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# Intrinsic dynamic and static nature of each HB in the multi-HBs between nucleobase pairs and its behavior, elucidated with QTAIM dual functional analysis and QC calculations $\dagger$ 

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#### Abstract

The intrinsic dynamic and static nature of each HB in the multi-HBs between nucleobase pairs ( $\mathrm{Nu}-\mathrm{Nu}^{\prime}$ ) is elucidated with QTAIM dual functional analysis (QTAIM-DFA). Perturbed structures generated using coordinates derived from the compliance constants $\left(C_{i i}\right)$ are employed for QTAIM-DFA. The method is called CIV. Two, three, or four HBs are detected for $\mathrm{Nu}-\mathrm{Nu}^{\prime}$. Each HB in $\mathrm{Nu}-\mathrm{Nu}^{\prime}$ is predicted to have the nature of CT-TBP (trigonal bipyramidal adduct formation through charge transfer (CT)), CT-MC (molecular complex formation through CT ), or $\mathrm{t}-\mathrm{HB}_{\mathrm{wc}}$ (typical HB with covalency), while the vdW nature is predicted for the $\mathrm{C}-\mathrm{H} \cdots \mathrm{X}$ interactions, for example. Energies for the formation of the pairs ( $\Delta E$ ) are linearly correlated with the total values of $C_{i i}^{-1}$ in $\mathrm{Nu}-\mathrm{Nu}^{\prime}$. The total $C_{i i}{ }^{-1}$ values are obtained by summing each $C_{i i}{ }^{-1}$ value, similarly to the case of Ohm's law for the parallel connection in the electric resistance. The total $\Delta E$ value for a nucleobase pair could be fractionalized to each HB, based on each $C_{i i}{ }^{-1}$ value. The perturbed structures generated with CIV are very close to those generated with the partial optimization method, when the changes in the interaction distances are very small. The results provide useful insights for better understanding DNA processes, although they are highly enzymatic.


## Introduction

Hydrogen bonds (HBs) are fundamentally important in all fields of chemical and biological sciences. ${ }^{1-9}$ The energy-lowering effect on the formation of HBs contributes to molecular association, and the formation of HBs controls the direction of the atoms taking part in HBs. One of the most important roles of HBs in biological sciences is the operation of the genetic code. ${ }^{10}$ The two helical chains of nucleotides in DNA associate through the multi-HBs between adenine-thymine ( $\mathrm{A}-\mathrm{T}$ ) and guaninecytosine (G-C) pairs, as proposed by Watson and Crick. ${ }^{3,10-13}$ The duplex DNA structure first opens and then closes in active proliferation at approximately room temperature, which is a typical event in DNA induced by the action of HBs. The processes must be highly enzymatically catalyzed. ${ }^{14}$ The multiHBs between A-T and G-C pairs are formed in close proximity in space and will mutually and strongly interact with each other. Therefore, clarifying the nature of each HB in the multi-HBs between nucleobase pairs ( $\mathrm{Nu}-\mathrm{Nu}^{\prime}$ ), containing $\mathrm{A}-\mathrm{T}$ and $\mathrm{G}-\mathrm{C}$,

[^0]is very important. The results will provide useful insights for better understanding DNA processes, although they are highly enzymatic. The ability to image the initial stage of the opening and closing of duplex DNA through a simple mechanism based on the nature of each HB in the multi-HBs between $\mathrm{Nu}-\mathrm{Nu}^{\prime}$ would be helpful. The basic behavior and stability of the duplex DNA structure should be closely related to the nature of each HB


Adenine (A)



Thymine ( $T$ )


Uracil (U)

Fig. 1 Molecular graphs for the nucleobases adenine (A), guanine (G), cytosine (C), thymine (T) and uracil (U), optimized with MP2/BSS-B'a (see Table 1 for BSS-B'a).

Table 1 Basis set systems (BSSs) employed for the calculations

| BSS | H, C, N, O | BSS | H, C, N, O |
| :--- | :--- | :--- | :--- |
| BSS-A | $6-311++\mathrm{G}(3 \mathrm{df}, 3 \mathrm{pd})$ | BSS-A $^{\prime}$ | $6-311+\mathrm{G}(3 \mathrm{df}, 3 \mathrm{pd})$ |
| BSS-B'a $^{\prime}$ | $6-311+\mathrm{G}(3 \mathrm{df}, 3 \mathrm{pd})^{a}$ | BSS-B $^{\prime} \mathrm{b}$ | $6-311+\mathrm{G}(3 \mathrm{df}, 3 \mathrm{pd})^{b}$ |
| BSS-C | $6-311++\mathrm{G}(3 \mathrm{df}, 3 \mathrm{p})$ | BSS-C $^{\prime}$ | $6-311+\mathrm{G}(3 \mathrm{df}, 3 \mathrm{p})$ |
| BSS-D | $6-311++\mathrm{G}(3 \mathrm{~d}, 3 \mathrm{p})$ | BSS-D $^{\prime}$ | $6-311+\mathrm{G}(3 \mathrm{~d}, 3 \mathrm{p})$ |

${ }^{a}$ The $6-311+\mathrm{G}(3 \mathrm{~d})$ basis set being employed for C . ${ }^{b}$ The $6-311+\mathrm{G}(\mathrm{d})$ basis set being employed for C .
in $\mathrm{Nu}-\mathrm{Nu}^{\prime}$. The ability to fractionalize the energy for the formation of $\mathrm{Nu}-\mathrm{Nu}^{\prime}$ from the components ( Nu and $\mathrm{Nu}^{\prime}$ ) to each HB in the multi-HBs between $\mathrm{Nu}-\mathrm{Nu}^{\prime}$ is also very interested. Such considerations led us to elucidate the dynamic and static nature of each HB in the multi-HBs between $\mathrm{Nu}-\mathrm{Nu}^{\prime}$, where the static nature of the interaction is calculated based on the fully optimized structure, while the dynamic nature is derived from the data of the perturbed structures around to the fully optimized one, which is explained later, again. Fig. 1 shows the structures of the nucleobases adenine (A), guanine (G), cytosine $(\mathrm{C})$, thymine ( T ) and uracil ( U ) as molecular graphs, which are calculated with MP2/BSS-B'a (see Table 1 for BSS-B'a). Indeed, there are some possibilities in the structures of $\mathrm{Nu}-\mathrm{Nu}^{\prime}$, and these possibilities, which control the functionalities of the pairs in the DNA chains, seem promising; ${ }^{15}$ however, the typical cases are discussed in this paper.

The QTAIM approach, introduced by Bader, ${ }^{16,17}$ enables us to analyze the nature of chemical bonds and interactions. ${ }^{18-29}$ The bond critical point (BCP, ${ }^{* 16,17,30}$ ) is an important concept in QTAIM corresponding to the point where $\rho(\boldsymbol{r})$ (charge density) reaches a minimum along the interatomic (bond) path, while it is a maximum on the interatomic surface separating the atomic basins. $\rho(\boldsymbol{r})$ at the BCP is denoted by $\rho_{\mathrm{b}}\left(\boldsymbol{r}_{\mathrm{c}}\right)$ in this paper, as are other QTAIM functions, such as the total electron energy density $H_{\mathrm{b}}\left(\boldsymbol{r}_{\mathrm{c}}\right)$, potential energy density $V_{\mathrm{b}}\left(\boldsymbol{r}_{\mathrm{c}}\right)$ and kinetic energy density $G_{\mathrm{b}}\left(\boldsymbol{r}_{\mathrm{c}}\right)$ at the BCP. A chemical bond or an interaction between A and B is denoted by $\mathrm{A}-\mathrm{B}$, which corresponds to the bond path (BP) in QTAIM. We use A-*-B for the BP, where the asterisk emphasizes the existence of a BCP in A-B. ${ }^{16,17,30}$ Eqn (1), (2) and (2') represent the relations among $G_{\mathrm{b}}\left(\boldsymbol{r}_{\mathrm{c}}\right), V_{\mathrm{b}}\left(\boldsymbol{r}_{\mathrm{c}}\right), H_{\mathrm{b}}\left(\boldsymbol{r}_{\mathrm{c}}\right)$, and $\nabla^{2} \rho_{\mathrm{b}}\left(\boldsymbol{r}_{\mathrm{c}}\right)$.

$$
\begin{gather*}
H_{\mathrm{b}}\left(\boldsymbol{r}_{\mathrm{c}}\right)=G_{\mathrm{b}}\left(\boldsymbol{r}_{\mathrm{c}}\right)+V_{\mathrm{b}}\left(\boldsymbol{r}_{\mathrm{c}}\right)  \tag{1}\\
\left(\hbar^{2} / 8 m\right) \nabla^{2} \rho_{\mathrm{b}}\left(\boldsymbol{r}_{\mathrm{c}}\right)=H_{\mathrm{b}}\left(\boldsymbol{r}_{\mathrm{c}}\right)-V_{\mathrm{b}}\left(\boldsymbol{r}_{\mathrm{c}}\right) / 2  \tag{2}\\
\left(\hbar^{2} / 8 m\right) \nabla^{2} \rho_{\mathrm{b}}\left(\boldsymbol{r}_{\mathrm{c}}\right)=G_{\mathrm{b}}\left(\boldsymbol{r}_{\mathrm{c}}\right)+V_{\mathrm{b}}\left(\boldsymbol{r}_{\mathrm{c}}\right) / 2
\end{gather*}
$$

Interactions are classified by the signs of $\nabla^{2} \rho_{\mathrm{b}}\left(\boldsymbol{r}_{\mathrm{c}}\right)$ and $H_{\mathrm{b}}\left(\boldsymbol{r}_{\mathrm{c}}\right)$. $H_{\mathrm{b}}\left(\boldsymbol{r}_{\mathrm{c}}\right)$ must be negative when $\nabla^{2} \rho_{\mathrm{b}}\left(\boldsymbol{r}_{\mathrm{c}}\right)<0$, as confirmed by eqn (2), since $V_{\mathrm{b}}\left(\boldsymbol{r}_{\mathrm{c}}\right)<0$ at all BCPs. Interactions are called shared shell (SS) interactions when $\nabla^{2} \rho_{\mathrm{b}}\left(\boldsymbol{r}_{\mathrm{c}}\right)<0$ and closed-shell (CS) interactions when $\nabla^{2} \rho_{\mathrm{b}}\left(\boldsymbol{r}_{\mathrm{c}}\right)>0 .{ }^{16}$ In particular, CS interactions are called pure CS (p-CS) interactions when $H_{\mathrm{b}}\left(\boldsymbol{r}_{\mathrm{c}}\right)>0$ and $\nabla^{2} \rho_{\mathrm{b}}\left(\boldsymbol{r}_{\mathrm{c}}\right)>0$. We call interactions where $H_{\mathrm{b}}\left(\boldsymbol{r}_{\mathrm{c}}\right)<0$ and $\nabla^{2} \rho_{\mathrm{b}}\left(\boldsymbol{r}_{\mathrm{c}}\right)>$ 0 regular CS (r-CS) interactions, which clearly distinguishes
these interactions from the p-CS interactions. The signs of $\nabla^{2} \rho_{\mathrm{b}}\left(\boldsymbol{r}_{\mathrm{c}}\right)$ can be replaced by those of $H_{\mathrm{b}}\left(\boldsymbol{r}_{\mathrm{c}}\right)-V_{\mathrm{b}}\left(\boldsymbol{r}_{\mathrm{c}}\right) / 2$ because $\left(\hbar^{2} /\right.$ $8 m) \nabla^{2} \rho_{\mathrm{b}}\left(\boldsymbol{r}_{\mathrm{c}}\right)=H_{\mathrm{b}}\left(\boldsymbol{r}_{\mathrm{c}}\right)-V_{\mathrm{b}}\left(\boldsymbol{r}_{\mathrm{c}}\right) / 2$ (see eqn (2)). Again, the details are explained later.

Experimental chemists have recently used QTAIM to explain their results by considering chemical bonds and interactions. Indeed, $H_{\mathrm{b}}\left(\boldsymbol{r}_{\mathrm{c}}\right)-V_{\mathrm{b}}\left(\boldsymbol{r}_{\mathrm{c}}\right) / 2=0$ corresponds to the borderline between the classic covalent bonds of SS and the noncovalent interactions of CS, but $H_{\mathrm{b}}\left(\boldsymbol{r}_{\mathrm{c}}\right)=0$ appears to be buried in the noncovalent interactions of CS. As a result, it is difficult to characterize the CS interactions of van der Waals (vdW) interactions, typical hydrogen bonds ( $\mathrm{t}-\mathrm{HBs}$ ), interactions in molecular complexes formed through charge transfer (CT-MCs), trihalide ions $\left(\mathrm{X}_{3}{ }^{-}\right)$, and interactions in trigonal bipyramidal adducts formed through CT (CT-TBPs), based on the signs of $H_{\mathrm{b}}\left(\boldsymbol{r}_{\mathrm{c}}\right)-V_{\mathrm{b}}\left(\boldsymbol{r}_{\mathrm{c}}\right) / 2$ and/or $H_{\mathrm{b}}\left(\boldsymbol{r}_{\mathrm{c}}\right)$. How can such CS interactions be classified and characterized effectively? It is essential for experimental chemists.

