Molecular Orbital (MO) theory. Here, we have systematically studied the Potential Energy Curves (PECs) of low lying high spin sigma states of  $C_2$ ,  $N_2$ ,  $Be_2$  and  $HC\equiv CH$  using several MO based techniques such as CASSCF, RASSCF and MRCI. The analyses of the PECs for the  $2S+1\Sigma_{g/u}$  (with  $2S + 1 = 1, 3, 5, 7, 9$ ) states of  $C_2$  and comparisons with those of relevant dimers and the respective wavefunctions were conducted. We contend that unlike in the case of  $N_2$  and  $HC\equiv CH$ , the presence of a deep minimum in the  $7\Sigma^+$  state of  $C_2$  and  $CN^+$  suggests a latent quadruple bonding nature in these two dimers. Our investigations reveal that the number of bonds in the ground state can be determined for 2<sup>nd</sup> row dimers by figuring out at what value of spin symmetry a purely dissociative PEC is obtained. For  $N_2$  and  $HC\equiv CH$  the purely dissociative PEC appears for the septet spin symmetry as compared to that for the nonet in  $C_2$ . This is indicative of a higher number of bonds between the two 2<sup>nd</sup> row atoms in  $C_2$  as compared to those of  $N_2$  and  $HC\equiv CH$ . Hence, we have struck a reconciliatory note between the MO and VB approaches. The evidence provided by us can be experimentally verified, thus providing the window so that the narrative can move beyond theoretical conjectures.

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## Introduction

Bonding in homodiatomic 2<sup>nd</sup> period elements constitutes the bedrock of our understanding of chemical bonding.<sup>1</sup> Though bonding in many of these homodiatomic species is well understood, the bonding situation in  $C_2$  presents an exceptionally enigmatic scenario. A routine inspection of molecular orbitals (Hartree Fock orbitals) of  $C_2$  would suggest that the bond order of  $C_2$  is 2.0 arising from the two  $\pi$  bonds.<sup>2</sup> However, decades back a typical Wiberg bond index computation conducted on  $C_2$  indicated the presence of four bonds.<sup>3</sup> The last few years have witnessed a steep spike in interest to comprehend the state of bonding in  $C_2$ .<sup>4–12</sup> This has led to intense debate on the aspect of quadruple bonding in  $C_2$ .<sup>13</sup> Shaik and co-workers have investigated the electronic structure of  $C_2$  within the VB manifold and have concluded that the bonding in  $C_2$  is best described as a case of quadruple bond and have went on to predict that the strength of the fourth bond is approximately 12 kcal mol<sup>-1</sup>.<sup>4,5</sup> These findings were contested by Frenking and co-workers who mainly disagreed on the approach adopted by Shaik of estimating the strength of the fourth bond.<sup>7,10</sup> There

have been several attempts by different groups which either concur or refute the presence of a fourth bond in  $C_2$ . The incongruity in views from different corners arise from the multi-reference nature of the  $\sigma$  orbitals in  $C_2$ .<sup>14</sup> Incidentally, MO based approaches have by and large refuted the case of quadruple bonding, with some exceptions.<sup>7,10</sup> Zhong and co-workers have tried to point out some similarities between the MO based and the VB approaches for  $C_2$  regarding a key orbital which was achieved through unitary transformation of CASSCF orbitals.<sup>15</sup> However, their final Effective Bond Order study with the same orbitals suggested that the dimer in question has a bond order of 2.15. Alternatively, magnetic shielding studies based on MO based approaches by Karadakov *et al.* suggested a “bulkier” bond compared to that of acetylene.<sup>11</sup> Nevertheless, it can be argued that there is no direct or definitive proof of the presence of four bonds in  $C_2$  from a MO standpoint. The lack of existence of reconciliation on this issue between the VB and the MO approaches still has kept the debate wide open and given the conflicting views it may not be unfair to comment that the overall understanding is still nebulous. Here we report our view point on the bonding in  $C_2$  by conducting extensive studies on potential energy curves of excited states of  $C_2$ , largely covering the cases of high spin states of  $C_2$  along with similar investigations on  $N_2$  and  $HC\equiv CH$ . Our findings provide clinching evidence in support of the presence of quadruple bonding in  $C_2$  or rather the ability to form two bonds by electrons in orbitals in

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$\sigma$  symmetry thus establishing reconciliation between MO and VB manifold of methods.

