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Theoretical and machine learning models for reaction-barrier predictions: acrylate and methacrylate radical reactions†

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We propose density functional theory (DFT)- and random forest (RF)-based theoretical and machine learning (ML) models, respectively, for predicting reaction barriers (ΔE_{TS}) using acrylate and methacrylate radical reactions as representatives. DFT is used to determine 100 transition state (TS) structures of both radicals, after which the obtained data are used to determine theoretical relationships (explained with Bell–Evans–Polanyi or Brønsted–Evans–Polanyi (BEP) and Marcus-like models) between ΔE_{TS} and stabilization energy of the product. Next, we construct several theoretical regression models for predicting ΔE_{TS} of the representative reactions based on our theoretical analyses, presenting an RF-based ML model that eases ΔE_{TS} predictions by circumventing time-consuming DFT calculations. These theoretical and RF-based ML approaches will accelerate the advancement of material development.

1. Introduction

Radical reactions are employed for diverse applications across various fields, including chemistry, medicine, and materials science, owing to the high reactivities and versatilities of radicals.^{1–3} Typically, these highly selective reactions proceed under relatively mild conditions. Furthermore, the high reactivities of radicals under mild conditions facilitate the extensive adoption of radical polymerization in the syntheses of various acrylic polymers with numerous applications in products, such as paints, adhesives, medical materials, plastics, and fibers.^{4–7}

Here, we discussed the radical reactions of acrylate (ACR) and/or methacrylate (MA), which are essential to the synthesis of acrylic polymers. Generally, the efficient development of acrylic polymers requires sophisticated and precise controls of the radical reactions, and the detailed mechanisms of these reactions must be understood. Furthermore, effective density functional theory (DFT)-based transition state (TS) searches are required for the investigation of radical reaction mechanisms. The DFT-based TS analyses of simple MA monomers⁸ and catalyzed radical polymerization products⁹ have been reported. However, the high computational cost of TS search complicates the calculations of many structures for material developments, as TS-structure determination typically requires numerous trials and errors together with highly expensive DFT calculations. To avoid heavy calculations, simple theoretical models have been employed for reaction analysis. For example, the Bell–Evans–Polanyi or Brønsted–Evans–Polanyi (BEP) model^{10–12} is widely utilized in catalysis reactions, which is based on an empirical linear relationship between the activation energy and the reaction enthalpy to predict the catalytic activity. Conversely, the utilization of databases and machine learning (ML) for efficient material development has garnered substantial interest in recent years.^{13–24} For example, in the experimental syntheses of polymers with desired thermal conductivities, ML was deployed for the screening of several promising materials among numerous candidates,²⁵ representing a promising approach for accelerating and advancing material development.

Therefore, in this study, we considered the radical reactions between ACR and MA. First, we performed DFT calculations to determine the TSs of these reactions, after which we discussed the general trends of the obtained computational results. Next, we confirmed that theoretical regression models for performing easy reaction-barrier predictions can be constructed based on the results, after which we constructed a random forest (RF)-based ML model for predicting their reaction barriers (ΔE_{TS}) using simple descriptors based on their chemical structures and circumventing complex DFT calculations.

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2. Computational methods

The structures of the reactants, products, and TSs were optimized using the B3LYP functional^{26,27} with Grimme's empirical dispersion²⁸ and the 6-31+G* basis set (B3LYP+D3/6-31+G*). All the DFT calculations were performed using the Gaussian16 package.²⁹ The ML algorithm (RF)³⁰ with scikit-learn liberally (version 0.22.1)³¹ was used to construct the ML model for predicting ΔE_{TS} . Here, the number of trees in the forest ($n_{\text{estimators}}$) was set to 30, and tenfold cross-validation was used to evaluate the models.

