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Representativeness of shorter measurement sessions in long-term indoor air monitoring

M. Maciejewska* and A. Szczurek

Indoor air quality (IAQ) considerably influences health, comfort and the overall performance of people who spend most of their lives in enclosed spaces. For that reason, there is a strong need to develop methods for IAQ assessment. The fundamental issue in quantitative determination of IAQ is the duration of measurements. Its inadequate choice may result in providing incorrect information and this potentially leads to wrong conclusions. The most exhaustive information may be acquired through long-term monitoring. However it is typically perceived as impractical due to time and cost load. The aim of this study was to determine whether long-term monitoring can be adequately represented by a shorter measurement session. There were considered three measurable quantities: temperature, relative humidity and carbon dioxide concentration. They are commonly recognized as indicatives for IAQ and may be readily monitored. Scaled Kullback-Leibler divergence, also called relative entropy, was applied as a measure of data representativeness. We considered long-term monitoring in a range from 1 to 9 months. Based on our work, representative data on CO₂ concentration may be acquired while performing measurements over 20 % of time dedicated to long-term monitoring. In case of temperature and relative humidity the respective time demand was 50 % of long-term monitoring. From our results, in indoor air monitoring strategies, there could be considered shorter measurement sessions, while still collecting data which are representative for long-term monitoring.

Introduction

Over last few decades indoor air quality (IAQ) has become an important problem in many countries [1-4]. Numerous studies have shown that it has serious influence on health, comfort and well-being of building occupants [5,6]. It has been linked to Sick Building Syndrome, reduced productivity in offices and impaired learning at schools [7]. For that reason, there is a strong need to develop methods for IAQ assessment [8,9]. The quality of indoor air may be defined in two ways: qualitatively, by descriptors such as, for example, odors perceived by the individual [10] or quantitatively, by entities which are determined on the basis of chemical or physical measurements [11,12]. A number of parameters can be used to characterize IAQ in quantitative manner e.g. temperature (T), relative humidity (RH), carbon dioxide (CO₂), volatile organic compounds (VOCs) and particles concentrations [13, 14].

The fundamental issue in IAQ assessment is the duration of a measurement session. It is also referred as the problem of adequate sampling period and frequency [15]. The selection of appropriate time for measurements has a principal significance for the realization of indoor air quality study objectives. An inadequate choice may result in large uncertainties or even in wrong information, potentially leading to incorrect conclusions [16]. This problem is associated with the temporal variability of indoor T, RH and contaminants concentration. Their variation is caused by the influence of numerous factors on indoor air. If it was constant, the temporal variability of IAQ would be negligible. Unfortunately, the factors change, moreover distinct factors alter in different time scales. The most obvious cycles of factors variation are seasonal and diurnal. The seasonal one is caused by meteorological conditions which exhibit quite predictable, annual pattern of change. Meteorological conditions influence indoor environment in a direct (building interaction with its surrounding, including air exchange) and indirect manner (seasonal changes in habits of building occupants). Diurnal cycle of variation is mostly caused by the building occupancy and usage patterns. They are translated to the emission of human related effluents e.g. CO₂, water vapour and heat gains. Still, there remains a considerable group of other, frequently unknown factors, which demonstrate yet different scales of temporal change. Some of them may be considered random.
All types of influences collectively translate into the variation of indoor air parameters [17, 18]. Hence, the adequate sampling duration is important in matching the overall pattern of variation. This is a prerequisite of collecting representative data. In practice, measurements can be both short- as well as long-term. Short-term measurements are a source of valuable information if variations of factors influencing indoor air are negligible. In real conditions, this kind of assumption is dubious. Hence, short-term measurements reflect temporary conditions that are not representative of longer time periods. The temporal variability of indoor parameters makes it very difficult to design strategies of IAQ monitoring based on short time period. On the other hand, long-term monitoring provides the capability of recognizing short-term fluctuations or the ‘extreme’ events, as well as observing long-term regularities. Although regarding indoor air assessment, the duration of monitoring considered as long-term has not been precisely defined, such measurements are typically viewed as impractical. The obvious major limitations are time and cost.

The problem of selecting duration for measurement sessions is not yet solved, although some recommendations have been made [15,19]. Considering chemical pollutants of indoor air, the guidelines refer to the exposure types e.g. acute, sub-chronic or chronic, and indicate the observation time necessary to perform the adequate risk assessment. In case of parameters like temperature, relative humidity and CO₂ concentration, such recommendations are lacking. This situation is very disadvantageous. The mentioned parameters well describe indoor conditions and slowly become the basic element in IAQ studies. The availability of relatively cheap devices which allow for continuous, real time measurements gives rise for very detailed characterization of temporal variability of indoor air. The applicability for this type of information is very broad, involving purely cognitive aspects, human comfort studies and building diagnostics [14,20].

