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We theoretically and numerically demonstrate a novel soft metamaterial based on silica-coated gold core/shell-structured nanorod fluids, which shows an electrically tunable negative refraction property in visible optical region under external electric field stimuli.
Electrically tunable negative refraction in core/shell-structured nanorod fluids

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We theoretically investigate an optical refraction behavior in a fluid system which contains silica-coated gold nanorods dispersed in silicone oil under an external electric field. Because of the formation of chain-like or lattice-like structure of dispersed nanorods along electric field, the fluid shows a hyperbolic equifrequency contour characteristic and, as a result, all-angle broadband optical negative refraction for transverse magnetic waves propagating can be realized. We calculate the effective permittivity tensor of the fluid and verify the analysis using finite element simulations. We also find that the negative refractive index can vary with the electric field strength and external field distribution. Under a non-uniform external field, the gradient refraction behavior can be realized.

Introduction

Metamaterials are a type of artificial materials, which have abnormal physical behaviour. Left-handed metamaterial (LHM), which has negative permittivity and permeability simultaneously, is one of the most important metamaterials.\(^1\) The electromagnetic wave in LHM shows reverse phase and group velocities, resulting in many abnormal electromagnetic behaviors, such as negative refraction, reversed Doppler shift, reversed Cherenkov radiation, and so on.\(^2,3\) These abnormal electromagnetic behaviors make metamaterials have potential uses in superlens imaging, highly sensitive sensor, invisible cloak, and so on.\(^4,5\)

In the past decade, many efforts have been devoted to develop various metamaterials with negative refraction.\(^6-10\) But most of the metamaterials are fabricated based on hard solid matter. For example, LHMs are usually fabricated by periodically arranging electric/magnetic resonator units (e.g. split ring resonator, fractal tree branches, Mie resonance clusters, etc.) on hard plastic or ceramic substrates. These hard metamaterials have advantages including good structure stability and easy top-down fabrication for large-size resonator units. However, the difficulties in the fabrication of solid metamaterials constantly increase with the decrease of unit sizes in order to be satisfied with higher frequency response. Although metamaterial-based optical invisible cloak have been fabricated by the top-down approach or other micro-nano machining equipment, the methods are often high-cost and complex. On the other hand, due to the “hard” characteristic of solid matter, the structures of solid metamaterials are difficult to be further changed or adjusted once they have been fabricated. As a result, the material’s behavior can’t rapidly response to the external environment change (such as different electromagnetic field frequencies and strength), that is, solid metamaterials are lack of adaptability. In addition, the flexibility of solid metamaterials is relatively lack and it will limit the processing of some special products, such as flexible “invisible cloak” that can be arbitrarily curved. Therefore, exploring new material systems and preparation methods are important to the developments and applications of metamaterials.

Different from hard matter, soft matter not only has good flexibility but also can change its structure and behavior under external stimuli. This change originates from the formation of long-range self-organized or ordered structure of molecules or particles in soft matter under external fields. Specially, the ordered structure in soft matter can be controllably adjusted through the external field strength and field distribution. Considering these characteristics, some recent researches have introduced soft matter into solid metamaterials in order to adaptively control the behaviors. For example, Zhao et al have immersed a kind of metamaterial with negative permeability in a nematic liquid crystal, realizing the adjustment of the magnetic response frequency under external field.\(^11\) We have immersed LHMs into electrorheological fluid (ER fluid) and achieved a continuous adjustment of left-handed transmission peak frequency by external electric field.\(^12\) In these studies, however, soft matter is only acted as additive or filler of solid metamaterials. In 2006, Zhao et al. have theoretically studied the optical negative group refraction effect in pure liquid crystal...
under external fields and designed a wedge sample to observe the negative group refraction by magnetic field stimuli.\(^{13,14}\) Khoo et al. have observed the change of refractive index from positive to negative at infrared frequencies in a fluid composed of silver nanoparticles dispersed in nematic liquid crystal.\(^{15}\) However, the narrow working temperature region largely limits the development of liquid crystal-based metamaterials.

Recently, Urzhumov et al. have theoretically proposed a new metafluid composed of tetrahedral clusters consisting of four gold nanospheres and, by simulations, demonstrated that this metafluid possessed not only negative permeability but also negative permittivity when the cluster concentration was enough high and the electric loss was enough low.\(^{16}\) However, it is relatively difficult to prepare the metafluid because it needs to not only obtain perfect nanoclusters but also ensure all clusters orientate toward a fixed direction. Huang et al have theoretically proposed a ferrofluid containing Fe\(_3\)O\(_4\)@Ag nanoparticles and numerically demonstrated a negative refraction when the nanoparticles formed chain-like structures by external magnetic fields.\(^{17}\) However, the use of ferrofluid in visible optical region is still limited due to the large absorption of Fe\(_3\)O\(_4\) or other magnetic materials as core. Furthermore, compared to an electric field that is applied only by thin-film electrodes, the magnetic field is relatively complex due to large-size of magnetic coils or poles, and this largely limits device design.