We proposed QTAIM dual functional analysis (QTAIM-DFA), based on the QTAIM approach, to classify and characterize the various CS interactions more effectively. ${ }^{31-36}$ QTAIM-DFA is very useful for experimental chemists to analyse their own chemical bond and interaction results based on their own expectations. In QTAIM-DFA, $H_{\mathrm{b}}\left(\boldsymbol{r}_{\mathrm{c}}\right)$ are plotted versus $H_{\mathrm{b}}\left(\boldsymbol{r}_{\mathrm{c}}\right)-V_{\mathrm{b}}\left(\boldsymbol{r}_{\mathrm{c}}\right) / 2$ at BCPs, which incorporates the classification of interactions by the signs of $\nabla^{2} \rho_{\mathrm{b}}\left(\boldsymbol{r}_{\mathrm{c}}\right)\left[=\left(8 m / \hbar^{2}\right)\left(H_{\mathrm{b}}\left(\boldsymbol{r}_{\mathrm{c}}\right)-V_{\mathrm{b}}\left(\boldsymbol{r}_{\mathrm{c}}\right) / 2\right)\right]$ and $H_{\mathrm{b}}\left(\boldsymbol{r}_{\mathrm{c}}\right)$ (see, eqn (2)). In this treatment, both axes of the plot are given by the common unit of energy. As a result, four-function calculations can be applied to analyze the plot, which leads to the analysis of the interactions in a unified form.

The signs of the first derivatives of $H_{\mathrm{b}}\left(\boldsymbol{r}_{\mathrm{c}}\right)-V_{\mathrm{b}}\left(\boldsymbol{r}_{\mathrm{c}}\right) / 2$ and $H_{\mathrm{b}}\left(\boldsymbol{r}_{\mathrm{c}}\right)\left(\mathrm{d}\left(H_{\mathrm{b}}\left(\boldsymbol{r}_{\mathrm{c}}\right)-V_{\mathrm{b}}\left(\boldsymbol{r}_{\mathrm{c}}\right) / 2\right) / \mathrm{d} r\right.$ and $\mathrm{d} H_{\mathrm{b}}\left(\boldsymbol{r}_{\mathrm{c}}\right) / \mathrm{d} r$, respectively, where $r$ is the HB distance) are used to characterize CS interactions in QTAIM-DFA, in addition to those of $H_{\mathrm{b}}\left(\boldsymbol{r}_{\mathrm{c}}\right)-V_{\mathrm{b}}\left(\boldsymbol{r}_{\mathrm{c}}\right) / 2$ and $H_{\mathrm{b}}\left(\boldsymbol{r}_{\mathrm{c}}\right)$. In our treatment, data from the perturbed structures around the fully optimized structures are employed, in addition to those from the fully optimized structures. Data from the fully optimized structure are analyzed using polar coordinate $(R, \theta)$ representation, which corresponds to the static nature of interactions. ${ }^{32-37}$ Each interaction plot, containing data from both the perturbed and fully optimized structures, includes a specific curve that provides important information about the interaction. This plot is expressed by $\left(\theta_{\mathrm{p}}, \kappa_{\mathrm{p}}\right)$, where $\theta_{\mathrm{p}}$ corresponds to the tangent line of the plot and $\kappa_{\mathrm{p}}$ is the curvature. $\theta$ and $\theta_{\mathrm{p}}$ are measured from the $y$-axis and the $y$-direction, respectively. The concept of the dynamic nature of interactions has been proposed based on $\left(\theta_{\mathrm{p}}, \kappa_{\mathrm{p}}\right) .^{32-37}$ We call $(R, \theta)$ and $\left(\theta_{\mathrm{p}}\right.$, $\kappa_{\mathrm{p}}$ ) QTAIM-DFA parameters (see Fig. 4 for the definition, as illustratively exemplified by $\mathrm{NH}^{*}-\mathrm{N}$ of $\mathrm{A}-\mathrm{A}$ ).

How can the perturbed structures for effective analysis with QTAIM-DFA be generated? Accordingly, we very recently proposed a highly reliable method to the generate perturbed structures for QTAIM-DFA. ${ }^{38}$ The method, which is called CIV, employs the coordinates derived from the compliance constants $C_{i i}$ for internal vibrations. ${ }^{39,40}$ Eqn (3) defines $C_{i j}$ as the partial second derivative of the potential energy due to an external force, where $i$ and $j$ refer to internal coordinates and the
external force components acting on the system $f_{i}$ and $f_{j}$ correspond to $i$ and $j$, respectively. ${ }^{39}$ While the off-diagonal elements $C_{i j}(i \neq j)$ in eqn (3) correspond to the compliance coupling constants, the diagonal elements $C_{i i}$ represent the compliance constants for an internal coordinate $i$.

$$
\begin{equation*}
C_{i j}=\partial^{2} E / \partial f_{i} \partial f_{j} \tag{3}
\end{equation*}
$$

The $C_{i j}$ value given in eqn (3) corresponds to a lower numerical value ( $i$ ) of a compliance constant representing a stronger bond ( $j$ ); that is, $C_{i j}$ measures the flexibility (or compliance) of a particular bond. The applications of CIV to CS interactions are substantially more effective than those to SS interactions in QTAIM-DFA. ${ }^{38}$ The $C_{i i}$ values and the coordinates corresponding to $C_{i i}\left(\boldsymbol{C}_{i}\right)$ were calculated using the Compliance 3.0.2 program, released by Grunenberg and Brandhorst. ${ }^{4-43}$ The dynamic nature of interactions based on the perturbed structures with CIV is described as the "intrinsic dynamic nature of interactions" since the coordinates are invariant to the choice of coordinate system. The mechanism for the formation the $\mathrm{Nu}-$ $\mathrm{Nu}^{\prime}$ pairs will also be clarified in more detail based on the $C_{i i}$ parameters.

QTAIM-DFA is applied to standard interactions, and rough criteria that distinguish the interaction in question from others are obtained. QTAIM-DFA has excellent potential for evaluating, classifying, characterizing, and understanding weak to strong interactions according to a unified form. ${ }^{32-37}$ QTAIM-DFA and the criteria are explained in the Appendix of the ESI using Schemes SA1-SA3, Fig. SA1, SA2, Table SA1 and eqn (SA1)(SA7). $\dagger$ The basic concept of the QTAIM approach is also explained.

Indeed, the understanding of HBs has been considerably growing recently, ${ }^{1-9,44,45}$ but evaluating, characterizing, and understanding the nature of each HB in multi-HBs, especially in nucleobase pairs, is inevitably needed to obtain a better understanding of DNA processes. How can the dynamic and static nature of each HB in the multi-HBs between $\mathrm{Nu}-\mathrm{Nu}^{\prime}$, where the multi-HBs are formed in close proximity in space and interact mutually and strongly with each other, be clarified? Grunenberg and Brandhorst calculated the strength of each HB of the multi-HBs in the A-T and C-G pairs by applying the compliance constants. ${ }^{42,46}$ The elucidation of the intrinsic dynamic and static nature of each HB in multi-HBs, exemplified by the acetic acid dimer and derivatives, was attempted by employing the perturbed structures generated with CIV to examine the effective applicability of QTAIM-DFA to the system. ${ }^{47}$

We consider QTAIM-DFA to be well suited to elucidate the nature of each HB in the multi-HBs between $\mathrm{Nu}-\mathrm{Nu}^{\prime}$ by employing the perturbed structures generated with CIV, with above discussion in mind. The method enables us to classify and characterize the nature of the interaction and the results will be very useful when experimental chemists analyze their own chemical bond and interaction results based on their own expectations. This is another purpose of this work. Weak interactions in $\mathrm{Nu}-\mathrm{Nu}^{\prime}$ may sometimes be called HBs in this paper, even if they should be assigned to other categories of
interactions. Herein, we present the results of investigations on the intrinsic dynamic and static nature of each HB in the multiHBs between nucleobase pairs. Each HB interaction in $\mathrm{Nu}-\mathrm{Nu}^{\prime}$ can be classified and characterized effectively with QTAIM-DFA, employing the perturbed structures generated with CIV. The criteria are employed in this process as reference. The behavior of the pairs is also discussed based on the nature.

## Methodological details in calculations

Calculations were performed employing the Gaussian 09 program package. ${ }^{48}$ Table 1 summarizes the basis set systems (BSSs) used in this paper. The Møller-Plesset second-order energy correlation (MP2) level ${ }^{49}$ was applied for the calculations, together with the DFT level of M06-2X. ${ }^{50,51}$ It was reported that MP2/BSS-A, MP2/BSS-A', MP2/BSS-B'a, and MP2/BSS-B'b gave excellent results of very similar quality in the evaluation of the nature of each HB in the acetic acid dimer and related species. ${ }^{47}$ Optimized structures were confirmed by all real frequencies in the possible cases. The reliability of the structures optimized with MP2/BSS-B'a was also examined by comparison with structures optimized with MP2/BSS-A'. The results of the frequency analysis were used to obtain the $C_{i i}$ values and the coordinates corresponding to $C_{i i}\left(\boldsymbol{C}_{i}\right)$. The M062X level was also employed, if necessary. The results obtained with MP2/BSS-B'a are mainly discussed in the text, while the results obtained with the other BSSs are mainly in the ESI. $\dagger$ BSSD or lower basis sets were employed for pre-optimizations.

Eqn (4) explains the method used to generate the perturbed structures with CIV. The $i$-th perturbed structure in question $\left(\boldsymbol{S}_{i w}\right)$ is generated by the addition of the coordinates $\boldsymbol{C}_{i}$, derived from $C_{i i}$, to the standard orientation of a fully optimized structure $\left(\boldsymbol{S}_{\mathrm{o}}\right)$ in the matrix representation. The coefficient $g_{i w}$ in eqn (4) controls the structural difference between $\boldsymbol{S}_{i w}$ and $\boldsymbol{S}_{\mathrm{o}}$ : $g_{i w}$ is determined to satisfy eqn (5) for $r$, where $r$ and $r_{\mathrm{o}}$ stand for the HB distances in the perturbed and fully optimized structures, respectively, and $a_{\mathrm{o}}$ is the Bohr radius $\left(0.52918 \AA\right.$ ). The $\boldsymbol{C}_{i}$ values of five digits are used to predict $\boldsymbol{S}_{i w}$.

The perturbed structures were also generated by the partial optimization method (POM) ${ }^{31,33}$ of the $Z$-matrix and/or ModRedundant types, ${ }^{34}$ where the HB distances in question $(r)$ in the perturbed structures were fixed to satisfy eqn (5).

$$
\begin{gather*}
\boldsymbol{S}_{i w}=\boldsymbol{S}_{\mathrm{o}}+g_{i w} \cdot \boldsymbol{C}_{i}  \tag{4}\\
r=r_{\mathrm{o}}+w a_{\mathrm{o}}\left(w=(0), \pm 0.025 \text { and } \pm 0.05 ; a_{\mathrm{o}}=0.52918 \AA\right)  \tag{5}\\
y=c_{\mathrm{o}}+c_{1} x+c_{2} x^{2}+c_{3} x^{3} \tag{6}
\end{gather*}
$$

QTAIM functions were calculated using the same method as in the optimizations, unless otherwise noted, and were analyzed with the AIM2000 ${ }^{52}$ and AIMAll ${ }^{53}$ programs. $H_{b}\left(\boldsymbol{r}_{\mathrm{c}}\right)$ is plotted versus $H_{\mathrm{b}}\left(\boldsymbol{r}_{\mathrm{c}}\right)-V_{\mathrm{b}}\left(\boldsymbol{r}_{\mathrm{c}}\right) / 2$ for five data points of $w=0, \pm 0.025$ and $\pm 0.05$ in eqn (5). Each plot is analyzed using a regression curve of the cubic function, as shown in eqn (6), where $(x, y)=\left(H_{b}\left(\boldsymbol{r}_{\mathrm{c}}\right)\right.$ $\left.-V_{\mathrm{b}}\left(\boldsymbol{r}_{\mathrm{c}}\right) / 2, H_{\mathrm{b}}\left(\boldsymbol{r}_{\mathrm{c}}\right)\right)\left(R_{\mathrm{c}}{ }^{2}\right.$ (square of the correlation coefficient) $>$ 0.99999 is typical). ${ }^{37}$

## Results and discussion

## Optimization of nucleobase pairs, $\mathbf{N u}-\mathbf{N u}^{\prime}$

The nucleobase pairs are optimized with various BSSs at the MP2 and M06-2X levels, where many results have been reported. ${ }^{54}$ The HB distances $(r(\mathrm{H}, \mathrm{B}) ; r)$ in $\mathrm{Nu}-\mathrm{Nu}^{\prime}$ optimized with MP2/BSS-A', MP2/BSS-B'a, MP2/BSS-B'b, M06-2X/BSS-A, M06-

2X/BSS-C and M06-2X/BSS-D are collected in Table S1 of the ESI. $\dagger$ Energies for the formation of $\mathrm{Nu}-\mathrm{Nu}^{\prime}$ from the components $\left(\mathrm{Nu}\right.$ and $\left.\mathrm{Nu}^{\prime}\right) \Delta E\left[=E\left(\mathrm{Nu}-\mathrm{Nu}^{\prime}\right)-\left(E(\mathrm{Nu})+E\left(\mathrm{Nu}^{\prime}\right)\right)\right]$ are evaluated with various methods. The $\Delta E_{\mathrm{ES}}$ and $\Delta E_{\mathrm{ZP}}$ values correspond to those on the energy surface and those containing the zero-point corrections, respectively. The values evaluated with MP2/BSS-B'a, MP2/BSS-B'b, M06-2X/BSS-A, M06-2X/BSS-C,