The problem of evaluating representativeness of continuous measurements is distinctive. In this case, in focus there is the time series of data and not an unordered set of measurement results. Our work contributes to the development of the relevant methodology [16,21]. The objective of this work is to determine whether long-term monitoring session can be represented by a shorter measurement period. We proposed to apply scaled relative entropy as a measure which may be used to evaluate the representativeness of data set in indoor air monitoring. One may find examples of applying Shannon Entropy, and a number of entropy derived measures, in environmental studies. They were willingly utilised for the selection of sampling points and air pollution monitoring network design [22-25]. The analysis focus on the spatial aspect of performing measurements. In this work we exploit the temporal domain. It was motivated by the fact that the presented approach may be helpful in designing indoor air monitoring strategies.

Methods

Assumptions

Our considerations are based on few assumptions. (1) Long-term monitoring is conducted to record variations of the measured quantity which occur over a period of at least 1 month. (2) Results of long-term as well as shorter measurements are presented in the form of time series. Shorter measurement period is represented by a suitable segment (subseries) of the long time series. (3) In our approach, time series is a source of data set describing an appropriate monitoring session. Data sets coupled with shorter monitoring sessions are subsets of data set collected during long monitoring session. (4) Representativeness of shorter monitoring sessions is reflected in representativeness of corresponding data subsets. In this work, representativeness of data subset means the ability to represent stochastic nature of T, RH and CO₂ concentration over long period of measurements. In other words, the representative data subset should reflect data structures and content of the original time series. (5) The length of shorter, representative period of measurements is determined by the time required to obtain time series representing long-term monitoring. (6) Various measures can be used to determine data representativeness. We focused our attention on Kullback-Leibler divergence, also known as the relative entropy. It was theoretically approved that Kullback-Leibler divergence is an appropriate means for determination of data representativeness [26].

Relative entropy

Assume \(p(x)\) and \(q(x)\) are probability distributions of random variable \(X\) in domain \(\chi\). In particular, let \(p(x)\) be the probability distribution of that variable for a sample and \(q(x)\) be the probability distribution for an entire, source data set. The distance between these probability distributions can be measured by the Kullback-Leibler divergence (K-L divergence) also known as the relative entropy [27]:

\[
D(p\|q) = \sum_{x \in \chi} p(x) \log \frac{p(x)}{q(x)}
\]

The divergence \(D(p\|q)\) is always nonnegative. The distance between sample distribution \(p(x)\) and the original distribution \(q(x)\) is short if they are similar. Exclusively, when the distributions match exactly relative entropy equals zero. In particular, this condition is fulfilled when the sample and source data set are the same. Bigger distance indicates less similarity between sample distribution \(p(x)\) and the original distribution \(q(x)\). Potentially, when the distributions are very different, relative entropy can be infinite.

When using logarithm base 2, relative entropy quantifies the distance between two distributions in information units. Namely it tells in bits how close it is from a candidate probability distribution \(p(x)\) to the model distribution \(q(x)\) [28]. In other words, Kullback and Leiber [29] proposed an information distance measure which has convincing statistical interpretation.

In statistics the K-L divergence is a measure derived to indicate the dissimilarity of two distributions [30]. It shall be mentioned, that this measure is not symmetric in \(p\) and \(q\), namely \(D(p\|q) \neq D(q\|p)\), and it does not satisfy the triangle inequality. This property makes relative entropy well suitable for the examination of data representativeness. This type of relation is also asymmetric. In general, one evaluates the representativeness of data subset with respect to an entire set, while the opposite reasoning is not justified.

It was theoretically demonstrated in [26] that a concept of relative entropy may be leveraged to quantify and compare the representativeness of data subset with respect to entire data set. Being inspired by this report we proposed to apply relative entropy for the examination of representativeness in indoor air monitoring.