3. Results and discussion

3.1. Trends of reaction barriers and energy of the products obtained *via* density functional theory calculations

In this study, we performed DFT calculations to determine ΔE_{TS} and the product–reactant energy difference (ΔE_{prod}) for the radical reactions between radical monomers X^\bullet and Y (Fig. 1(a) and (b)), where X and Y represent ACR and/or MA monomers, respectively, including an acrylic acid/methacrylic acid (Fig. 1(c)). Notably, X^\bullet was generated by adding a hydrogen radical to the monomer X . Additionally, the calculated ΔE_{TS} and ΔE_{prod} values of all the combinations of the 10 monomers, *i.e.*, 100 reactions, are listed in Table S1 ESI.† Here, the energy references were set to the pre-reactant complex of each system. For example, the lowest ΔE_{TS} (3.9 kcal mol^{−1}) was obtained from the reaction between an

ethylcyclohexyl ACR radical and methacrylic acid. In comparison, the highest ΔE_{TS} (10.1 kcal mol^{−1}) was obtained from the reaction between a γ -butyrolactone MA radical and γ -butyrolactone ACR. These radical reactions can be categorized into four types based on whether the chemical structures of reactants X^\bullet and Y correspond to ACR or MA, respectively.

Table 1 presents the average ΔE_{TS} and ΔE_{prod} for each category. In this study, we categorized acryl and methacrylic acids as ACR and MA, respectively. As presented in Table 1, ΔE_{TS} and ΔE_{prod} tended to be lower when the reactant radical (X) was ACR rather than MA. For example, in the case of $Y = \text{ACR}$, average ΔE_{TS} values of 5.4 and 7.2 kcal mol^{−1} were obtained for $X = \text{ACR}$ and MA, respectively. Conversely, average ΔE_{prod} values of −16.6 and −11.6 kcal mol^{−1} were obtained for $X = \text{ACR}$ and MA, respectively. Additionally, we observed a decreasing trend in ΔE_{prod} of the Y species. These trends were roughly consistent with those reported in the literature^{1,32} and could be explained by the stabilities of the radical species: as ACR^\bullet and MA^\bullet are secondary and tertiary radicals, respectively, the latter would be more stable than the former owing to hyperconjugation.

To gain an insight into the trend of the computational results, we plotted the datasets with ΔE_{TS} and ΔE_{prod} on the vertical and horizontal axes (Fig. 2). In the figure, the colors and markers correspond to the reaction categories. Namely, the blue and red colors represent ACR^\bullet and MA^\bullet for the reactant radical (X^\bullet), respectively. Conversely, the circle (●) and cross (×) markers represent ACR and MA for the monomer (Y),