Table 2 QTAIM functions and QTAIM-DFA parameters for each hydrogen bond in nucleobase pairs evaluated with MP2/BSS-B'a ${ }^{a}$

| AH-*-B in $\mathrm{Nu}-\mathrm{Nu}{ }^{b}$ (symmetry: no. ${ }^{\text {. }}$ ) | $\begin{aligned} & \rho_{\mathrm{b}}\left(\boldsymbol{r}_{\mathrm{c}}\right) \\ & \left(e a_{\mathrm{o}}^{-3}\right) \end{aligned}$ | $\begin{aligned} & c \nabla^{2} \rho_{\mathrm{b}}\left(\boldsymbol{r}_{\mathrm{c}}\right)^{d} \\ & (\mathrm{au}) \end{aligned}$ | $H_{\mathrm{b}}\left(\boldsymbol{r}_{\mathrm{c}}\right)$ <br> (au) | $R^{e}$ <br> (au) | $\begin{aligned} & \theta^{f} \\ & \left({ }^{\circ}\right) \end{aligned}$ | $\begin{aligned} & C_{i i}^{g} \\ & \left(\AA \mathrm{Adyn}^{-1}\right) \end{aligned}$ | $\begin{aligned} & \theta_{\mathrm{p}}^{h} \\ & \left({ }^{\circ}\right) \end{aligned}$ | $\begin{aligned} & \kappa_{\mathrm{p}}^{i} \\ & \left(\mathrm{au}^{-1}\right) \end{aligned}$ | Predicted nature |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathrm{N}-*-\mathrm{HN}$ in A-T ( $\left.C_{1}: 1\right)$ | 0.0498 | 0.0094 | -0.0143 | 0.0171 | 146.7 | 3.12 | 182.4 | 8.5 | $\begin{aligned} & \text { r-CS/CT- } \\ & \text { TBP } \end{aligned}$ |
| NH-*-O in A-T ( $\left.C_{1}: 2\right)$ | 0.0291 | 0.0114 | -0.0012 | 0.0115 | 95.9 | 5.78 | 145.4 | 115.3 | $\mathrm{r}-\mathrm{CS} / \mathrm{t}-\mathrm{HB}_{\mathrm{wc}}$ |
| CH-*-O in $\mathrm{A}-\mathrm{T}\left(C_{1}: 3\right)$ | 0.0059 | 0.0025 | 0.0007 | 0.0026 | 74.5 | 16.31 | 80.6 | 64.9 | p-CS/vdW |
| N-*-HN in A-T ( $\left.C_{s}: 4\right)$ | 0.0498 | 0.0094 | -0.0143 | 0.0171 | 146.7 | 3.12 | 182.4 | 8.5 | $\begin{aligned} & \text { r-CS/CT- } \\ & \text { TBP } \end{aligned}$ |
| NH-*-O in $\mathrm{A}-\mathrm{T}\left(C_{s}: 5\right)$ | 0.0291 | 0.0114 | -0.0012 | 0.0115 | 95.9 | 5.78 | 145.4 | 115.2 | $\mathrm{r}-\mathrm{CS} / \mathrm{t}-\mathrm{HB}_{\mathrm{wc}}$ |
| CH-*-O in A-T ( $C_{s}: 6$ ) | 0.0059 | 0.0025 | 0.0007 | 0.0026 | 74.5 | 16.30 | 80.6 | 64.9 | p-CS/vdW |
| NH-*-O in C-G ( $C_{1}: 7$ ) | 0.0449 | 0.0134 | -0.0096 | 0.0165 | 125.5 | 3.20 | 169.9 | 11.4 | r-CS/CT-MC |
| $\mathrm{N}-*-\mathrm{HN}$ in $\mathrm{C}-\mathrm{G}\left(C_{1}: 8\right)$ | 0.0377 | 0.0099 | -0.0062 | 0.0117 | 122.2 | 2.15 | 175.0 | 30.8 | r-CS/CT-MC |
| O-*-HN in C-G ( $C_{1}: 9$ ) | 0.0305 | 0.0118 | -0.0017 | 0.0119 | 98.2 | 4.08 | 148.3 | 101.3 | $\mathrm{r}-\mathrm{CS} / \mathrm{t}-\mathrm{HB}_{\mathrm{wc}}$ |
| NH-*-N in A-A ( $\left.C_{1}: 10\right)$ | 0.0289 | 0.0093 | -0.0018 | 0.0095 | 100.9 | 5.74 | 158.6 | 99.6 | r-CS/CT-MC |
| $\mathrm{N}-*-\mathrm{HC}$ in $\mathrm{A}-\mathrm{A}\left(C_{1}: 11\right)$ | 0.0119 | 0.0045 | 0.0013 | 0.0047 | 74.1 | 17.10 | 75.6 | 55.8 | p-CS/vdW |
| $\mathrm{N}-*-\mathrm{HN}$ in A-C $\left(C_{1}: 12\right)$ | 0.0391 | 0.0101 | -0.0071 | 0.0123 | 125.1 | 3.70 | 174.1 | 23.3 | r-CS/CT-MC |
| NH-*-O in A-C ( $\left.C_{1}: 13\right)$ | 0.0364 | 0.0135 | -0.0042 | 0.0141 | 107.5 | 3.72 | 158.2 | 40.2 | r-CS/CT-MC |
| $\mathrm{N}-*-\mathrm{HN}$ in A-G ( $\left.C_{1}: 14\right)$ | 0.0424 | 0.0098 | -0.0091 | 0.0133 | 132.8 | 3.52 | 178.6 | 22.5 | r-CS/CT-MC |
| NH-*-O in A-G ( $\left.C_{1}: 15\right)$ | 0.0361 | 0.0125 | -0.0044 | 0.0133 | 109.5 | 4.45 | 161.0 | 45.0 | r-CS/CT-MC |
| CH-*-HN in A-G ( $C_{1}: 16$ ) | 0.0056 | 0.0026 | 0.0009 | 0.0027 | 71.1 | 29.31 | 78.8 | 111.5 | p-CS/vdW |
| $\mathrm{N}-*-\mathrm{HN}$ in A-U $\left(C_{1}: 17\right)$ | 0.0500 | 0.0093 | -0.0145 | 0.0172 | 147.2 | 3.10 | 182.6 | 8.2 | $\begin{aligned} & \text { r-CS/CT- } \\ & \text { TBP } \end{aligned}$ |
| NH-*-O in A-U ( $\left.C_{1}: 18\right)$ | 0.0289 | 0.0114 | -0.0011 | 0.0115 | 95.5 | 5.79 | 141.5 | 117.4 | $\mathrm{r}-\mathrm{CS} / \mathrm{t}-\mathrm{HB}_{\mathrm{wc}}$ |
| CH-*-O in A-U ( $\left.C_{1}: 19\right)$ | 0.0060 | 0.0025 | 0.0007 | 0.0026 | 74.5 | 16.06 | 80.3 | 77.2 | p-CS/vdW |
| $\mathrm{N}-*-\mathrm{HN}$ in $\mathrm{C}-\mathrm{C}\left(C_{1}: 20\right)$ | 0.0488 | 0.0099 | -0.0134 | 0.0167 | 143.5 | 2.63 | 180.6 | 2.3 | $\begin{aligned} & \text { r-CS/CT- } \\ & \text { TBP } \end{aligned}$ |
| NH-*-O in $\mathrm{C}-\mathrm{C}\left(C_{1}: 21\right)$ | 0.0421 | 0.0131 | -0.0079 | 0.0153 | 121.0 | 3.86 | 168.1 | 17.8 | r-CS/CT-MC |
| O-*-HC in $\mathrm{C}-\mathrm{C}\left(C_{1}: 22\right)$ | 0.0050 | 0.0021 | 0.0006 | 0.0022 | 73.0 | 14.60 | 82.4 | 61.9 | p-CS/vdW |
| $\mathrm{N}-*-\mathrm{HN}$ in C-T ( $\left.C_{1}: 23\right)$ | 0.0406 | 0.0096 | -0.0083 | 0.0127 | 130.7 | 4.77 | 178.1 | 24.7 | r-CS/CT-MC |
| NH-*-O in C-T ( $C_{1}: 24$ ) | 0.0348 | 0.0125 | -0.0037 | 0.0130 | 106.5 | 4.81 | 158.6 | 55.6 | r-CS/CT-MC |
| $\mathrm{O}^{*}-\mathrm{O}$ in $\mathrm{C}-\mathrm{T}\left(C_{1}: 25\right)$ | 0.0026 | 0.0013 | 0.0006 | 0.0014 | 67.2 | 32.13 | 86.3 | 344.8 | p-CS/vdW |
| $\mathrm{N}-*-\mathrm{HN}$ in $\mathrm{C}-\mathrm{U}\left(C_{1}: 26\right)$ | 0.0410 | 0.0096 | -0.0085 | 0.0129 | 131.6 | 4.73 | 178.5 | 23.5 | r-CS/CT-MC |
| NH-*-O in C-U ( $C_{1}: 27$ ) | 0.0347 | 0.0125 | -0.0036 | 0.0130 | 106.2 | 4.79 | 158.4 | 55.9 | r-CS/CT-MC |
| O-*-O in C-U ( $C_{1}: 28$ ) | 0.0028 | 0.0014 | 0.0006 | 0.0015 | 67.9 | 30.90 | 87.4 | 325.6 | p-CS/vdW |
| NH-*-O G-G ( $C_{i}: 29$ ) | 0.0500 | 0.0136 | -0.0124 | 0.0184 | 132.4 | 2.86 | 172.4 | 7.6 | r-CS/CT-MC |
| O-*-HN G-G ( $C_{i}: 30$ ) | 0.0083 | 0.0044 | 0.0015 | 0.0046 | 71.6 | 12.98 | 73.0 | 10.5 | p-CS/vdW |
| $\mathrm{N}-*-\mathrm{HN}$ in G-T ( $\left.C_{1}: 31\right)$ | 0.0416 | 0.0100 | -0.0087 | 0.0133 | 130.8 | 3.90 | 177.1 | 19.7 | r-CS/CT-MC |
| NH-*-O in G-T ( $\left.C_{1}: 32\right)$ | 0.0335 | 0.0123 | -0.0030 | 0.0127 | 103.7 | 4.92 | 155.6 | 68.9 | r-CS/CT-MC |
| NH-*-O in G-U ( $C_{1}: 33$ ) | 0.0419 | 0.0138 | -0.0072 | 0.0155 | 117.5 | 3.09 | 165.7 | 21.7 | r-CS/CT-MC |
| O-*-HN in G-U ( $C_{1}: 34$ ) | 0.0404 | 0.0127 | -0.0070 | 0.0145 | 118.8 | 4.32 | 167.5 | 22.2 | r-CS/CT-MC |
| NH-*-O in T-T ( $\left.C_{1}: 35\right)$ | 0.0375 | 0.0129 | -0.0051 | 0.0139 | 111.4 | 4.29 | 163.5 | 34.1 | r-CS/CT-MC |
| O-*-HN in T-T ( $\left.C_{1}: 36\right)$ | 0.0375 | 0.0129 | -0.0051 | 0.0139 | 111.4 | 4.29 | 163.5 | 34.1 | r-CS/CT-MC |
| NH-*-O in T-T ( $\left.C_{i}: 37\right)$ | 0.0375 | 0.0129 | -0.0051 | 0.0139 | 111.4 | 4.29 | 163.5 | 34.1 | r-CS/CT-MC |
| NH-*-O in T-U ( $\left.C_{1}: 38\right)$ | 0.0381 | 0.0130 | -0.0054 | 0.0141 | 112.6 | 4.17 | 164.2 | 30.9 | r-CS/CT-MC |
| O-*-HN in T-U ( $\left.C_{1}: 39\right)$ | 0.0366 | 0.0128 | -0.0046 | 0.0136 | 109.8 | 4.42 | 162.5 | 38.2 | r-CS/CT-MC |
| NH-*-O in U-U ( $\left.C_{1}: 40\right)$ | 0.0373 | 0.0129 | -0.0050 | 0.0138 | 111.1 | 4.29 | 163.3 | 34.5 | r-CS/CT-MC |
| O-*-HN in U-U ( $C_{1}: 41$ ) | 0.0373 | 0.0129 | -0.0050 | 0.0138 | 111.1 | 4.29 | 163.3 | 34.5 | r-CS/CT-MC |
| NH-*-O in U-U ( $\left.C_{s}: 42\right)$ | 0.0373 | 0.0129 | -0.0050 | 0.0138 | 111.1 | 4.29 | 163.3 | 34.5 | r-CS/CT-MC |
| O-*-HN in U-U ( $C_{s}: 43$ ) | 0.0373 | 0.0129 | -0.0050 | 0.0138 | 111.1 | 4.29 | 163.3 | 33.6 | r-CS/CT-MC |
| ${ }^{a}$ See Table 1 for BSS-B'a. ${ }^{b}$ Data are given at the BCPs. ${ }^{c}$ Numbers given for the interactions are the same as those in Fig. 2 and $4 .{ }^{d} c \nabla^{2} \rho_{\mathrm{b}}\left(\boldsymbol{r}_{\mathrm{c}}\right)=H_{\mathrm{b}}\left(\boldsymbol{r}_{\mathrm{c}}\right)$ <br> $-V_{\mathrm{b}}\left(\boldsymbol{r}_{\mathrm{c}}\right) / 2$, where $c=\hbar^{2} / 8 m .{ }^{e} R=\left(x^{2}+y^{2}\right)^{1 / 2}$, where $(x, y)=\left(H_{\mathrm{b}}\left(\boldsymbol{r}_{\mathrm{c}}\right)-V_{\mathrm{b}}\left(\boldsymbol{r}_{\mathrm{c}}\right) / 2, H_{\mathrm{b}}\left(\boldsymbol{r}_{\mathrm{c}}\right)\right) .{ }^{f} \theta=90^{\circ}-\tan ^{-1}(y / x) .{ }^{g}$ Defined in eqn $(3)$ in the text. ${ }^{h} \theta_{\mathrm{p}}=90^{\circ}$ <br> $-\tan ^{-1}(\mathrm{~d} y / \mathrm{d} x) .{ }^{i} \kappa_{\mathrm{p}}=\left\|\mathrm{d}^{2} y / \mathrm{d} x^{2}\right\| /\left[1+(\mathrm{d} y / \mathrm{d} x)^{2}\right]^{3 / 2}$. |  |  |  |  |  |  |  |  |  |