In this work, we design a novel soft metamaterial based on the fluid of silica-coated gold (Au@SiO\(_2\)) nanorods (NRs) in silicone oil and theoretically demonstrate a tunable negative refraction behavior in visible region under external electric field stimuli. Here, Au NRs are chosen as core because they are easy to prepare and have low loss and strong surface plasmon resonance (SPR) in visible light region, which can induce a big negative real part and small positive imaginary part of permittivity. Coating Au NRs with low-permittivity silica shell will protect the system from dielectric breakdown and large current leakage, and simultaneously provide a good compatibility and similar refractive index with silicone oil. The elongated nanorod morphology is employed because it can enhance interparticle interaction by stronger long-axis polarization and weaken Brownian motion, and thus improve the stability of electric field-induced ordered structure.\(^{18}\) Without electric fields, the Au@SiO\(_2\) NRs are randomly distributed in the fluid. When external electric fields are applied, NRs polarize and orient to form chain-like or lattice-like ordered structure after the electrostatic energy exceeds the thermal energy and, as a result, the permittivity of fluid becomes strongly anisotropic. We simulate the achievement conditions of negative refraction of transverse magnetic (TM) light in this strongly anisotropic fluid and the dependence of refractive behavior on particle concentration, electric field strength, incident wavelength, and so on. Based on the negative refraction behavior, we also study the near-field imaging of the fluid-based superlens and its smart zoom behavior as a function of electric field strength. We finally use a non-uniform electric field to induce a gradient negative refractive index in the fluid and analyse its change with electric field distribution.

**Theory**

The proposed fluid is made of Au@SiO\(_2\) NRs dispersed in silicone oil. Without electric fields, the Au@SiO\(_2\) NRs are randomly dispersed in the fluid. So the fluid is isotropic medium, and the permittivity of the fluid can be described as:

\[
\begin{pmatrix}
\varepsilon_{xx} & 0 & 0 \\
0 & \varepsilon_{yy} & 0 \\
0 & 0 & \varepsilon_{zz}
\end{pmatrix}
\]

where \(\varepsilon_{xx} = \varepsilon_{yy} = \varepsilon_{zz}\). When the external electric fields are applied along \(z\) axis, the long-axis of Au@SiO\(_2\) NRs start to orientate along electric field direction due to polarization-induced interparticle attraction.\(^{19}\) Meanwhile, the fluid tends to transform from an isotropic medium into an anisotropic one.\(^{20}\) Thus, the permittivity of fluid also becomes anisotropic, i.e. \(\varepsilon_{xx} \neq \varepsilon_{yy} \neq \varepsilon_{zz}\). As the electric field increases and exceeds a critical value \(E_c\), Au@SiO\(_2\) NRs will orientate and arrange into a gap-spanning ordered chain and even lattice structure between electrodes.\(^{20}\) Especially, according to the phase transition mechanism of electro-responsive or magneto-responsive smart fluids,\(^{21}\) the phase transition of nanorod fluid is of a second order transition. With the increase of applied electric field, the structure of the fluid changes gradually from isotropy to anisotropy, and as a result, the permittivity of fluid gradually changes from isotropy to anisotropy with the electric field strength. Thus, we can realize a controllable adjustment about the structure and dielectric properties of fluid through electric field.

When a beam of incident light enters into this field-induced structured fluid from free space, its refractive behavior will be influenced by the change of dielectric properties of fluid. If the light wavelength (\(\lambda\)) is much larger than the radius (\(r\)) of Au NRs, the length (\(l\)) of Au NRs, the thickness (\(d_{\text{shell}}\)) of silica shell, and the lattice distance (or the gap between nanorods) (\(d\)), the whole fluid system can be regarded as an effective uniaxial medium and the optical axis is along the nanorod chain or lattice oriented direction, i.e. the direction of applied electric field (\(z\)-axis direction). We can utilize the dynamic Maxwell-Garnett (M-G) theory to calculate the effective permittivity of the global fluid because M-G theory is very simple and effective for the calculation of dielectric parameters of a mixture system of inclusions immersed in a host medium.\(^{22,23}\) Of course, the weakness of M-G theory is to cause inaccurate results or deviations from actual situation when the volume fraction of inclusions is very large or the geometry dimension of the inclusion particles is comparable to the wavelength of insert
electro-magnetic wave. However, the diameter of the inclusion particles in our proposed fluid of Au@SiO$_2$ NRs (r = 10 nm, l = 60 nm for Au core, and 3 nm for SiO$_2$ shell, which will be described in detail later) is much smaller than the wavelength of insert electro-magnetic wave, which is satisfied with the assumptions of M-G theory. Thus, under electric fields, the permittivity of fluid parallel to electric field (\(\varepsilon_{xx}\)) and perpendicular to electric field (\(\varepsilon_{zz}=\varepsilon_{yy}\)) can be given by following M-G equations (1) and (2):\(^{22,23}\)