The quintessential signature of bonding between two atoms lies in the presence of a well-defined discrete minimum in the potential energy curve (PEC) plotted against the interatomic distance of the two atoms. For instance, in the case of  $H_2$  ground electronic state the PEC shows the presence of a distinct minimum and has a significant dissociation energy. This is due to the presence of a  $\sigma$  bond arising from  $1\sigma_g^2$  in the  $H_2$  molecule.<sup>16</sup> If this electron pair in the bonding orbital is broken and one of the electrons is promoted to the corresponding  $1\sigma_u$  one would find that the bonding stabilization is negated and as a result a dissociative PEC with no minimum is obtained for the  $^3\Sigma_u^+$  state. Hence one can create a high spin state corresponding to the rupture of a bond that generates a PEC without a minimum. This seemingly simple argument may be extended to multiply bonded species as well, albeit with some intrinsic limitations which are discussed later. Multiple bonding between two atoms in the ground state would certainly leave footprints on the excited state PECs. Unlike  $H_2$ ,  $N_2$  which is multiply bonded exhibits high spin triplet and quintet states with respective minima.<sup>17</sup> This becomes more evident when one compares and contrasts spectra of diatomics with and without multiple bonding.<sup>18</sup>

Hence, in the multiply bonded diatomic species one may generate high spin states by breaking bonding electron pairs within the valence orbitals and promoting electrons from a particular bonding orbital to the corresponding antibonding orbital such that for a particular high spin state a set of bonding orbital/s and respective antibonding have single occupation with parallel spins. This would lead to high spin states of  $^{2S+1}\Sigma^+$  states. A step by step procedure can be adopted such that particular types of bonding may be negated by the proper choice of orbital symmetry. If specific orbitals are chosen to generate high spin electronic states, one can gradually generate electronic states which eliminate bonds one by one and reach a high spin state which would correspond to the total absence of any bonds between the two atoms resulting to a purely dissociative state. The no. of bonds in the ground state of the diatomic species would determine at which high spin state the purely dissociative PEC would be reached. For instance, in  $H_2$  which has only one bond the triplet state is purely dissociative, whereas for  $N_2$  which is known to have three bonds the septet state is purely dissociative.<sup>16,17</sup>

## Results and discussion

As the bonding in  $C_2$  is suspect, we decided to investigate the nature of the PECs of high spin states of  $C_2$  and see at which spin state a purely dissociative state is obtained. A cursory estimate obtained from the primary Hartree Fock (HF) MO picture of the  $C_2$  would suggest that  $C_2$  has only two  $\pi$  bonds.<sup>2</sup> The four electrons in the  $\sigma$  orbitals would appear not to contribute to bonding as there are two electrons in a bonding orbital and two electrons in an antibonding orbital. It must be noted that the  $Be_2$  dimer with the same  $\sigma$  electron population and no  $\pi$  electron shows a total absence of bonding at HF and CASSCF levels of theory.<sup>19</sup> Hence, one is tempted to infer that  $C_2$  will have just two  $\pi$  bonds arising from the two  $\pi$  electrons. A full valence CASSCF calculation on the ground electronic state of  $C_2$  reveals the multireference character of  $C_2$ . The dominant configuration state functions (CSFs) are with 70.9% and 13.6% contributions. The 2<sup>nd</sup> CSF may suggest that it can contribute two  $\sigma$  bonds in the ground electronic state of  $C_2$  (see Table 1). The no. of bonds arising from  $\sigma$  orbitals is the bone of contention, whereas there is no debate as to the presence of two  $\pi$  bonds in the ground electronic state. VB theory predictions from Shaik and others suggest that indeed two  $\sigma$  bonds are present. Due to the multi-reference character of  $C_2$ , simple MO based electronic structure theories are inadequate to predict the no. of  $\sigma$  bonds. However, as discussed earlier the high spin state PECs can be investigated to gauge the bonding situation in the ground electronic state. Using a full valence CASSCF for  $C_2$ ,  $N_2$ ,  $CN^+$  and  $HC\equiv CH$  we have investigated the dissociation PECs for high spin  $\Sigma$  states (see Fig. 1(a), (b) and (c) for  $C_2$ ,  $N_2$  and  $CN^+$  and Fig S1 in ESI† for acetylene respectively). The high spin  $\Sigma$  states are created in such a way that it would have lesser number of electron pairs in bonding orbitals. Such an exercise immediately reveals that for  $N_2$  (Fig. 1(b)) and  $HC\equiv CH$  (Fig S1†) the lowest lying  $^7\Sigma_u^+$  state is purely dissociative. This would suggest that only three electron pairs contribute to bonding in the ground electronic state and when all of them are disrupted to create a high spin state of septet symmetry it leads to a purely dissociative PEC. Intriguingly, the same exercise with  $C_2$  reveals a purely dissociative PEC is obtained for the  $\Sigma$  state of nonet spin symmetry, with  $^9\Sigma_u^+$  displaying a distinct minimum. The trends immediately suggest that probably the no. of electron pairs contributing to the bonding in the ground state of  $C_2$  is