Fig. 2 Molecular graphs for nucleobase pairs ( $\mathrm{Nu}-\mathrm{Nu} u^{\prime}$ ), with the contour plots of $\rho(r)$, evaluated with MP2/BSS-B'a. The numbers for the bonds are the same as those in Fig. 4 and Table 2. Bond critical points (BCPs) are denoted by red dots, ring critical points (RCPs) are denoted by yellow dots and bond paths (BPs) are denoted by pink lines. Oxygen, nitrogen, carbon and hydrogen atoms are in red, blue, black and gray, respectively. Contour plots are drawn on the planes containing at least one side of the HB interaction. The contours ( $e a_{0}{ }^{-3}$ ) are at $2^{\prime}(l= \pm 8, \pm 7, \ldots$ and 0 ).
and M06-2X/BSS-D are collected in Table S2 of the ESI, $\dagger$ which also contains the $\Delta E_{\text {ES }}$ values evaluated with MP2/BSS-A'. The results for the $C_{1}$ structures of $\mathrm{Nu}-\mathrm{Nu}^{\prime}\left(\mathrm{Nu}-\mathrm{Nu}^{\prime}\left(C_{1}\right)\right)$ are mainly employed for the discussion, and the results for A-T $\left(C_{s}\right), \mathrm{T}-\mathrm{T}$ $\left(C_{i}\right)$ and $\mathrm{U}-\mathrm{U}\left(C_{s}\right)$ are essentially the same as those for the corresponding $C_{1}$ pairs. In the case of $\mathrm{G}-\mathrm{G}$, it is optimized as the $C_{i}$ structure (see Table 2). The optimized structures are not shown in the figures, but they can be found in the molecular graphs drawn on the structures optimized with MP2/BSS-B'a (see Fig. 2).

The $r(\mathrm{H}, \mathrm{B})$ values in $\mathrm{Nu}-\mathrm{Nu}^{\prime}$ evaluated with the various methods are plotted versus those evaluated with MP2/BSS-A'. The plot is shown in Fig. S1 of the ESI. $\dagger$ This plot gave very good correlations, as shown in the figure. The high similarities in $r(\mathrm{H}$, B) correspond to the high similarities of the structures of $\mathrm{Nu}-$ $\mathrm{Nu}^{\prime}$ optimized with the methods employed in the calculations. ${ }^{43,46,55}$ The similarities are excellent, especially for the structures optimized with MP2/BSS-B'a, MP2/BSS-B'b and MP2/ BSS-A ${ }^{\prime}$, although frequency analysis could not be performed on those with MP2/BSS-A'.

Molecular graphs with contour plots of $\rho(r)$ and negative Laplacians around HBs in $\mathrm{Nu}-\mathrm{Nu}^{\prime}$
Fig. 2 illustrates the molecular graphs with the contour plots of $\rho(\boldsymbol{r})$ for $\mathrm{Nu}-\mathrm{Nu}^{\prime}$ drawn on the structures optimized with MP2/


Fig. 3 Negative Laplacians of $\rho(r)$ for the A-T and C-G pairs, calculated with MP2/BSS-B'a. Positive and negative areas are in blue and red lines, respectively

BSS-B'a. As expected, a BP with a BCP is clearly detected for each HB. These BCPs seem well located at the (three-dimensional) saddle points of $\rho(\boldsymbol{r})$. Fig. 3 shows the negative Laplacians of $\rho(r)$, exemplified by A-T and C-G. All BCPs for HBs exist in the blue area (the outsides of the red area) in the figure, which means that the HBs in A-T and C-G are all classified as CS interactions.

For example, the $\Delta E_{\mathrm{ES}}$ values evaluated with MP2/BSS-B'a are $-70.3,-70.6$ and $-123.5 \mathrm{~kJ} \mathrm{~mol}^{-1}$ for A-T, A-U and C-G, respectively. The values for A-T and A-U are very close to each other due to their structural similarity. ${ }^{13}$ The $\Delta E_{\mathrm{ZP}}$ values are plotted versus the $\Delta E_{\text {ES }}$ values calculated with MP2/BSS-B'a. The plot, which is shown in Fig. S2 of the ESI, $\dagger$ gives a very good correlation $\left(\Delta E_{\mathrm{ZP}}=0.968 \Delta E_{\mathrm{ES}}+1.80:{R_{\mathrm{c}}}^{2}=0.9993\right)$. Therefore, either $\Delta E_{\mathrm{ES}}$ or $\Delta E_{\mathrm{ZP}}$ can be employed for the discussion of the energy terms.

The nature of each HB in multi-HBs of $\mathrm{Nu}-\mathrm{Nu}^{\prime}$ will be clarified by employing QTAIM-DFA.

## Survey of the $\mathbf{H B}$ interactions in $\mathbf{N u}-\mathbf{N u}^{\prime}$

The HB interactions in $\mathrm{Nu}-\mathrm{Nu}^{\prime}$ seem straight, considering the BPs corresponding to the HBs shown in Fig. 2. To examine the linearity of the HB interactions further, the lengths of the BPs $\left(r_{\mathrm{BP}}\right)$ in question and the corresponding straight-line distances


Fig. 4 Plots of $H_{b}\left(r_{\mathrm{c}}\right)$ versus $H_{b}\left(r_{\mathrm{c}}\right)-V_{\mathrm{b}}\left(r_{\mathrm{c}}\right) / 2$ for each HB in $\mathrm{Nu}-\mathrm{Nu}^{\prime}$, evaluated with MP2/BSS-B'a. For the whole picture (a) and the magnified image for the pure CS region (b). The numbers for the interactions are the same as those in Fig. 2 and Table 2, respectively. Two streams appear in the plots of (a) by $\mathrm{NH}-{ }^{*}-\mathrm{N}$ and $\mathrm{NH}-{ }^{*}-\mathrm{O}$, which are shown by the solid and hollow marks, respectively. The definitions of $(R, \theta)$ and $\left(\theta_{\mathrm{p}}, \kappa_{\mathrm{p}}\right)$ are also illustrated.
( $R_{\mathrm{SL}}$ ) are calculated for $\mathrm{Nu}-\mathrm{Nu}^{\prime}$. The values evaluated with MP2/ BSS-B'a are collected in Table S3 of the ESI, $\dagger$ together with the differences between them $\left(\Delta r_{\mathrm{BP}}=r_{\mathrm{BP}}-R_{\mathrm{SL}}\right)$. The magnitudes of $\Delta r_{\mathrm{BP}}$ are less than $0.072 \AA$ for the BPs; therefore, all HBs in $\mathrm{Nu}-$ $\mathrm{Nu}^{\prime}$ can be approximated by straight lines, except for $\mathrm{CH}-*-\mathrm{HN}$ in A-G ( $\Delta r_{\mathrm{BP}}: 0.1972 \AA$ ).

QTAIM functions are calculated at each BCP on the BP corresponding to each HB in $\mathrm{Nu}-\mathrm{Nu}^{\prime}$. Table 2 collects the $\rho_{\mathrm{b}}\left(\boldsymbol{r}_{\mathrm{c}}\right)$, $H_{\mathrm{b}}\left(\boldsymbol{r}_{\mathrm{c}}\right)-V_{\mathrm{b}}\left(\boldsymbol{r}_{\mathrm{c}}\right) / 2\left(=\left(\hbar^{2} / 8 m\right) \nabla^{2} \rho_{\mathrm{b}}\left(\boldsymbol{r}_{\mathrm{c}}\right)\right)$ and $H_{\mathrm{b}}\left(\boldsymbol{r}_{\mathrm{c}}\right)$ values evaluated with MP2/BSS-B'a, ${ }^{46}$ where each HB in a nucleobase pair is numbered in the order of decreasing $\rho_{\mathrm{b}}\left(\boldsymbol{r}_{\mathrm{c}}\right)$ values. $H_{\mathrm{b}}\left(\boldsymbol{r}_{\mathrm{c}}\right)$ is plotted versus $H_{\mathrm{b}}\left(\boldsymbol{r}_{\mathrm{c}}\right)-V_{\mathrm{b}}\left(\boldsymbol{r}_{\mathrm{c}}\right) / 2$ for the data shown in Table 2, together with those data from the perturbed structures generated with CIV. Fig. 4 illustrates the plots. Fig. 4 a shows the whole picture of the plots, and Fig. 4b presents the magnified plots that appeared in the p-CS region of $H_{\mathrm{b}}\left(\boldsymbol{r}_{\mathrm{c}}\right)-V_{\mathrm{b}}\left(\boldsymbol{r}_{\mathrm{c}}\right) / 2>$ 0 and $H_{\mathrm{b}}\left(r_{\mathrm{c}}\right)>0$. The data (points) in Fig. 4 are divided into three groups: (a) $\mathrm{NH}^{*}-\mathrm{N}$ appeared in the r-CS region of $H_{\mathrm{b}}\left(\boldsymbol{r}_{\mathrm{c}}\right)-$ $V_{\mathrm{b}}\left(\boldsymbol{r}_{\mathrm{c}}\right) / 2>0$ and $H_{\mathrm{b}}\left(\boldsymbol{r}_{\mathrm{c}}\right)<0$, (b) NH-*-O appeared in the r-CS region and (c) very weak $\mathrm{O}^{-*}-\mathrm{O}$ and $\mathrm{CH}-{ }^{*}-\mathrm{X}(\mathrm{X}=\mathrm{O}, \mathrm{N}$ and HN ) interactions appeared in the p-CS region, where the weaker NH-*-O interaction in G-G (30) is also contained. The three groups are called $G(\mathbf{A}), G(B)$ and $G(\mathbf{C})$, respectively, here. Relative to those from $G(\mathbf{B})$, data from $G(\mathbf{A})$ appear more on the left and lower sides overall. The results would show that interactions in $G(\mathbf{A})$ are stronger than those corresponding to $G(\mathbf{B})$ as a whole. As shown later, interactions in $G(\mathbf{C})$ are predicted to have the vdW nature. The QTAIM-DFA parameters of $(R, \theta)$ and $\left(\theta_{\mathrm{p}}, \kappa_{\mathrm{p}}\right)$ are calculated for each HB in $\mathrm{Nu}-\mathrm{Nu}^{\prime}$ by analyzing each plot shown in Fig. 4, according to eqn (SA3)-(SA6) of the ESI. $\dagger$ The ( $\theta_{\mathrm{p}}, \kappa_{\mathrm{p}}$ ) values calculated with CIV should be denoted by ( $\theta_{\mathrm{p}: \text { CIV }}, \kappa_{\mathrm{p}: \text { CIV }}$ ); however, we will use $\left(\theta_{\mathrm{p}}, \kappa_{\mathrm{p}}\right)$ in place of $\left(\theta_{\mathrm{p}: \text { CIV }}\right.$, $\left.\kappa_{\mathrm{p}: \text { CIV }}\right)$ for simplification of the notation. Table 2 collects the ( $R$, $\theta)$ and $\left(\theta_{\mathrm{p}}, \kappa_{\mathrm{p}}\right)$ values evaluated with MP2/BSS-B'a, together with the $C_{i i}$ values related to the perturbed structures. Similar results calculated with the various methods other than MP2/BSS-B'a are collected in Tables S4 and S5 of the ESI. $\dagger$

Each HB in $\mathrm{Nu}-\mathrm{Nu}^{\prime}$ is classified and characterized based on the ( $R, \theta, \theta_{\mathrm{p}}$ ) values evaluated with MP2/BSS-B'a. The results are discussed in the following.