\[
\begin{align*}
\varepsilon_{zz} &= (1 - \phi_\theta)\varepsilon_t + \phi_\theta\varepsilon_g \\
\varepsilon_{xx} &= \varepsilon_t + \frac{\phi_\theta\varepsilon_t (\varepsilon_g - \varepsilon_t)}{\varepsilon_t (1 - \phi_\theta) (\varepsilon_g - \varepsilon_t) s_x}
\end{align*}
\]

Here \(\phi_\theta\) is the volume fraction of Au NRs, \(\varepsilon_g\) and \(\varepsilon_t\) are relative permittivity of Au NRs and surrounding medium respectively. Because the dielectric properties of SiO$_2$ shell are similar to silicone oil, we can take them as a homogenous medium with the same permittivity of \(\varepsilon_t = 2.13\) (\(\varepsilon_t = n_t^2\), \(n_t = 1.46\) is the refractive index\(^{24}\)). Thus, the fluid can be regard as a simple two-phase system containing ellipsoidal inclusions (Au NRs) immersed in host medium (silicone oil). We simplify a single Au NR as a prolate ellipsoid and its long axis is along the direction of external electric field. Hence, \(g_x\) and \(g_z\) are the effective depolarization factors of an individual Au NR perpendicular to the electric field and parallel to the electric field, respectively. Depolarization factors reflect the influence of shape anisotropy of particles on dielectric properties. They satisfy a simple geometry rule of \(g_x + 2g_z = 1\) and \(g_z\) can be obtained by an approximate equation (3):\(^{25}\)

\[
g_z = \frac{\varepsilon_g}{2\varepsilon_t} \left( \ln \frac{1 + e}{1 - e} - 2e \right)
\]

where \(e = \sqrt{1 - (2r/d)^2}\) is the eccentricity. The permittivity (\(\varepsilon_g\)) of Au NRs in equations (1) and (2) is derived from Drude model, i.e. \(\varepsilon_g = 1 - \frac{\alpha_0^2}{\omega (\omega + i\gamma_d)}\) (where \(\alpha_0 = 1.37 \times 10^{16}\) s$^{-1}$ is bulk plasmon frequency of Au and \(\gamma_d = 4.08 \times 10^{13}\) s$^{-1}$ is the collision frequency).\(^{26}\) Thus, according to equations (1) and (2), when the volume fraction of Au is sufficient, this electric field-induced anisotropic fluid can possess \(\varepsilon_{xx}=\varepsilon_{yy}>0\) and \(\varepsilon_{zz}<0\), which can be called as “indefinite medium.”\(^{27}\)

According to Maxwell’s equations, the dispersion relation for the TM light with the magnetic field component polarized in the y-axis and the electric field component located in the x-z plane propagating in such indefinite medium follows the equation (4):

\[
k^2_{zz}/\varepsilon_{zz} + k^2_{xx}/\varepsilon_{xx} = \omega^2/c^2
\]

where \(\omega\) is the speed of light in vacuum, \(k_x\) and \(k_z\) are the wave vectors along x-axis and z-axis in such medium, respectively. Equation (4) indicates that the equifrequency contour of such medium is hyperbolic in the \((k_x, k_z)\) plane. Thus, the time-averaged Poynting vector \(\mathbf{S}\) in x-axis and z-axis can be written as \(S_x = \frac{k_x}{\varepsilon_{xx}} S_{xx} S_{zz}, \quad S_z = \frac{S_{zz}}{\varepsilon_{xx}} S_{xx},\) respectively.\(^{28}\) The refractive angles (\(\vartheta\)) of wave vector \(\mathbf{k}\) and Poynting vector \(\mathbf{S}\) are given by equation (5) and equation (6), respectively:\(^{28}\)

\[
f_k = \tan^{-1}(k_x / k_z)
\]

\[
\phi_k = \tan^{-1}(S_x / S_z) = \tan^{-1}[(k_x \varepsilon_{xx})/(k_z \varepsilon_{zz})]
\]