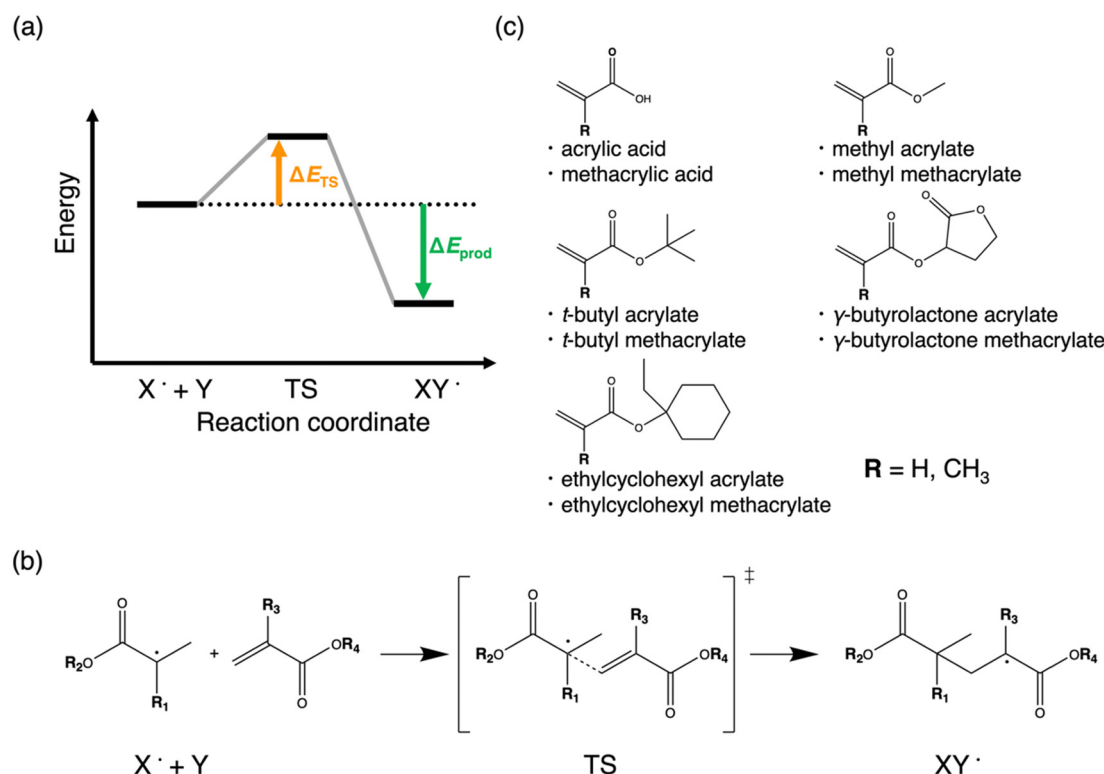


Fig. 1 (a) Energy diagram and (b) scheme of the radical reaction between ACR and/or MA: $X^\bullet + Y \rightarrow XY^\bullet$, where X^\bullet represents the radical monomer. (c) Target reactant monomers of acrylic acid, ACR, methacrylic acid, and MA. Here, acrylic and methacrylic acids were categorized as ACR and MA, respectively.



Table 1 Average ΔE_{TS} and ΔE_{prod} (kcal mol⁻¹) for each category. Their standard deviations are listed in parentheses

Category ($X^\bullet + Y$)	Average ΔE_{TS}	Average ΔE_{prod}
ACR [•] + ACR	5.4 (0.6)	-16.6 (1.2)
ACR [•] + MA	4.5 (0.3)	-18.4 (1.6)
MA [•] + ACR	7.2 (1.3)	-11.6 (1.5)
MA [•] + MA	7.3 (1.0)	-12.6 (1.3)

respectively. Some trends can be easily confirmed from Fig. 2 (see also Table 1). For example, the computational results were largely clustered into two regions, *i.e.*, $X = \text{ACR}$ and MA , which were represented by blue and red colors, respectively; ΔE_{TS} and ΔE_{prod} were influenced by the stability of the reactant radical (X^\bullet). Moreover, the stability of the reactant in the product (Y) caused a slight difference between \bullet and \times within the blue and red regions. In these reactions, ACR[•], a secondary radical, was more unstable than MA[•], a tertiary radical. Thus, in the reactant, $X^\bullet = \text{ACR}^\bullet$ was relatively more reactive, *i.e.*, its ΔE_{TS} value became lower than that of $X^\bullet = \text{MA}^\bullet$.

Furthermore, ΔE_{prod} , which is the relative energy between the product and reactant, tended to become unstable when $X^\bullet = \text{ACR}^\bullet$; thus a larger ΔE_{prod} would be obtained compared with the case of $X^\bullet = \text{MA}^\bullet$, following the polymerization reaction. Similarly, the decreasing tendency of Y would also be due to the stability of the product (XY^\bullet). After the polymerization reaction, the radical moves from X to Y , and hence, the secondary radical ($Y = \text{ACR}$) yields more unstable products than $Y = \text{MA}$. However, radical reactions are affected by the stability of the radical species and other factors, such as steric hindrance and electronic effects, especially with the increasing polymer-chain length. For example, we could not ignore the steric hindrance effects from the methyl groups on the radical reactions of $\text{MA}^\bullet + \text{MA}$. In practical cases, other factors, such as the solvents and electronic effects from the side chains, must be considered. Although the precise assessments of all the aforementioned factors were challenging, our assessments of ΔE_{TS} and ΔE_{prod} offered a valuable overview of radical reactions.

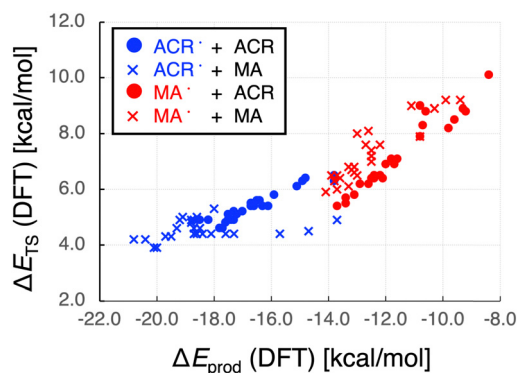


Fig. 2 Scatter plot of the ΔE_{TS} with respect to the DFT-obtained ΔE_{prod} . The blue and red colors represent ACR[•] and MA[•] for the reactant radical (X^\bullet), respectively. Conversely, \bullet and \times represent ACR and MA for the reactant monomer (Y), respectively.