Table 1 Percentage of the major contributing CSFs for the five spin states of  $C_2$  at equilibrium

Spin multiplicity	Major contributing CSF	Percentage
$^1\Sigma_g^+$	$ 2\sigma_g^2 2\sigma_u^2 1\pi_{ux}^2 1\pi_{uy}^2\rangle$	70.9
	$ 2\sigma_g^2 3\sigma_g^2 1\pi_{ux}^2 1\pi_{uy}^2\rangle$	13.6
$^3\Sigma_u^+$	$ 2\sigma_g^2 2\sigma_u^1 1\pi_{ux}^1 1\pi_{uy}^1 3\sigma_g^1\rangle$	86.3
	$ 2\sigma_g^2 1\pi_{ux}^2 1\pi_{uy}^2 2\sigma_u^2 1\pi_{gx}^1\rangle +  2\sigma_g^2 1\pi_{ux}^1 1\pi_{uy}^2 2\sigma_u^2 1\pi_{gx}^1\rangle$	3.6
$^5\Sigma_g^+$	$ 2\sigma_g^2 2\sigma_u^1 1\pi_{ux}^1 1\pi_{uy}^2 3\sigma_g^1 1\pi_{gx}^1\rangle +  2\sigma_g^2 2\sigma_u^1 1\pi_{ux}^2 1\pi_{uy}^1 3\sigma_g^1 1\pi_{gy}^1\rangle$	81.7
	$ 2\sigma_g^2 2\sigma_u^2 1\pi_{ux}^1 1\pi_{uy}^1 1\pi_{gx}^1 1\pi_{gy}^1\rangle$	6.7
$^7\Sigma_u^+$	$ 2\sigma_g^2 2\sigma_u^1 1\pi_{ux}^1 1\pi_{uy}^1 3\sigma_g^1 1\pi_{gx}^1 1\pi_{gy}^1\rangle$	97.8
	$ 2\sigma_g^2 2\sigma_u^1 1\pi_{ux}^1 1\pi_{uy}^1 3\sigma_g^2 1\pi_{gx}^1 1\pi_{gy}^1\rangle$	0.5
$^9\Sigma_g^+$	$ 2\sigma_g^2 2\sigma_u^1 1\pi_{ux}^1 1\pi_{uy}^1 3\sigma_g^1 1\pi_{gx}^1 1\pi_{gy}^1 3\sigma_u^1\rangle$	100.0





Fig. 1 PECs corresponding to the (a) five spin states of  $C_2$ , (b) four spin states of  $N_2$ ,<sup>17a</sup> (c) five spin states of  $CN^+$  and (d) five spin states of  $BN$ .

more compared to those for  $N_2$  and  $HC\equiv CH$ . Our hypothesis is based on the fact that on promoting electrons to the high spin states eliminate bonds. With that one needs to go the nonet spin state ( ${}^9\Sigma_g^+$ ) for the purely dissociative state. It suggests that four bonding pairs have to be disrupted to decimate any form of bonding between the two atoms.