The effective refractive index of the indefinite medium is \(\eta = \sin\theta / \sin\vartheta\), \(\theta\) is the incident angle of TM light. Because of the continuity of tangential component (\(k_z\)) of wave vector, the hyperbolic equifrequency contour, and the causality theorem, the sign of \(\eta\) will flip at the interface between air and medium when an TM light is incident from air to such indefinite medium\(^{17,28,29}\). Thus, the real part of \(\phi_k\), is negative (i.e. \(\Re(\phi_k) < 0\)) and a negative refraction occurs. Obviously, the effective refractive index \(\eta\) is negative according to \(\eta = \sin\theta / \sin\vartheta\). Therefore, we can achieve all-angle negative refraction in this electric field-induced structured nanoparticle fluid that has an anisotropic permittivity (\(\varepsilon_{xx} = \varepsilon_{yy} > 0\), \(\varepsilon_{zz} < 0\)).

Based on the theory analysis above, we employ the Au@SiO$_2$ NRs with size parameters of \(r = 10\) nm, \(l = 60\) nm for Au core, and \(d = 5\) nm for SiO$_2$ shell as dispersal phase. The dimension of NRs is satisfied with the assumptions of M-G theory and is easy to be prepared by wet-chemical methods.\(^{30,31}\)

To realize \(\varepsilon_{zz}<0\) in this fluid (e.g. at 564 THz), the volume fraction (\(\phi_\theta\)) of Au needs to be larger than 0.13 according to equation (1). We choose two typical volume fractions of \(\phi_\theta = 0.15\) and 0.19 in order to calculate expediently. Thus, the total volume fraction (\(\phi\)) of the Au@SiO$_2$ NRs is 0.39 and 0.51 by \(\phi = \frac{1}{1 + 2d_{shell}/(r + d_{shell})^2} \phi_\theta = 0.38\phi_\theta\). According to reported phase diagrams of particle suspension system under electric fields,\(^{32,33}\) the nanorods orientate along electric fields and tend to arrange into body centered tetragonal (BCT) lattice at \(\phi = 0.39\) as showed in Fig.1 (a) and (c), while the nanorods tend to form hexagonal lattice at \(\phi = 0.51\) as showed in Fig.1 (b) and (d). Fig. 2 shows the calculated permittivities of the fluid at the \(\phi = 0.39\) and \(\phi = 0.51\) according to equations (1) and (2), respectively. It can be found that, in a broad frequency region, the real part of \(\varepsilon_{zz}\) is negative and the real part of \(\varepsilon_{xx}\) (or \(\varepsilon_{yy}\)) is positive. Therefore, it is possible to achieve all-angle negative refraction in this electric field-induced structured nanoparticle fluid.
To confirm these, we numerically simulate our design in the following section.

Fig. 1  The schematic illustration of cross-section of the lattice perpendicular to the electric field for the fluid of Au@SiO$_2$ NRs at $\phi = 0.39$ (a) and $\phi = 0.51$ (b). The schematics of the lattice of the chains formed by Au@SiO$_2$ NRs at $\phi = 0.39$ (c) and $\phi = 0.51$ (d). The rectangular unit cells are adopted in the finite-element simulations for $\phi = 0.39$ and $\phi = 0.51$ respectively.

Fig. 2 (a) Real part and (b) image part of the effective permittivity parallel to the electric field for the fluid of Au@SiO$_2$ NRs in silicone oil; (c) Real part and (d) image part of the effective permittivity perpendicular to the electric field for the fluid of Au@SiO$_2$ NRs in silicone oil. (black line: $\phi = 0.39$; red line: $\phi = 0.51$).

Numerical simulations and discussions

Electric field-induced formation of negative refractive index

The software COMSOL Multiphysics 4.3 is used to numerically demonstrate our design based on the finite element method (FEM). Firstly, in the simulation, two kinds of fluids of Au@SiO$_2$ NRs in silicone oil with particle volume fraction $\phi = 0.39$ and $\phi = 0.51$ are employed based on the theory analysis above. According to reported phase diagrams, under electric fields the NRs orientate and tend to arrange into BCT lattice as showed in Fig.1 (a) and (c) at $\phi = 0.39$, while the NRs tend to form hexagonal structure as showed in Fig.1 (b) and (d) at $\phi = 0.51$. Secondly, we regard the silica shell of NRs and surrounding silicone oil as a homogeneous medium due to their similar permittivity and optical property. The size of NRs is $r=10$ nm and $l=60$ nm in simulation, which is much smaller than the incident light wavelength. This is in accordance with the assumptions of theory above. Thirdly, we set the sample thickness (i.e. the gap between electrodes) as 1050 nm that is larger than wavelength. Both sides of the sample are air. We ignore the electrode’s thickness and surface or anchoring effects from electrodes for nanorod orientation. The nanorod orientation is only controllably adjusted by external electric fields. Finally, we take a transverse magnetic (TM) Gaussian beam (magnetic field component is polarized along the y–axis, electric field component located in the x-z plane) with a width of 1000 nm as incident light and the incident angle ($\theta$) is $30^\circ$. The distance between light source and sample is 700 nm.