3.2. Relationships between the reaction-barrier and product-reactant energy difference

In the BEP model, the potential energy surfaces of the reactant and product were assumed to be two intersecting linear functions, and the ΔE_{TS} (Δy_{TS}) was obtained using coefficients of a and b as follows,

$$\Delta y_{\text{TS}} = a\Delta y_{\text{prod}} + b \quad (1)$$

We confirmed that ΔE_{TS} (Δy_{TS}) represents a linear relationship with respect to the relative energy of the product (Δy_{prod}). In computational results in Fig. 2, we observed a BEP linear relationship between ΔE_{TS} and ΔE_{prod} (see also Fig. S3(a) in the ESI[†]).

Conversely, we noticed that the relationship between ΔE_{TS} and ΔE_{prod} seems to be described as a downwardly convex quadratic curve from the computational results in Fig. 2 (see also Fig. S3(b) in the ESI[†]). The chemical meaning behind this relationship may be explained using the Marcus-like model.^{33,34} In the model, the potential energy surfaces of the reactant and product were assumed to be parabolic functions with the same coefficient (c), described as $y_{\text{reac}} = cx^2$ and $y_{\text{prod}} = c(x - x_{\text{prod}})^2 + \Delta y_{\text{prod}}$, respectively (Fig. 3). Here, x and y represent the reaction coordinate and potential energy, respectively. The y surface of the product deviates by Δy_{prod} from the most stable energy of the reactant at the reaction coordinate (x_{prod}). From these assumptions, the ΔE_{TS} (Δy_{TS}) was obtained from the intersection of these parabolas, as follows,

$$\Delta y_{\text{TS}} = \frac{(\Delta y_{\text{prod}} + \lambda)^2}{4\lambda} \quad (2)$$

Here, $\lambda \equiv cx_{\text{prod}}^2$. Furthermore, Δy_{TS} represents a quadratic function with respect to the relative energy of the product (Δy_{prod}). The Marcus-like model could explain the quadratic relationship between ΔE_{TS} and ΔE_{prod} (Fig. 2). Thus, the relationship between ΔE_{TS} (Δy_{TS}) and ΔE_{prod} (Δy_{prod}) is obtained by assuming the simple analytic function for the potential energy surface of the reactant and product in the Marcus-like model.

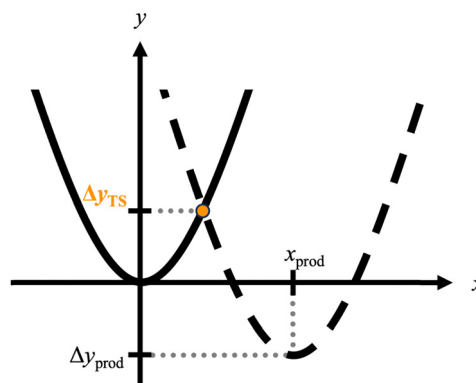


Fig. 3 Model potential energy surfaces of the reactant and product represented by solid and dashed lines, respectively. These energy surfaces are approximately described as parabolic functions. The reaction barrier in this case is described as $\Delta y_{\text{TS}} = \frac{(\Delta y_{\text{prod}} + \lambda)^2}{4\lambda}$, where $\lambda \equiv cx_{\text{prod}}^2$. Thus, Δy_{TS} exhibits a quadratic relationship with Δy_{prod} .



This theoretical treatment is similar to that in the BEP model. However, linear and quadratic functions are assumed to describe the potential energy surfaces in the BEP and Marcus-like models, respectively.

3.3. Prediction of reaction-barriers: theoretical models based on the product-reactant energy difference

The BEP and Marcus-like models may be useful for constructing a simple regression model to significantly ease the predictions of ΔE_{TS} for those radical reactions. Here, we proposed the following two theoretical regression models using ΔE_{prod} (and its squared value) based on BEP and Marcus-like models:

$$\Delta E_{TS} = 0.42\Delta E_{prod} + 12.3 \quad (3)$$

$$\Delta E_{TS} = 0.0341\Delta E_{prod}^2 + 1.44\Delta E_{prod} + 19.5 \quad (4)$$

Furthermore, we compared ΔE_{TS} predicted by these theoretical models and DFT-based calculations (Fig. 4(a) and (b)). We obtained determination coefficients (R^2) of 0.83 and 0.88 from the BEP and Marcus-like models, respectively. To evaluate the predictive performance of these models, we also calculated the

Akaike's Information Criterion (AIC).³⁵ We obtained AICs of 648.9 and 618.0 for the BEP and Marcus-like regression models. In the AIC analysis, a model with a lower criterion value is considered to be superior. Thus, the regression model of eqn (4) based on the Marcus-like theory may show slightly better predictive performance for the reaction barrier. In fact, the BEP-based regression model tends to slightly underestimate ΔE_{TS} in the high value region, as seen in Fig. 4(a).