Relative entropy as a measure of representativeness

In order to achieve the goal of this work, the study procedure was applied which consisted of several steps.
1. Measurement of T, RH and CO₂ concentration over long period of time and acquisition of data in the form of time series.
2. Determination of entire data sets representing long-term monitoring. The long-term monitoring was characterized by the duration \( t_{\text{active}} \).
3. The choice of the length of shorter measurement sessions. They were characterized by the durations \( t_i \in \{ t_1, t_2, \ldots, t_{\text{active}} \} \).
4. Determination of data subsets characterized by the same \( t_i \). They were extracted from the data set describing long term monitoring using moving time window technique. Consequently, each \( t_i \) indicated many measurement periods of the same length and, respectively many data subsets of the same size.
5. Calculation of:
   - relative entropy \( D_{t_i} \) between entire data set and an individual subset, which was \( t_i \) long;
   - average relative entropy \( \bar{D}_{t_i} \) for all data subsets, which originated from \( t_i \) long measurement sessions;
   - average relative entropy \( \bar{D}_{\text{max}} \) associated with the shortest time period, \( t_i \) long;
   - ratio:
     \[
     I_{t_i} = \frac{D_{t_i}}{\bar{D}_{\text{max}}} \tag{2}
     \]
\( I_{t_i} \) indicates scaled relative entropy for data subsets, which are \( t_i \) long.

Due to its construction (eq. 2), scaled relative entropy belongs to the range \( I_{t_i} \in [0,1] \). Hence, this index allows for comparing representativeness in various circumstances of indoor air monitoring, particularly regarding various measured parameters and measurement time periods.

**Scope of analysis**

The representativeness of indoor air monitoring data collected during shorter measurement session may be different depending on:
- monitored quantity;
- duration of long-term monitoring;
- time of year;
- type of indoor space;

In this respect we investigated:
1. There were two periods of observation in lecture theatre and computer laboratory.
2. We analysed data from the time series of various lengths. They were associated with 1, 2, 3, 4, 5, 6, 7, 8 and 9 months of continuous indoor air monitoring.
3. There were observed long-term monitoring periods differently located in calendar. Namely, there were examined: 9 individual months, 8 distinct 2-months periods, 7 distinct 3-months periods etc. and finally one 9-months period.
4. Identical study was performed for two indoor spaces: lecture theatre and computer laboratory.

**Experimental**

We studied the representativeness of indoor air monitoring based on measurements performed in two enclosed spaces. There were lecture theatre and computer laboratory at the university. Their brief description is presented in Table 1.

<table>
<thead>
<tr>
<th>Feature</th>
<th>Lecture theater</th>
<th>Computer laboratory</th>
</tr>
</thead>
</table>

The applied measuring instruments were based on a sensor technique. Their measuring characteristics are given in Table 2. The instruments are dedicated to continuous measurements of temperature, relative humidity and carbon dioxide concentration.

In the rooms, the instruments were located away from the direct influence of occupants, about 1 m over the floor. Two identical measuring devices were used in our study, one per indoor space. The analysed data consisted of 2 min average values of T, RH and CO₂ concentration.

**Table 2 Measuring characteristics of sensor devices applied for indoor air monitoring.**

<table>
<thead>
<tr>
<th>Characteristics</th>
<th>Temperature</th>
<th>Relative humidity</th>
<th>CO₂ concentration</th>
</tr>
</thead>
<tbody>
<tr>
<td>accuracy</td>
<td>± 0.2°C ± 0.15% measured value</td>
<td>± 50 ppm ± 3% measured value</td>
<td>± 2% (10...90% RH)</td>
</tr>
<tr>
<td>resolution</td>
<td>0.1 °C</td>
<td>0.1 %</td>
<td>1 ppm</td>
</tr>
</tbody>
</table>

**Table 3 Time periods of observation.**

<table>
<thead>
<tr>
<th>Indoor space</th>
<th>Monitoring time period</th>
</tr>
</thead>
<tbody>
<tr>
<td>computer laboratory</td>
<td>August 2012 – April 2013 (9 months)</td>
</tr>
<tr>
<td>lecture theater</td>
<td>October 2013 – June 2014 (9 months)</td>
</tr>
</tbody>
</table>

The monitoring was carried out over 9 months. The periods of observation in lecture theatre and computer laboratory were non-overlapping (see Table 3).

**Results and discussion**

In Fig. 1 we show the data collected in lecture theater during 9 month long indoor air monitoring. The data displayed in Fig. 2 were collected in computer laboratory, in November 2012. Based on the two figures, measured parameters of indoor air exhibited strong temporal variations. A type of rhythm could be recognized in these changes. The basic and most distinctive cycle of variation was 1 day. It could be attributed to the daily rhythm of human activity and meteorological conditions. The first factor determined variation of CO₂ concentration (Fig. 2). In case of temperature and relative humidity, their rapid changes...
caused by human presence were imposed over more smooth ones, associated with other impacts (Fig. 2). Other cycle of indoor air parameters variation was week-long. The distinctness of this cycle was dependent on the actual lack of people in the monitored spaces on the weekends. As shown in Fig.1 and Fig.2, temperature and relative humidity variation occurred also in time scales longer than one day and shorter than one week. They may be attributed to the variation of outdoor and indoor factors. However, pinpointing particular ones would be very difficult. As shown in Fig. 1 the seasons of the year relatively weakly influenced the measured parameters. There was recognizable decrease of relative humidity in a winter season. Also the increase of temperature during summertime could be noticed.