At particle volume fraction $\phi = 0.39$, the electric field-induced chains tend to form BCT fashion as shown in Fig. 1(a) and (c). We set the lattice constant as $d=60$ nm ($\phi = (2\pi r^2) / d^2$) and employ $n_x \times n_y \times n_z = 45 \times 1 \times 15$ unit cells in the simulation box. The number of NRs is 1350. The periodic boundary condition is used along x-axis and y-axis. Fig. 3(a) shows the cross-sectional view of the power flow in the x-z plane. It clearly shows a negative refraction. Fig. 3(b) shows the simulation by replacing the fluid system with a homogenous slab whose effective permittivity is calculated from equation (1) and equation (2). It can be found that the negative refraction is in accordance with the result in Fig. 3(a) so that it is reasonable to use the dynamical Maxwell-Garnett theory to analyse the system. The realization of negative refraction in the fluid of Au@SiO$_2$ NRs can be attributed to the fact that the value of $\varepsilon_{zz}$ changing from positive to negative under electric fields.

Fig. 3 Simulation results for the fluid of Au@SiO$_2$ NRs in silicone oil at $\Phi = 0.39$ under electric fields: (a) the time average power flow at incident wave frequency
At particle volume fraction \( \phi = 0.51 \), the electric-field-induced chains tend to form hexagonal fashion as shown in Fig. 1(b) and (d).\(^{36}\) We set the lattice constant as \( a = 40 \text{ nm} \) (\( \phi = (2n^3)/\sqrt{3}a^3 \)) due to higher particle concentration and employ \( n_x \times n_y \times n_z = 40 \times 1 \times 15 \) unit cells in the simulation box. The number of NRs is 1200. The periodic boundary condition is also used along the x-axis and y-axis. Fig. 4 demonstrates the negative refraction still appears in the system with hexagonal structure because the fluid system still can be regarded as an effective uniaxial medium in this situation. The simulation result replacing the fluid structure with a homogenous slab in Fig. 4(b) is still agreement with that in Fig. 4(a). However, it should be pointed out that, because the refractive index is determined by the concentration of Au core and the frequency of incident light, the value of refractive index of the Poynting vector in Fig. 4 is different from that in Fig. 3, because the volume fraction of Au core is larger for \( \phi = 0.51 \).

Furthermore, from Fig. 3 we can find the refractive angle of the time average power flow becomes larger as the incident light frequency increases from 400 THz to 600 THz. The variation of the refractive angle is induced by the change of the permittivity of the system at different incident wave frequencies. Because the real part of the permittivity of Au follows the Drude model, \( \varepsilon_{zz} \) tends to gradually become zero with the increase of the incident wave frequency at the certain concentration of Au NRs. At 700 THz, Fig. 3 shows there is a strong absorption. According to above calculation, the real part of \( \varepsilon_{zz} \) is nearly zero at the incident wave frequency 700 THz when \( \phi = 0.51 \). This frequency corresponds to the so-called longitudinal SPR.\(^{34}\) For a Drude metal with \( \varepsilon_{gg} = 1 - f_{P}^2/f^2 \) and \( f_{p} \) the plasmon frequency, one has the so-called longitudinal SPR frequency \( f_{l} = f_{p}(1 + (\phi_{g}^{-1} - 1)\varepsilon_{l})^{-1/2} \). Thus, \( f_{l} \) is very sensitive to the volume fraction \( \phi_{g} \) and permittivity \( \varepsilon_{l} \) of the surrounding medium. The smaller the \( \varepsilon_{l} \), the higher the longitudinal SPR frequency \( f_{l} \). Obvious, decreasing the volume fraction of the Au NRs results in a redshift of the longitudinal SPR frequency. Therefore, in Fig. 4, the strong absorption corresponding to the so-called longitudinal appears at the incident wave frequency 600 THz due to reducing of the concentration of Au NRs. Because \( \varepsilon_{zz} \) is positive at 700 THz when \( \phi = 0.39 \), the refractive angle is positive at 700 THz, which is presented in Fig. 3. We can change this frequency by adjusting the lattice constant of the system, i.e., the volume fraction of Au NRs.