## Nature of each $\mathbf{H B}$ in multi-HBs of $\mathbf{N u}-\mathbf{N u}^{\prime}$

It is instructive to survey the criteria shown in Scheme SA3 and Table SA1 of the ESI $\dagger$ before a detailed discussion. While $\theta$ classifies interactions, $\theta_{\mathrm{p}}$ characterizes them. The criteria tell us that $45^{\circ}<\theta<180^{\circ}\left(0<H_{\mathrm{b}}\left(\boldsymbol{r}_{\mathrm{c}}\right)-V_{\mathrm{b}}\left(\boldsymbol{r}_{\mathrm{c}}\right) / 2\right)$ for the CS interactions and that $180^{\circ}<\theta<206.6^{\circ}\left(H_{\mathrm{b}}\left(\boldsymbol{r}_{\mathrm{c}}\right)-V_{\mathrm{b}}\left(\boldsymbol{r}_{\mathrm{c}}\right) / 2<0\right)$ for the SS interactions. ${ }^{37}$ The CS interactions are subdivided into $45^{\circ}<\theta<$ $90^{\circ}\left(H_{\mathrm{b}}\left(\boldsymbol{r}_{\mathrm{c}}\right)>0\right)$ for the p-CS interactions and $90^{\circ}<\theta<180^{\circ}$ $\left(H_{\mathrm{b}}\left(\boldsymbol{r}_{\mathrm{c}}\right)<0\right)$ for the r-CS interactions. In the p-CS region of $45^{\circ}<\theta$ $<90^{\circ}$, the interactions will be characterized as the vdW type when $45^{\circ}<\theta_{\mathrm{p}}<90^{\circ}\left(45^{\circ}<\theta<75^{\circ}\right)$, whereas they will be considered typical hydrogen bonds ( $\mathrm{t}-\mathrm{HB}$ ) with no covalency ( $\mathrm{t}-$ $\left.\mathrm{HB}_{\mathrm{nc}}\right)$ when $90^{\circ}<\theta_{\mathrm{p}}<125^{\circ}\left(75^{\circ}<\theta<90^{\circ}\right)$, where $\theta=75^{\circ}$ and $\theta_{\mathrm{p}}$ $=125^{\circ}$ are tentatively given for $\theta_{\mathrm{p}}=90^{\circ}$ and $\theta=90^{\circ}$, respectively. The CT interactions appear in the r-CS region of $90^{\circ}<\theta<$
$180^{\circ}$. The interactions with the $\mathrm{t}-\mathrm{HB}_{\mathrm{wc}}$ ( $\mathrm{t}-\mathrm{HB}$ with covalency) nature appear in the range of $125^{\circ}<\theta_{\mathrm{p}}<150^{\circ}\left(90^{\circ}<\theta<115^{\circ}\right)$, where $\left(\theta, \theta_{\mathrm{p}}\right)=\left(115^{\circ}, 150^{\circ}\right)$ are tentatively given as the borderline between the $\mathrm{t}-\mathrm{HB}_{\mathrm{wc}}$ and CT-MC (molecular complex formation through CT) interactions. The borderline between CT-MC and CT-TBP (TBP adduct formation through CT) interaction types is defined by $\left(\theta, \theta_{\mathrm{p}}\right)=\left(150^{\circ}, 180^{\circ}\right)$, where $\theta=150^{\circ}$ is tentatively given corresponding to $\theta_{\mathrm{p}}=180^{\circ}$. As a result, the $(\theta$, $\left.\theta_{\mathrm{p}}\right)$ values of $\left(75^{\circ}, \mathbf{9 0}^{\circ}\right),\left(\mathbf{9 0}^{\circ}, 125^{\circ}\right),\left(115^{\circ}, \mathbf{1 5 0}^{\circ}\right),\left(150^{\circ}, \mathbf{1 8 0}^{\circ}\right)$ and $\left(\mathbf{1 8 0}^{\circ}, 190^{\circ}\right)$ correspond to the borderlines between $\mathrm{vdW} / \mathrm{t}-\mathrm{HB}_{\mathrm{nc}}$, $\mathrm{t}-\mathrm{HB}_{\mathrm{nc}} / \mathrm{t}-\mathrm{HB}_{\mathrm{wc}}, \mathrm{t}-\mathrm{HB}_{\mathrm{wc}} / \mathrm{CT}-\mathrm{MC}, \mathrm{CT}-\mathrm{MC} / \mathrm{CT}-\mathrm{TBP}$ and CT-TBP/Covw (weak covalent bond) interactions, respectively.

The parameters given in bold are superior to those given in plain font in the classification and characterization of interactions, where those in plain font are given as the tentative ones. The classic chemical bonds of SS interactions $\left(180^{\circ}<\theta\right)$ are not detected in the HBs collected in Table 2. As a result, each HB in $\mathrm{Nu}-\mathrm{Nu}^{\prime}$ can be classified and characterized using the ( $\theta, \theta_{\mathrm{p}}$ ) values in place of $\left(R, \theta, \theta_{\mathrm{p}}\right)$. If the data of an HB appear in the CTTBP region, for an example, the HB interaction is recognized to have the CT-TBP nature.

The $\left(\theta, \theta_{\mathrm{p}}\right)$ values are $\left(67.2-74.5^{\circ}, 73.0-87.4^{\circ}\right)$ for $\mathrm{O}^{-}-\mathrm{O}$ in $\mathrm{C}-\mathrm{U}(28)$ and $\mathrm{C}-\mathrm{T}(25)$; $\mathrm{CH}-*-\mathrm{O}$ in $\mathrm{C}-\mathrm{C}(22), \mathrm{A}-\mathrm{T}(3,6)$ and $\mathrm{A}-\mathrm{U}$ (19); CH-*-HN in A-G (16); CH-*-N in A-A (11); and the weaker O-*-HN in G-G (30). Therefore, the interactions are classified as p-CS interactions $\left(45^{\circ}<\theta<90^{\circ}\right)$ and characterized as having the vdW nature ( $45^{\circ}<\theta_{\mathrm{p}}<90^{\circ}$ ), which is denoted by p-CS/vdW. The $\angle$ NHO angle for the weaker $\mathrm{NH}^{-*}-\mathrm{O}$ interaction in G-G (30) is $135.0^{\circ}\left(\ll 180^{\circ}\right)$; therefore, it is much weaker than expected. The $\mathrm{NH}^{-}-\mathrm{O}$ interactions in $\mathrm{A}-\mathrm{T}(2,5)$ and $\mathrm{A}-\mathrm{U}(18)$ along with the weaker $\mathrm{O}^{*}-\mathrm{HN}$ in $\mathrm{C}-\mathrm{G}(9)$ are predicted to be r$\mathrm{CS} / \mathrm{t}-\mathrm{HB}_{\mathrm{wc}}$ since the $\left(\theta, \theta_{\mathrm{p}}\right)$ values are $\left(95.5-98.2^{\circ}, 141.5-148.3^{\circ}\right)$ $\left(90^{\circ}<\theta ; \theta_{\mathrm{p}}<150^{\circ}\right)$, although the weaker $\mathrm{NH}-*-\mathrm{O}$ in $\mathrm{C}-\mathrm{G}$ (9) seems fairly close to the borderline area with r-CS/CT-MC, of which $\left(\theta, \theta_{\mathrm{p}}\right)=\left(98.2^{\circ}, 148.3^{\circ}\right)$. The $\mathrm{NH}^{*}-\mathrm{O}$ interactions in $\mathrm{A}-\mathrm{C}$ (13), A-G (15), C-C (21), C-T (24), C-U (27), G-G (29), G-T (32), G-U $(33,34)$, T-T $(35,36,37)$, T-U $(38,39)$ and U-U $(40,41,42$, 43 ), together with the stronger $\mathrm{NH}^{-}-\mathrm{O}$ in $\mathrm{C}-\mathrm{G}(7)$, are predicted to have the r-CS/CT-MC nature since the $\left(\theta, \theta_{\mathrm{p}}\right)$ values are $\left(103.7-132.4^{\circ}, 155.6-172.4^{\circ}\right)\left(150^{\circ}<\theta_{\mathrm{p}}<180^{\circ}\right)$. On the other hand, the $\left(\theta, \theta_{\mathrm{p}}\right)$ values for $\mathrm{N}-*-\mathrm{HN}$ in A-T $(1,4)$, A-U (17) and C-C (20) are (143.5-147.2 ${ }^{\circ}$, 180.6-182.6 ${ }^{\circ}$ ); therefore, the interactions are predicted to have the r-CS/CT-TBP nature ( $\theta_{\mathrm{p}}>180^{\circ}$ ), while the $\mathrm{NH}^{*}-\mathrm{N}$ interactions in A-A (10), A-C (12), A-G (14), $\mathrm{C}-\mathrm{T}$ (23), C-U (26) and G-T (31) along with the weaker NH-*-N in C-G (8) are predicted to be of the r-CS/CT-MC nature since $(\theta$, $\left.\theta_{\mathrm{p}}\right)=\left(100.9-132.8^{\circ}, 158.6-178.6^{\circ}\right)\left(150^{\circ}<\theta_{\mathrm{p}}<180^{\circ}\right)$. The NH-*-N interactions in A-G (14), C-T (23) and C-U (26) seem fairly close to the borderline area with r-CS/CT-TBP $\left(\theta_{\mathrm{p}}=180^{\circ}\right)$ since the $\theta_{\mathrm{p}}$ values are $178.6^{\circ}, 178.1^{\circ}$ and $178.5^{\circ}$, respectively, which are fairly close to $180^{\circ}$. The results are summarized in Table 2. The nature of each HB in the multi-HBs between $\mathrm{Nu}-\mathrm{Nu}^{\prime}$, calculated with MP2/BSS-B'a, together with the number, is illustrated in Fig. S3 of the ESI. $\dagger$

The total orders for $\mathrm{NH}^{-}-{ }^{*} \mathrm{~N}$ and $\mathrm{NH}^{*}-\mathrm{O}$, based on $\theta$ and $\theta_{\mathrm{p}}$, are shown in eqn (7) and (8), respectively. The NH $\cdots \mathrm{N}$ interactions are again demonstrated to be stronger than the $\mathrm{NH} \cdots \mathrm{O}$


Fig. 5 Plots of $\theta$ versus $\rho_{\mathrm{b}}\left(r_{\mathrm{c}}\right)$ for each HB in $\mathrm{Nu}-\mathrm{Nu}^{\prime}$, calculated with MP2/BSS-B'a. While data for $\mathrm{G}(\mathrm{A})$ of $\mathrm{NH}-*-\mathrm{N}$ are shown by black solid circles, those for $\mathrm{G}(\mathrm{B})$ of $\mathrm{NH}^{*}-\mathrm{O}$ are by red solid circles, together with those for $\mathrm{G}(\mathrm{C})$ of $\mathrm{CH}-*-\mathrm{X}(\mathrm{X}=\mathrm{O}, \mathrm{N}$ and HN$)$ and $\mathrm{O}-*-\mathrm{O}$ by blue hole circles. The numbers for the interactions are the same as those in Table 2 and Fig. 4.
interactions, overall. The orders shown in eqn (7) and (8) are similar with each other, although the similarity is not necessarily. These results would arise from the specific nature of each HB in multi-HBs of $\mathrm{Nu}-\mathrm{Nu}^{\prime}$. The applicability of QTAIM-DFA, which employs the perturbed structures generated with CIV, is also demonstrated to elucidate the nature of each HB of the multi-HB system in $\mathrm{Nu}-\mathrm{Nu}^{\prime}$. There are some differences, however. The differences in the orders are shown by italic. The differences seem large for NH-*-O (G-G: 29), NH-*-N (C-G: 8), NH-*-O (A-C: 13) and NH-*-N (A-A: 10), among them, as shown by italic.