The periodic changes of measured quantities are one of elements, which are taken under consideration when establishing the duration of measurements assuring the acquisition of representative data. It is typically assumed that grasping regularities which occur in a particular time scale requires that measurements extended over it. Potentially, if longer period may be considered as composed of shorter cycles, the measurement of one cycle could be viewed as sufficient. The results obtained in this work contribute with a new observations in this respect.

Maximum relative entropy gives an idea about the information distance between the least representative data subset and entire data set. In our work, least representative data sets were the shortest ones, i.e. obtained during 30 min long measurement sessions. The information distance between them and data sets acquired in course of long-term monitoring (1, 2, 3 up to 9 months) was from 1 to 4 bits. Maximum relative entropy varied as a function of: (1) measured quantity, (2) duration of long-term monitoring, (3) time of year and (4) type of indoor space. Hence, using scaled relative entropy, which refers relative entropy to its maximum was fully justified and allowed to achieve the comparison between various scenarios of indoor air monitoring.

The maximum distance separates data set collected during long-term monitoring and data subset associated with the shortest monitoring period. In this case scaled relative entropy equals 1. When extending the duration of measurement session to maximum, i.e. making it equal to the period of long-term monitoring, the two data sets become identical and the distance between them drops down to zero.

Basically, the relationship between the scaled relative entropy and the length of measurement session demonstrates the gain in data representativeness which is achieved by increasing the duration of monitoring. The relationship may take various forms. In Fig. 3 we display essential features of the advantageous and disadvantageous relationship regarding the easiness of achieving the information gain. In Fig. 4a scaled relative entropy rapidly decreases when increasing the duration of monitoring from shortest. Once this tendency stops, further extension of measurement session allows for a very limited gain in data representativeness. If this form of relationship is observed for real data, short periods of observation are sufficient to achieve good representativeness in indoor air monitoring. It is advantageous. The disadvantageous form of relationship is displayed in Fig. 4b. In this case scaled relative entropy decreases very slowly when increasing the duration of monitoring from shortest. Once this tendency is broken, the achievable gain in data representativeness increases. But, it
takes until longer periods of time. If the relationship for true data reminds of the one displayed in Fig. 4b, long periods of observation are necessary to achieve good representativeness in indoor air monitoring. This is unfavourable and in extreme cases, it may be best to perform complete long-term monitoring.

In Fig. 5 to Fig. 7 we display the exemplary results of analysis performed in this work. The plots represent the relationships between the length of measurement session and scaled relative entropy. The presented results refer to long-term monitoring lasting 1 month, but they well illustrate the essential features of the complete set of outcomes involving long-term monitoring periods of 1, 2, 3, …, up to 9 months.

For the data analysed in this work, the shape of relationships between the length of measurement session and scaled relative entropy was advantageous. Although the relationships were not identical. The differences revealed the dependency of indoor air monitoring representativeness on a number of factors like: monitored quantity, time of year as well as the type of indoor space. Regarding measured quantity, the most advantageous dependency was observed in case of carbon dioxide (Fig. 4a and Fig. 5a). For this component of air scaled relative entropy exhibited much more rapid decline compared with temperature (Fig. 4b and Fig. 5b) and relative humidity (Fig. 4c and Fig. 5c). The gain in representativeness achieved by extending measurement session was also sensitive to the time of year. The bunches of lines displayed in Fig. 4 and Fig. 5 indicate there was a discrepancy between the results of analysis when long-term monitoring was performed in distinct months of year. Also the type of indoor space shall be considered as non-neutral for the representativeness in indoor air monitoring. In our study there were noticed differences between the results obtained for lecture theatre and computer lab, compare Fig. 3 and Fig. 4.

![Fig. 4](image-url)

*Fig. 4* Scaled relative entropy for CO$_2$, temperature and relative humidity as a function of measurement session duration in lecture theatre, assuming long-term monitoring was 1 month long.

![Fig. 5](image-url)

*Fig. 5* Scaled relative entropy for CO$_2$, temperature and relative humidity as a function of measurement session duration in computer laboratory, assuming long-term monitoring was 1 month long.