On the other hand, the permittivity match between the shell of core/shell-structured NRs and the surrounding medium is important to negative refraction behavior. Fig. 5 compares the negative refraction behaviors of the nanorod fluid with different permittivity match between the shell of NRs and the surrounding medium. For example, \( \varepsilon_{shell} \approx -2.13 \) for Au@SiO\(_2\) NRs, \( \varepsilon_{shell} \approx -2.40 \) for Au@Al\(_2\)O\(_3\) NRs, and \( \varepsilon_{shell} \approx -6.50 \) for Au@TiO\(_2\) NRs. It can be found that the transmission level declines as the permittivity difference between the shell of NRs and the surrounding medium becomes large. When \( \varepsilon_{shell} \approx -6.50 \) for Au@TiO\(_2\) NRs in silicone oil, a longitudinal SPR occurs in the fluid which leads to a strong absorption and the negative refraction behavior disappears. Therefore, we suggest using silica (\( \varepsilon_{shell} \approx -2.13 \)) as coating shell to not only provide an effectively insulating effect for Au core but also provide a good permittivity match between the shell of NRs and the surrounding medium (\( \varepsilon_{f} \approx -2.13 \) for silicone oil, which depends on molecular structure) in order to obtain the optimal negative refraction effect.

The shape anisotropy or aspect ratio of NRs also has an effect on negative refraction behaviour. Fig. 6 shows the simulate results of the negative refraction behavior as a function of the aspect ratio of...
NRs at the same Au volume fraction. It is found that the negative refraction angle of the time average power flow becomes larger as the increase of shape anisotropy or aspect ratio of NRs. This can be ascribed to the fact that the increase of the shape anisotropy or aspect ratio enhances the permittivity anisotropy of the fluid. Therefore, to enhance negative refraction behavior, we had best use NRs with large aspect ratio as disperse phase.

In addition, it should point out that the negative refraction in the nanorod fluid is tolerant to the deviation of perfect lattice. Compared to the negative refraction effect in fluid with perfect lattice (Fig. 7(a)), the negative refraction effect in Fig. 7(b) is still strong even if there are some defects (such as small-size nanorod aggregations) whose size is significantly smaller than the incident light wavelength. This can be attributed to the fact that the structured fluid containing small-size defects is still regarded as the effective media approximation. If there appear some large-size defects (such as large-size nanorod aggregations) in the lattice, however, the optical transmission and negative refraction behavior are compressed (see Fig. 7(c)) because the large-size defects can act as scattering centers and the effective media approximation is not valid. Therefore, to enhance negative refraction behavior, we should make electric field–induced nanorod arrangement as perfect or effective as possible and try to avoid large-size nanorod aggregations. One effective way to avoid nanorod aggregations is to increase colloidal repulsive force of NRs by surface charge in preparation.

Electric field strength dependence of negative refractive index and electrically tunable image

In this section, we analyze the dependence of the refractive index on electric field strength in detail. Without electric fields, the particles are randomly distributed and the fluid is isotropic, whose permittivity \( \varepsilon_{xx} = \varepsilon_{yy} = \varepsilon_{zz} = \varepsilon_{\text{eff}} \) can be approximately calculated by the Looyenga formula as shown in equation (7)

\[
\varepsilon_{\text{eff}} = \left( \frac{\varepsilon_1^{1/3} + \varepsilon_\infty^{1/3} - \varepsilon_\text{g}^{1/3}}{\varepsilon_1^{1/3} - \varepsilon_\text{g}^{1/3}} \right)^3
\]

When the external electric fields are applied along z-axis, the long-axis of Au@SiO\(_2\) NRs starts to orientate along electric field direction. Meanwhile, the fluid tends to transform from isotropic medium into anisotropic medium. As the external electric field strength increases, NRs form chains along the direction of the field, which cause the system to be more anisotropic. It has been demonstrated that the permittivity of particle fluid is linearly dependent on electric field strength when the particles start to orientate and align \(^{36}\). When the electric field exceeds a critical value \( E_c \), all particles have form gap-spanning chain structure between electrodes and, as a result, the whole system can be regarded as an effect uniaxial medium whose permittivity become constant. Thus, the relation of the fluid permittivity parallel to electric field \( \varepsilon_{xx} \) can be expressed by equation (8) and the fluid permittivity perpendicular to electric field \( \varepsilon_{xx,yy} \) can be expressed by equation (9):