Further inspection of the excited state PECs of  $C_2$  along with the dominant CSF at the minimum at the PEC is instructive. The lowest lying  ${}^3\Sigma_u^+$  is dominated by a CSF which has the presence of two  $\pi$  bonds (see Table 1). The electron distribution in  $\sigma$  orbitals in  ${}^3\Sigma_u^+$  space avoids any significant participation from configurations which will have simultaneous double occupation in  $2\sigma_g$  and  $3\sigma_g$  orbital. Also, the dissociation energy of the  ${}^3\Sigma_u^+$  state is lower compared to that of the  ${}^1\Sigma_g^+$  state by about 27 kcal mol (see ESI Table S1†). The dominant CSF of  ${}^3\Sigma_u^+$  at the PEC minimum,  $|2\sigma_g^2 2\sigma_u^1 1\pi_u^4 3\sigma_g^1\rangle$  may be viewed as an excitation from the CSF  $|2\sigma_g^2 2\sigma_u^0 1\pi_u^4 3\sigma_g^2\rangle$  (the 2<sup>nd</sup> most important CSF present in the ground state minimum) (see Fig. 2), which is essentially breaking a bonding pair of  $3\sigma_g$  and putting parallel spins in  $3\sigma_g$  and  $2\sigma_u$ . It may be argued that  ${}^3\Sigma_u^+$  essentially maintains  $2\pi$  bonds and eliminates plausibly one  $\sigma$  bond, which shows up in the decrease of the dissociation energy. The  ${}^5\Sigma_g^+$  is riddled with strong signatures of avoided crossings. State averaging by including four low lying  ${}^5\Sigma_g^+$  states

were used to draw up the PECs (see ESI, Section S1†). The nature of the dominant CSF suggests the elimination of only one  $\pi$  bond and one  $\sigma$  bond w.r.t that of the ground state, using the same line of argument which was employed in the previous case. The dominant CSF at the minimum of the PEC for the lowest lying septet  $\Sigma$  state,  ${}^7\Sigma_u^+$  shows a total absence of  $\pi$  bonding but the presence of a deep minimum indicates stability conferred from the electron pair in  $2\sigma_g$  orbital. The dominant septet state CSF at the  ${}^7\Sigma_u^+$  PEC minimum shows a marked absence of  $\pi$  bonding, yet this is the third state which has one  $\sigma$  bond w.r.t that of the ground electronic state as it has parallel spins in  $3\sigma_g$  and  $2\sigma_u$  akin to the dominant CSF in the PEC minima of  ${}^3\Sigma_u^+$  and  ${}^5\Sigma_g^+$  states. As mentioned earlier, all of the bonding interactions are annihilated at the  ${}^9\Sigma_g^+$  state leading to a purely dissociative PEC. This fact is in conjunction with the fact that  ${}^7\Sigma_u^+$  state PEC has a distinct minimum which strongly indicates that the ground electronic state of  $C_2$  has four bonds, two  $\sigma$  and two  $\pi$ .

Though we have noted earlier that for  $N_2$  and acetylene purely dissociative state appears only when one reaches the septet spin state ( ${}^7\Sigma_u^+$ ), unfortunately the corresponding  ${}^5\Sigma_g^+$  state PECs do not show presence of distinct minima. Further investigations reveal that the situation for the  ${}^5\Sigma_g^+$  state is extremely complex as it involves significant contribution from







Fig. 3 (a) PEC corresponding to the RASSCF computation for the quintet spin state of  $C_2$  and (b) PECs corresponding to the RASSCF computation for the two septet spin states of  $C_2$  along with their dominant configurations.

with that of  $HC\equiv CH$ , with the bond in  $C_2$  being weaker than that of  $HC\equiv CH$ .<sup>7</sup> In the light of our findings this may stand as a contradiction. Here we must reiterate that the dominant determinant of the ground state of  $C_2$  contributes only two  $\pi$  bonds. Hence, analysis of force constants is likely to reflect the attribute of primarily the dominant determinant of  $C_2$  and may not serve as a good metric for this purpose. The latent quadruple nature of the bond in  $C_2$  and  $CN^+$  can only be recognized through the PECs of the excited states.