In all analysed cases, the relationship between scaled relative entropy and the length of measurement session was advantageous (see Fig. 3a). Based on this observation, it is justifiable to think about the scale of possible reduction in measurement effort dedicated to long-term indoor air monitoring. This may be judged by determining the measurement time which is sufficient to provide the representative information.

For this purpose, we proposed to establish a cut-off value for scaled relative entropy. It would indicate the satisfactory information distance between the data collected during the actual measurement session and the complete data set, potentially collectable over long-term monitoring. The cut-off value points at the duration of measurement session which is required to provide representative measurement data. The data would be considered representative for long-term monitoring in the sense of this criterion.

While setting cut-off level we were guided by the approaches that are valid for quantities exhibiting exponential change. In such cases, it is widely accepted to recognize 90 % of the saturation value as its sufficient estimate. We may observe in Fig. 4 and Fig. 5 that scaled relative entropy demonstrates nearly exponential decay with the increasing duration of monitoring session. The range of scaled relative entropy is $[0,1]$ and its saturation level is zero. Hence, the cut-off value was 0.1. With this threshold and for indoor spaces examined in this work, the information distance between long-term data set and representative data set was between 0.05 and 0.4 bits. Of course, different cut-off values may be chosen if justified.

In Fig. 6 and Fig. 7 we present the fraction of long-term monitoring which was sufficient to obtain representative data. The representativeness is equivalent to $I=0.1$. 

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Based on Fig.6 and Fig.7, there was a considerable difference between the duration of measurement session which provides representative data on CO$_2$ compared with temperature and relative humidity. In general, long-term monitoring of CO$_2$ could be well represented by the measurement sessions covering less than 20% of the designated time (Fig.6a and Fig.7a). Taking an example of 1 month long-term monitoring, this percentage corresponds to 6 days. Although the basic and very stable cycle of CO$_2$ variation was 1 day (Fig.1 and Fig.2), clearly a month cannot be considered as a series of approximately identical days. We demonstrated that more days have to be covered with measurements to get data representative for longer time periods. Their number grows respectively with the duration of the target period.

Delivering representative data on temperature and relative humidity requires longer monitoring compared with CO$_2$. As shown in Fig. 6 and Fig. 7, the representatives equivalent to $I=0.1$ is achieved when the measurement session extends over more than 30 to 50% of long-term monitoring. Taking an example of 1 month, the mentioned percentage corresponds to about two weeks. Hence, in case of temperature and relative humidity neither daily, nor weekly cycle may be considered repeatable. The structure of their temporal variation is much more complex and longer measurements are needed for reflecting it in a sufficient manner.

Clearly, when considering temperature and relative humidity, possible reduction of monitoring time was not as spectacular as in case of CO$_2$, although still substantial. This may indicate, that the parameters belong to distinct groups of quantities, which characterise indoor air in different ways and require individual monitoring strategies.

**Conclusions**

This work was dedicated to the representativeness in indoor air monitoring. We analysed this problem in time domain. The study concentrated on the following measurable quantities: temperature, relative humidity and carbon dioxide concentration. They are commonly recognized as indicative for the quality of indoor air, and their measurements gradually become the basis in IAQ assessment. Hence, it is very important to assure that the collected data are representative.

We proposed scaled relative entropy as a tool to examine data representativeness in indoor air monitoring. It was applied to evaluate how representative the shorter measuring session is for long-term monitoring. The method is generic and may be utilized for other measurable quantities as well.

Based on the obtained results, the measurement session providing multivariate data was not equally representative for distinct parameters of indoor air. In particular, shorter time was needed to collect the adequate data on CO$_2$ (20% of long-term monitoring), compared with temperature and relative humidity (30 to 50% of long-term monitoring). This observation may stimulate the development of innovative indoor air monitoring strategies. In particular, the new hardware or software solutions may be proposed which allow for establishing the individual time scheduling of temperature, relative humidity and CO$_2$ measurements. The additional positive effect would come from energy saving during measurements.

We demonstrated that it is possible to replace long-term monitoring with shorter measurements, while still collecting representative data. As shown, the representativeness of measurement session depends on: measured quantity, anticipated duration of long-term monitoring, time of year when the measurements are performed, as well as the type of indoor space. All these factors shall be taken into consideration when designing indoor air monitoring strategies in individual cases. But, it is our idea to perform the analysis of scaled entropy for various sets of data from long-term indoor air monitoring. Based on many results some general recommendations could be possibly formulated.
In our opinion, this work may be very helpful in planning the diagnostic studies in buildings.

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