\[
\begin{align*}
\varepsilon_{xx} & = \begin{cases} 
\varepsilon_{\text{eff}} = \left( \frac{\varepsilon_1^{1/3} + \varepsilon_\infty^{1/3} - \varepsilon_\text{g}^{1/3}}{\varepsilon_1^{1/3} - \varepsilon_\text{g}^{1/3}} \right)^3, & E = 0 \\
\varepsilon_{\text{eff}} + k_1E, & 0 < E < E_c \\
\varepsilon_{\text{eff}} + k_2E, & E > E_c
\end{cases} \\
\varepsilon_{xx,yy} & = \begin{cases} 
\varepsilon_{\text{eff}} = \left( \frac{\varepsilon_1^{1/3} + \varepsilon_\infty^{1/3} - \varepsilon_\text{g}^{1/3}}{\varepsilon_1^{1/3} - \varepsilon_\text{g}^{1/3}} \right)^3, & E = 0 \\
\varepsilon_{\text{eff}} + k_1E, & 0 < E < E_c \\
\varepsilon_{\text{eff}} + k_2E, & E > E_c
\end{cases}
\end{align*}
\]

In these equations, \( k_1 \) and \( k_2 \) are constant. In experiment realization, the critical field strength \( E_c \) is the key to the formation of chains of NRs. Here, \( E_c \) can be estimated by the
ratio of electric field energy to thermal energy as shown in equation (10):

\[
\frac{1}{3} \pi \varepsilon_0 \delta t (K_L) E_{\phi}^2 \times E_{\phi}^2 k_B T
\]

where \( K_L \) is the Clausius-Mossotti factor given by \( K_L = \frac{\varepsilon_B - \varepsilon_0}{\varepsilon_B + \varepsilon_0} \). Here, \( \varepsilon_0 = 1 + i \frac{\omega}{\sigma} \) represents the permittivity of Au NRs at applied low-frequency AC electric field with angle frequency \( \omega \), \( \sigma \) is the conductivity of gold. The reason we choose AC electric field is that the dielectrophoresis of Au@SiO\(_2\) NRs will be decreased and thus the chains are more stable. So the critical field strength can be estimated as by equation (11):

\[
E_c \approx \frac{9k_B T}{2\pi \varepsilon_0 \varepsilon \Re(\varepsilon_L)}
\]

At \( T = 300 \) K and \( \omega = 50 \) kHz, the critical field value is about \( 2.39 \times 10^6 \) V/m. Thus, we need to apply at least a voltage of 2.51 V on a 1050 nm-thick fluid system in the reality experiment. At the same time, the voltage is independent of particle volume fraction. So when the electric field strength reaches \( 2.39 \times 10^6 \) V/m, almost all Au@SiO\(_2\) NRs form chains. Fig. 8 gives the calculated value of permittivity of fluid at the incident wave frequency of 564 THz as a function of electric field strength according to equations (8) and (9). It can be found that the real part of permittivity of the fluid changes from isotropic to anisotropic when the electric field strength is lower than \( 2.39 \times 10^6 \) V/m and then becomes constant when the electric field strength is higher than \( 2.39 \times 10^6 \) V/m. At the same time, the image part of the permittivity parallel and perpendicular to \( z \)-axis decreases with the increase of anisotropic ordered structure.

Fig. 8 The dependence of permittivity of the fluid of Au@SiO\(_2\) NRs in silicone oil on applied electric field strength at the incident wave frequency of 564 THz. (a) Real and (b) imaginary part of the permittivity at \( \phi = 0.51 \). The permittivity of the system is changing from isotropic (\( \varepsilon_{\text{eff}} = 1.47 + 1.86i \)) to anisotropic (\( \varepsilon_{xx} = 3.96 + 0.01i, \varepsilon_{yy} = -0.99 + 0.03i \)) in the electric field region between 0 and \( E_c \); (c) Real and (d) imaginary part of the permittivity at \( \phi = 0.39 \). The permittivity of the system is changing from isotropic (\( \varepsilon_{\text{eff}} = 1.70 + 1.49i \)) to anisotropic (\( \varepsilon_{xx} = 3.43 + 0.01i, \varepsilon_{yy} = -0.27 + 0.03i \)) in the electric field region between 0 and \( E_c \).

Fig. 9 shows the time average power flow simulation at the incident optical frequency of 564 THz and \( \phi = 0.51 \) as a function of electric field strength. It can be found that the loss of incident wave gradually decreases and the negative refraction is realized at sufficient high field strength. This can be attributed to that the structure of the system will transform from isotropic fluid to a uniaxial medium with the increasing of electric field strength, the loss of incident wave will decline, according to the imaginary part of the permittivity decreasing in Fig. 8.