For both $\mathrm{NH}^{-*}-\mathrm{N}$ and $\mathrm{NH}-^{*}-\mathrm{O}$, based on $\theta$ :
NH-*-N (A-U: 17) $\geq$ NH-*-N $(A-T: 1)>N H-*-N(C-C: 20)>$ NH-*-N (A-G: 14) $\geq N H_{-}^{*}-O(G-G: 29) \geq N H-*-N(C-U: 26)$ $\geq \mathrm{NH}-{ }^{*}-\mathrm{N}(\mathrm{G}-\mathrm{T}: 31) \geq N H_{-}^{*}-N(C-T: 23)>N H_{-}^{*}-O(C-G: 7)$ $\geq N H^{*}-N(A-C: 12)>N H{ }^{*}-N(C-G: 8)>N H-*-O(C-C: 21)$ $>$ NH-*-O (G-U: 34) > NH-*-O (G-U: 33) > NH-*-O (T-U: 38) $>\mathrm{NH}-{ }^{*}-\mathrm{O}$ (T-T: 35,36$) \geq \mathrm{NH}-{ }^{*}-\mathrm{O}$ (U-U: 40, 41) $>\mathrm{NH}-$ *-O (T-U: 39) $\geq \mathrm{NH}-{ }^{*}-\mathrm{O}(\mathrm{A}-\mathrm{G}: 15)>N H-*-O(A-C: 13)>$ $\mathrm{NH}-{ }^{*}-\mathrm{O}(\mathrm{C}-\mathrm{T}: 24) \geq \mathrm{NH}-{ }^{*}-\mathrm{O}(\mathrm{C}-\mathrm{U}: 27)>\mathrm{NH}^{-}-\mathrm{O}(G-T: 32)>$ $N H-*-N(A-A: 10)>\mathrm{NH}-*-\mathrm{O}(\mathrm{C}-\mathrm{G}: 9)>\mathrm{NH}-*-\mathrm{O}(\mathrm{A}-\mathrm{T}: 2) \geq$ $\mathrm{NH}-*-\mathrm{O}(\mathrm{A}-\mathrm{U}: 18) \gg \mathrm{NH}-{ }^{-} \mathrm{O}$ (G-G: 30)

For both $\mathrm{NH}^{-*}-\mathrm{N}$ and $\mathrm{NH}^{-}-\mathrm{O}$, based on $\theta_{\mathrm{p}}$ :

$$
\begin{aligned}
& \mathrm{NH}-*-\mathrm{N}(\mathrm{~A}-\mathrm{U}: 17) \geq \mathrm{NH}-{ }^{*}-\mathrm{N}(\mathrm{~A}-\mathrm{T}: 1)>\mathrm{NH}-{ }^{*}-\mathrm{N}(\mathrm{C}-\mathrm{C}: 20)> \\
& \mathrm{NH}-*-\mathrm{N}(\mathrm{~A}-\mathrm{G}: 14) \geq N H-^{*}-N(C-U: 26) \geq N H-*-N(C-T: 23) \\
& >\mathrm{NH}-{ }^{*}-\mathrm{N}(\mathrm{G}-\mathrm{T}: 31)>N H-*-N(C-G: 8)>N H-^{*}-N(A-C: 12) \\
& >N H-*-O(G-G: 29)>N H-^{*}-O(C-G: 7)>\mathrm{NH}-*-\mathrm{O}(\mathrm{C}-\mathrm{C}: 21)
\end{aligned}
$$

 and (12) ${ }^{a}$

| Entry | Correlation | $a$ | $b$ | c | Correlation with $n$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | $R$ vs. $\rho_{\mathrm{b}}\left(\boldsymbol{r}_{\mathrm{c}}\right)$ | 0.388 | $-0.003$ | 0.963 | Fig. S4 (G(A): 10 ) |
| 2 | $R$ vs. $\rho_{\mathrm{b}}\left(\boldsymbol{r}_{\mathrm{c}}\right)$ | 0.320 | 0.002 | 0.993 | Fig. S4(G(B): $20^{\text {b }}$ ) |
| 3 | $R$ vs. $\rho_{\mathrm{b}}\left(\boldsymbol{r}_{\mathrm{c}}\right)$ | 0.355 | 0.001 | 0.992 | Fig. S4 (G(C): $7^{\text { }}$ ) |
| 4 | $\theta$ vs. $\rho_{\mathrm{b}}\left(r_{\mathrm{c}}\right)$ | 2110.5 | 42.5 | 0.985 | Fig. 5 (G(A): 10) |
| 5 | $\theta$ vs. $\rho_{\mathrm{b}}\left(\mathrm{r}_{\mathrm{c}}\right)$ | 1811.6 | 43.4 | 0.989 | Fig. 5 (G(B): 19) |
| 6 | $\Delta E v s .\left(1 / C_{i i}\right)_{\mathrm{Nu}-\mathrm{Nu}}{ }^{\prime}$ | -121.1 | -7.52 | 0.954 | Fig. 6 (15) |
| 7 | $\Delta E v s .\left(1 / C_{i i}\right)_{\mathrm{Nu}-\mathrm{Nu}}{ }^{\prime}$ | -136.6 | -0.02 | 0.956 | Fig. 6 (14 ${ }^{\text {d }}$ |
| 8 | $\left(w^{\prime} / w\right)_{\text {POM }} v s .\left(w^{\prime} / w\right)_{\mathrm{CIV}}$ | 1.021 | -0.001 | 0.9997 | Fig. 7 (15) |
| 9 | $\left(w^{\prime} / w\right)_{\text {POM }}$ vs. $\left(w^{\prime} / w\right)_{\text {CIV }}$ | 1.046 | -0.004 | 0.997 | Fig. S11 (15) |

${ }^{a}$ Evaluated with MP2/BSS-B'a. ${ }^{b}$ Data from weaker NH-*-O of G-G (30) being added to G(B). ${ }^{c}$ Omitting the data from weaker NH-*-O of G-G (30).
${ }^{d}$ Omitting the data from C-G.
$>$ NH-*-O (G-U: 34) > NH-*-O (G-U: 33) > NH-*-O (T-U: $38)>$ NH-*-O $(\mathrm{T}-\mathrm{T}: 35,36)>\mathrm{NH}-*-\mathrm{O}(\mathrm{U}-\mathrm{U}: 40,41)>\mathrm{NH}-$ *-O (T-U: 39) > NH-*-O (A-G: 15) $>N H-*-N(A-A: 10)>$ NH-*-O (C-T: 24) $\geq \mathrm{NH}-*-\mathrm{O}(\mathrm{C}-\mathrm{U}: 27) \geq \mathrm{NH}^{-}{ }^{-}-\mathrm{O}(A-C: 13)$ $>$ NH-*-O (G-T: 32) > NH-*-O (C-G: 9) > NH-*-O $(\mathrm{A}-\mathrm{T}: 2)>$ NH-*-O (A-U: 18) > NH-*-O (G-G: 30)

After elucidation of the nature of each HB in $\mathrm{Nu}-\mathrm{Nu}^{\prime}$, the next extension is to consider the behavior of the HBs.

## Relations between $\boldsymbol{R}, \boldsymbol{\theta}, \boldsymbol{\theta}_{\mathrm{p}}$ and $\rho_{\mathrm{b}}\left(\boldsymbol{r}_{\mathrm{c}}\right)$ for each HB in $\mathbf{N u}-\mathrm{Nu}^{\prime}$

Relations between the QTAIM-DFA parameters of $\left(R, \theta, \theta_{\mathrm{p}}\right)$ and QTAIM functions, such as $\rho_{\mathrm{b}}\left(\boldsymbol{r}_{\mathrm{c}}\right)$, are examined, first.

Good correlations are detected for the relations. The $R$ values are plotted versus $\rho_{\mathrm{b}}\left(\boldsymbol{r}_{\mathrm{c}}\right)$ for each HB in $\mathrm{Nu}-\mathrm{Nu}^{\prime}$, as shown in Fig. S4 in the ESI. $\dagger$ The plot can be analyzed as three correlations of $G(\mathbf{A}), G(\mathbf{B})$, and $G(\mathbf{C})$, which are closely related to the plot shown in Fig. 4. The data point for the weaker $\mathrm{NH}^{-*}$-O in $\mathrm{G}-\mathrm{G}(30)$ is just on the correlation line for $\mathrm{G}(\mathbf{B})$; therefore, it is added to $G(B)$ in the analysis. The correlations are shown in Table 3 (entries 1-3).

The results seem to promise similar relations between the parameters. Fig. 5 shows the plot of $\theta$ versus $\rho_{\mathrm{b}}\left(\boldsymbol{r}_{\mathrm{c}}\right)$. The plot is analyzed as three correlations for $\mathrm{G}(\mathbf{A})$ of $\mathrm{NH}^{*}-\mathrm{N}, \mathrm{G}(\mathbf{B})$ of $\mathrm{NH}^{-}$ *-O and $\mathrm{G}(\mathbf{C})$ of vdW interactions. The correlations are shown in Table 3 (entries 4 and 5), except for the very poor correlation for $\mathrm{G}(\mathbf{C})$, which is given in the figure. The plot of $\theta$ versus $R$ is illustrated in Fig. S5 of the ESI. $\dagger$ The plot is also analyzed as two correlations, similar to the case of the plot in Fig. 5. The correlations are given in the figure.

Good linear correlations are not found in the plots of $\theta_{\mathrm{p}}$ versus $\rho_{\mathrm{b}}\left(\boldsymbol{r}_{\mathrm{c}}\right)$ and $\theta_{\mathrm{p}}$ versus $R$. The plot of $\theta_{\mathrm{p}}$ versus $\theta$ also does not give a good linear correlation. Instead, the relation between $\theta_{\mathrm{p}}$ and $\theta$ is analyzed using a cubic function as a regression curve. The correlation was much improved when analyzed as two correlations, which are shown in Fig. S6 of the ESI. $\dagger$ The correlations are given in the figure.

## Relations between $\Delta E$ and $C_{i i}$ for $\mathbf{N u}-\mathbf{N u}^{\prime}$

We reported a good inverse correlation between $\Delta E$ and $C_{i i}$ $\left(\Delta E \cdot C_{i i}=\right.$ constant $)$ for the neutral mono-HB species. ${ }^{45}$ Therefore, eqn (9) is expected to hold between $\Delta E$ and $1 / C_{i i}$ in the multi-HB system of $\mathrm{Nu}-\mathrm{Nu}^{\prime}$, where $1 / C_{i i}$ should be the total values for $\mathrm{Nu}-\mathrm{Nu}^{\prime}$, together with $\Delta E$. How can the total values of $1 / C_{i i}$ for the multi- HBs of $\mathrm{Nu}-\mathrm{Nu}^{\prime}$ be calculated from the value of each HB in $\mathrm{Nu}-\mathrm{Nu}^{\prime}$ ? Eqn (10) is applied for the purpose, where $\left(1 / C_{i i}\right)_{\mathrm{Nu}-\mathrm{Nu}^{\prime}}$ is the total value of $1 / C_{i i}$ for a nucleobase pair and $\left(1 / C_{i i}\right)_{\mathrm{Nu}-\mathrm{Nu}^{\prime}: k}$ is the $1 / C_{i i}$ value for the $k$-th HB in the nucleobase pair. The $\left(1 / C_{i i}\right)_{\mathrm{Nu}-\mathrm{Nu}^{\prime}: k}$ values of the vdW interactions are also contained in the summation.

$$
\begin{gather*}
\Delta E=a\left(1 / C_{i i}\right)+b  \tag{9}\\
\left(1 / C_{i i}\right)_{\mathrm{Nu}-\mathrm{Nu}^{\prime}}=\Sigma_{k}\left(1 / C_{i i}\right)_{\mathrm{Nu}-\mathrm{Nu}^{\prime}: k} \tag{10}
\end{gather*}
$$

The $\Delta E$ values are plotted versus $\left(1 / C_{i i}\right)_{\mathrm{Nu}-\mathrm{Nu}^{\prime}}$ for $\mathrm{Nu}-\mathrm{Nu}^{\prime}$ in Fig. 6. A (very) good correlation was obtained for the plot, which is shown in Table 3 (entry 6). In this case, a $y$-intercept value ( $b$ in eqn (9)) very close to zero is obtained ( $b=0.02 \mathrm{~kJ} \mathrm{~mol}^{-1}$ ) if data from $\mathrm{C}-\mathrm{G}$ are omitted from the correlation, although the correlation seems not very improved. The correlation is shown in Table 3 (entry 7). The inverse proportion also holds for the multi-HB system of $\mathrm{Nu}-\mathrm{Nu}^{\prime}$ in this case. The constant value (in $\Delta E \cdot C_{i i}=$ constant $)$, as the averaged value of $\Delta E \cdot C_{i i}$ for $\mathrm{Nu}-\mathrm{Nu}^{\prime}$, is evaluated to be -137.04 without $\mathrm{C}-\mathrm{G}$. The constant value for $\mathrm{Nu}-\mathrm{Nu}^{\prime}(-137.04)$ is close to but somewhat smaller than that reported for the neutral mono-HB species $(-165.64)$ in magnitude. ${ }^{45}$ The constant value for all $\mathrm{Nu}-\mathrm{Nu}^{\prime}$ is evaluated to be -135.96 , which is very close to that without the data from C-G. The results show that the compliance constants $\left(C_{i i}\right)$ are closely related to $\Delta E$ for the formation of not only the neutral mono-HB species but also the multi-HB system of $\mathrm{Nu}-\mathrm{Nu}^{\prime}$. A similar mechanism would be operative in both processes of $\Delta E$ and $C_{i i}$ in the multi-HB systems of $\mathrm{Nu}-\mathrm{Nu}^{\prime}$. Eqn (10) reminds us that the total value of resistance of a parallel connection should be calculated for each one according to Ohm's law for the electric resistance of resistors connected in parallel. ${ }^{56}$

Table 4 Fractionalization of the total values of $\Delta E$ for $\mathrm{Nu}-\mathrm{Nu}^{\prime}$ to each $\mathrm{HB}\left(\Delta E_{\mathrm{e}}\right)$, calculated with MP2/BSS- $\mathrm{B}^{\prime} \mathrm{a}^{a}$

| $\mathrm{Nu}-\mathrm{Nu}^{\prime}(s y m)$ | $\Delta E$ | $\Delta E_{\mathrm{e}}\left(\right.$ no. $\left.^{b}\right)$ | $\Delta E_{\mathrm{e}}\left(\mathrm{no}^{b}{ }^{b}\right)$ | $\Delta E_{\mathrm{e}}\left(\mathrm{no} .^{b}\right)$ |
| :--- | ---: | :--- | :--- | :--- |
| A-T $\left(C_{1}\right)$ | -70.3 | $-40.6(1)$ | $-21.9(2)$ | $-7.8(3)$ |
| $\mathrm{C}-\mathrm{G}\left(C_{1}\right)$ | -123.5 | $-37.7(7)$ | $-56.2(8)$ | $-29.6(9)$ |
| A-A $\left(C_{1}\right)$ | -35.4 | $-26.5(10)$ | $-8.9(11)$ |  |
| A-C $\left(C_{1}\right)$ | -73.1 | $-36.6(12)$ | $-36.5(13)$ |  |
| A-G $\left(C_{1}\right)$ | -80.5 | $-42.1(14)$ | $-33.3(15)$ | $-5.1(16)$ |
| A-U $\left(C_{1}\right)$ | -70.6 | $-40.8(17)$ | $-21.9(18)$ | $-7.9(19)$ |
| C-C $\left(C_{1}\right)$ | -102.6 | $-55.1(20)$ | $-37.6(21)$ | $-9.9(22)$ |
| C-T $\left(C_{1}\right)$ | -64.0 | $-29.9(23)$ | $-29.7(24)$ | $-4.4(25)$ |
| C-U $\left(C_{1}\right)$ | -64.9 | $-30.3(26)$ | $-29.9(27)$ | $-4.6(28)$ |
| G-G $\left(C_{i}\right)$ | -117.1 | $-48.0(29)$ | $-10.6(30)$ |  |
| G-T $\left(C_{1}\right)$ | -65.4 | $-36.5(31)$ | $-28.9(32)$ |  |
| G-U $\left(C_{1}\right)$ | -74.2 | $-43.3(33)$ | $-30.9(34)$ |  |
| T-T $\left(C_{1}\right)$ | -60.0 | $-30.0(35)$ | $-30.0(36)$ |  |
| T-U $\left(C_{1}\right)$ | -59.9 | $-30.8(38)$ | $-29.1(39)$ |  |
| U-U $\left(C_{1}\right)$ | -59.8 | $-29.9(40)$ | $-29.9(41)$ |  |

${ }^{a}$ The values are given in $\mathrm{kJ} \mathrm{mol}{ }^{-1}$. ${ }^{b}$ The number for each HB , containing the vdW interaction, is the same as that given in Table 2.