Though our approach possibly brings in an avenue to understand the bonding situation in  $C_2$ , a probable question may arise that whether this hypothesis can be applied to comprehend the number of bonds in all diatomic systems and what are its intrinsic limitations. Naturally, one is inclined to ask whether this technique can be extended to molecules with triplet ground states like  $O_2$  and  $B_2$ . Here we first discuss the case of  $O_2$  to illustrate how this approach can be tailored to address cases where the ground state is  $^3\Sigma_g^-$  within the framework of traditional MO theory. In the case of the  $^3\Sigma_g^-$  ground state of  $O_2$ , a formal bond order of 2.0, one from  $\sigma$  and the other from  $\pi$  is assigned from the electronic distribution  $[core]2\sigma_g^2 2\sigma_u^2 3\sigma_g^2 1\pi_u^4 1\pi_g^2$ . One can generate a higher spin state by breaking the bonding pair in the  $3\sigma_g^2$  bonding orbital and promoting an electron to  $3\sigma_u$ , thus eliminating the  $\sigma$  bond. In order to disrupt the bonding arising from  $\pi$  orbitals one needs to excite a single electron from  $\pi$  to  $\pi^*$  creating a configuration of the type  $\pi_u^3 \pi_g^3$  from  $\pi_u^4 \pi_g^2$  of the ground state. This would eliminate the single  $\pi$  bond without ascending on the spin ladder. This is obviously different from the molecules with singlet ground state  $^1\Sigma_g^+$  that has  $\pi$  bonds (consider the case of  $N_2$  or  $HC\equiv CH$ ). Armed with this simple information one would expect to obtain  $^3\Sigma_u^+$  with a minimum (conventional bond order = 1) and a  $^5\Sigma_g^+$  with a purely dissociative curve (conventional bond order = 0) (see Fig. S6(b) and S7(b) in ESI<sup>†</sup>). Indeed the findings from the previously reported CASSCF/MRCI PECs of high spin  $\Sigma$  states show that the approach holds good, it must be emphasized that for triplet ground state diatomic systems arising from  $\pi_u^3 \pi_g^3$  and  $\pi_u^4 \pi_g^2$

configurations moving the bonding electron to the antibonding electron would not create a sigma state with higher spin symmetry.<sup>21</sup> If this may seem confusing one may modify our strategy from a different viewpoint. Associated with these two configurations is a low-lying singlet  $^1\Delta_g$  configuration. From the lowest lying singlet state bonding pairs have to be broken to create high spin sigma states and the corresponding wavefunctions and their respective PECs have to be inspected to arrive at a proper conclusion regarding the number of bonds present in them (see Fig. S8 in ESI<sup>†</sup>). This tailored strategy would yield two bonds for  $O_2$  and a single bond in  $B_2$ . While our approach appears to be simple it would be prudent to add a cautionary note, particularly for the quintet states of p-block elements. As has been discussed earlier, we find that for the p-block elements the quintet  $\Sigma$  state PECs are fraught with multiple avoided crossings as the dominating configurations can arise from electron bond pair breaking of the  $\sigma$  orbital or from  $\pi$  orbital. One has to ensure that adequate state-averaging is conducted to reveal the true nature of the state PECs of these dimers. Admittedly, the approach has to be tested further on metal dimer systems to see whether proper inferences can be drawn from analogous high spin state PECs regarding the number of bonds in their respective ground states.

## Conclusions

In summary we provide overwhelming evidence which brings out the quadruple bonding nature in  $C_2$  and  $CN^+$  with two  $\sigma$  and two  $\pi$  bonds. However, our approach indicates that  $BN$ , which is isoelectronic to  $C_2$  at the most has three bonds. Additionally, we suggest that for both  $CN^+$  and  $C_2$ , a  $^7\Sigma^+$  state exists which has a clear distinct and deep minimum, which opens up a window for experimental verification.

## Conflicts of interest

There are no conflicts to declare.