Because of the anisotropic permittivity of the system under external electric field, the fluid is a kind of indefinite medium. Image of uniaxial indefinite material has been demonstrated in the theory and experiment. Therefore, we also numerically demonstrate that such a fluid system can be used for imaging. Fig. 10 shows the time-averaged power flow in the 3D finite-element simulation for the incident wave of 564 THz in the fluid with \( \phi = 0.51 \). In the FEM simulation depicted in Fig. 8, the point source of light located 200 nm in front of a 1050 nm-thick fluid system. It can be found that a clearly elongated focus point emerges when the wave traverse the fluid. Because the effective refraction index of the indefinite material is angle-dependent, there is an aberration of the imaging.

Fig. 9 The time average power flow simulation based on the effective medium approximation for the fluid of Au@SiO\(_2\) NRs in silicone oil at \( \phi = 0.51 \) according to the permittivity calculated in Fig. (5) at the incident wave frequency of 564 THz under different external electric field strength of (a) \( E = 0 \), (b) \( E = 2 \times 10^6 \) V/m, and (c) \( E = 2.39 \times 10^6 \) V/m.
The deviation of the focus distance in Fig. 10(a) and (b) results from the interaction of the internal units in the fluid system. The image is realized by the hyperbolic equifrequency surface. We can change the permittivity of the system by adjusting the external electric field strength to realize the subwavelength and magnified imaging. When the permittivity of the fluid system changes with external electric field strength, the focus of fluid lens system will change with the electric field strength. This situation is plotted in Fig. 11, in which the fluid system is replaced by a homogenous medium. As shown in Fig. 11, the distance between the focus and the fluid lens becomes small as the electric field strength increases. Thus, the fluid system can be used as a smart fluid lens whose focus distance can be adjusted via external field strength.

Electric field distribution dependence of negative refractive index and gradient negative refractive index

Besides the electric field strength, the distribution of electric field will influence the structure of NRs or the lattice parameter in gap-spanning chain structure. For example, when a non-uniform radial electric field is applied, the Au NRs will form radial chain-like structure and, as a result, the lattice constant $d$ or distance between chains will be continuously changed. Thus the concentration of the Au NRs will not be uniform in this situation and it should be a function of coordinate. From equations (1) and (2), we can find that the permittivity of the system is determined by the volume fraction of Au NRs when other conditions are constant. The local concentration of Au NRs will gradually decreases along $z$-axis when a non-uniform radial electric field is applied, which causes $\varepsilon_{zz}$ gradually to decrease along $z$-axis. Therefore, the refractive angle of the time average power flow will increase along $z$-axis and the graded negative refraction can be obtained.

In the simulation of the fluid system under the non-uniform radial electric field, the number of unit cells along $x$-axis decreases from 40 to 28, indicating that the volume fraction of NRs decreases from 0.51 to 0.36. The number of unit cells along $z$-axis remain 15. The degree of concentration decline is proportional to the non-uniformity of the electric field. Replacing the system by a homogenous slab, we firstly simulated the refractive behavior under different non-uniform radial electric fields based on the effective medium approximation as shown in Fig. 12. It shows the refractive deflection effect increases with the increase of the non-uniformity of electric field. Fig. 13 compares the negative refraction results by finite element simulations under a uniform field and a non-uniform electric field. It is found that the negative refraction of time average power flow is graded when the lattice constant $d$ is gradually increased due to the change of external electric field distribution due to the gradient permittivity.
Fig. 13 The cross-sectional view of finite element simulations: (a) The negative refraction of time average power flow is graded when lattice constant $d$ is gradually increase due to the change of external electric field distribution. (b) The negative refraction of time average power flow at uniform electric field. The electric field-induced chains displaced in hexagonal structure.

Conclusions

In summary, we have designed a fluid system containing Au@SiO$_2$ NRs in silicone oil, which can realize all-angle broadband negative refraction at optical frequencies under external electric fields because of the formation of chain-like or lattice-like structure of dispersed NRs along electric fields. We calculate the effective permittivity tensor of the fluid and verify that, in a broad spectra region, the real part of $\varepsilon_{xx}$ is negative and the real part of $\varepsilon_{yy}$ (or $\varepsilon_{zz}$) is positive. Thus, we can realize all-angle negative refraction in the fluid system because of the hyperbolic equifrequency surface and the boundary condition for electromagnetic waves at the interface between a uniaxial medium and an isotropic medium. The refractive index can vary with the electric field strength and external field distribution because that the effective permittivity tensor of the fluid can change with electric fields. Based on this point, we further demonstrate a near-field imaging in the fluid-based superlens under uniform electric field and a negative refractive index gradient under non-uniform electric field. This novel fluid with electrically tunable negative refraction not only proposes a new strategy to fabricate soft metamaterials but also provides a foundation for the future development of smart metamaterial devices, such as adaptive fluid-based super lens, flexible "cloak", and so on.

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Notes and references