3.4. Prediction of reaction-barriers: a random forest-based machine learning model based on simple descriptors from the reactant monomers

The theoretical models established in eqn (3) and (4) can ease the estimation of ΔE_{TS} ; however, it still requires DFT calculations to determine ΔE_{prod} . Therefore, we constructed an ML model for predicting ΔE_{TS} while circumventing DFT calculations. To employ the ML approach, we first established some reaction-related descriptors (feature vectors). To do this, we examined several physicochemical properties of the reactant monomers, which were estimated from several group contribution methods implemented in OpenBabel (version 3.3.1)³⁶ and

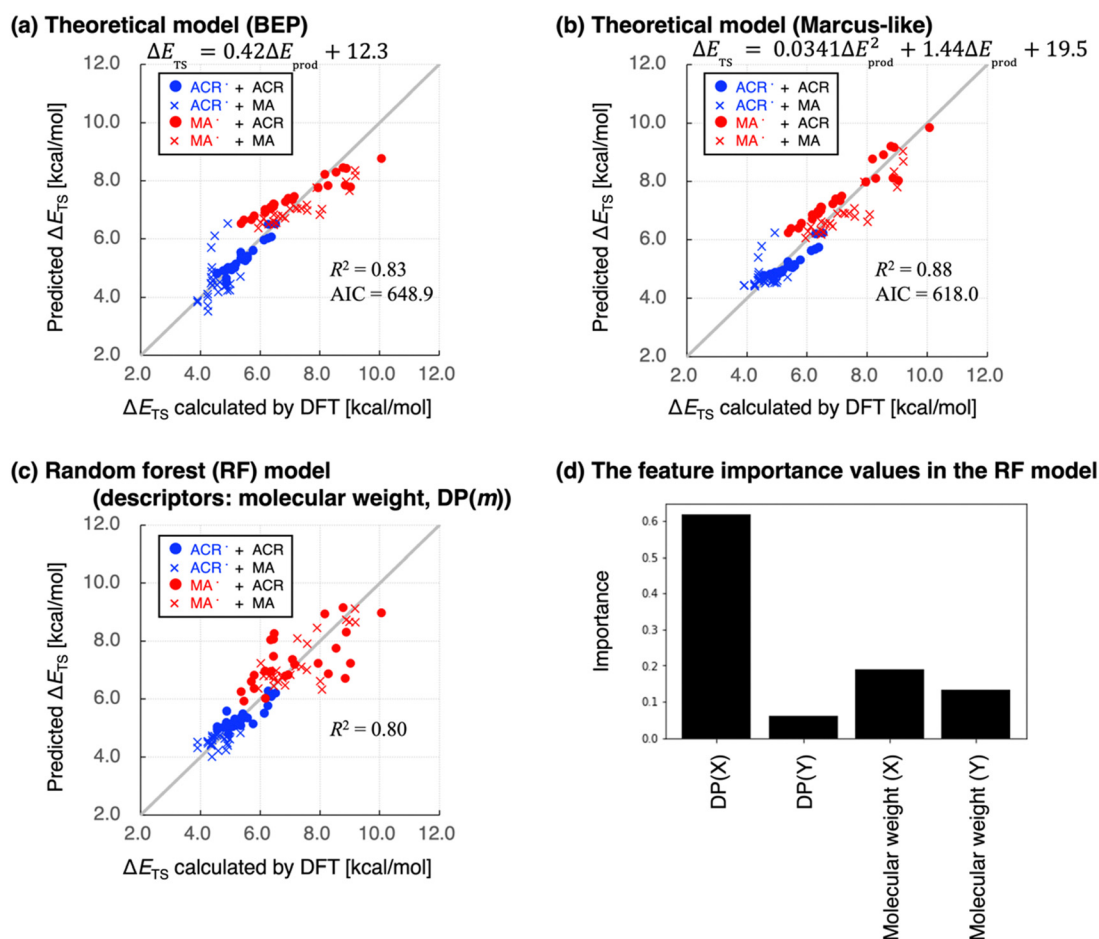
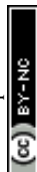