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## References

- (a) R. C. Fischer and P. P. Power, *Chem. Rev.*, 2010, **110**, 3877–3923; (b) P. R. Schreiner and H. P. Reisenauer, *Angew. Chem., Int. Ed.*, 2009, **48**, 8133–8136.
- I. N. Levine, *Quantum Chemistry*, Allyn and Bacon, 2nd edn, 1974, p. 321, table 13.2.
- P. V. R. Schleyer and P. Maslak, *Tetrahedron Lett.*, 1993, **34**, 6387–6390.
- P. Su, J. Wu, J. Gu, W. Wei, S. Shaik and P. C. Hiberty, *J. Chem. Theory Comput.*, 2011, **7**, 121–130.
- S. Shaik, D. Danovich, W. Wu, P. Su, H. S. Rzepa and P. C. Hiberty, *Nat. Chem.*, 2012, **4**, 195–200.
- (a) S. Shaik, H. S. Rzepa and R. Hoffmann, *Angew. Chem., Int. Ed.*, 2013, **52**, 3020–3033; (b) D. Danovich, S. Shaik, H. S. Rzepa and R. Hoffmann, *Angew. Chem., Int. Ed.*, 2013, **52**, 5926–5928.
- G. Frenking and M. Hermann, *Angew. Chem., Int. Ed.*, 2013, **52**, 5922–5925.
- D. Danovich, P. C. Hiberty, W. Wu, H. S. Rzepa and S. Shaik, *Chem. –Eur. J.*, 2014, **20**, 6220–6232.
- D. W. O. Sousa and M. A. C. Nascimento, *J. Chem. Theory Comput.*, 2016, **12**, 2234–2241.
- M. Hermann and G. Frenking, *Chem. –Eur. J.*, 2016, **22**, 4100–4108.
- P. B. Karadakov and J. Kirsopp, *Chem. –Eur. J.*, 2017, **23**, 12949–12954.
- J. M. Matxain, F. Ruipérez, I. Infante, X. Lopez, J. M. Ugalde, G. Merino and M. Piris, *J. Chem. Phys.*, 2013, **138**, 151102.
- (a) S. Shaik, H. S. Rzepa and R. Hoffmann, *Angew. Chem., Int. Ed.*, 2013, **52**, 3020–3033; (b) L. T. Xu and T. H. Dunning Jr, *J. Chem. Theory Comput.*, 2014, **10**, 195–201; (c) S. Shaik, D. Danovich, B. Braidia and P. C. Hiberty, *Chem. –Eur. J.*, 2016, **22**(52), 18977–18980; (d) M. Piris, X. Lopez and J. M. Ugalde, *Chem. –Eur. J.*, 2016, **22**(12), 4109–4115.
- P. F. Fougere and R. K. Nesbet, *J. Chem. Phys.*, 1966, **1**, 285–298.
- R. Zhong, M. Zhang, H. Xu and Z. Su, *Chem. Sci.*, 2016, **7**, 1028–1032.
- K. K. Lange, E. I. Tellgren, M. R. Hoffmann and T. Helgaker, *Science*, 2012, **337**, 327–331.
- (a) A. Engels-Putzka and M. Hanrath, *Mol. Phys.*, 2009, **107**, 143–155; (b) R. S. Mulliken, *J. Chem. Phys.*, 1962, **37**, 809–813; (c) H. Werner and P. J. Knowles, *J. Chem. Phys.*, 1991, **94**(2), 1264–1270.
- (a) G. S. Scuseria, T. P. Hamilton and H. F. Schaefer III, *J. Chem. Phys.*, 1990, **92**, 568–573; (b) W. D. Laidig, P. Saxe and R. J. Bartlett, *J. Chem. Phys.*, 1987, **86**, 887–907.
- J. M. Matxain, F. Ruipérez and M. Piris, *J. Mol. Model.*, 2013, **19**, 1967–1972.
- A. L. Padellec, J. B. A. Mitchell, A. Al-Khalili, H. Danared, A. Källberg, Å. Larsen, S. Rosén, M. Ugglas, L. Vikor and M. Larsson, *J. Chem. Phys.*, 1999, **110**(2), 890–901.
- (a) H. Liu, D. Shi, J. Sun, Z. Zhu and Z. Shulin, *Spectrochim. Acta, Part A*, 2014, **124**, 216–229; (b) H. Partridge, C. W. Bauschlicher Jr, S. R. Langhoff and P. R. Taylor, *J. Chem. Phys.*, 1991, **95**(11), 8292–8300; (c) H. Lefebvre-Brion, H. P. Liebermann, J. M. Amero and G. J. Vázquez, *J. Chem. Phys.*, 2016, **144**(14), 144302.