The total contributions of $\Delta E$ and $C_{i i}$ should be calculated as the summations of the contributions from each HB. As a result, it is expected that the $\Delta E$ value for a nucleobase pair can be fractionalized to each HB in the multi- HB system of the $\mathrm{Nu}-\mathrm{Nu}^{\prime}$. Based on the good relation with eqn (9) and (10) shown in Fig. 6 (see entry 6 or 7 in Table 3 ), the $\Delta E$ value for a nucleobase pair is expected to be fractionalized to each $\mathrm{HB}\left(\Delta E_{\mathrm{e}}\right)$ by the ratio of $1 /$ $C_{i i}$ of each HB , according to eqn (11), where $\Delta E_{\mathrm{e}: 1}$ and $\left(1 / C_{i i}\right)_{\mathrm{Nu}}$ $\mathrm{Nu}^{\prime}: 1$ stand for the fractionalized energy to the first HB and for the $1 / C_{i i}$ value of the first HB in the $\mathrm{Nu}-\mathrm{Nu}^{\prime}$, respectively. The results are collected in Table 4.

$$
\begin{equation*}
\Delta E_{\mathrm{e}: 1}: \Delta E_{\mathrm{e}: 2}: \ldots=\left(1 / C_{i i}\right)_{\mathrm{Nu}-\mathrm{Nu}^{\prime}: 1}:\left(1 / C_{i i}\right)_{\mathrm{Nu} u-\mathrm{Nu}^{\prime}: 2} 2 \ldots \tag{11}
\end{equation*}
$$



Fig. 6 Plot of $\Delta E$ versus $\left(1 / C_{i j}\right) N u-N u^{\prime}$ in $N u-N u^{\prime}$, calculated with MP2/ BSS-B'a.

Similar relation is also observed in the di-HB system of acetic acid dimer and the related species. The results are explained in Fig. S7 and Table S6 of the ESI. $\dagger$

The mutual interactions between HBs must also be of very importance in the multi-HBs of $\mathrm{Nu}-\mathrm{Nu}^{\prime}$, which would be clarified through the detailed analysis of $C_{i j}(i \neq j)$ for the multiHBs. ${ }^{46 a}$

## Relations among the total values of $\boldsymbol{R}, \boldsymbol{\theta}, \boldsymbol{\theta}_{\mathrm{p}}$ and $\boldsymbol{\Delta E}$ for $\mathbf{N u}-\mathbf{N u}^{\prime}$

What are the relations among the total values of $R, \theta, \theta_{\mathrm{p}}$ and $\Delta E$ for $\mathrm{Nu}-\mathrm{Nu}^{\prime}$ ? The total values of $P_{\mathrm{Nu}-\mathrm{Nu}^{\prime}}\left(=R_{\mathrm{Nu}-\mathrm{Nu}}{ }^{\prime}, \theta_{\mathrm{Nu}-\mathrm{Nu}^{\prime}}\right.$ and $\left.\theta_{\mathrm{p}: \mathrm{Nu}-\mathrm{Nu}^{\prime}}\right)$ of $\mathrm{Nu}-\mathrm{Nu}^{\prime}$ are necessary for the analysis. The total values are calculated according to eqn (12), where $P_{\mathrm{Nu}-\mathrm{Nu}^{\prime}: k}$ is the $P_{\mathrm{Nu}-\mathrm{Nu}^{\prime}}$ value for each HB in $\mathrm{Nu}-\mathrm{Nu}^{\prime}$. The $P_{\mathrm{Nu}-\mathrm{Nu}^{\prime}: k}$ values from the vdW interactions are also contained in eqn (12).

$$
\begin{equation*}
P_{\mathrm{Nu}-\mathrm{Nu}^{\prime}}=\Sigma_{k} P_{\mathrm{Nu}-\mathrm{Nu}^{\prime}: k} \tag{12}
\end{equation*}
$$

The $\Delta E$ values are plotted versus $R_{\mathrm{Nu}-\mathrm{Nu}^{\prime}}, \theta_{\mathrm{Nu}-\mathrm{Nu}^{\prime}}$ and $\theta_{\mathrm{p}: \mathrm{Nu}}$ $\mathrm{Nu}^{\prime}$, and the plots are shown in Fig. S8-S10 of the ESI. $\dagger$ The correlation is greatly improved by analyzing the plot as two or three correlations instead of one correlation. The correlations are shown in the figure. The correlation for $\Delta E$ versus $\theta_{\mathrm{p}: \mathrm{Nu}-\mathrm{Nu}^{\prime}}$ seems poorer than that for $\Delta E$ versus $\theta_{\mathrm{Nu}-\mathrm{Nu}}$.

It is also instructive to clarify the structural feature in the perturbed structures of $\mathrm{Nu}-\mathrm{Nu}^{\prime}$ to discuss the behavior of each HB of $\mathrm{Nu}-\mathrm{Nu}^{\prime}$ in more detail, which is examined in the following.

## Structural feature in the perturbed structures of $\mathbf{N u}-\mathbf{N u}^{\prime}$

How can the perturbed structures of $\mathrm{Nu}-\mathrm{Nu}^{\prime}$ generated with CIV and POM be simply and effectively visualized? Eqn (13)-(15) are applied to a tri-HB system for the purpose. Subscribes 1,2 and 3 in eqn (13)-(15) correspond to the first, second and third HBs in tri-HBs of $\mathrm{Nu}-\mathrm{Nu}^{\prime}$, while $k(=1,2$ and 3$)$ designates the role of each HB in the calculations. In eqn (13), $r_{k 1}$ will be $r_{11}$ when $k=$ 1 , which means that the first HB in $\mathrm{Nu}-\mathrm{Nu}^{\prime}$ is selected as the major HB and therefore is fixed in POM. In this case, relative to $r_{11}, r_{12}$ for the second HB in eqn (14) and $\boldsymbol{r}_{13}$ for the third HB in eqn (15) $(k=1)$ are the minor HBs, which are (partially) optimized in POM. Similarly, eqn (14) defines $r_{22}$ with $k=2$, and eqn (15) does $r_{33}$ with $k=3$, where the second and third HBs are selected as the major interactions, respectively, for $\mathrm{Nu}-\mathrm{Nu}^{\prime}$. Compared to $r_{22}$, the $r_{21}$ and $r_{23}$ values are the minor HBs, while compared to $r_{33}, r_{31}$ and $r_{32}$ are the minor HBs. The $w_{k 1}, w_{k 2}$ and $w_{k 3}$ values are calculated according to eqn (13)-(15), where $w_{11}$, $w_{22}$ and $w_{33}$ are the fixed values. Eqn (13) and (14) with $k=1$ and 2 are applied to the di-HB system of $\mathrm{Nu}-\mathrm{Nu}^{\prime}$.

$$
\begin{align*}
& r_{k 1}=r_{k 1 \mathrm{o}}+w_{k 1} a_{\mathrm{o}}  \tag{13}\\
& r_{k 2}=r_{k 2 \mathrm{o}}+w_{k 2} a_{\mathrm{o}}  \tag{14}\\
& r_{k 3}=r_{k 3 \mathrm{o}}+w_{k 3} a_{\mathrm{o}} \tag{15}
\end{align*}
$$

$k=1,2$ and 3.


Fig. 7 Plot of $\left(w_{i j}^{\prime} / w_{i i}\right)_{\text {POM }}$ versus $\left(w_{i j}^{\prime} / w_{i i}\right)_{\text {CIV }}$ for each HB of multi- HB system in $\mathrm{A}-\mathrm{T}, \mathrm{C}-\mathrm{G}, \mathrm{A}-\mathrm{A}, \mathrm{U}-\mathrm{U}$ and $\mathrm{T}-\mathrm{T}$, calculated at $w_{i i}=0.05$ with MP2/BSS-B'a.

The structural feature in the perturbed structures of $\mathrm{Nu}-\mathrm{Nu}^{\prime}$ is examined by dividing them into four groups, $G(\mathbf{A T}), G(\mathbf{C G})$, $\mathrm{G}(\mathbf{A A})$ and $\mathrm{G}(\mathbf{T T}) .{ }^{57} \mathrm{Nu}-\mathrm{Nu}^{\prime}$ of $\mathrm{A}-\mathrm{T}, \mathrm{C}-\mathrm{G}, \mathrm{A}-\mathrm{A}$ and $\mathrm{T}-\mathrm{T}$ are the typical members of the groups, respectively. The feature is discussed by employing A-T, C-G, A-A and T-T, together with U-U. The feature in UU will supply a small structural difference from that in TT, although UU belongs to G(TT).
The values of $\quad\left(w_{i j}^{\prime} / w_{i i}\right)_{\mathrm{CIV}}$ and $\left(w_{i j}^{\prime} / w_{i i}\right)_{\text {POM }}(i, j=1,2$ and/or 3) are calculated for each HB in A-T, C-G, A-A, T-T and U-U with MP2/BSS-B'a at $w_{i i}=0.05$ by applying in eqn (13)-(15). ${ }^{32-38}$ The values are collected in Table S7 of the ESI. $\dagger$ Small differences in $\left(w_{i j}^{\prime} / w_{i i}\right)$ between T-T and U-U are detected. A positive value of $w_{i j}^{\prime} / w_{i i}$ implies that the minor (HB) interaction in $\mathrm{Nu}-\mathrm{Nu}^{\prime}$ moves in the same direction as the major interaction. On the other hand, relative to the major interaction, the minor HB interaction moves in the inverse direction for negative $w_{i j}^{\prime} / w_{i i}$ values. Compared to that of the major interaction, the magnitudes in the movement of the minor HB interactions would be negligible when the $w_{i j}^{\prime} / w_{i i}$ values are close to zero. Fig. 7 shows the plot of $\left(w_{i j}^{\prime} / w_{i i}\right)_{\mathrm{POM}} \operatorname{versus}\left(w_{i j}^{\prime} / w_{i i}\right)_{\mathrm{CIV}}$ for the HB interactions. The plot gave an excellent correlation, which is shown in Table 3 (entry 8). The results show that the perturbed structures generated with CIV and POM are very close to each other, approximately at $w_{i i}=0.05$, in the multi- HB system of $\mathrm{Nu}-\mathrm{Nu}^{\prime}$, as well as in the case of the mono-HB system. ${ }^{45}$

What happens if the $\mathrm{H} \cdots \mathrm{B}$ distance $(r(\mathrm{H}, \mathrm{B}))$ in each HB of $\mathrm{Nu}-\mathrm{Nu}^{\prime}$ is elongated further, where $\Delta r(\mathrm{H}, \mathrm{B})\left(=r(\mathrm{H}, \mathrm{B})-r_{\mathrm{o}}(\mathrm{H}\right.$, $\mathrm{B})$ ) is defined as the difference in $\mathrm{H} \cdots \mathrm{B}$ distance between the perturbed structure and the fully optimized structure. Relative to that of M06-2X/BSS-A, the reliability of M06-2X/BSS-C ${ }^{\prime}$ is confirmed for the optimizations. That is, the $r_{\mathrm{o}}(\mathrm{H}, \mathrm{B})$ values calculated with M06-2X/BSS-C' differ from the corresponding
values calculated with M06-2X/BSS-A by less than $0.01 \AA$ in magnitude (see Table S1 of the ESI $\dagger$ ). Therefore, these perturbed structures are calculated with POM by fixing the $r(\mathrm{H}, \mathrm{B})$ distances in question in the wider range of $-0.05 \AA \leq \Delta r(\mathrm{H}, \mathrm{B})$ $\leq 0.50 \mathrm{~A}$ for all HBs in A-T, C-G, A-A, T-T and U-U with M06-2X/BSS-C ${ }^{\prime}$ for improved calculation cost. The results are summarized in Table S8 of the ESI $\dagger$ in the $\left(w_{i j}^{\prime} / w_{i i}\right)_{\mathrm{POM}}$ form. The $\Delta E_{\mathrm{ESps}}\left(=E_{\mathrm{ESps}}-E_{\mathrm{ESo}}\right)$ values are also calculated for each HB in A-T, C-G, A-A, T-T and U-U based on the partially optimized structures. The $E_{\mathrm{ESps}}$ values are the energies of the perturbed structures at $r(\mathrm{H}, \mathrm{B})$ on the energy surface, and the $E_{\text {ESo }}$ values are those for the fully optimized structures. The magnitudes of the differences between $\Delta E_{\mathrm{ESps}}$ calculated with M06-2X/BSS-C ${ }^{\prime}$ and those calculated with M06-2X/BSS-A are less than $0.3 \mathrm{~kJ} \mathrm{~mol}^{-1}$ for $\mathrm{A}-\mathrm{T}, \mathrm{C}-\mathrm{G}, \mathrm{A}-\mathrm{A}, \mathrm{T}-\mathrm{T}$ and $\mathrm{U}-\mathrm{U}$ if the corresponding values are compared at $\Delta r(\mathrm{H}, \mathrm{B})=0.025 \AA$ (see Table S 3 of the ESI $\dagger$ ). The results again support the reliability of M06$2 \mathrm{X} / \mathrm{BSS}^{\prime} \mathrm{C}^{\prime}$ relative to M06-2X/BSS-A in the optimizations.