Fig. 4 Comparisons of ΔE_{TS} values predicted via DFT-based calculations and the theoretical models (a) BEP model in eqn (3) and (b) Marcus-like model in eqn (4). (c) Comparisons of ΔE_{TS} predicted via DFT-based calculations and the RF model using four descriptors; the dummy parameter ($DP(m)$) and molecular weight of each reactant. Here, the blue and red colors represent ACR• and MA• for the reactant radical (X^\bullet), respectively. Conversely, ● and × represent ACR and MA for the reactant monomer (Y), respectively. (d) Feature-importance values in the RF-based model.



RDkit (vers. 2021.03.5)³⁷ libraries. Here, for simplicity, we used the properties of the “X monomer,” not X^\bullet itself. Additionally, we considered a dummy parameter ($DP(m)$) to represent ACR or MA, using m to specify X or Y. For example, $DP(X) = 0$ represented $X = \text{ACR}$, and $DP(Y) = 1$ was used to describe $Y = \text{MA}$. We employed the tenfold cross-validation technique to evaluate the performance of ML models. In the cross-validation technique, data are divided into several groups in the first step. Then, a group is used for model evaluation and the others for model training. This operation is carried out while replacing the groups for the model evaluation. As a first trial, we obtained a model with $R^2 = 0.84$ to predict ΔE_{TS} using RF with 50 descriptors, using the cross-validation technique. For each reactant species, 24 descriptors from the group contribution method and one $DP(m)$ were used (S3 in the ESI†). Moreover, after multiple trials using the RF-based feature-importance guideline, we finally obtained a simpler model (Fig. 4(c)) with $R^2 = 0.80$ using 4 descriptors, where only the molecular weight and $DP(m)$ for each reactant species were utilized. As shown in Fig. 4(c), we compared the RF- and DFT-based ΔE_{TS} predictions.

Here, we examined another approach to predict ΔE_{TS} . We first predict ΔE_{prod} using a RF-based ML model. Then, ΔE_{TS} is estimated using the BEP and Marcus-like regression models (eqn (3) and (4)), respectively. From this procedure, we can predict ΔE_{TS} values with R^2 of 0.75 and 0.76 for the BEP and Marcus-like models, respectively (see also S6 in the ESI†). Thus, we confirmed the validity of the BEP and Marcus-like models. Conversely, data related to the ΔE_{prod} property are easier to collect compared with data related to ΔE_{TS} . Therefore, the collaborative approach between theoretical and ML models may be useful to construct a more convenient means for estimating the reaction energy.

We demonstrated the feature-importance values of the descriptors obtained using RF (Fig. 4(d)). Even without the DFT calculations, the results revealed that the ML model achieved an accuracy that was comparable with that of the DFT-based theoretical model (eqn (2)). Among the feature-importance values (Fig. 4(d)), $DP(X)$ was the most significant. It might be related to the reactant stability, as it determined whether the reactant radical was a secondary or tertiary one. Similarly, $DP(Y)$ was related to the product stability but less significant. This trend correlates with the results in Table 1 and Fig. 2. In this study, we employed the same basic chemical framework for the reactant molecules, and only side chains are different, as shown in Fig. 1(b). When the molecular weight descriptor together with $DP(m)$ to distinguish ACR or MA is given, we can obtain some information about side chains related to the bulkiness (steric effect) of the monomer. For example, acrylic acid and ethylcyclohexyl methacrylate molecules have molecular weights of 72.1 and 196.3, respectively. Here, ethylcyclohexyl methacrylate has a bulkier side chain. ML models may use such information to predict the reaction barrier. Thus, $DP(m)$ and molecular weight descriptors may comprise some chemical features, such as the stability and bulkiness of the radical.