The perturbed structures of A-T, C-G, A-A, T-T and $\mathrm{U}-\mathrm{U}$ are also generated by employing CIV with M06-2X/BSS-C' in a wider range of $-0.1 \leq w_{i i} \leq 1.0$ ( $c f .:-0.05 \AA \leq \Delta r \leq 0.50 \AA$ for POM). The $w_{i j}$ values of the minor HBs are calculated, corresponding to $w_{i j}$ at $w_{i i}=0.05$ for the $\mathrm{Nu}-\mathrm{Nu}^{\prime}$. The results are also summarized in Table S8 of the ESI $\dagger$ in the $\left(w_{i j}^{\prime} / w_{i i}\right)_{\text {CIV }}$ form. The $\left(w_{i j}^{\prime} / w_{i i}\right)_{\mathrm{POM}}$ values are plotted versus $\left(w_{i j}^{\prime} / w_{i i}\right)_{\text {CIV }}$ calculated at $w_{i i}=0.05$ with M06-2X/BSS-C' ${ }^{\prime}$, as shown in Fig. S11 of the ESI. $\dagger$ The plot also gives a very good correlation, which is shown in Table 3 (entry 9). The quality of the correlation based on M06-2X/BSS-C ${ }^{\prime}$ is noticeably the same as that of the correlation based on MP2/ BSS-B'a.

The $\Delta E_{\text {ESps }}$ values are plotted versus a wide range of $-0.05 \AA$ $\leq \Delta r(\mathrm{H}, \mathrm{B}) \leq 0.50 \AA$ and $-0.1 \leq w_{i i} \leq 1.0$ for each HB in A-T, CG, A-A, T-T and U-U evaluated with POM and CIV, respectively. The plot is illustrated in Fig. S12 of the ESI, $\dagger$ where $r(\mathrm{H}, \mathrm{B})$ in the $x$ axis with POM is replaced by $w_{i i}$. As shown in the figure, the differences in $\Delta E_{\text {ESps }}$ between the structures evaluated with CIV and those evaluated with POM are negligible at approximately $w_{i i}<0.2$. Indeed, the $\Delta E_{\mathrm{ESps}}$ curves evaluated with CIV show a similar trend as those evaluated with POM for $w_{i i}<0.3$, but overall, the curves begin to grow rather exponentially for $w_{i i}>$ 0.4 as $w_{i i}$ increases. The results show that the perturbed structures generated with POM and CIV are very similar for $w_{i i}<0.2$ and similar for $0.2<w_{i i}<0.3$ but become different for $0.4<w_{i i}$.

The gradient for $\Delta E_{\text {ESps }}$ is largest for $\mathrm{N}-\mathrm{H} \cdots \mathrm{N}$ in $\mathrm{C}-\mathrm{G}$, which must be the reflection of the largest magnitude of $\Delta E_{\mathrm{ES}}$ for $\mathrm{C}-\mathrm{G}$ $\left(-117.2 \mathrm{~kJ} \mathrm{~mol}^{-1}\right)$ among A-T, C-G, A-A, T-T and U-U. The gradient for $\Delta E_{\mathrm{ESps}}$ decreases in the order shown in eqn (16). The order seems to not necessarily reflect the strength of each HB in the $\mathrm{A}-\mathrm{T}$ and $\mathrm{C}-\mathrm{G}$ pairs.
$\mathrm{N}-\mathrm{H} \cdots \mathrm{N}(\mathrm{C}-\mathrm{G}) \gg \mathrm{N}-\mathrm{H} \cdots \mathrm{O}(\mathrm{C}-\mathrm{G}) \geq \mathrm{O} \cdots \mathrm{H}-\mathrm{N}(\mathrm{C}-\mathrm{G})>$ $\mathrm{N}-\mathrm{H} \cdots \mathrm{N}(\mathrm{A}-\mathrm{T})>\mathrm{N}-\mathrm{H} \cdots \mathrm{O}(\mathrm{U}-\mathrm{U})>\mathrm{N}-\mathrm{H} \cdots \mathrm{O}(\mathrm{T}-\mathrm{T})>\mathrm{N}-\mathrm{H} \cdots \mathrm{N}$ $(\mathrm{A}-\mathrm{A}) \approx \mathrm{N}-\mathrm{H} \cdots \mathrm{O}(\mathrm{A}-\mathrm{T}: j=1)>\mathrm{C}-\mathrm{H} \cdots \mathrm{N}(\mathrm{A}-\mathrm{A})>\mathrm{N}-\mathrm{H} \cdots \mathrm{O}$ $(\mathrm{A}-\mathrm{T}: j=3)$

The gradient increased when POM or CIV is applied to the central $\mathrm{N}-\mathrm{H} \cdots \mathrm{N}$ interaction for both the A-T and C-G pairs. The
behavior of $\Delta E_{\mathrm{ESps}}$ evaluated with POM may correspond to that in the initial stage for the scission of $\mathrm{Nu}-\mathrm{Nu}^{\prime}$ to Nu and $\mathrm{Nu}^{\prime}$ under the simple mechanism for each HB. Such large $\Delta E_{\text {ESps }}$ values must be effectively decreased by the enzyme-catalyzed reactions in vivo at approximately room temperature. However, it is helpful to understand the behavior of HBs in Nu$\mathrm{Nu}^{\prime}$ through a simple mechanism.

Indeed, the behavior of HBs, containing those of multi-HBs in $\mathrm{Nu}-\mathrm{Nu}^{\prime}$, will be revealed in more detail, if the magnitudes in the movement of HBs is directly investigated. The NVT ensemble method seems typical one of such methods. ${ }^{58}$ The predicted nature will change depending on the quality of the calculation levels, especially for weak HBs. However, the results in the framework of QTAIM-DFA with CIV should be reasonable, if calculated with MP2/BSS-B'a.

## Conclusions

The intrinsic dynamic and static nature of each HB in the multiHBs of $\mathrm{Nu}-\mathrm{Nu}^{\prime}$ is elucidated with QTAIM-DFA by employing the perturbed structures generated using the coordinates derived from the compliance constants $C_{i i}$. The method is called CIV. The initial stage of the opening or closing of the duplex DNA structure and the stability can be understand based on the nature of the interactions through the simple mechanism. In QTAIM-DFA, $H_{\mathrm{b}}\left(\boldsymbol{r}_{\mathrm{c}}\right)$ is plotted versus $H_{\mathrm{b}}\left(\boldsymbol{r}_{\mathrm{c}}\right)-V_{\mathrm{b}}\left(\boldsymbol{r}_{\mathrm{c}}\right) / 2$ for the data of each HB at the BCP in $\mathrm{Nu}-\mathrm{Nu}^{\prime}$, containing those from the perturbed structures generated with CIV. The plot consists of three groups of data: $\mathrm{G}(\mathbf{A})$ of $\mathrm{NH}^{-*}-\mathrm{N}, \mathrm{G}(\mathbf{B})$ of $\mathrm{NH}^{-*}-\mathrm{O}$ and $\mathrm{G}(\mathbf{C})$ of the very weak interactions of the vdW type. The plot is analyzed to give the QTAIM-DFA parameters of $(R, \theta)$ and $\left(\theta_{\mathrm{p}}\right.$, $\kappa_{\mathrm{p}}$ ), which correspond to the static and intrinsic dynamic nature, respectively. Each $\mathrm{NH}^{-*}-\mathrm{N}$ in $\mathrm{G}(\mathbf{A})$ is predicted to have the nature of r-CS/CT-MC to r-CS/CT-TBP, and each NH-*-O in $\mathrm{G}(\mathbf{B})$ is of the nature of $\mathrm{r}-\mathrm{CS} / \mathrm{t}-\mathrm{HB}_{\mathrm{wc}}$ to $\mathrm{r}-\mathrm{CS} / \mathrm{CT}-\mathrm{MC}$. The results show that $\mathrm{NH}^{-}{ }^{*}-\mathrm{N}$ in $\mathrm{G}(\mathbf{A})$ is stronger than $\mathrm{NH}^{-}{ }^{*}-\mathrm{O}$ in $\mathrm{G}(\mathbf{B})$ overall. It is demonstrated that the total values of $\Delta E_{\mathrm{ES}}$ are (directly) proportional to $\left(1 / C_{i i}\right)_{\mathrm{Nu}-\mathrm{Nu}}$, the total values of $1 / C_{i i}$ for $\mathrm{Nu}-\mathrm{Nu}^{\prime}$, where $\left(1 / C_{i i}\right)_{\mathrm{Nu}-\mathrm{Nu}^{\prime}}$ is calculated by $\Sigma_{k}\left(1 / C_{i j}\right)_{\mathrm{Nu}-\mathrm{Nu}^{\prime}: k}$, similar to Ohm's law for a parallel connection. The results demonstrate that $\Delta E_{\mathrm{ES}}$ values are closely related to $C_{i i}$ values. As a result, the total value of $\Delta E_{\mathrm{ES}}$ for $\mathrm{Nu}-\mathrm{Nu}^{\prime}$ can be fractionalized to each HB in multi-HB systems, even if the HBs in multi-HB systems, containing the weak vdW interactions, are formed in close proximity in space and interact mutually and strongly with each other. The results of the fractionalizations are shown in Table 4. The differences in the perturbed structures generated with POM and CIV are negligible for $w_{i i}<0.2$ and very small for $0.2<w_{i i}<0.3$, but they become larger for $0.4<w_{i i}$.

Many multi-HB systems play a crucial role in the chemical and biological sciences, not only in vitro but also in vivo. Each HB in such multi-HB systems will interact mutually and strongly with each other due to their close proximity in space. It is of very interest if the proposed method can open the door to elucidate each HB in such multi-HB systems, although some devices seem necessary for the effective analysis.

## Conflicts of interest

The authors declare no conflict of interest.

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$\mathrm{Nu}^{\prime}$ and $\left(1 / C_{i i}\right)_{\mathrm{Nu}-\mathrm{Nu}^{\prime}: k}$ in eqn (10) correspond to $R_{\mathrm{T}}{ }^{-1}$ and $R_{\mathrm{i}}{ }^{-1}$, respectively, for the parallel resistor network, while $\Delta E$ and $\Delta E_{\text {e:i }}$ in eqn (11) do to $I_{\mathrm{T}}$ and $I_{\mathrm{i}}$, respectively. Namely, the total and each $C_{i i}$ in the multi-HB system between $\mathrm{Nu}-\mathrm{Nu}^{\prime}$ work as if they are $R_{\mathrm{T}}$ and $R_{\mathrm{i}}$ in the parallel resistor network, according to the Ohm's low, while the total and each energies for $\mathrm{Nu}-\mathrm{Nu}^{\prime}\left(\Delta E\right.$ and $\Delta E_{\text {e:i }}$, respectively) act as if they are $I_{\mathrm{T}}$ and $I_{\mathrm{i}}$, respectively. The energy for the strong mutual interactions between each HB in the multi-HBs between $\mathrm{Nu}-\mathrm{Nu}^{\prime}$ would be reasonably fractionalized to each HB based on $1 / C_{i i}$, which would be the reason for eqn (9)-(11) to be hold.
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    $\dagger$ Electronic supplementary information (ESI) available: QTAIM-DFA approach, computational data, and the fully optimized structures given by Cartesian coordinates, together with total energies of the nucleobase pairs $\left(\mathrm{Nu}-\mathrm{Nu}^{\prime}\right)$. See DOI: 10.1039/d0ra01357a