4. Conclusion

In this study, we analyzed 100 radical reactions determined by combining ten types of ACRs and/or MAs whose TSs were assessed using DFT. To analyze calculation data, we employed the BEP model. In addition, the computational results revealed the quadratic relationship between ΔE_{TS} and ΔE_{prod} , and this relationship was explained with a Marcus-like model. Based on these theoretical analyses, we constructed theoretical regression models; BEP and Marcus-like models yield $R^2 = 0.83$ and 0.88 to predict ΔE_{TS} . However, these models still require time-consuming DFT-based calculations to obtain ΔE_{prod} . Therefore, we constructed an ML model with $R^2 = 0.80$ without DFT calculations using simple monomer descriptors, namely the dummy parameter ($DP(m)$) and molecular weight. We believe that our theoretical and ML approaches for radical reaction predictions will benefit future material developments.

Author contributions

Makito Takagi: data curation, investigation, methodology, software, visualization, writing – original draft, and writing – review & editing. Tomomi Shimazaki: methodology, project administration, resources, software, supervision, and writing – review & editing. Osamu Kobayashi: methodology and writing – review & editing. Takayoshi Ishimoto: project administration, supervision, and writing – review & editing. Masanori Tachikawa: funding acquisition, project administration, resources, supervision, and writing – review & editing.

Data availability

The data supporting this article have been included as part of the ESI.†

Conflicts of interest

There are no conflicts to declare.

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References

- 1 T. Otsu, *J. Synth. Org. Chem. Jpn.*, 1970, **28**, 1183–1196.



- 2 G. Moad, E. Rizzardo and S. H. Thang, *Polymer*, 2008, **49**, 1079–1131.
- 3 T. Pirman, M. Ocepek and B. Likozar, *Ind. Eng. Chem. Res.*, 2021, **60**, 9347–9367.
- 4 U. Ali, K. J. B. Abd Karim and N. A. Buang, *Polym. Rev.*, 2015, **55**, 678–705.
- 5 S. C. Ligon-Auer, M. Schwentenwein, C. Gorsche, J. Stampfl and R. Liska, *Polym. Chem.*, 2016, **7**, 257–286.
- 6 N. Ballard and J. M. Asua, *Prog. Polym. Sci.*, 2018, **79**, 40–60.
- 7 M. S. Zafar, *Polymers*, 2020, **12**, 2299.
- 8 I. Degirmenci, V. Aviyente, V. Van Speybroeck and M. Waroquier, *Macromolecules*, 2009, **42**, 3033–3041.
- 9 A. Debuigne, C. Michaux, C. Jérôme, R. Jérôme, R. Poli and C. Detrembleur, *Chem. – Eur. J.*, 2008, **14**, 7623–7637.
- 10 R. P. Bell, *Proc. R. Soc. London, Ser. A*, 1936, **154**, 414–429.
- 11 M. Evans and M. Polanyi, *Trans. Faraday Society*, 1936, **32**, 1333–1360.
- 12 M. M. Montemore and J. W. Medlin, *Catal. Sci. Technol.*, 2014, **4**, 3748–3761.
- 13 W. A. Warr, *Mol. Inform.*, 2014, **33**, 469–476.
- 14 K. Jorner, A. Tomberg, C. Bauer, C. Sköld and P. O. Norrby, *Nat. Rev. Chem.*, 2021, **5**, 240–255.
- 15 J. A. Keith, V. Vassilev-Galindo, B. Q. Cheng, S. Chmiela, M. Gastegger, K. R. Mueller and A. Tkatchenko, *Chem. Rev.*, 2021, **121**, 9816–9872.
- 16 R. X. Wang, X. L. Fang, Y. P. Lu and S. M. Wang, *J. Med. Chem.*, 2004, **47**, 2977–2980.
- 17 J. J. Irwin, T. Sterling, M. M. Mysinger, E. S. Bolstad and R. G. Coleman, *J. Chem. Inf. Model.*, 2012, **52**, 1757–1768.
- 18 A. Jain, S. P. Ong, G. Hautier, W. Chen, W. D. Richards, S. Dacek, S. Cholia, D. Gunter, D. Skinner, G. Ceder and K. A. Persson, *APL Mater.*, 2013, **1**, 011002.
- 19 S. Kim, P. A. Thiessen, E. E. Bolton, J. Chen, G. Fu, A. Gindulyte, L. Y. Han, J. E. He, S. Q. He, B. A. Shoemaker, J. Y. Wang, B. Yu, J. Zhang and S. H. Bryant, *Nucleic Acids Res.*, 2016, **44**, D1202–D1213.
- 20 M. Nakata and T. Shimazaki, *J. Chem. Inf. Model.*, 2017, **57**, 1300–1308.
- 21 R. Jose and S. Ramakrishna, *Appl. Mater. Today*, 2018, **10**, 127–132.
- 22 C. Draxl and M. Scheffler, *J. Phys.: Mater.*, 2019, **2**, 036001.
- 23 M. Nakata, T. Shimazaki, M. Hashimoto and T. Maeda, *J. Chem. Inf. Model.*, 2020, **60**, 5891–5899.
- 24 T. R. Lane, D. H. Foil, E. Minerali, F. Urbina, K. M. Zorn and S. Ekins, *Mol. Pharm.*, 2021, **18**, 403–415.
- 25 S. Wu, Y. Kondo, M. A. Kakimoto, B. Yang, H. Yamada, I. Kuwajima, G. Lambard, K. Hongo, Y. B. Xu, J. Shiomi, C. Schick, J. Morikawa and R. Yoshida, *NPJ Comput. Mater.*, 2019, **5**, 66.
- 26 A. D. Becke, *J. Chem. Phys.*, 1993, **98**, 5648–5652.
- 27 C. T. Lee, W. T. Yang and R. G. Parr, *Phys. Rev. B: Condens. Matter Mater. Phys.*, 1988, **37**, 785–789.
- 28 S. Grimme, J. Antony, S. Ehrlich and H. Krieg, *J. Chem. Phys.*, 2010, **132**, 154104.
- 29 M. J. Frisch, G. W. Trucks, H. B. Schlegel, G. E. Scuseria, M. A. Robb, J. R. Cheeseman, G. Scalmani, V. Barone, G. A. Petersson, H. Nakatsuji, X. Li, M. Caricato, A. V. Marenich, J. Bloino, B. G. Janesko, R. Gomperts, B. Mennucci, H. P. Hratchian, J. V. Ortiz, A. F. Izmaylov, J. L. Sonnenberg, D. Williams-Young, F. Ding, F. Lipparini, F. Egidi, J. Goings, B. Peng, A. Petrone, T. Henderson, D. Ranasinghe, V. G. Zakrzewski, J. Gao, N. Rega, G. Zheng, W. Liang, M. Hada, M. Ehara, K. Toyota, R. Fukuda, J. Hasegawa, M. Ishida, T. Nakajima, Y. Honda, O. Kitao, H. Nakai, T. Vreven, K. Throssell, J. A. Montgomery Jr., J. E. Peralta, F. Ogliaro, M. J. Bearpark, J. J. Heyd, E. N. Brothers, K. N. Kudin, V. N. Staroverov, T. A. Keith, R. Kobayashi, J. Normand, K. Raghavachari, A. P. Rendell, J. C. Burant, S. S. Iyengar, J. Tomasi, M. Cossi, J. M. Millam, M. Klene, C. Adamo, R. Cammi, J. W. Ochterski, R. L. Martin, K. Morokuma, O. Farkas, J. B. Foresman and D. J. Fox, *Gaussian 16, Revision A.03*, Gaussian, Inc., Wallingford CT, 2016.
- 30 L. Breiman, *Mach. Learn.*, 2001, **45**, 5–32.
- 31 F. Pedregosa, G. Varoquaux, A. Gramfort, V. Michel, B. Thirion, O. Grisel, M. Blondel, P. Prettenhofer, R. Weiss, V. Dubourg, J. Vanderplas, A. Passos, D. Cournapeau, M. Brucher, M. Perrot and E. Duchesnay, *J. Mach. Learn. Res.*, 2011, **12**, 2825–2830.
- 32 K. S. Anseth, C. M. Wang and C. N. Bowman, *Polymer*, 1994, **35**, 3243–3250.
- 33 R. A. Marcus, *J. Chem. Phys.*, 1956, **24**, 966–978.
- 34 R. A. Marcus, *Rev. Mod. Phys.*, 1993, **65**, 599–610.
- 35 D. G. Brooks, *Technometrics*, 1989, **31**(2), 270–271.
- 36 N. M. O'Boyle, M. Banck, C. A. James, C. Morley, T. Vandermeersch and G. R. Hutchison, *J. Cheminform.*, 2011, **3**, 33.
- 37 RDKit: Open-Source Cheminformatics Software. (<https://www.rdkit.org>).

